Mapping of nonuniformly distributed mine fields

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Land mine fields must essentially be regular according to certain basis decided by the battle field engineers. But sometimes irregularities in the distribution of the mines within the field could be planned or randomly introduced like those mines thrown by airplanes. The mine distribution may follow certain functional basis or may be represented by a random function.

شمل هذا البحث تناول حالات متنوعة لحقول الغام غير منتظمة التوزيع وأمكن الحصول على نماذج لشكل المجال الكهربي المرتد منها، مثل الحقول ذات التغير المتزايد خطيا أو لوغاريتميا أو طبيعيا في المسافة بين الألغام، ومثل الحقول ذات التغير المتناقص خطيا أو لاخطيا في المسافة بين الألغام. أمكن من خلال دراسة الحالة الا خيره تم رصد حالة التوزيع المتساوي الفرق كهربيا في الوجه بين المجالات الكهربية المرتدة من الألغام وهى تحدث عندما يتبع موقع اللغم القاعدة $Y_n = D\sqrt{n}$ آي الجذر التربيعي لرقمه المتسلسل. ويكون طيف مجالها الكهربي منتظم الشكل، وعدد القيم العظمى لمنحنياته يساوى عدد الألغام الملتقطة. تلا ذلك دراسة تأثير التغير المتأرجح في المسافة بين الألغام سواء بمقدار ثابت أو باتباع دالة مثلثية على شكل المجال الكهرومغناطيسى للحقل، ووجد أن هذه الحالة اقرب إلى الواقع وتعطى نتائج اقرب لنتائج الحقل المنتظم. أيضا تم إدخال إشارات من خرج مولد عشوائي لتمثل تغيرا غير مقنن في المسافة بين الألغام لدراسة تأثير هذا التغير على شكل المجال الكهربي لحقل منتظم خاصة حول الرنين الأساسي وعلاقة ثوابت هذا التغير بالنتائج التي تم الحصول عليها. وقد أمكن صياغة نظام رياضي لتحديد المواقع الغير منتظمه للألغام باستخدام مميز الوجه مع الاشارة المرتده من الحقل الرادارى.

Keywords: Pattern factor, Resonance frequency, Resonance tale

1. Introduction

Irregularly distributed land mine fields can be classified into fields following functional changes or fields whose distributions do not follow functional changes. In the next section both kinds are discussed. In the first kind the distance between every two adjacent mines contains two terms one is constant, the other is represented by different possible forms of alternating changes. In the second kind a randomly disturbed distribution is considered. The relation between any deviation in the regular mine locations and the total field pattern of the whole mine field is studied. At the end, a formulation of a direct relation between the taken measurements and the exact mine location is obtained.

2. Mine distributions with mathematical functions

From a previous study carried out on the regular mine fields [1], frequencies of ptimum detection (resonance frequencies) f_{0i} were related to the regular distance between any two adjacent mines D, for an antenna located at a height h, through,

$$f_{0i} = \frac{i\,c\,h}{D^2}\,, \qquad (1)$$

where c is the light velocity in air. The Pattern Factor PF which represents the normalized received field strength of the whole mine field with respect to the field from the central mine at resonance is given by [1-2], as:

$$PF = 1 + 2\sum_{i=1}^{n} \frac{h^3 e^{-2jk_0(R_i - h)}}{{R_i}^3}\,, \qquad (2)$$

where

$$R_i = \sqrt{{Y_i}^2 + h^2}\,. \qquad (3)$$

k_0 is the wave number $2\pi / \lambda_0$, R_i as shown in fig. 1 is the range of the i^{th} mine from the radar, Y_i is the distance between the i^{th} mine and the central mine and n is the number of mines at one side of the radar foot print. At the resonance frequency, a uniform mine field has *PF* given by,

$$[PF]_{max} = 2n+1 \,, \tag{4}$$

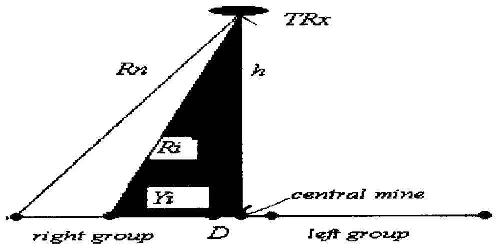

Fig. 1. The geometry of the mine detection problem using monostatic radar.

where $(2n+1)$ is the total number of mines illuminated by the radar.

In the case of nonuniform distributions, the smallest change in the distance *D* is defined as δ . *D* could be considered as a multiple of this minor distance δ . The operating resonance frequency will be calculated according to δ. All other distances will satisfy the condition of resonance, however the maximum value of *PF* will be equal to the total number of the actually existing mines. It should be noted that there is a peak identifying the existence of a mine when we use a bistatic radar moving over the mine field. The radar height in this case must be low enough to match the lower value of the minor distance, and high enough to verify the condition $h \gg 20n\,\delta$ [1].

2.1. Linear alternation in mine spacing

The simple function representing the alternating change in the mine's spacing can be represented as follow,

$$D_i = D_0 + (-1)^i \delta \,, \tag{5}$$

where D_i is the distance between the i^{th} mine and its neighbor, δ is the expected deviation in mines distant apart D_0. This alternating change in the mine spacing distribution stated in eq. (5) is called a triangular distribution. The pattern factors against frequency for δ =0.1 and 0.05 are shown in fig. 2.

2.2. Sinusoidal alternation in the mine spacing

If a sinusoidal fraction is added to a constant distance D_0, the i^{th} mine location can be written as:

$$Y_i = nD_0 + \delta \sin\left(\frac{2\pi i}{N}\right) . \tag{6}$$

This function is shown as in the fig. 3. Fig 4 show the values of *PF* for differen values of δ and sine arguments.

2.3. Mine spacing with equal phase difference between wave returns from different mines

In a uniform mine field, the resonance frequency helped in constructing a coherent mine field return. There is a case in the nonuniform mine distributions where the phase difference between the wave return from adjacent mines is always constant at any working frequency and equals 2π at resonance and π at midresonance [1]. The following nonuniform case of mine distribution is expressed as:

$$Y_i = D_0 \sqrt{i} \,. \tag{7}$$

The phase of the reflected signal from the i^{th} mine is given as:

$$\phi_i = 2k_0 (R_i - h) = 2k_0 [\sqrt{h^2 + (iD_0)^2} - h] = k_0 \frac{iD_0^{\,2}}{h} \,. \tag{8}$$

Thus,

$$\phi_{i+1} = k_0 \frac{(i+1)D_0^{\,2}}{h} \,. \tag{9}$$

The difference in phase between the i^{th} mine and its neighbor is:

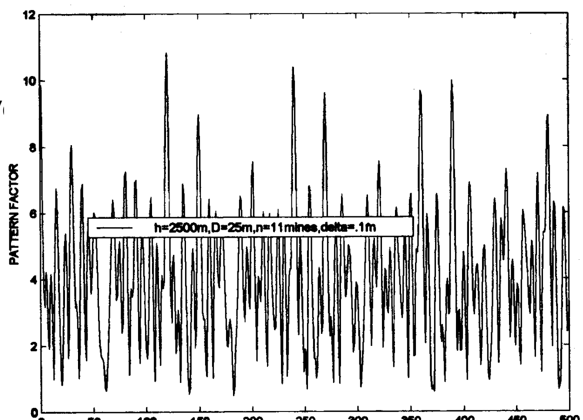

Fig. 2-a. The *PF* against frequency for a triangular atternation.

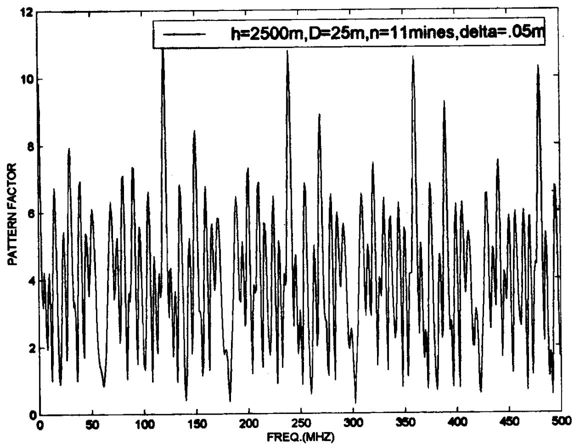

Fig. 2-b. The *PF* against frequency for a triangular atternation.

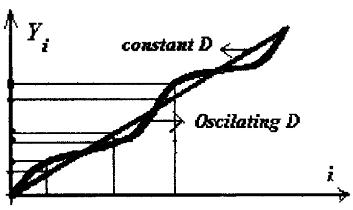

Fig. 3. The sinusoidal alternation in the mine spacing.

$$\Delta\phi_i = \phi_{i+1} - \phi_i = k_0 \frac{D_0^{\,2}}{h} = \frac{2\pi\ f\ D^2}{ch} = \frac{2\pi\ f}{f_0} = \alpha\ f . \tag{10}$$

Analyzing the result of this case, we found that the phase difference between every two subsequent mines is always equal and independent of the mine location. It changes linearly with the frequency, and α is a constant equals to $2\pi / f_0$. At the resonance frequencies the phase difference equals 2π. The *PF* of this distribution of mines includes a number of regular ripples equals to the number of mines. Thus, for a number of mines n we have a *PF* having $(n\text{-}1)$ ripples between any two subsequent resonance frequencies. Also their locations are at frequencies equal to,

$$f_i = \frac{i\ f_0}{n} . \tag{11}$$

The *PF* of this distribution gets its maximum level, which equals to the total number of mines $(2n+1)$ only at the resonance frequency. Fig. 5 show the *PF* for this case.

3. Mines with randomly disturbed distributions

In this case, we assume that a random fraction is superimposed upon the mine spacing distance. This can express the mistakes, accidentally, happened in the mine cultivation. Thus, the distance between the mine and the radar projection, Y_i, can be expressed by:

$$Y_i = iD + \delta \text{ where } \delta = \text{normrnd}[\mu,\sigma,(1\quad 1)] , \tag{12}$$

$$Y_i = \text{normrnd}[iD + \mu,\sigma,(1\quad 1)] , \tag{13}$$

where μ is the average value of the added random fraction and σ is its variance. Simulation of the added random fraction is obtained by a random signal generator. By studying the effect of σ on the level of *PF* and the location of the resonance for an undisturbed field, it is found that when $\sigma > 0.1\,D$ the resonance position is greatly dislocated, and *PF* is greatly deteriorated. The results are shown in fig. 6.

4. Mine spacing determination by a stationary radar

The case of a nonuniform mine field detection presents a great challenge. It is created due to the unknown distance between any two adjacent mines leading to unknown resonance frequency for detection [3]. The solution of this problem can be overcome by regarding the change in the amplitude of the received field strength from mines when the beam of the used antenna becomes wider. For this purpose array antenna is used. The beamwidth of an array of M linear dipoles is given by [4]:

$$2\theta = \frac{.886\lambda}{Md} , \tag{14}$$

where d is the distance between any two dipoles, and λ is the wavelength. Let us assume a set of PF measurements $S_1, S_2, ..S_n$ for n mines, where S_1 is the PF when the antenna beam includes only the central mine and the first mine adjacent to it, S_2 is the PF when the beam extends to include the nearest subsequent mine to S_1 and S_n is the PF when the beam extends to include all the group of mines. These values of $S_1, S_2, ..S_n$ can be expressed as:

$$S_1 = 1 + \frac{h^3 e^{-2jk_0(R_1 - h)}}{R_1^{\,3}} , \tag{15}$$

$$S_2 = S_1 + \frac{h^3 e^{-2jk_0(R_2 - h)}}{R_2^{\,3}} , \tag{16}$$

$$S_n = S_{n-1} + \frac{h^3 e^{-2jk_0(R_n - h)}}{R_n^{\,3}} . \tag{17}$$

Solving the above set of equations given that $R_1 \cong R_2 \cong ... \cong R_n \cong h$ in the denominators , k_0 is the wave number resonating with the average mine distant apart, we can get the unknown locations for n mines as follows:

$$S_n - S_{n-1} = |S_n - S_{n-1}| e^{-j\phi} = e^{-2jk_0(R_n - h)} , \tag{18}$$

where

$$R_n = \sqrt{Y_n^{\,2} + h^2} \quad \text{or} \quad (R_n - h) = \frac{Y_n^{\,2}}{2h} . \tag{19}$$

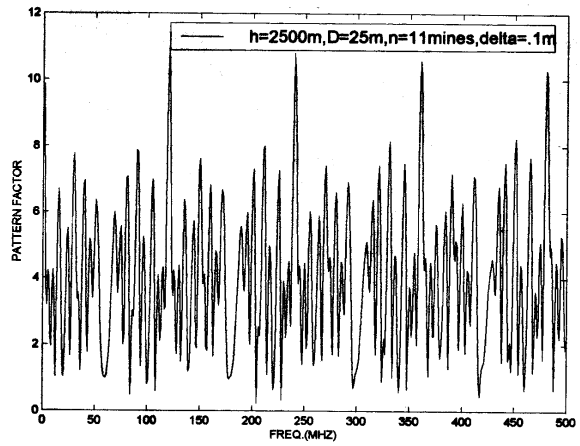

Fig. 4-a. The PF against frequency for a sinusoidal alternation

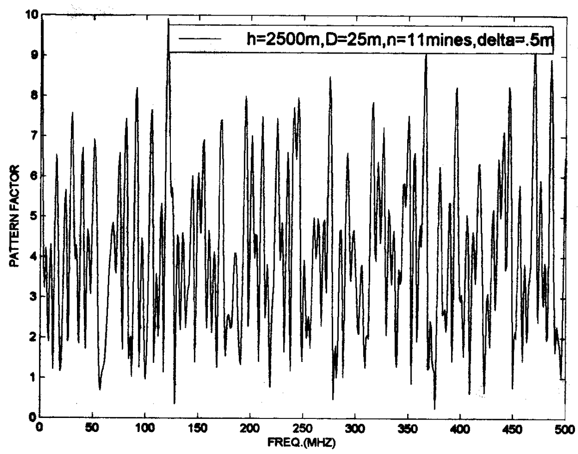

Fig. 4-b. The PF against frequency for a sinusoidal alternation.

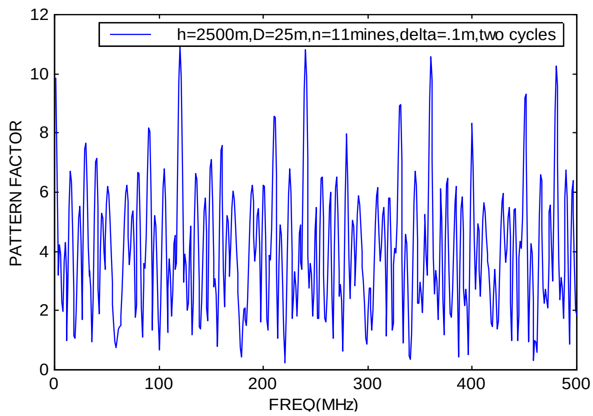

Fig. 4-c. The *PF* against frequency for a sinusoidal alternation.

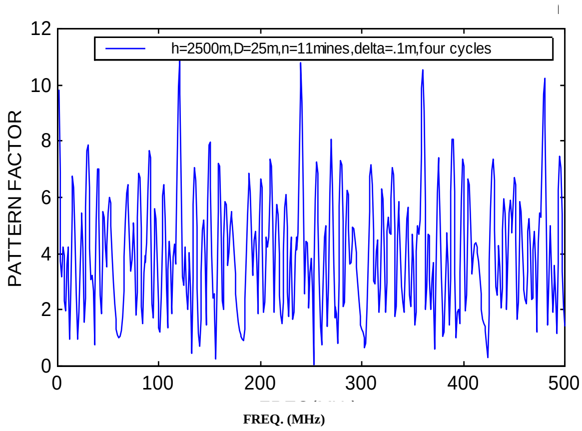

Fig. 4-d. The *PF* against frequency for a sinusoidal alternation.

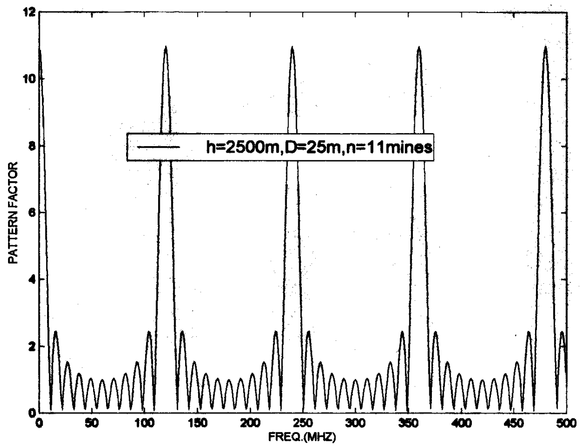

Fig. 5-a. The *PF* against frequency for an equiphase mine distribution.

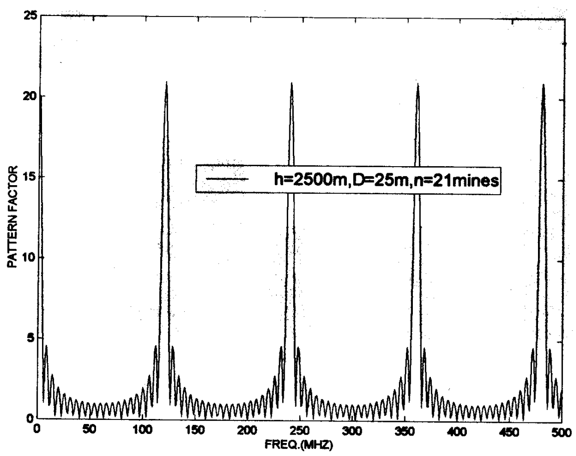

Fig. 5-b. The *PF* against frequency for an equiphase mine distribution.

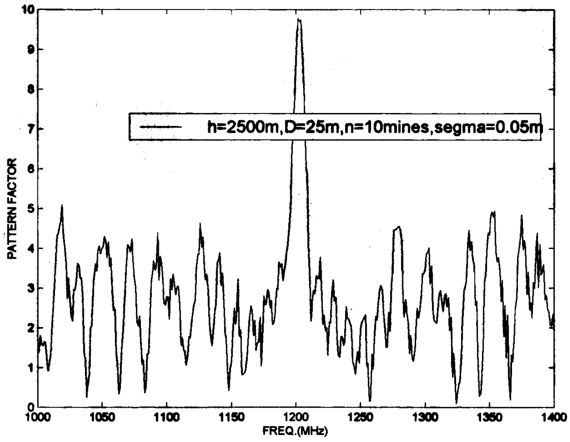

Fig. 6-a. The *PF* against frequency for an added random shift to the mine spacing .

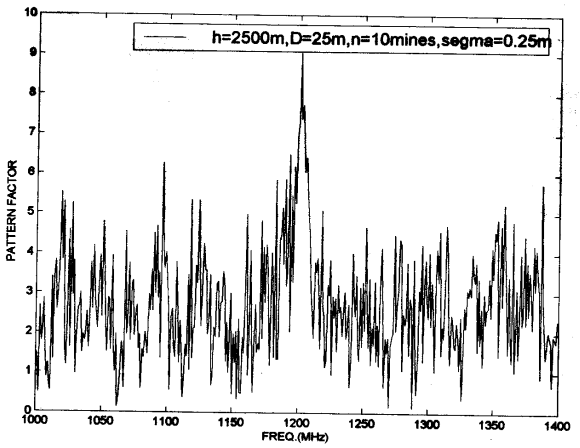

Fig. 6-b. The *PF* against frequency for an added random shift to the mine spacing .

The beam extension step is preferred to be less than the expected average mine distance, otherwise we will include more than one mine in one jump. With respect to the array antenna this demand can be achieved by making the array dipoles number as large as possible then,

$$Y_n = \sqrt{\frac{h}{k_0}\phi_n} \; . \tag{20}$$

Using a phase discriminator to measure the phases ϕ_n and according to eq. (20) the mine location Y_n can be obtained. Another direct way for getting the mine location is through a matrix formulation expressing the array of measurements as follows [5-6]:

$$\begin{bmatrix} S_1 \\ S_2 \\ \downarrow \\ S_n \end{bmatrix} = \begin{bmatrix} 1+e^{-2jk_0(R_1-h)} & 0 & 0 & 0 \\ 1+e^{-2jk_0(R_1-h)} & e^{-2jk_0(R_2-h)} & \rightarrow & 0 \\ \downarrow & \downarrow & \rightarrow & 0 \\ 1+e^{-2jk_0(R_1-h)} & e^{-2jk_0(R_2-h)} & \rightarrow & e^{-2jk_0(R_n-h)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \downarrow & \downarrow & \rightarrow & 0 \\ 1 & 1 & \rightarrow & 1 \end{bmatrix} \begin{bmatrix} 1+e^{-2jk_0(R_1-h)} \\ e^{-2jk_0(R_2-h)} \\ \downarrow \\ e^{-2jk_0(R_n-h)} \end{bmatrix}. \quad (21)$$

Sending the unitary matrix into the other side of eq. (21), then through its eign values we can apply the result obtained in eq. (20).

5. Conclusions

Irregular alternation by adding a little fraction to the equidistant mines spacing doesn'*t* give a resonance results like the equidistant mines results. In this case, the *PF* is less than its value for equidistance distribution, and depends on the value of δ and the argument of the sinusoidal variation. For both linear and sinusoidal alternation added upon the equidistance spacing, the *PF* deterioration increases with the increase of δ. The increase of δ also causes a shift in the frequencies of maximum *PF* which creeps to the first resonance when δ becomes lower than 2% of D. Equiphase mines distribution gives results for *PF* which can be identified only at resonance and approximately level around one or little more than one all over the rest of the frequency band. Resonance tale phenomena of the regular mine fields and the relation of its beamwidth with the number of mines and mines spacing at a fixed height helped us in the identification of the mine field parameters whose irregularity level is very small (δ <0.1 D). The field measurements using a stationary radar, whose antenna beam can be expanded could be used to formulate a direct form for the mine location in a nonuniform mine distribution.

References

[1] N.H. Ismail, H. N. Khairallah and M.A. Metwally, "Mine Detection and Mine Field Mapping", Nineteenth National Radio Science Conference Alexandria, March, 14-21 (2002).

[2] K.K Mei, and T.M. Kvam, "Numerical ParamStudy of Electromagnetic Wave Scattering by Buried Dielectric Land Mines", Final report. Berkeley, CA: Geo Electromagnetics, Inc., p. 95 (1983).

[3] T.G. Engel, W.C. Nunnally and N. B. VanKirk, "Airborne Ground-Penetrating Radar System to Detect Surface and Subsurface Land Mines", pp. 586-595 (1994)

[4] A.G. Yarovoy, A. Schukin , A.L.I. Kaploun, A.L. Mintz, L.P. Ligthart, and Marshall R. Bradley, "Antenna system for UWB GPR for Landmine Detection", Univ. of Technology, Delft, Netherlands. Planning Systems Inc. Slidell, LA, USA, pp. 692-699 (1997).

[5] X. Xiaoyin, and E.L. Miller,"Statistical method to localize buried landmines from GPR array measurement", Northeastern Univ.Boston, MA, USA, pp.742-753 (2001).

[6] B.M. Duston, and D. Lang, "Statistical processing of ground-penetrating radar signals for mine detection", BAE Systems, San Diego, CA, USA, pp.494-502 (1999).

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