Optimum estimation of single-phase inverter's switching angles using genetic based algorithm

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Programmed PWM technique requires the solution of a set of non-linear trigonometric equations in order to obtain the switching angles as well as controlling the fundamental component of inverter output voltage. Normally iterative methods are used to solve these equations, where divergence problems are highly arise as the initial values of the switching angles have to be close to the correct values of an exact solution. This paper proposes a new method for the solution of the programmed PWM equations using genetic based algorithm. The proposed GA technique estimates the optimum switching angles for PWM single-phase inverters, such that a pre-selected harmonics are minimized, or ideally eliminated, as well as controls the fundamental component. The proposed GA technique is applied to a bi-polar PWM scheme to estimate 3 angles and to minimize 2 harmonics as well as controlling the fundamental component. It is also applied to uni-polar PWM scheme to estimate 3, 4, and 5 angles in order to minimize 2, 3, and 4 harmonics respectively besides the control of the fundamental component. The estimated angles are applied to an inverter circuit as look-up tables in order to verify its validity through output waveforms, harmonic analysis, THD, and FFT.

يتطلب الحل الأمثل لتقنية تعديل عرض النبضة لدوائر العاكس حل مجموعة من المعادلات المثلثية الغير خطية وذلك للحصول علي زوايا الفتح والغلق وللتحكم في قيمة المركبة الرئيسية لجهد الخرج. وتستخدم عادة طرق تكرارية لحل تلك المعادلات حيث تظهر مشكلة عدم التقارب أو عدم الوصول للحل وذلك بسبب أن القيم الابتدائية المفروضة لزوايا الفتح والغلق لابد أن تكون قريبة جدا من الحل الدقيق وهذا ما يصعب تحقيقه. يقدم هذا البحث طريقة جديدة تعتمد علي تقنية الجينات للحصول علي قيم زوايا الفتح والغلق المثلي لتقنية تعديل عرض النبضة لدوائر العاكس أحادية الوجه ذات جهد الخرج أحادي وثنائي القطبية.

Keywords: Index terms-harmonic elimination, Genetic Algorithm, Programmed PWM, Bi-polar PWM, Uni-polar PWM, Single-phase inverters

1. Introduction

Programmed PWM techniques optimize a particular object function such as to obtain minimum losses, to reduced torque pulsations (in drive systems) or for selective elimination of harmonics. They are the most effective means of obtaining high performance results, and they are often referred to as "*optimum PWM*" techniques [1]. This technique offers some interesting advantages such as:

1. About 50% reductions in inverter switching frequency.
2. Higher voltage gain due to over modulation, therefore higher utilization of power conversion process.
3. High quality of output voltage and current, small ripple in dc link current, reduction in size of dc link filter components.
4. Reduction in switching losses, especially in high power applications.

However, it has some drawbacks such as:

1. Computational difficulties (lower-output frequency range, large number of PWM switching instants).
2. Sometimes the system is trapped in a local minimum.

In optimum PWM technique, the fundamental component is set to a required amplitude while the low-order harmonics are set to zero. This is the most common approach in electric drives since low-order harmonics are the most detrimental to motor performance. In other applications, like active harmonic filters or control of electromechanical systems, harmonics are set to non-zero values. This task of designing a PWM waveform using the Fourier series coefficients to match those of a desired waveform has been the subject of many papers [2-5]. Often, the Newton iteration method or an unconstrained optimization approach is used to solve the system of nonlinear equations. Those methods are computationally intensive for on-line calculations and the storage of off-line calculations leads to high memory requirements. Recent results from the research community show two approaches to real-time implementation of an approximate optimal PWM. One approach is to fine tune conventional PWM techniques like regular-sampled [6] or space-vector methods [7] to approximate programmed PWM switching patterns. Another approach is to simplify the nonlinear harmonic elimination equations [8] in order to obtain real-time approximate solutions using modern digital signal processors.

Genetic Algorithms (GAs) have recently received much attention as robust stochastic search algorithms for solving optimization problems. GAs have been applied to various power system problems with promising results [9-11]. This paper introduces a new digital method based on GAs for eliminating harmonics in a switching converter. The problem is formulated as an estimation problem. The proposed GA based technique uses digital samples of the voltage waveform to estimate the switching angles in an electrical cycle. These switching angles are used for turning the switches on and off in a full bridge inverter, controlled by either a bi-polar or a uni-polar PWM control schemes, so as to produce a desired fundamental amplitude while eliminating a sepcific harmonics.

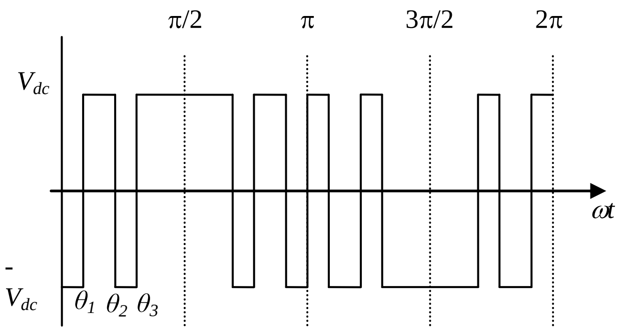

Fig.1. Bi-polar PWM waveform.

2. Problem formulation

In this section, the mathematical formulation of the problem is presented for both of bipolar and unipolar cases.

2.1. Bi-polar PWM scheme

Fig. 1 shows the standard bi-polar PWM waveform with a 3 switching angles θ_1, θ_2, and θ_3. The Fourier series expansion of such a voltage waveform is given by:

$$V(\omega t) = \frac{4V_{dc}}{\pi}\left\{\sum_{n=1,3,5,\ldots}^{\infty} -\frac{sin(n\omega t)}{n} \left[1-2cos(n\theta_1)+2cos(n\theta_2)-2cos(n\theta_3)\right]\right\}. \quad (1)$$

Given a desired fundamental voltage V_1, the problem is to find the switching angles that satisfy the following equations:

$$\left.\begin{aligned} &1-2cos(\theta_1)+2cos(\theta_2)-2cos(\theta_3)=-m \\ &1-2cos(3\theta_1)+2cos(3\theta_2)-2cos(3\theta_3)=0 \\ &1-2cos(5\theta_1)+2cos(5\theta_2)-2cos(5\theta_3)=0 \end{aligned}\right\}. \quad (2)$$

Where m is the modulation index, and is given by:

$$m = \frac{V_1}{\left(\frac{4V_{dc}}{\pi}\right)} \; . \quad (3)$$

2.2. Uni-polar PWM scheme

In this case, the Fourier series expansion of the voltage waveform shown in fig. 2 is given by:

$$V(\omega t) = \left\{\sum_{n=1,3,5,..}^{\infty} \frac{4V_{dc}}{n\pi} \left\{cos(n\theta_1)-cos(n\theta_2)+cos(n\theta_3)\right\} sin(n\omega t)\right\} \quad (4)$$

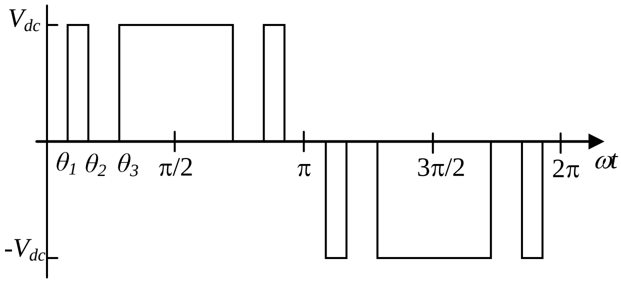

Fig. 2. Uni-polar PWM waveform.

Again, the problem is to find the switching angles that satisfy the following set of equations:

$$\left.\begin{aligned}\cos(\theta_1)-\cos(\theta_2)+\cos(\theta_3)&=m\\ \cos(3\theta_1)-\cos(3\theta_2)+\cos(3\theta_3)&=0\\ \cos(5\theta_1)-\cos(5\theta_2)+\cos(5\theta_3)&=0\end{aligned}\right\} \quad (5)$$

In a matrix form, each of the above-mentioned PWM schemes can be described by the following compact equation:

$$Z(x)=F(x)+e\,, \quad (6)$$

where,

Z(x) is a 3x1 vector represents the equation constants,
F(x) is a 3x1 information vector,
x are the parameters to be estimated (switching angles), and
e is a 3x1 vector represents the error.

This formulated estimation problem will be solved using a simple genetic algorithm. The goal is to find the best estimate for the parameters (angles) that minimizes the error (e).

3. Genetic algorithms

GAs are adaptive search procedures for optimization and learning. The concepts of the algorithms are based on natural selection and natural population genetics. They involve survival of the fittest among string structures. In every generation, a new set of strings is generated using bits and pieces of the fittest previous strings. They efficiently exploit historical information to speculate on new search points with expected improved performance [9]. GAs differ than other conventional optimization techniques in many ways. They use the objective function itself and not the gradient, they search from a population of strings and not single point and they work with a coding of the parameter set, not with the parameter themselves. Because of these reasons, and others, GAs are considered as an attractive alternative optimization technique.

GA is a simple algorithm, starts with random generation of a population. A population consists of a set of strings. Usually, the string size ranges between 50-1000. The population may be of any size according to the accuracy required. The population size remains constant throughout the whole process. A string in GAs may be divided into a number of sub-strings. The number of sub-strings, usually, equals to the number of the problem variables. The problem variables are coded using suitable coding system. In this study binary coding system is used. In addition to coding and fitness evaluation, the simple GA is composed of another three basic operations; Reproduction, Crossover and Mutation. Each string of the old population goes through these three steps before a new population is generated [12].

3.1. Fitness function

The Fitness Function (FF) is one of the key elements of GAs as it determines whether a given potential solution will contribute its elements to future generation through the reproduction process. The FF should be able to provide a good measure of the quality of the solution and should differentiate between the performance of different strings. The objective here is to minimize the estimation error (*e*) in determining the switching angles such that some selected harmmonics are minimized or put equal to zero. Two different functions are proposed to evaluate the quality of the solution.

3.1.1. Sum of Square errors fitness function (SS)

By squaring the individual errors (e_i) of eq. (6) and adding them together, we can find the root mean square error for N equations as:

$$F_{sum} = \sqrt{\frac{\sum_{i=1}^{N} e_i^2}{N}} \; . \qquad (7)$$

Since the objective of GAs is to maximize the objective function, it is necessary to map the error square function (F_{sum}) into minimization fitness function (SS) as:

$$SS = \frac{1}{\left(F_{sum} + \Delta\right)} \; . \qquad (8)$$

Where Δ is a small constant (Δ=0.00001 in this work) to avoid overflow problems if F_{sum} goes to zero.

It is important to mention here that this formulation assumes that the error of eq. (6) is a Gaussian white sequence with known covariance as,

$$E\left\{e(i)e(j)^T\right\} = \begin{cases} 0 & ; i \neq j \\ R(i) & ; i = j \end{cases} \; . \qquad (9)$$

3.1.2. Absolute Error fitness function (AE)

In this case, the fitness function is set to minimize the summation of the individual absolute errors. Thus we can write *AE* as:

$$AE = \frac{1}{\sum_{i=1}^{N} |e_i| + \Delta} \; . \qquad (10)$$

This formulation assumes that the error is exponentially distributed. In the presented study, the two fitness functions are used in testing. It is found that AE fitness function gives more accurate results. Indeed the small number of eqs. (3 in bi-polar PWM, and 3-5 in uni-polar PWM) makes the reultant error has a random distribution and not gaussian. Therefore results presented in the next section are those obtained using AE fitness function only.

3.2. Reproduction

As mentioned above, every time of a duration of a search, each string in the population is evaluated using FF. After the evaluation, according to the fitness, strings with higher fitness (parents) are stochastically selected from the old population and copied according to the probability determined by a parent selection technique. The parent selection can carried out in a variety of ways. In this study, the process is carried out using roulette wheel technique [9].

3.3. Crossover

Crossover is a genetic step in which the members of the population obtained after reproduction process are randomly mated according to pre-specified probability. Each pair mutually interchanges a portion of bits. The position at which the interchange starts is selected randomly. In this way, new strings are generated to form the new population. Crossover can occur at a single position or at a number of different positions. In this study, single position crossover is used.

3.4. Mutation

After crossover, the population passes through another genetic process called mutation. In this process randomly selected bits of a randomly selected strings are changed from 0 to 1 and vice versa. This process occurs according to pre-specified probability, usually less than 5% of bits are changed in this process. Mutation process is used to escape from probable local optimum.

A GA toolbox package for Matlab is used to perform the study, “FlexGA version 1-CynapSys.LLC”. This code initializes a random sample of indiviuals with different parameters to be optimized using the GA algorithm approach. Based on experience, trial and error, the GA operator probabilities and sizes are selected [12]. A population size of 100 was found to enough for our problem. The crossover probability used was 0.6 and mutation was 0.02. The coding system used in all tests was the binary system.

4. BI-Polar PWM scheme

The above-mentioned GA is applied to the bi-polar PWM single-phase inverter described by the set of eq. (2) in order to obtain the optimum switching angles such that the 3rd and 5th harmonics are minimized (or ideally put equal to zero). Therefore, the estimation process is required to obtain three switching angles θ_1, θ_2, and θ_3 such that the 3rd and 5th harmonics are eliminated as well as the fundamental component is controlled. This computation was done as m was incremented between 0 and 1, where only solutions are found between m=0.05 and 0.85. The variation of the optimum estimated values of the switching angles θ_1, θ_2, and θ_3 versus m is shown in fig. 3.

The harmonic components are calculated using the estimated switching angles. The corresponding Total Harmonic Distortion (THD) was computed out to the 21st harmonics according to

$$THD = \sqrt{\frac{V_3^2 + V_5^2 + V_7^2 + \cdots V_{21}^2}{V_1^2}} \, . \tag{11}$$

and is plotted versus m in fig. 4. Since the number of the eliminated harmonics is quite low, the value of the THD is expected to be quite high especially at low values of m.

Since the *rms* value of the output voltage at any m is given by:

$$V_{rms} = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{V^2(\omega t) d\omega t} = V_{dc} \, . \tag{12}$$

This voltage is constant irrespective of the value of m, for each given set of eliminated harmonics of the proposed algorithm. Therefore, the THD is constant and is only being shifted in the frequency spectrum.

In order to verify the validity of the estimated values of the switching angles, the estimated switching angles are converted to the time domain, implemented as look-up tables, and used to trigger an inverter circuit simulated on Simulink. Although a great number of simulation results are obtained, only a case is selected (with m=0.8) for demonstration and verification. The output waveform of the bipolar PWM scheme together with the fundamental component for m = 0.8 are shown in fig. 5. The FFT analysis is carried out where, the corresponding harmonic spectrum is shown in fig. 6. As estimated, the 3rd and 5th harmonics are eliminated and the 7th is highly reduced, however there is a few percent of harmonic content in the 9th, 11th, and 13th. Therefore, at this modulation index, the THD due to the remaining harmonics is over 80% as previously shown in fig. 4. The validity of the estimated switching angles for several values of m are also verified.

5. Uni-polar PWM scheme

Since bi-polar PWM scheme cannot produce a zero output voltage, it has a higher harmonic content. However, uni-polar PWM scheme can inherently produce a zero output voltage, and therefore has a lower harmonic content. Therefore, it is worth noting to consider the application of the proposed GA technique to minimize the 3rd and 5th harmonics in uni-polar PWM scheme to compare the results with those obtained from bi-polar PWM scheme.

The proposed GA technique is used to estimate the switching angles θ_1, θ_2, and θ_3 such that the 3rd and 5th harmonics are minimized according to the set of eq. (5). The variation of the estimated switching angles versus m is shown in fig. 7. These angles are also used to determine the harmonic analysis in order to obtain the THD. The variation of the THD, calculated up to the 21st harmonic, versus m is shown in fig. 8. Compared to that of bi-polar PWM, shown in fig. 4, the THD is significantly reduced during the entire range of m. The verification of the estimated switching angles is illustrated in the output waveforms in fig. 9, where a value of m=0.5 is selected for illustration. The harmonic spectrum of fig. 10 verifies the validity of the estimated angles where the 3rd and 5th harmonics are completely eliminated. However, a THD above 80% was found and verified from fig. 8 due to the high content

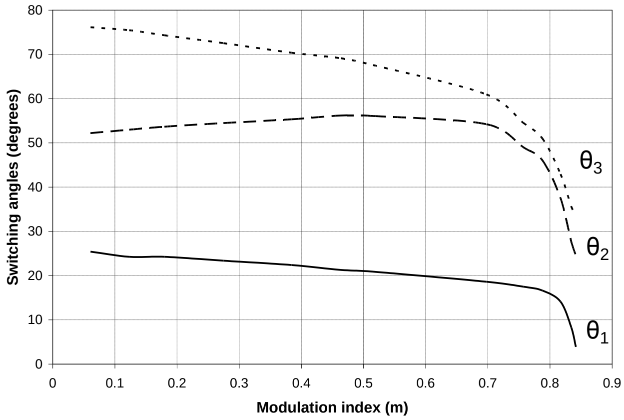

Fig. 3. Bi-polar PWM switching angles versus m (Elimination of the 3rd and 5th harmonics).

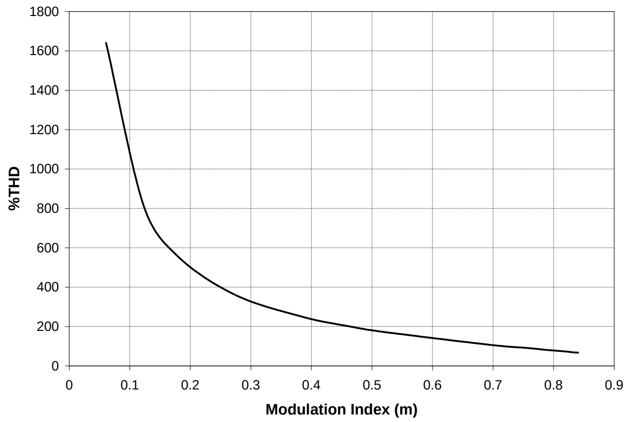

Fig. 4. THD of Bi-polar PWM versus m (Elimination of the 3rd and 5th harmonics).

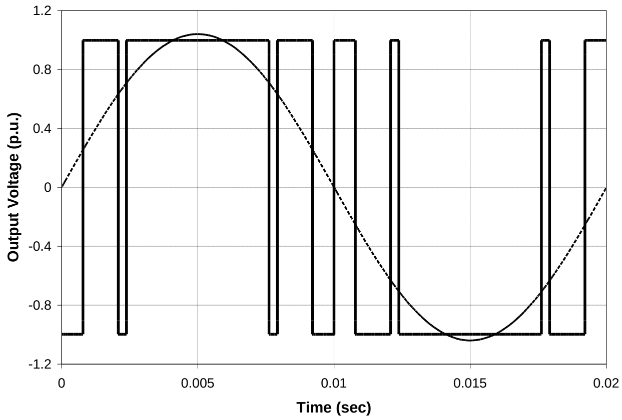

Fig. 5. Bi-polar PWM output waveform and its fundamental component (m=0.8, Elimination of the 3rd and 5th harmonics).

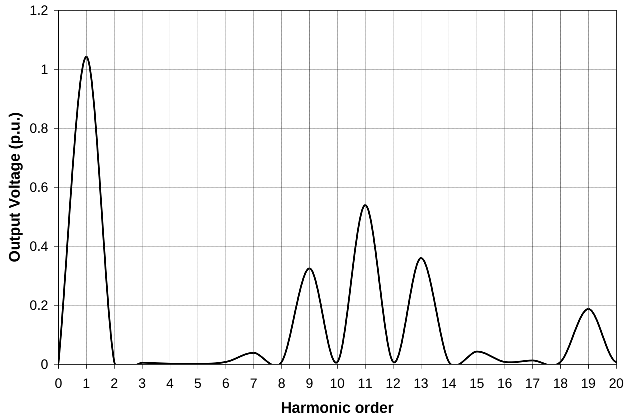

Fig. 6. Harmonic spectrum of output waveform of bi-polar PWM (m=0.8, Elimination of the 3rd and 5th harmonics).

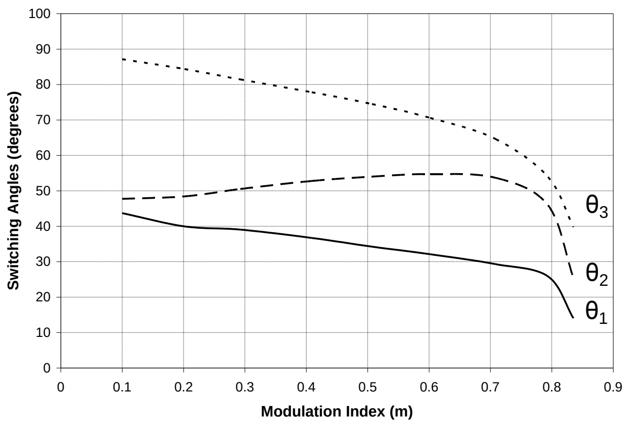

Fig.7. Uni-polar PWM switching angles versus m (Elimination of the 3rd and 5th harmonics).

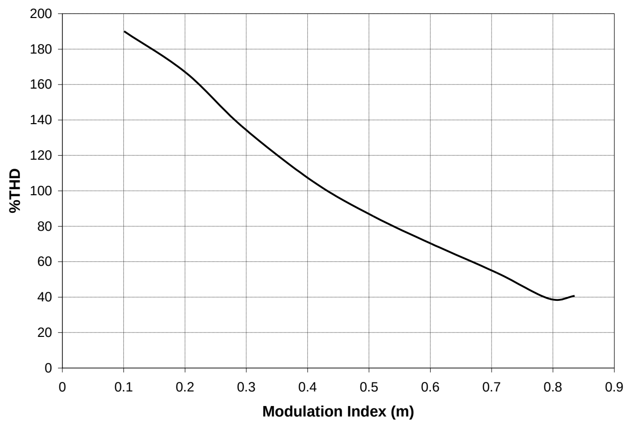

Fig.8. THD of Uni-polar PWM scheme versus m (Elimination of the 3rd and 5th harmonics).

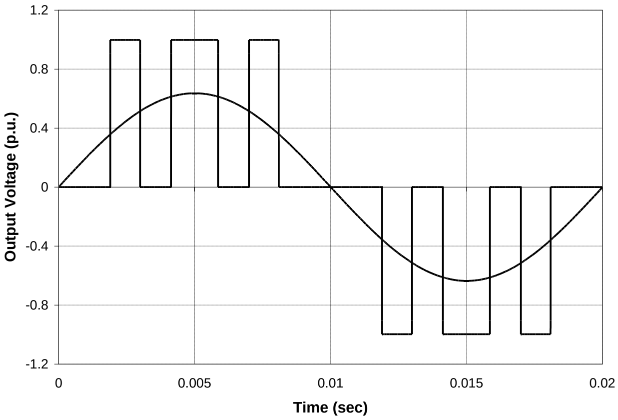

Fig.9. Uni-polar PWM output waveform and its fundamental component (m=0.5, elimination of the 3rd and 5th harmonics).

Fig.10. Harmonic spectrum of output waveform of uni-polar PWM (m=0.5, Elimination of the 3rd and 5th harmonics).

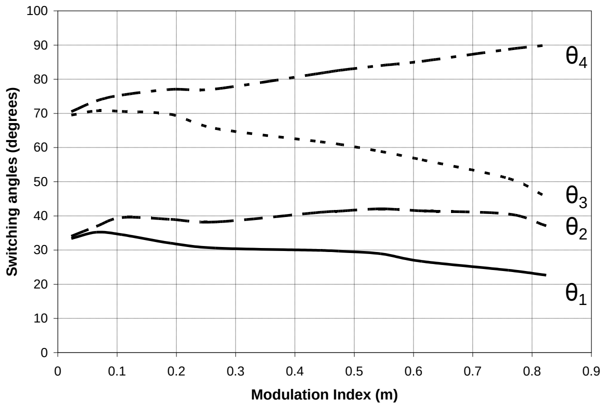

Fig.11. Uni-polar PWM switching angles versus m (elimination of the 3rd, 5th, and 7th harmonics).

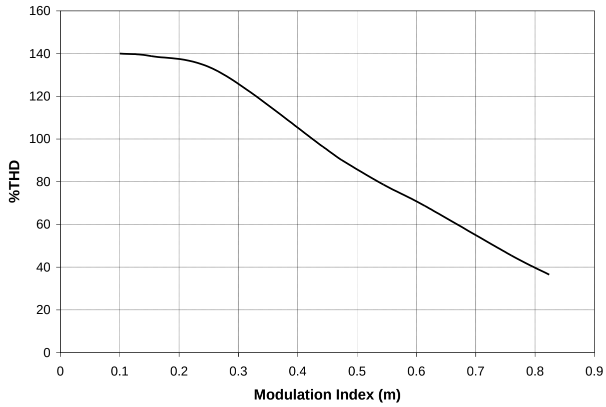

Fig.12. THD of Uni-polar PWM scheme versus m (elimination of the 3rd, 5th, and 7th harmonics).

of higher harmonic order specially the 7th harmonic.

The application of the proposed GA technique to uni-polar PWM technique is extended to the of minimization of the 3rd, 5th, and 7th harmonics, where the estimation of 4 angles θ_1, θ_2, θ_3, and θ_4 is required. The variation of the estimated switching angles and the corresponding THD (calculated up to the 21st harmonics) versus m are shown in figs. 11 and 12, respectively. The output waveform together with its fundamental component are shown in fig. 13 for m=0.6, and fig. 14 its corresponding FFT. As shown, the 3rd, 5th, and 7th harmonics are eliminated. However, there is a few percent of harmonic content in the 9th to 19th order. Therefore, the THD due to the remaining harmonics is about 70%as indicated in fig. 12.

The soundness of the proposed GA technique is tested again for the estimation of 5 switching angles θ_1, θ_2, θ_3, θ_4, and θ_5 in order to minimize the 3rd, 5th, 7th, 9th harmonics. The complete set of figures describing this case is shown in figs. 15 to 18. The THD is continuing to improve and getting smaller due to the increased number of eliminated harmonics. The case shown in figs. 17 and 18 shows that the four harmonics are completely eliminated, and the THD of about 40% is found due to the higher harmonic content. Further reduction of the THD is obviously predicted when the number of eliminated

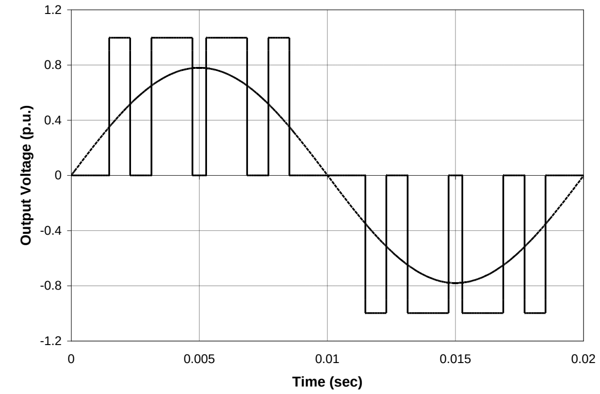

Fig.13. Uni-polar PWM output waveform and its fundamental component (*m*=0.6, Elimination of the3rd, 5th, and 7th harmonics).

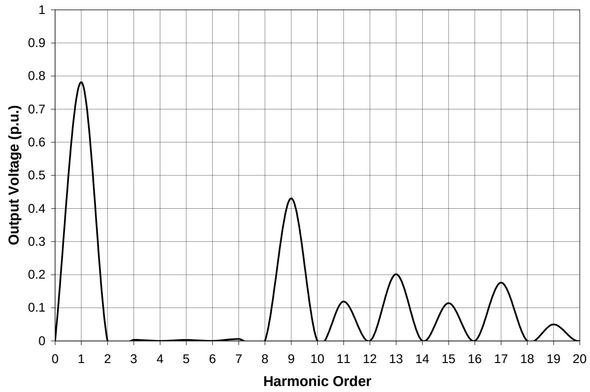

Fig.14. Harmonic spectrum of output waveform of uni-polar PWM (*m*=0.6, Elimination of the 3rd, 5th, and 7th harmonics).

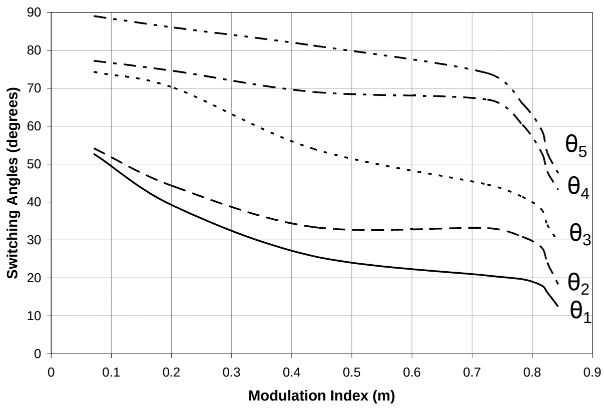

Fig.15. Uni-polar PWM switching angles versus m (elimination of the 3rd, 5th, 7th, and 9th harmonics).

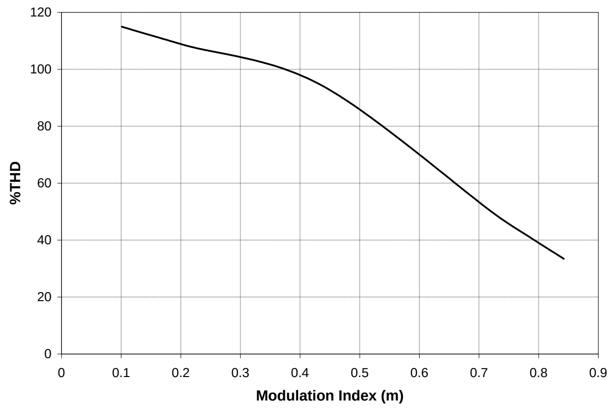

Fig.16. THD of Uni-polar PWM scheme versus m (elimination of the 3rd, 5th, 7th, and 9th harmonics).

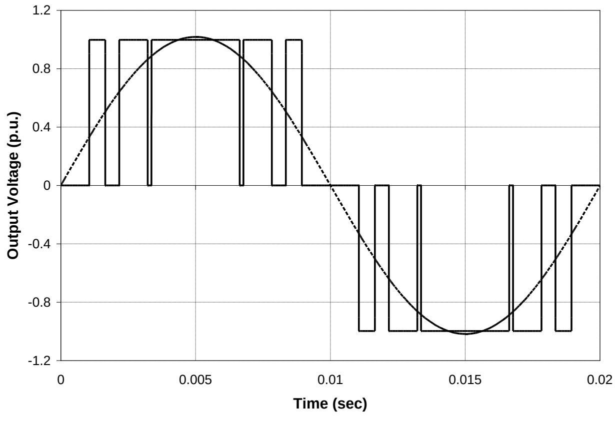

Fig.17. Uni-polar PWM output waveform and its fundamental component (*m*=0.8, elimination of the 3rd, 5th, 7th, and 9th harmonics).

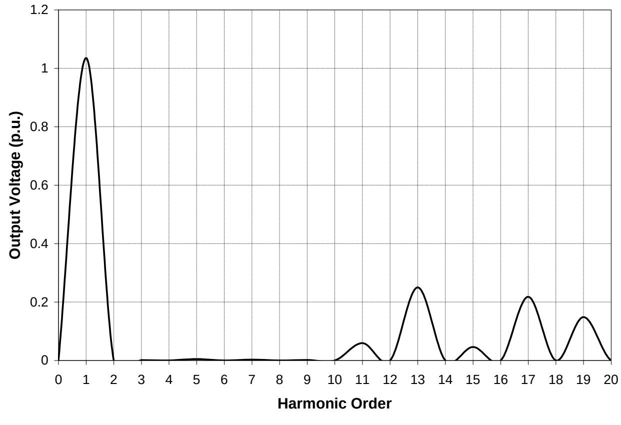

Fig.18. Harmonic spectrum of output waveform of uni-polar PWM (*m*=0.8, elimination of the 3rd, 5th, 7th, and 9th harmonics).

harmonics is increased. This, will in turn, increases the number of the estimated switching angles.

6. Conclusions

A new method of determining the switching angles of the programmed PWM single-phase inverter circuit based on GAs is presented. The problem is formulated as an estimated problem. GA technique is used to find the optimum switching angles estimation based on minimization of a suitable fitness function. The proposed technique has been applied to both bi-polar and uni-polar PWM control schemes. The estimation of up to 5 switching angles per quarter cycle has been performed, while minimizing a pre-selected number of harmonics and, at the same time, controlling the fundamental component. The validity of the estimated angles has been verified from the THD and the FFT of the resultant waveforms, where a complete elimination of the minimized harmonics is achieved.

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