Optimal dimensions of gravity concrete dams

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Storing water, by constructing dam across a river, in great reservoirs to keep water for man needs, agriculture and hydropower generation is an old idea that has become true particularly in the arid countries. The type and the size of dam are decided according to site hydrology, geology, materials of construction, spillway size and location etc. Concrete gravity dams are widely used recently all over the world although the construction of such dams needs skilled labor and special materials of construction. Stability of a gravity dam mainly depends upon own weight of the dam which should be big enough to resist forces mobilized from hydrostatic pressure acting on the face of the dam and uplift pressure acting underneath the base of the dam. Stability of a dam against overturning and lateral movement (sliding) must be assured. Also, gravity dams should be founded on hard rock foundation to resist the overstressing caused by the acting loads. The worst case of loading occurs when the reservoir is full; tail water elevation is low; seismic shock acts from upstream to downstream direction; the silt zone is full, so, the silt lateral pressure is maximum; and finally, a wave shock acts at the maximum upstream water level. The conventional methods of gravity dam design depend upon the background of the designer in some basic sciences such as hydraulics, soil mechanics and structure engineering. No attempt has been made for searching the least cost design of gravity concrete dam and at the same time satisfying all the safety requirements. On the basis of random search technique, this paper describes a design methodology for solid gravity concrete dam. This method gives the proper dimensions of gravity dam that satisfy all constraints required for dam safety with minimum volume of material that reduces construction cost.

إن فكرة تخزين المياه فى خزانات كبيره وذلك بإنشاء سد مقام على نهر لاستخدامات الإنسان والزراعة وتوليد الكهرباء، كانت فكره من قديم الزمان وأصبحت الآن حقيقة وبالأخص في المناطق القاحلة يتحدد نوع وحجم السد حسب جغرافية وهيدرولوجية المنطقة المراد إقامة السد بها. وأصبحت السدود الكتلية الخرسانية شائعة الاستخدام في كل أنحاء العالم بالرغم من أن إنشاء هذه النوعية من السدود يتطلب عماله ذات مهارات خاصه وأيضا مواد إنشائية ذات مواصفات خاصة. إن أتزان السدود الكتلية يعتمد أساسا على وزن المنشأ الذي يجب أن يكون كافيا لمقاومة عزوم الانقلاب نتيجةً القوى المتولده من ضغوط المياه أمام السد وأيضا قوى الرفع أسفل قاعدة السد ولذلك يجب التأكد من أن معامل الأمان للسد المنشأ ضد الإنقلاب والزحزحه الأفقية فى الحدود المسموح بها. أيضا يجب التأكد من أن طبقة الأساس الكتلى من النوع الصخرى وذلك لتتحمل الضغوط الشديده المتولده أسفل قاعده السد. أقصى حاله تحميل للسد لغرض التصميم يمكن أن نعتبرها عندما يكون منسوب المياه فى الخزان أمام السد عند أقصاه ومنسوب المياه فى الخلف أقل ما يمكن مع إعتبار هزة أرضية قادمة من ناحية أمام السد. أيضا قوى الضغط المتولده من الطمى أمام السد عند أقصى حاله لها وأخيرا الصدمة الناتجه من الأمواج فى الخزان أمام السد وهى مؤثرة عند أقصى منسوب، فى أكبر قيمه لها. الطرق التقليدية لتصميم السدود الكتلية تعتمد على خلفية المصمم فى بعض فروع العلوم الهندسية مثل الهيدروليكا ميكانيكا التربة والاساسات والهندسة الإنشائية. الأبعاد الأساسية للسد يجب أن تختار بدقة متناهيه لتفى بكل متطلبات التصميم والأمان. ليس هناك أى إتجاه فى التصميم لإيجاد التصميم الأمثل للسد الأقل فى التكاليف والذى يضمن فى نفس الوقت كافة متطلبات الأمان للمنشأ. فى هذه الورقة نتناول وصف طريقة مبنية على تكنيك الأرقام العشوائية وذلك لتصميم السد الكتلى المنشأ من الخرسانة. هذا الطريقة تعطى الأبعاد المثلى (فى صورة منحنيات تصميمية) التى تعطى أقل حجم للسد وذلك يعنى أقل تكاليف للمنشأ وفى نفس الوقت تضمن كل متطلبات الأمان للسد.

Keywords: Gravity concrete dam, Dam objective function, Safety constraints, Random search technique, Dam design charts

1. Introduction

Since the old centuries, man has known the importance of water as the source of life. Also, man found that all the ancient civilizations existed near the water resources. So, the idea of storing water in great reservoirs by constructing high dams has become true since that time. Historically, the dam built at Sadd-el-Kafra, Egypt, around 2600 BC [1], is generally accepted as the oldest known dam of real significance.

The construction of gravity dams ranks the earliest and the most fundamental of civil engineering activities. Irrespective of size and type, dams demonstrate great complexity in their load response and their interactive relationship with site hydrology and geology.

A gravity dam should have sufficient weight so as to withstand the forces and the overturning moment caused by water impounded in the reservoir behind it. The dam transfers the acting loads to the foundation by cantilever action and hence hard soil foundation is prerequisite for gravity dam. The United States Bureau of Reclamation (USBR) [2], specifies normal, extreme load and other load combinations for the design purpose. The design criteria, governing the structural competence of a gravity dam, is based on the idea that the dam is considered to be made of a number of vertical cantilevers, which act independently of each other.

The cost of constructing a gravity dam mainly depends upon the amount of concrete required for constructing such dam. So, the objective function of design of a gravity dam will be the volume of such dam. To obtain a functional design, safety constraints must be satisfied to fulfill the gravity dam design criteria.

2. Main loads acting on gravity dam

A gravity dam derives its stability from the forces of gravity of the material in its section, and hence the name. The gravity dam should has sufficient weight so as to withstand the forces and the overturning moment caused by water impounded in the reservoir behind it. Dam transfers the acting loads (see fig. 1) to the foundation by cantilever action and hence good foundation is prerequisited. Forces causing stability are the own weight of the dam and the thrust of the tail water. Forces causing instability of dam are reservoir water pressure, uplift forces acting underneath the base of dam, forces due to waves action in the reservoir side, silt pressure, seismic forces, wind pressure and temperature stresses.

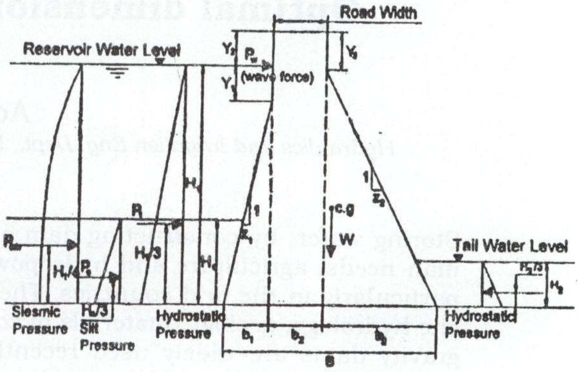

Fig. 1 Main loads acting on gravity dam.

3. Loading combinations

Design should be based on the most adverse combination of probable load condition, but should include only those loads having reasonable probability of simultaneous occurrence. The USBR [2], specifies the normal and the extreme load combination as below:

1. Normal load combination, which include maximum reservoir elevation with appropriate dead load, silt (if applicable), normal uplift, minimum temperature (if applicable) and tail water.
2. Extreme load combination, which include normal design reservoir elevation with appropriate dead load, silt load (if applicable), uplift pressure, minimum temperature (if applicable), minimum tail water height and maximum credible earthquake. In the present paper the above two conditions were taken into account but the effect of the temperature was unconsidered.

4. Design criteria

The essential criteria governing the structural competence of a gravity dam is based on that the dam is considered to be made of a number of vertical cantilevers that act independently of each other. According to the USBR [2,3] the following assumptions for the gravity dam analysis are made:

1. Material in the foundation and the body of dam is isotropic and homogeneous.
2. Stresses in the foundation and the body of the dam are within the elastic limits.

3. No movements are caused in the foundation due to transfer of the loads.
4. The dam and the foundation behave as one unit, the joint being perfect.
5. The stability analysis of the dam is based on considering a slice of the dam one-meter thick.

Fig. 1 shows the loads acting on the body of a gravity dam for the case of the extreme load combination. For that load combination and for the other cases of loading the profile of the dam must demonstrate an acceptable margin of safety with regard to overturning, translation (sliding) and soil overstressing and material failure.

4.1. Safety against overturning

A factor of safety with respect to overturning F_o can be expressed in terms of the moment operating about the downstream toe of any horizontal plane. By definition F_o is the ratio of the stabilizing moment to the overturning moment about the toe of the dam. Values of F_o in excess of 1.25 [3] may generally be regarded as acceptable, but $F_o \geq 1.5$ is desirable [4].

4.2. Safety against sliding

Sliding stability is a function of loading pattern and of the resistance to transnational displacement, which can be mobilized on any plane. Factor of safety against sliding F_s at any section, can be defined as the ratio of the forces which make the dam tend to slide and the forces acting in the opposite direction. Dam resistance to sliding at any plane within the body of the dam will be a function of the shear resistance mobilized in the concrete mass. At the base, concrete-rock bond and the resulting interface shear strength is the critical factor. F_s ranges between 1.5-4.0 [3] depending on the case of loading, the type of the construction materials and the type of soil foundation.

4.3. Safe stresses

Gravity dam is analyzed by the gravity method of stress analysis. Gravity stress analysis derives from the elastic theory and is applied to two-dimensional vertical cantilever. In addition to the above assumptions the stress analysis makes the following two assumptions:

1. Vertical stresses on horizontal planes vary linearly between upstream and downstream.
2. Variation in horizontal shear stress across horizontal planes is parabolic.

An advanced method, of gravity stress analysis, has given by USBR [2] and Jensen [5]. The primary vertical normal stresses σ_z act on any horizontal plane are determined by applying the equation of cantilever under combined axial and bending loads (see fig. 1) by the following equation:

$$\sigma_z = -\frac{\sum F_v}{B}\left(1 \pm \frac{6e}{B}\right). \tag{1}$$

In which $\sum F_v$ is the summation of the vertical forces; B is the base width and *e* is the eccentricity (distance between resultant of all forces acting on the base and center of the base). The negative sign gives the vertical stress σ_{zd} at the toe of dam and the positive sign gives the vertical stress σ_{zu} at the heel of the dam. The major principal stress σ_1 and the minor principal stress σ_2 at the upstream and the downstream sides of the dam can be written as:

$$\sigma_{1u} = \sigma_{zu}(1+\tan^2\varphi_u) - p_w \tan^2\varphi_u; \tag{2-a}$$

$$\sigma_{2u} = p_w, \tag{2-b}$$

$$\sigma_{1d} = \sigma_{zd}(1+\tan^2\varphi_d), \tag{3-a}$$

$$\sigma_{2d} = 0 \text{ (if no tail water).} \tag{3-b}$$

In which φ_u and φ_d are the angles between upstream face and the vertical; and the downstream exit and the vertical respectively. P_w is the hydrostatic pressure acting at plane level. The horizontal shear stresses τ_u and τ_d (at upstream and downstream respectively) at any plane, can be given by the following two equations:

$$\tau_u = (p_w - \sigma_{zu}) \tan\varphi_u \quad , \tag{4}$$

$$\tau_d = \sigma_{zd} \tan\varphi_d \ . \tag{5}$$

Vertical normal stresses σ_{zu}, σ_{zd}, σ_{1u}, σ_{2u}, σ_{1d} and horizontal shear stresses τ_u and τ_d should be lying within the allowable stresses according to the code standards.

5. Objective function

The cost of constructing a gravity concrete dam mainly depends upon the amount of concrete required for constructing such dam. So, the objective function of design F for a gravity dam will be the volume of such dam. Referring to fig. 1 the objective function F can be written as:

$$F = \text{Minimize } 0.5b_1 (H_1-y_1)\, z_1 + b_2 (H_1+y_2) + 0.5b_3 (H_1+y_2-y_3)\, z_2. \tag{6}$$

The objective function F was estimated for a slice of dam one-meter width. Eq.(6) gives F which is written in an explicit form including the main dimensions of the gravity dam z_1, z_2, y_1, y_2, y_3 and b_2.

7. Design constraints

To obtain a functional design, the following constraints must be satisfied to fulfill the gravity dam design criteria.

6.1. Free board constraint (c_1)

Free board y_2 is the vertical distance between the top of the dam and full supply level in the reservoir. Free board y_2 should be greater than or equal to the wave height h_w (in meter). Davis and Sorenson [6] have given h_w for (fetch< 20 km) as follows:

$$h_w = 0.34f^{0.5} + 0.76 - 0.26f^{0.25}, \tag{7-a}$$

in which f is the fetch in kilometer. For (fetch> 20 km) h_w can written as:

$$h_w = 0.032(uf)^{0.5} + 0.76 - 0.24f^{0.25}, \tag{7-b}$$

in which u is the wind velocity in (km h^{-1}).

Denoting f_w is the minimum free board which can written as:

$$f_w = 0.75h_w + c^2/2g, \tag{7-c}$$

in which, c, is the wave propagation velocity ($c=1.5+2\ h_w$) in (m s^{-1}) and g is the gravity acceleration. So, the constraint c_1 can be written as:

$$f_w/\ y_2 \leq 1.0, \tag{8}$$

6.2. Road width constraint (c_2)

Road width b_2 should be greater than or equal to the road width suggested by standards and codes R_w. So, the constraint c_2 can be written as:

$$R_w/b_2 \leq 1.0. \tag{9}$$

6.3. Overturning safety constraint (c_3)

The factor of safety against overturning F_o should be greater than or equal to a certain value suggested by the designer F_{os}. So, the constraint c_3 can be written as:

$$F_{os}/F_o \leq 1.0. \tag{10}$$

6.4. Sliding safety constraint (c_4)

The factor of safety against sliding F_s should be greater than or equal to a certain value suggested by the designer F_{ss}. So, the constraint c_4 can be written as:

$$F_{ss}/\ F_s \leq 1.0. \tag{11}$$

6.5. Stresses safety constraints (c_5, c_6 , c_7 and c_8)

For a certain type of concrete with a certain mixture of aggregate, cement, water content and special additives the codes give the allowable normal compression stress as σ_{zs} and the allowable shear stress τ_s. So, for safety requirements the constraints c_5, c_6, c_7 and c_8 can be written respectively as:

$$\sigma_{zu}/\ \sigma_{zs} \leq 1.0, \tag{12-a}$$

$$\sigma_{1u}/\ \sigma_{zs} \leq 1.0, \tag{12-b}$$

$$\sigma_{zd}/\ \sigma_{zs} \leq 1.0, \tag{13-a}$$

$$\sigma_{zd}/\ \sigma_{zs} \leq 1.0, \tag{13-b}$$

$$\tau_u/\ \tau_s \leq 1.0, \tag{14}$$

$$\tau_d / \tau_s \leq 1.0 \,. \qquad (15)$$

6.6. Buoyancy constraint (c_9)

At any section in the dam body the percentage of the uplift pressure from the total uplift is decided by the designer [7]. Denoting W is the weight of the dam above that section and U is the uplift force acting underneath that section. The factor of against buoyancy F_b can defined as the ratio between W and U. Let the decided factor of safety against buoyancy $\dot{F}_b$. So, the constraint c_9 can be written as:

$$\dot{F}_b / F_b \leq 1.0, \qquad (16)$$

7. Design algorithm

The gravity dam design boils down to minimization of the objective function F given by Equation (6) subjected to the constraints given by eqs. (8 - 16). On the basis of random search technique [8] a design methodology for a gravity concrete dam was prepared. The design variables (V_1, V_2, V_3, V_n) constitute the design vector V. An initial random design vector was chosen by:

$$V^{(o)} = V_L^{(o)} + [V_U^{(o)} - V_L^{(o)}]R_n \,. \qquad (17)$$

In which V_L and V_U are the lower and the upper limits of V respectively; R_n is a uniformly distributed random number lying between 0 and 1;and the superscript o denotes the initial value. The initial random design was checked for all the constraints. If any of the constraints is violated, a new random design was considered. The process was repeated till al the constraints were satisfied. The objective function F of this feasible design was obtained. The process was repeated for another feasible design. If the present objective function F was less than the previously obtained feasible design, the present design vector was retained by naming it as V_s. The process was repeated for a large number of times to get the least cost feasible design vector. The search was refined by reducing the range of V by:

$$V_L^{(r+1)} = V_s^{(r)} - 0.45[V_U^{(r)} - V_L^{(r)}], \qquad (18)$$

$$V_U^{(r+1)} = V_s^{(r)} + 0.45[V_U^{(r)} - V_L^{(r)}], \qquad (19)$$

in which r is the number of cycles. The process was repeated for several cycles till the percentage difference between the optimal design of two successive cycles was small.

8. Numerical example

A computer program was written by FORTRAN language to handle the optimization process. To check the used method, the following data was givin as input data:

1. Reservoir water elevation (H_1) = 250.0 m. ,
2. Tail water height (H_2) = 20.0 m. ,
3. Silt zone height (H_s) =30.0 m. ,
4. Fetch = 20.0 km. ,
5. Decided road width (R_w) = 20.0 m. ,
6. Gravitational acceleration (g) = 9.81 m s^{-2},
7. Unit weight of water = 9.81 kN m^{-3},
8. Unit weight of concrete = 23.0 kN m^{-3},
9. Unit weight of silt (saturated)=18.0 kN m^{-3},
10. Angle of internal friction (silt) =30.0°,
11. Rock friction coefficient (tan φ_c) = 1.8,
12. Cohesion = 2.0 MN m^{-2},
13. Concrete allowable normal compressive stresses = 11800.0 kN m^{-2},
14. Concrete allowable tensile stresses = 1200.0 kN m^{-2},
15. Concrete allowable shear stresses = 1180.0 kN m^{-2},
16. Seismic pressure factor = 0.63,
17. Seismic horizontal acceleration coefficient = 0.25 and
18. Seismic vertical acceleration coefficient = 0.1.
19. Using the above data, the optimal dimensions of the gravity dam was obtained. Fig. 2 shows the output dimensions, this solution has been selected from about 350000 design, on the other hand, all the design criteria constraints, describing safety of this problem, have been satisfied. Figs. 3 and 4 show the variation of the objective function and constraints with cycles. A glance to fig. 3, one notices that the design algorithm converging speedily.

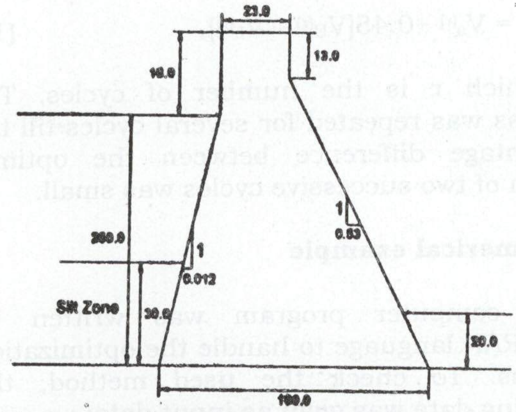

Fig. 2 Optimal dimensions of gravity dam.

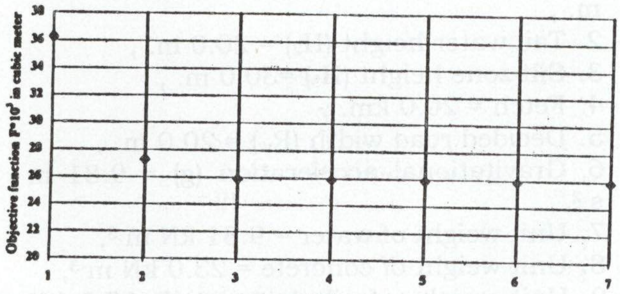

Fig. 3. Relationship between objective function F and cycles.

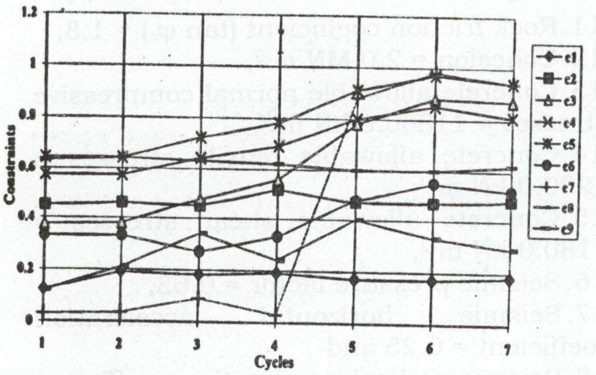

Fig.4. Variation of constraints with cycles.

9. Design charts

The upstream face slope z_1, the downstream exit slope z_2, the upstream vertical distance y_1, the free board y_2, the downstream vertical distance y_3 and the road width b_2 are the main parameters affecting the gravity dam design. As mentioned before that in the field of dam design the designer should have some specials tools, not available to all the designers, to get the cabipilities to design a gravity dam. A set of design charts was prepared to obtain the optimal dimensions of a concrete gravity dam if the reservoir water elevation, H_1, is known. Figs. (5, 6, 7 and 8) show the variation of z^*_1 and z^*_2 with H_1, the variation of y^*_2 with H_1, the variation of b^*_2 with H_1 and the variation of y^*_3 with H_1. The symbol * denoting the optimal value. The parametric study shows that the parameter y_1, as it is not restricted by tight constraint, vanished during the minimization process, so, it can be eliminated from the parameters list.

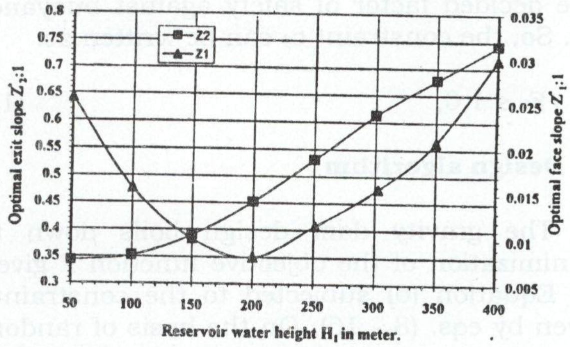

Fig. 5. Variation of z'_1 and z'_2 with H_1.

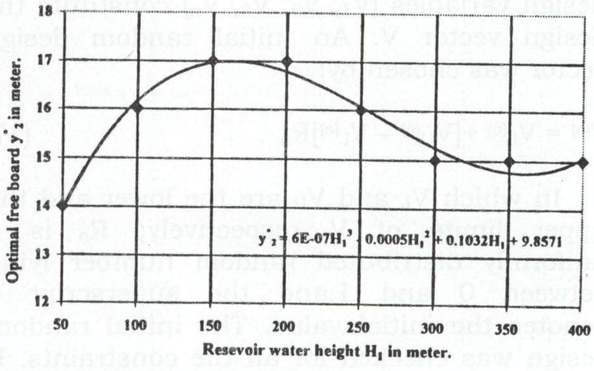

Fig. 6. Variation of y'_2 with H_1.

10. Conclusions

On the basis of random search technique, this study describes a design methodology for solid gravity concrete dam. The method gives the proper dimensions of dam which guarantees its volume to be minimum (i.e. least cost design) while on the other hand this design satisfy all the constraints required for dam safety. The parametric study shows that the parameter y_1, as it is not restricted by any tight constraint, vanished during the minimization process, so it could be eliminated from the list of the dam parameters. A set of design charts was prepared to obtain the optimal dimensions of concrete gravity dam if the reservoir water elevation H_1 is known.

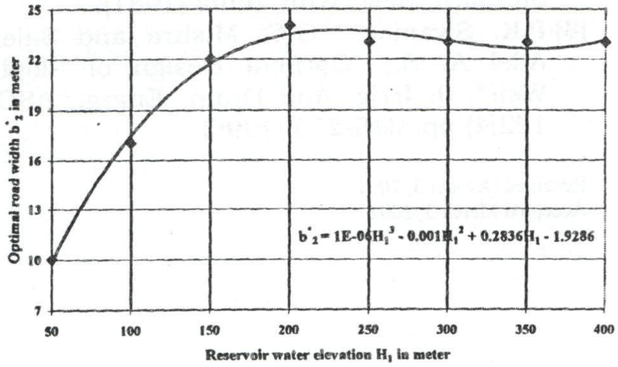

Fig. 7. Variation of b_2 with H_1.

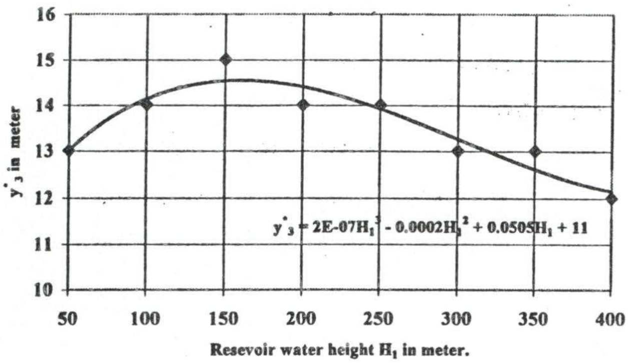

Fig. 8. Variation of y^*_3 with H_1.

Nomenclature

B	base width,
b_1	upstream base width portion,
b_2	road width,
b^*_2	optimal road width,
b_3	downstream base width portion,
c	wave propagation velocity,
$c_1, c_2,\dots,c_9$	constraints,
e	eccentricity,
F	objective function,
F_b	factor of safety against buoyancy,
F'_b	decided F_b,
F_s	factor of safety with respect to sliding,
F_{ss}	suggested F_s,
F_o	factor of safety with respect to overturning,
F_{os}	suggested F_o,
F_v	vertical forces,
f	fetch,
f_w	minimum free board,
g	gravitational acceleration,
H_1	reservoir water elevation,
H_2	tail water height,
H_s	silt zone height,
h_w	wave height,
p_w	hydrostatic pressure,
R_n	random number,
R_w	standard road width,
r	number of cycles,
U	uplift pressure,
u	wind velocity,
$V_1, V_2, V_3,\dots V_n$	design variables,
V	design vector,
V_L	lower limit of variable,
V_U	upper limit of variable,
y_1	upstream vertical distance,
y_2	free board,
y^*_2	optimal y_2,
y_3	downstream vertical distance,
y^*_3	optimal y_3,
z_1	dam face slope,
z^*_1	optimal z_1,
z_2	dam exit slope,
z^*_2	optimal z_2,
σ_z	vertical normal stress,
σ_{zd}	downstream vertical normal stress,
σ_{1d}	major downstream principal stress,
σ_{2d}	minordownstream principal stress,
σ_{zu}	upstream vertical normal stress,
σ_{1u}	major upstream principal stress,
σ_{2u}	minor upstream principal stress,
φ_c	angel of internal friction of rock,
τ_d	shear stress at downstream,
τ_s	allowable shear stress, and
τ_u	shear stress at upstream.

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