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Fatigue life prediction for woven-roving glass fiber reinforced-polyester composite using SWT parameter or the modified fatigue strength ratio

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The validity of using the Smith-Watson-Topper (SWT) parameter and the modified fatigue strength ratio (*Ψ*) to obtain an experimental master curve was checked in this work. The experimental fatigue tests were conducted on thin-walled tubular specimens woven-roving Glass Fiber Reinforced Polyester (GFRP), under combined bending and torsional moments, in-phase, with two fibre orientations ($[\pm45]_2$ and $[0,90]_2$), at different negative stress ratios, R = - 1, -0.75, -0.5, -0.25, 0. The experimental data showed that the master S-N curve can be obtained using on of the parameters.

فى هذة المقالة اختبرت صلاحيةاستخدام كل من Smith–Wasson–Topper (SWT) و معامل مقياس متانة الكلال(*Ψ*) وذلك لايجاد منحنى الاجهاد–العمرفى الحالةالعامة. اجريت اختبارات الكلال على عينات اسطوانية رفيعة السمك مصنعة من البوليستر المدعم بالألياف الزجاجية المنسوجة شبكيا عند زوايا اتجاه الالياف [صفر، ٩٠] و[٤٥، -٤٥] والمؤثر عليه بإجهادات مركبه مكونه من عزوم إنحناء وإلتواء بنفس الطور فى وجود أحمال متغيرة مختلفة R= - ١ ، - ٠,٧٥ ، - ٠,٥ ، - ٠,٢٥ ، صفر. و قد تبين صلاحية استخدام اى من المعاملين فى حالة الدراسة الحالية.

Keywords: Fatigue, Glass fiber, Combined moments, Fluctuating stresses, Modified fatigue strength ratio

1. Introduction and review

The Smith-Watson-Topper (SWT) parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ proposed by Smith et al. [1] has allowed a reduction in the number of tests needed to estimate the effects of mean stress on the fatigue life of metals. They found that, when plotting the fatigue life against the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$, the data from a test series with variable mean stress or R ratios $\left(\frac{\sigma_{\min}}{\sigma_{\max}}\right)$ tended to plot a single curve. It was subsequently possible to characterize the mean stress effect on fatigue life for a given metal with only a single fatigue test (usually fully reversed, R = -1).

Mohamed N.A. [2] tried to check the validity of the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ for composite materials subjected to torsional moments with different R. He tested woven-roving glass fiber-reinforced polyester specimens with $[0, 90]_2$ and $[\pm45]_2$ fiber orientations under torsional fatigue loading with negative stress ratios (R); R=-1, -0.75, -0.5, 0. He plotted the SWT parameter for each fiber orientations against the number of cycles to failure. Also, he used the data of Sharara A.I. [3] since the specimens had nearly the same specifications as those of his work, being woven-roving Glass Fiber Reinforced Polyester (GFRP) tubular specimens with volume fraction (V_f) ranging from 50% to 64% and two fiber orientations $[0, 90]_2$ and $[\pm45]_2$ were tested under uniaxial bending stress with the same stress ratios. He found that the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ is valid for woven-roving glass fiber-reinforced polyester specimens with $[0, 90]_2$ and $[\pm45]_2$ fiber orientations under negative stress ratios for both works, i.e. performing only the completely reversed fatigue test (R=-1) and using the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ is sufficient to find out the strength of the materials under any negative stress ratio.

Sauer J.A. et al. [4] examined unreinforced axially loaded polystyrene samples, at several tensile mean stress values. They made two

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groups of tests; the first one was with constant amplitude stress and different mean stresses resulting in varying maximum stresses, and the second group was with constant maximum stress with different combinations of mean and amplitude stresses. When plotting the test results, they used three forms of S-N curves; they used (σ_{max}), (σ_a) and ($\sqrt{\sigma_{max}\sigma_a}$) as the ordinate versus the number of cycles to failure as abscissa. The plots indicated that, using the ($\sqrt{\sigma_{max}\sigma_a}$), which is the SWT parameter, is slightly better than using (σ_{max}) and both are better than (σ_a). This was because ($\sqrt{\sigma_{max}\sigma_a}$) succeeded in showing the effect of mean stress in both types of tests.

One of the important targets of the fatigue work is to find an effective measure for fatigue data analysis to establishing the master S-N relationship that is independent of both stress ratio and fiber orientation in order to predict the fatigue at any arbitrary stress ratio or fiber orientation.

The simplest non-dimensional scalar measure for fatigue strength can be defined as $\psi = \frac{\sigma_{max}}{\sigma_b}$ Which is usually called the fatigue strength ratio. Where σ_{max} is the maximum fatigue strength, which is a function of number of cycles to failure, and σ_b is the static strength.

The fatigue strength ratio ψ becomes an effective measure for fatigue data analysis, which was confirmed by Basquin [5] for the fatigue behavior of metals and by Awerbuch and Hahn [6] for the off-axis fatigue behavior of unidirectional graphite/ epoxy composites at room temperature.

In order to incorporate the sensitivity to different modes of loading, the stress ratio R was used by Kawai [7], He decomposed σ_{max} into two parts as

$$\sigma_{max} = \sigma_a + \sigma_m .$$

Where σ_a and σ_m represent the alternating stress and mean stress, respectively, and are expressed as

$$\sigma_a = \frac{1}{2}(1-R)\sigma_{max}, \quad \sigma_m = \frac{1}{2}(1+R)\sigma_{max}$$

Using σ_a and σ_m, Kawai confirmed that the static failure condition $\sigma_{max} = \sigma_b$ is expressed as $\frac{\sigma_a}{(\sigma_b - \sigma_m)} = 1$. And by analogy with ψ, given by the pervious equation, therefore he defined a non-dimensional scalar quantity ψ as

$$\Psi = \frac{\sigma_a}{\sigma_b - \sigma_m}$$

or with the help of R, ψ can be expressed as

$$\Psi = \frac{\frac{1}{2}(1-R)\psi}{1-\frac{1}{2}(1+R)\psi} .$$

The non-dimensional scalar measure ψ was called the modified fatigue strength ratio.

As for fatigue behavior of metals, it is demonstrated by Landgraf [8] that the mean stress effect can be accounted for using the modified fatigue strength ratio ψ in the fatigue data analysis. Kawai [8] studied the validity of ψ for the off-axis fatigue behavior of unidirectional composites, using the experimental results of El Kadi and Ellyin [9] conducted on a unidirectional glass/epoxy composites with 0°, 19°, 45°, 71°, 90° fiber orientations at variety of stress ratios, R = -1, 0, 0.5 as plots of the maximum fatigue stress σ_{max} and modified fatigue strength ratio ψ against the number of reversals to failure on logarithmic scales, they observed that:

- The off-axis S-N relationship plotted using the maximum fatigue stress σ_{max} depends on both fiber orientations and stress ratios.
- The modified fatigue strength ratio ψ eliminated the fiber orientation dependence as well as the mean stress dependence.

Finally they confirmed that the modified fatigue strength ratio ψ becomes a unified strength measure to cope with the mean stress effect as well as the fiber orientation effect on the off-axis fatigue behavior of unidirectional composites undergoing constant amplitude stress cycling over a range of stress ratios.

2. Experimental work

2.1. Specimens

2.1.1. Materials

Woven-roving E-glass fibers and polyester resin, with trade name of "siropol 8330", were used to produce the used specimens. Table 1 shows the properties of the tested materials [2, 3, 10-12]. This resin was prepromoted with Cobalt Naphthenate (6%solution), as an accelerator in percentage of 0.2 % by volume, and Methyl Ethyl Ketone (MEK) peroxide as a catalyst in a percentage of 2% by volume, depending on room temperature. The volume fraction (V_f), in the present work, ranges from 55% to 65% was used; because this range has proved its suitability to ensure specimens with good strength, good adhesion between fibers and matrix, and acceptable mechanical properties.

2.1.2. Shape and dimensions

Thin-walled tubular specimens were used for the experimental work to ensure having a plane uniform stress. Fig. 1 shows the dimensions of the used specimens. These dimensions are similar to those used by pervious investigators [2, 3, 12]. To avoid the failure of some specimens at the end of the gauge length, beneath the grippers, two wooden plugs were inserted into the specimens from both ends. And an elastic sleeve was shrinked on the outer surface at both ends.

2.2. Testing machine

A strain controlled testing machine, previously designed by Abouelwafa M.N. et al. [13] and used by other researchers [2, 3, 10-12] in similar work, was used. It is a constant speed machine of 525 rpm (8.75 Hz); and capable of performing pure torsion, pure bending, or combined torsion and bending (in-phase or out-of-phase) fatigue tests. Fig. 2 shows a general layout for the machine, the loading systems (torsion and bending) are independent, and have the facility to apply different mean stresses.

2.3. Stress state

The global stresses resulted from bending and torsional moments may be found from the following equations:

$$\sigma_x = \frac{My}{I}\,,\ \sigma_y = 0.0\,,\ \tau_{xy} = \frac{Tr}{J}\,.$$

Where:

M	is the applied bending moment ($M = M_m + M_a \sin(\omega t)$),
T	is the applied torque ($T = T_m + T_a \sin(\omega t)$),
$M_m, M_a,$ $T_m and\ T_a$	are the mean and amplitude of bending and torsional moments, respectively,
(ωt)	is the rotating angle,
I	is the second moment of area,
J	is the second polar moment of area, and
d_o, d_i	are the outer and inner diameters of the specimen, respectively.

$$I = \frac{\pi}{64}(d_0^4 - d_i^4)\,,\ J = \frac{\pi}{32}(d_0^4 - d_i^4)\,,$$

$$Y = r = \frac{d_o}{2}\,.$$

The local stresses for [0, 90]$_2$ specimens were $\sigma_1 = \sigma_x, \sigma_2 = 0, and \sigma_6 = \tau_{xy}$, while for [±45]$_2$ specimens were,

$$\sigma_1 = \frac{\sigma_x}{2} + \tau_{xy}, \sigma_2 = \frac{\sigma_x}{2} - \tau_{xy}, and \sigma_6 = -\frac{\sigma_x}{2}$$

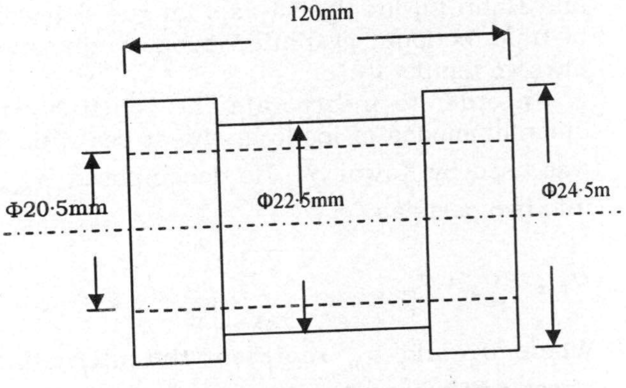

Fig. 1. Dimensions of used specimens.

groups of tests; the first one was with constant amplitude stress and different mean stresses resulting in varying maximum stresses, and the second group was with constant maximum stress with different combinations of mean and amplitude stresses. When plotting the test results, they used three forms of S-N curves; they used ($\sigma_{\max}$), (σ_a) and ($\sqrt{\sigma_{\max}\sigma_a}$) as the ordinate versus the number of cycles to failure as abscissa. The plots indicated that, using the ($\sqrt{\sigma_{\max}\sigma_a}$), which is the SWT parameter, is slightly better than using ($\sigma_{\max}$) and both are better than (σ_a). This was because ($\sqrt{\sigma_{\max}\sigma_a}$) succeeded in showing the effect of mean stress in both types of tests.

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3. Test results

Most researchers use the stress Ratio (R) when dealing with the effect of mean stress under different loading conditions [2, 3, 10-12]. The stress R is defined to be the ratio between the minimum to the maximum applied stresses. Specimens were fatigue tested in-phase. For each orientation $[0, 90]_2$ and $[\pm 45]_2$, the data points were used to plot the corresponding S-N curves on a semi-log scale, being fitted using the power low Max. Stress = $a N^b$.

The data points of the completely reversed pure bending tests of $[0, 90]_2$ and $[\pm 45]_2$ were plotted in figs. 3, while fig. 4 shows the data points of completely reversed pure torsion. Fig. 5 shows the corresponding S-N curve for $[0, 90]_2$ with different stress ratios (R = -1, -0.75, -0.5, -0.25, 0) at the ratio of the flexural stress (A) to the torsional shear stress (B), A/B= 2, while fig. 6 is for $[\pm 45]_2$ at A/B=1. The two constants (a) and (b), of the used power law, $\sigma_{max} = a N^b$, for $[0, 90]_2$ and $[\pm 45]_2$ specimens are found in tables 2 and 3, respectively.

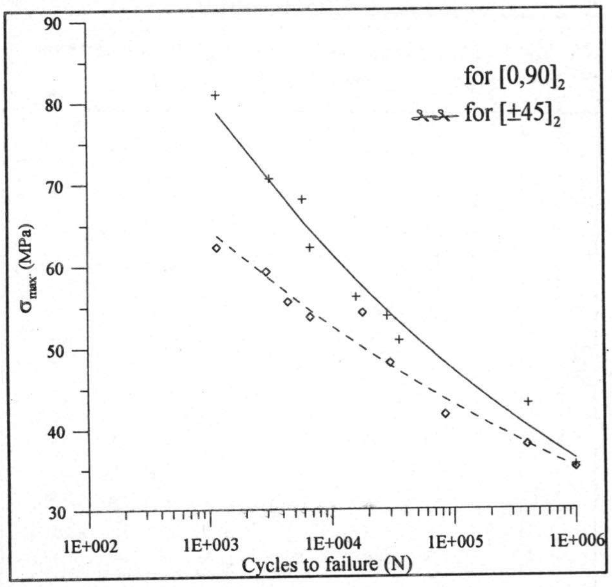

Fig. 3. Completely reversed pure bending S-N curve of $[0, 90]_2$ and $[\pm 45]_2$ specimens.

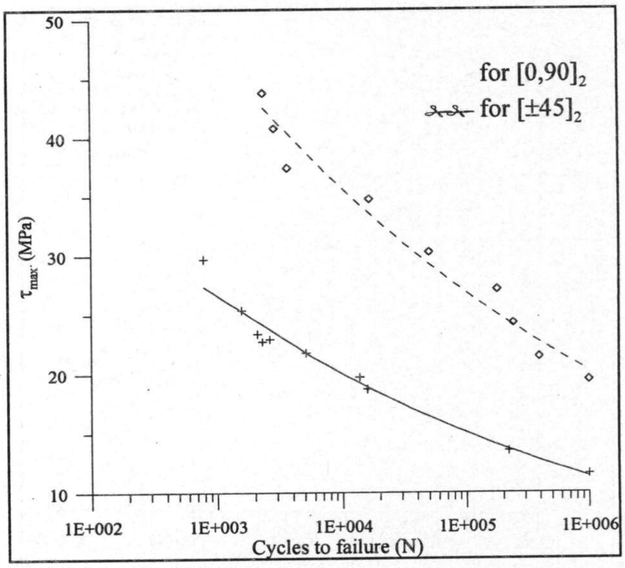

Fig. 4. Completely reversed pure torsion S-N curve of $[0, 90]_2$ and $[\pm 45]_2$ specimens.

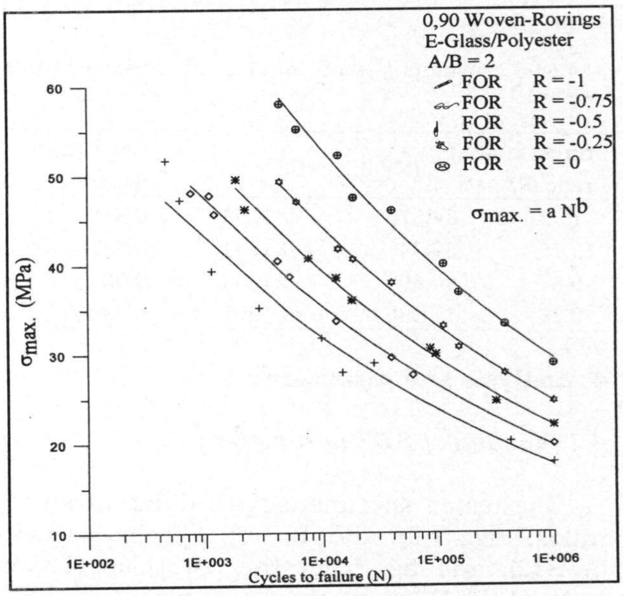

Fig. 5. S-N curve for $[0, 90]_2$ specimens at all stress ratios with A/B=2.

Table 2
Fatigue constants (a) and (b) of $[0, 90]_2$ specimens with A/B=2

Stress ratio (R)	a. (MPa)	b.	Correlation factor
-1	103.93	-0.1286	0.9748
-0.75	113.92	-0.1271	0.9967
-0.5	126.11	-0.1266	0.994
-0.25	141.85	-0.1267	0.9965
0	170.17	-0.1265	0.9837

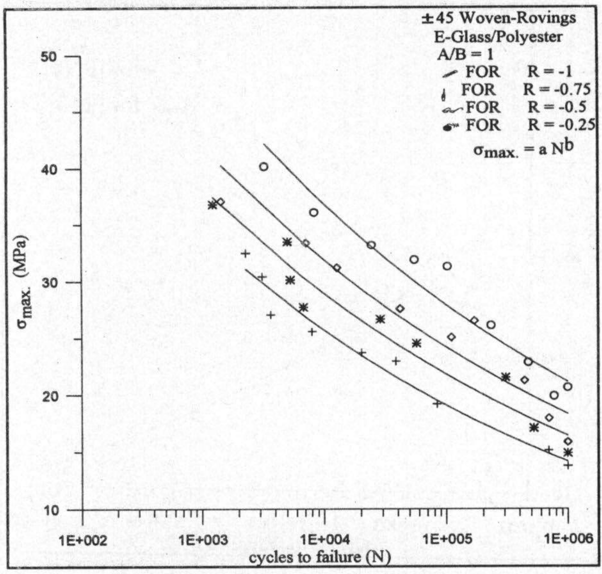

Fig.6. S-N curve for $[\pm45]_2$ specimens at all stress ratios with A/B=1

Table 3
Fatigue constants (a) and (b) of $[\pm45]_2$ specimens with A/B=1

Stress ratio (R)	(a), (MPa)	(b)	Correlation factor
-1	84.578	-0.1294	0.978
-0.75	89.94	-0.1231	0.937
-0.5	96.66	-0.1204	0.9042
-0.25	111.86	-0.1207	0.9363

4. Analysis and discussion

4.1. Validity of SWT parameter

The tested specimens with different stress ratios; Fig. 5 for the $[0, 90]_2$ specimens with (A/B=2), and fig. 6 for the $[\pm45]_2$ specimens with (A/B=1) were used to plot the SWT parameter for each of the two fiber orientations, against the number of cycles to failure. The values of SWT parameter at different stress ratios (R) were obtained from the following equation:

$$SWT = \sqrt{(\sigma_{\max}\sigma_a)}\,.$$

This formula can be deduced to the following form:

$$SWT = \sigma_{\max}\sqrt{\frac{1-R}{2}}\,.$$

Figs. 7 and 8 show these results for the $[0, 90]_2$ and the $[\pm45]_2$ specimens, respectively. Using the power-law form: $SWT = a_1\ N^{b1}$ for fitting these data points, the values of the constants, (a_1) and (b_1), are given in table 4.

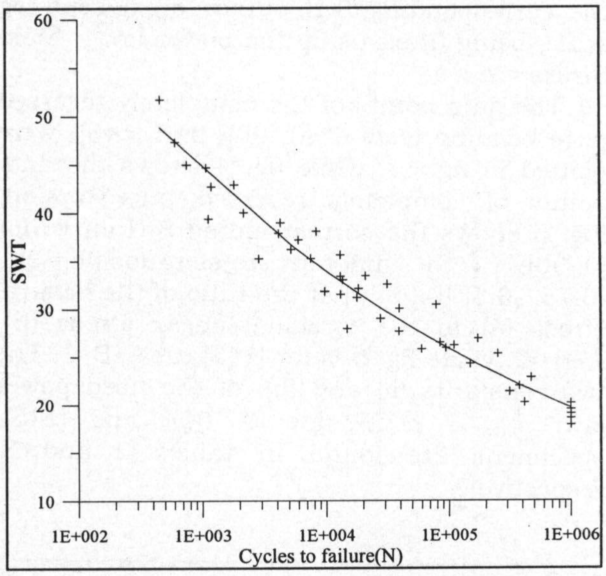

Fig. 9. SWT parameter for the $[0, 90]_2$ specimens with (A/B=2.)

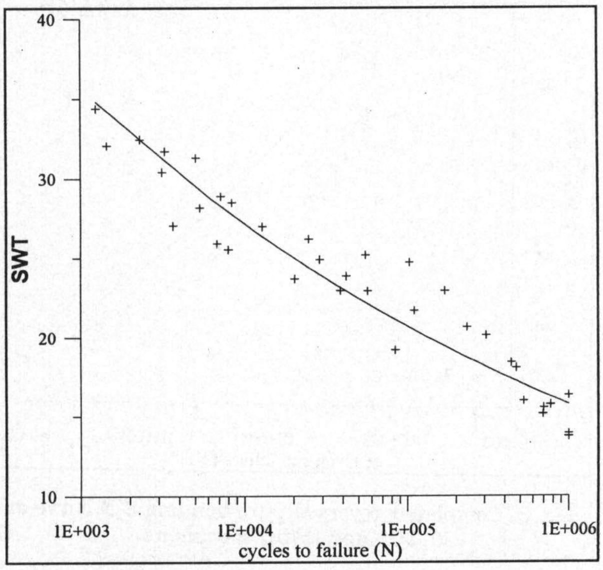

Fig. 10. SWT parameter for the $[\pm45]_2$ specimens with (A/B=1).

Table 4
Values of constants (a_1) and (b_1) for the SWT parameter, SWT = $a_1 N^{b1}$

Specimen type	(a_1), MPa	(b_1)	Correlation factor
$[0,90°]_2$ (A/B=2)	101.09	-0.1185	0.9498
$[\pm 45]_2$ (A/B=1)	81.17	-0.1188	0.9071

Comparing the values of the two constants (a_1) and (b_1), table 4, to the corresponding two constants (a) and (b) of the completely reversed test (R=-1) for corresponding fiber orientation resulted in a good promising result. This means that the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ can be used to predict fatigue life in the case of woven-roving GFRP with $[0, 90]_2$ and $[\pm 45]_2$ orientations under combined bending and torsional fatigue loading with negative stress ratios. Performing only the completely reversed (R=-1) fatigue test and using the SWT parameter will be sufficient to find out the life of the material under any negative stress ratio.

4.2. Modified fatigue strength ratio (Ψ)

4.2.1. Validity for the present work

The fatigue tests with different stress ratios; for the $[0, 90]_2$ specimens with (A/B=2) and for the $[\pm 45]_2$ specimens with (A/B=1) were used to plot the modified fatigue strength ratio (Ψ) for each of the two fiber orientations against the number of cycles to failure, using the power-law form: $\Psi = a_2 N^{b2}$ for fitting these data points, as shown in figs. 11 and 12.

Table 5 gives the values of the curve fitting constants (a_2) and (b_2) with acceptable correlation factors.

4.2.2. Validity for previous workers

To check the validity of the modified fatigue strength ratio (Ψ) for other loading conditions, data was obtained from Mohamed N.A. [2] and Sharara A. [3]. Their specimens had nearly the same specifications as those used in this work. Mohamed N.A. [2] tested his specimens under uniaxial torsional stress, while the specimens of Sharara A. [3] were fatigue tested under uniaxial bending stress, both with the same stress ratios as in this work. Theses data were used to plot the modified fatigue strength ratio (Ψ) against the number of cycles to failure, as shown in figs. 13 to 16, and resulting in acceptable correlation factors, table 5.

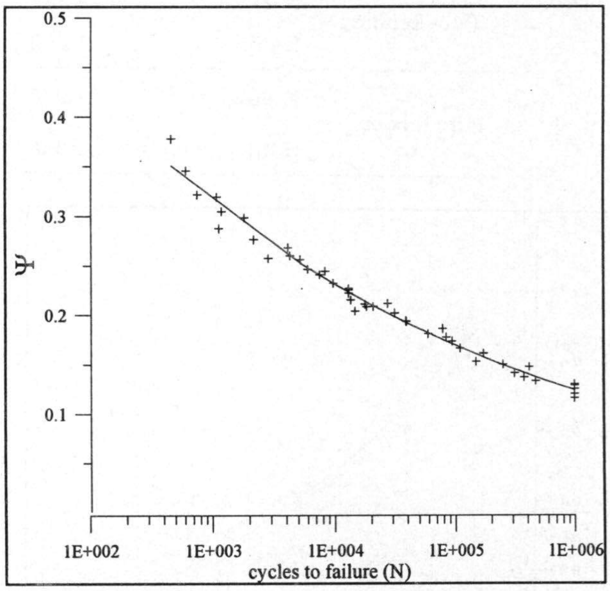

Fig. 11. Modified fatigue strength ratio (Ψ) for the $[0, 90]_2$ specimens with (A/B=2).

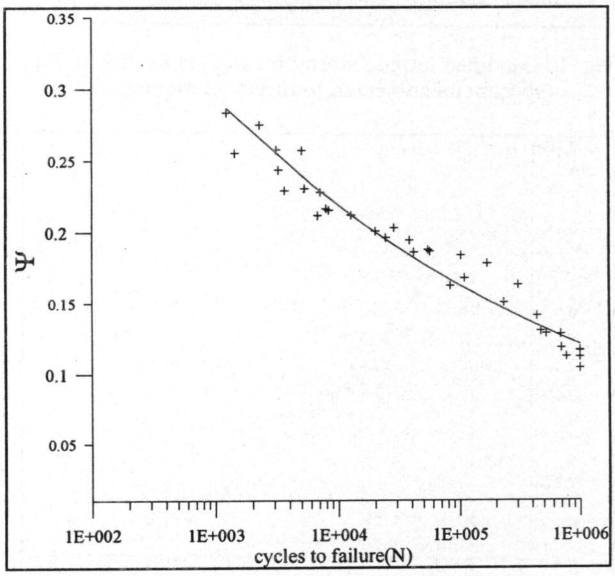

Fig. 12. Modified fatigue strength ratio (Ψ) for the $[\pm 45]_2$ specimens with (A/B=1).

Table 5
Values of constants (a_2) and (b_2) for the modified fatigue strength ratio: $\Psi = a_2 N^{b2}$

Test	Specimen type	(a_2), MPa	(b_2)	(a/s_u)	Correlation factor
(A/B=2)	$[0,90]_2$	0.8044	-0.1352	0.78	0.9872
(A/B=1)	$[\pm45]_2$	0.7184	-0.1289	0.75	0.9425
Pure bending	$[0,90]_2$	1.119	-0.134	1.1	0.9342
	$[\pm45]_2$	1.203	-0.136	1.14	0.916
Pure torsion	$[0,90]_2$	1.273	-0.113	1.28	0.9153
	$[\pm45]_2$	1.346	-0.1302	1.36	0.9856

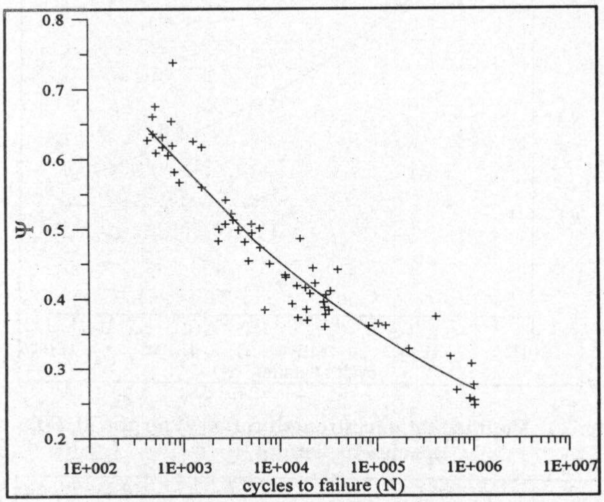

Fig. 13. Modified fatigue strength ratio (Ψ) for the [0,90]₂ specimens subjected to torsional moments.

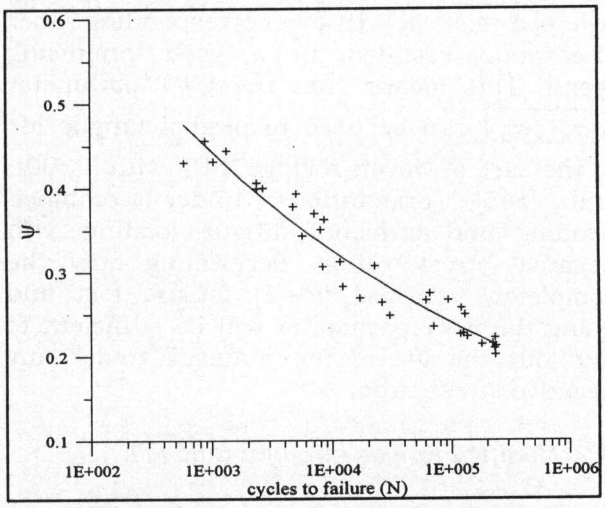

Fig. 15. Modified fatigue strength ratio (Ψ) for the [0, 90]₂ specimens subjected to bending moments.

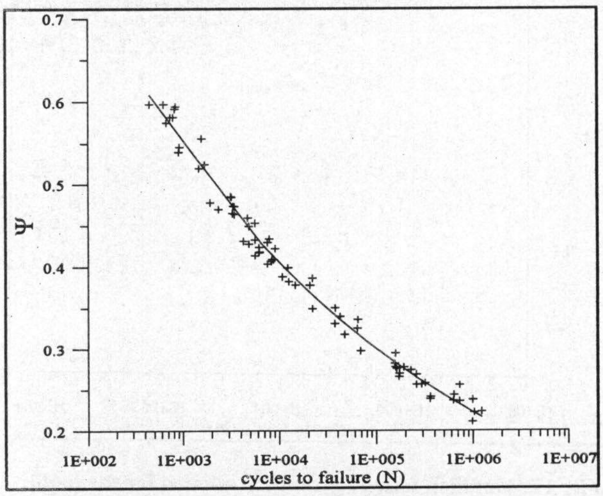

Fig. 14. Modified fatigue strength ratio (Ψ) for the [±45]₂ specimens subjected to torsional moments.

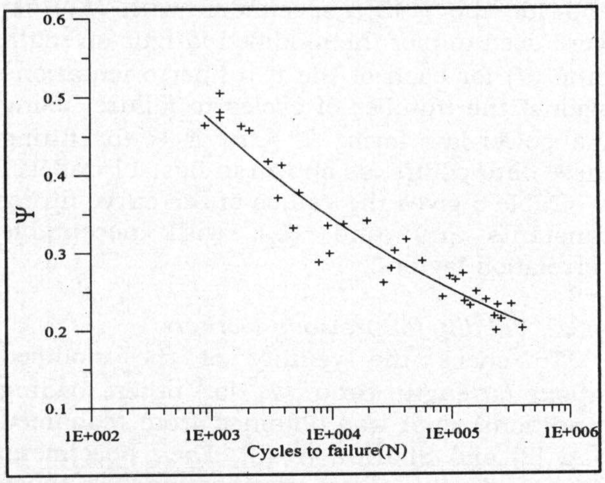

Fig. 16. Modified fatigue strength ratio (Ψ) for the [±45]₂ specimens subjected to bending moments.

From table 5 we can notice that:
- The deviation in the values of (b_2), at different loading conditions for both fiber orientations $[0, 90]_2$ and $[\pm 45]_2$, is negligible. So it may be considered to be material constant:

$b_2 = - 0.12955$
- The value of (a_2) was found to be nearly equal to the ratio (a/s_u).

Where:

s_u is the static strength in fiber direction

a is the the constant of ($\sigma_{max.} = a\ N^b$) and its value is determined according to the type of loading from:

1. Completely reversed pure bending test for tests under pure bending moments with different mean stresses.
2. Completely reversed pure torsion test for tests under pure torsional moments with different mean stresses.
3. Completely reversed combined bending and torsional moments with (A/B=2) for tests under combined bending and torsional moments with different mean stresses.

This means that the modified fatigue strength ratio (Ψ) has become a useful measure for establishing the master S-N relationship for woven-roving GFRP under uniaxial bending stress, unaxial torsional stress, and combined bending and torsional fatigue moments with $[0, 90]_2$ and $[\pm 45]_2$ orientations over a range of stress ratio.

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