Pipe-drainage spacing design considering subsurface evaporation

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In this paper the problem of draining a top clay layer above an impermeable stratum by a system of pipe drains is considered. In designing the spacing between drain pipes the effect of evaporation taking place from the subsoil water-table is taken into account. A rational drain discharge equation is combined with a general subsurface evaporation one to study the unsteady movement of the subsoil water-table. It is found that in some cases subsurface evaporation affects the rate of depression of the water-table substantially. An implicit drain spacing design equation is presented and a computer program is provided to give numerical solutions of this equation. The results of two practical numerical examples show that introducing the effect of subsurface evaporation results in more economical drain spacings with corresponding savings that may exceed 50%.

فى هذا البحث تم دراسة صرف طبقة تعلو طبقة غير منفذة بواسطة نظام من المصارف المغطاة أخذا فى الاعتبار تأثير البحر تحت السطحي على سطح المياه الأرضية (المياه التحت سطحية) وقد استخدمت معادلة تخص التصرف بالمصرف وأخرى للبخر تحت السطحي لدراسة الحركة الغير مستقرة لمنسوب المياه وتأثير ذلك على تصميم المسافة بين المصارف المغطاة بعد عمل برنامج بسيط على الحاسب، ولقد اتضح ذلك بمثالين عدديين تطبيقيين وتم التوصل الى النتائج والتي توضح انه بزيادة البخر تزداد المسافة بين المصارف واتضح أيضا أنه في بعض الحالات يلعب البخر دورا كبيرا فى تصميم المصارف ويكون لذلك آثرا كبيرا من الناحية الاقتصادية في التوفير في تكاليف الحلقيات بنسبة تصل إلى ٥٠%.

Keywords: Pipe-drainage, Subsurface evaporation, Drain dischange

1. Introduction

As early as 1862 the problem of subsurface drainage has been investigated using different lines of approach [1, 2]. Many investigators have provided rational formulas for designing pipe drainage systems considering the case of a clay layer overlying an impermeable barrier, fig. 1. In their analyses most of the researchers neglect the effect of evaporation which takes place from the subsurface water-table [2-4]. In fact subsurface evaporation may play an important role in designing drainage systems especially in hot climatic regions where evaporation is considerable [5-7] . Hammad [8] and Hathoot [9] considered the effect of evaporation in their drain–spacing design equations and both of them used his own drain- discharge equation. Considered the effect of evaporation but used local evaporation data of the central region of Saudi Arabia [7]. In this paper a rational drain-discharge equation together with a general evaporation equation are used to provide a design algorithm for pipe drainage spacing taking into account the effect of evaporation.

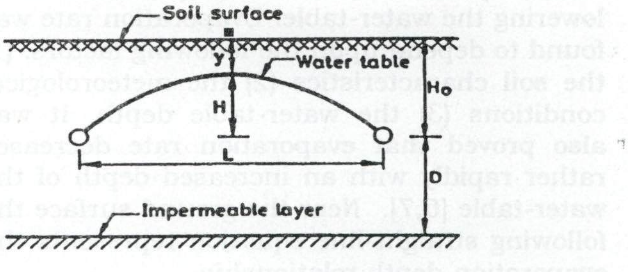

Fig. 1. Definition sketch.

2. Drain-discharge equation

Lovell and Youngs [2], fitted the Hooghoudt's equivalent depth equation [1] over the range of q/K=0.01 to 0.1 to the power relationship:

$$\frac{2H}{L}=\left(\frac{q}{K}\right)^{1/a}, \tag{1}$$

in which q= the steady-state rain fall rate ,K=hydraulic conductivity of soil, H=water-table height above drains midway between two drains and L=the spacing between drains. The coefficient a depends upon the soil thickness below drains relative to drain spacing and is given by,

$$a=\left(\frac{2D}{L}\right)^{2D/L}, \qquad 0\leq 2D/L\leq 0.35, \tag{2}$$

and

$$a=1.36, \qquad 2D/L>0.35, \tag{3}$$

in which D=soil depth below drains. For convenience eq. (1) is put in the form:

$$Q_d=KL\left(\frac{2H}{L}\right)^{a}, \tag{4}$$

in which Q_d=the discharge reaching each unit length of drain.

3. Subsurface evaporation

Actual measurements [6,7] have shown that evaporation plays an important role in lowering the water-table. Evaporation rate was found to depend upon the following factors: (1) the soil characteristics (2) the meteorological conditions (3) the water-table depth. It was also proved that evaporation rate decreases rather rapidly with an increased depth of the water-table [6,7]. Near the ground surface the following straight-line equation represents the evaporation-depth relationship,

$$q_e=q_o(1-CY), \tag{5}$$

in which q_e = the evaporation from a water-table at a depth Y from ground surface, q_o = the evaporation at ground surface, and C=constant depending on the soil characteristics and meteorological conditions which may be determined experimentally. For convenience eq. (5) is put in the form

$$q_e=q_o[1-C(H_o-H)], \tag{6}$$

in which H_o=pipe drain depth below ground surface [5]. It is worthy to note that, for a certain region, q_o varies along the year and for safe design of a drainage system the minimum value of q_o is to be considered [11].

4. Water-table depression

The unsteady movement of the water-table has been conventionally assumed to be the same as a continuous succession of steady states with the flux through the water-table assumed uniform and given by the drain discharge rate divided by the surface area [3, 12, 13] Youngs [13]). Hammad, [8], reported that the subsoil water-table is, for the larger part of its length, nearly horizontal with more or less local depressions over the drain pipes. Combining eqs. (4,6), the differential equation describing the unsteady movement of the water-table, taking into account the effect of evaporation is,

$$-\frac{dH}{dt}\mu L=Q_d+Q_e, \tag{7}$$

in which μ=drainable porosity of soil ,t=time ,and Q_e=evaporation taking place between two drains per unit drain length. Substitution from eqs. (4, 6) into eq. (7) yields,

$$\frac{dH}{dt}=-\left\{K\left(\frac{2H}{L}\right)^{a}+q_o[1-C(H_o-H)]\right\}\frac{1}{\mu}. \tag{8}$$

In fig. 2 is shown the rate of water-table depression ratio, - (dH/dt)/K ,versus the head ratio, H/H_o, for various evaporation ratios, q_o/K. It is evident that the rate of depression of the water-table substantially increases as evaporation increases. This means that introducing the effect of subsurface evaporation is expected to provide economical spacings between drains.

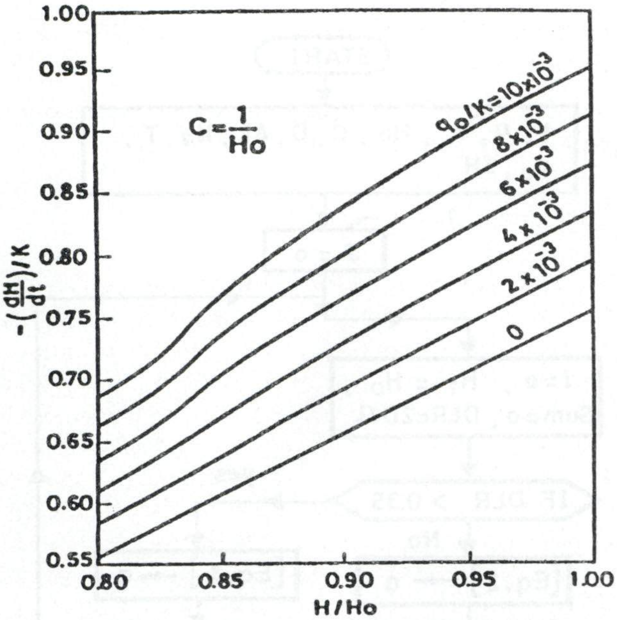

Fig.2. Rate of water depression ratio versus head ratio.

5. Drain spacing

The worst case in humid regions is that in which the full depth of soil is water logged after a heavy rain. The function of the drainage system is to clear the root zone of plant in a specified period, T, depending on the kind of plant. The period T is the critical drainage period [8]. In general a drainage system should be designed so as to lower the water-table from H_1 to H_2 in a given time T. Integration of eq. (8) yields,

$$T = \int_{H_2}^{H_1} \frac{\mu dH}{K\left(\frac{2H}{L}\right)^a + q_o\left[1 - C(H_{o-H})\right]} . \qquad (9)$$

The proper spacing between drains is that which when substituted into the right-hand side of eq. (9) the integral in that side equals T. For convenience eq. (9) is put in the form

$$T = \int_{H_2}^{H_1} f(H) dH , \qquad (10)$$

in which

$$f(H) = \frac{\mu}{K\left(\frac{2H}{L}\right)^a + q_o\left[1 - C(H_o - H)\right]} . \qquad (11)$$

It is evident that the integral of eq. (9) is to be evaluated numerically and the proper value of L should be found through a trial and error procedure.

6. Computer program

A computer program has been prepared to design the spacing between drain tubes taking into account the effect of subsurface evaporation. A detailed flow-chart of the program is given in fig. 3. The following algorithm is considered in designing the computer program

1- The following data should be known in advance: μ ,C,D, H_1 , H_o , H_2,q_o,T and K.

2- A reasonable value of L is assumed to be used in preliminary calculations.

3- A spacing increment ΔL ,in the order of 10 m and a head increment ΔH ,in the order of 0.001 m are assumed.

4- The value of a is calculated according to eq. (2) or eq. (3).

5- The value of f(H) as given by eq. (11) is calculated and called $f(H)_2$.

6- A new value of $H_1$is obtained by subtracting ΔH from the preceding H_1.

7- $f(H)_2$is renamed $f(H)_1$ and the value of $f(H)_2$ is calculated through step 5.

8- The elementary time is calculated from

$$\Delta T = 0.5[f(H)_1 + f(H)_2]\Delta H . \qquad (12)$$

9- Steps from 5 through 8 are repeated and values of ΔT are continuously summed up until the final value of H_1 is equal to H_2. In this stage, $\Sigma \Delta T$ is called T_2.

10- T_2 is then compared with T and if the absolute value of the difference is less than an acceptable small number, say 0.001, the assumed value of L is the required design spacing, otherwise a new value of L is assumed by adding ΔL to the initially assumed value(this is only for the second trial).

11- The first spacing is called L_1 and the second L_2 the time is called T_1 and another time T_2 is to be estimated by applying steps 4 through 10 considering the initial value of H_1 given in the data.

12- Instead of adding ΔL, as in the second trial the following equation is used to obtain a new trial spacing

$$L = (T - T_2)\left[\frac{L_1 - L_2}{T_1 - T_2}\right] + L_2 . \tag{13}$$

13- Steps 4 through 12 are repeated until the resulting time T_2 is practically equal to T and in this case the corresponding spacing L is the required design spacing.

Example 1: It is required to design the spacing between drain tubes for the following data: μ=0.05, K= 0.1m/day, H_o= 1.8m, C=0.56, D= 20.0m, H_1=1.5m, T=2.0 days, and q_o=0.005 m/day.

Solution: Applying the program of fig.3, assuming L=30.0m, ΔL=10.0m,and ΔH=0.001 m , we get the results of successive trials as given in table 1.

As it is important to know the effect of considering subsurface evaporation the same calculations are performed but neglecting evaporation and the results are shown in table 2.

Table 1
Successive trials for example 1 (q_o=0.005)

L	2D/L	A	T
30.000000	1.333333	1.36	1.564709
40.000000	1.000000	1.36	1.881525
43.739540	0.914504	1.36	1.977219
44.629810	0.896262	1.36	1.998477
44.693580	0.894983	1.36	1.999979

Table 2
Successive trials for example 1 (q_o =0.0)

L	2D/L	A	T
30.000000	1.333333	1.36	3.009099
40.000000	1.000000	1.36	4.449931
22.996420	1.739401	1.36	2.096088
22.302310	1.793536	1.36	2.010515
22.217010	1.800422	1.36	2.000065

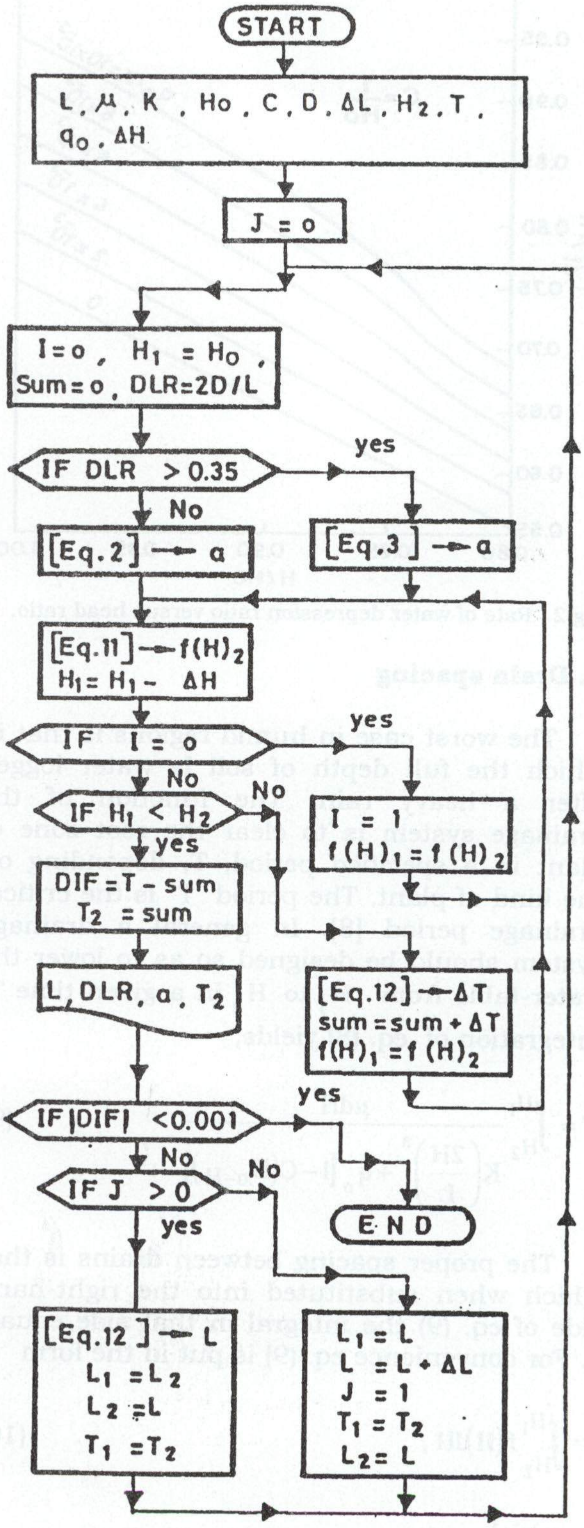

Fig.3. Flow chart of the computer program.

It is evident from tables 1 and 2 that introducing the effect of evaporation has the effect of increasing the spacing between drains about 50%.

Example 2: It is required to design the spacing between drains for the same data of example 1 but for D=2.5m,T=1.5 days.

Solution: Starting by L=30.0m and assuming ΔL=10.0m and ΔH=0.001, the results are given in table 3.

Table 3
Successive trials of example 2(q_o=0.005m/day)

L	2D/L	A	T
30.000000	0.166666	1.483673	1.787012
40.000000	0.125000	1.542211	2.225679
23.457160	0.213155	1.438594	1.419033
25.117650	0.199063	1.450390	1.517754
24.819020	0.201458	1.448279	1.500236

As in example 1 ,the results in the case of neglecting evaporation are estimated and given in table 4.

In this example about 44% increase of spacing is achieved as subsurface evaporation is considered.

Table 4
Successive trials of example 2(q_o=0.0)

L	2L/D	A	T
30.000000	0.166666	1.483673	3.955844
40.000000	0.125000	1.542211	7.017145
21.977780	0.227503	1.428048	2.242949
19.173200	0.260781	1.408659	1.783596
17.441710	0.286669	1.397906	1.533189
17.212220	0.290491	1.396609	1.501834
17.198790	0.290718	1.396534	1.500014

7. Conclusions

The computer program provided for designing the spacing between drain tubes considering subsurface evaporation is simple and practical. The required degree of accuracy of the results can be attained by suitably choosing the head increment ΔH and the allowable difference between the specified time period, T, and the final estimated time , sum. In some cases it has been found that subsurface evaporation plays an important role in designing the spacing between drain tubes. The design spacing increases as evaporation increases. Through the solution of two practical numerical examples it has been found that introducing the effect of evaporation may lead to double the spacing for which evaporation has been neglected.

Notations

The following symbols are used in this paper:

a	is the quantity defined by Eqs. (2) or (3),
C	is the constant contained in eq. (5)
D	is the depth of clay layer below drains,
F(H)	is the quantity defined by eq. (11),
H	is the water-table height above drains midway between drains,
H_o	is the depth of drains below ground Surface,
H_1	is the initial water-table height,
H_2	is the final water-table height,
K	is the hydraulic conductivity of clay,
L	is the spacing between drains,
Q_d	is the total drain discharge per unit length of drain,
Q_e	is the evaporation taking place between two drains per unit length of drain,
q_d	is the Q_d/L,
q_e	is the Q_e/L,
q_o	is the evaporation at ground surface,
T	is the specified time period depending on kind of plant,
t	is the time,
Y	is the depth of water-table below ground surface, and
μ	is the drainable porosity of clay.

Computer program notations

DIF = T- sum,
DLR = 2D/L,
sum = $\Sigma \Delta T$,
ΔH = head increment,
ΔT = time increment given by eq. (12), and
ΔL =drain spacing increment.

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References

[1] D. Kirkham,"Steady state theories for land drainage", J. Irrig. and Drain Div., ASCE, Vol. (92), pp. 19-39 (1966)

[2] C.J. Lovell, and E.G. Youngs, "A comparison of steady-state land drainage equations", J. Agric. Engrg. Res., Vol. (9), pp. 1-21 (1984).

[3] J.V. Schilfgaarde, ed. Drainage for agriculture Am. Soc. of Agronomy ,Madison, Wis., Vol. (17), pp. 115-143 (1974).

[4] H.M. Hathoot, "Theory of pipe drainage assisted by mole drainage", J. Irrig. And Drain. Engrg. ASCE, Vol. 124(2), pp. 102-107 (1998).

[5] S.F. Averianov, "Influence of irrigation systems on ground water regime", Akad. Nayk., USSR (in Russian) (1956).

[6] J. R. Philip, "The physical principles of soil water movement during the irrigation cycle", 3rd. Congress, ICID, R7, III (1956).

[7] F.S. Mohammad, "Effect of evaporation on water-table drawdown under hot climatic conditions",Dirasat, Vol. 20 (2), pp. 16-33 (1993).

[8] H. Y. Hammad, "Depth and spacing of tile drain systems", J. Irrig. and Drain.Div. ASCE, 88 (IR1), pp. 15-33 (1962).

[9] H.M. Hathoot, "Effect of evaporation on subsurface drainage", Fac.Of Engrg., Alexandria Univ., Alexandria, Egypt, XIX(1), pp. 65-75 (1980).

[10] H.M. Hathoot, Al-Amoud,A.I., Mohammad,F.S. and H.M. Abo-Ghobar, "Design criteria of drain tube systems in the central region of Saudi Arabia", J. King Saud Univ., Engrg. Sci., Vol. 5(2), pp. 191-212 (1993).

[11] H.M. Hathoot, "New formulas for determining discharge and spacing of subsurface drains", ICID Bull., Vol. 28(2), pp. 82-86 (1979).

[12] E.C. Childs, Introduction to the physical basis of soil water phenomena, John Wiley & Sons, London, U.K (1969).

[13] E.G. Youngs, "A simple drainage equation for predicting water-table drawdowns", J. Agric. Engrg. Res. Vol. 31, pp. 321-328 (1985).

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