Mathematical model for bars formation and movement

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There has been a very scarce data concerning bar (submerged sand dunes parallel to the shore in the surf zone and off-shore zone) characteristics along the Nile delta coast, as well as, a lack of relating the bars to the most prevailing dynamic actions along the shore. Due to the importance of bars formation, characteristics and movement, intensive seasonal field data were collected and recorded since 1980 up to 1998 along the Nile delta coast. These field data included beach profile data, wave characteristics both in deep and shallow water, water level variations, grain size, and wind speed and direction. In the present study the Nile delta coast is considered as three zones named; Rosetta zone, Burullus zone and Ras El Bar zone. Each zone has its own characters and parameters due to the most prevailing dynamic forces, profile shape, sediment characteristics and sediment transport quantities. Depending on these very reliable parameters and characters, a computer program was developed to predict and evaluate the profile shape in each zone under the effect of the dynamic forces of waves, sediment characters, and water level variation to know the bar formation and bar characteristics along the required zone. The developed program is verified and checked with the actual data. Group of curves were obtained to simplify the use of the study results.

علـــى مــــر عشرات السنين تواجه شواطئ دلتا النيل في مصر مشكلة نقص البيانات الخاصة بخصائص القرون البحرية (bars) وكذلك عدم ربط هذه الخصائص بالعوامل البحرية الديناميكية المؤثرة على الشواطئ المصرية. و نظــرا لأهمية خصائص و حــركة هذه القرون فقد تم تجميع بيانات مكثفة عنها طوال الفترة من ١٩٨٠ إلى ١٩٩٨ و هذه البيانات تتضمن خصائص الأمواج فـــي المــناطق العميقة و المناطق الضحلة و التغير في منسوب سطح البحر و حجم حبيبات القاع و سرعة واتجاه الرياح المؤثرة. وقــد تم التعامل مع شواطئ الدلتا على اعتبار أنها ثلاث مناطق هي منطقة رشيد ،منطقة البرلس (وسط الدلتا) و منطقة رأس البر اذ أن كــل منطقة يحكمها خصائصها المميزة من حيث شكل القطاعات البحرية و العوامل الديناميكية المؤثرة. وقد تم عمل برنامج كمبــيوتر يعــتمد علـــى هذه البيانات الحقلية المكثفة وذلك للتنبؤ بشكل القطاعات البحرية تحت تأثير مختلف العوامل البحرية من أمواج و تغير فى منسوب سطح البحر و خصائص حبيبات القاع .وقد تم التحقق من عمل البرنامج وذلك باستخدام البيانات الحقلية المــتاحة وتســهيلا لاستخدام ما تم التوصل إليه تم استنتاج مجموعة من المنحنيات يستعان بها للتنبؤ بالخصائص المختلفة للقرون البحرية تحت تأثير القوي الديناميكية المختلفة واحتمالات تكونها و حركتها وانتقالها.

Keywords: Mathematical model, Bar formation, Bar movement, Bar characteristics, Nile delta coast

1. Introduction

A mathematical computer model was developed to obtain the characteristics of bar and bar formation and to predict macro scale beach profile changes with emphasis on the main morphological features to evaluate the relation between sand bar characteristics versus waves and sediment.

The development of this mathematical model begins with the assumption that the nearshore profile can be described by a monotonic function of depth (h) which increases in the seaward direction, x, across the actively modified portion of the profile according to the relationship $h=Ax^{2/3}$.

The cross shore area was divided into three zones of cross-shore sand transport relations named, prebreaking zone, breaker zone and broken wave zone.

The model consists of three submoduls; one for predicting waves characteristic in the deep water from meteorological data and the second is for irregular wave deformation along the profile using probability distribution function of wave heights. The third part of the model concerns the calculation of sediment transport rates along the profile and then interpretation of bed topography using the calculated total sediment transport.

A linear regression analysis was used to enhance the cross-shore sand transport equations used in the model to be capable to predict the evolution of sand bars. From the model obtained results, graphs were established to predict the inner and outer bars characteristics under any given deep and breaker wave conditions.

2. General flow-chart for the presented new model

Fig. 1. shows the sequence of the submodels for the new presented work starting by calculating the deep water wave characteristics using the available wind data up to calculating the sediment transport quantity for each zone and bars formation along the cross shore beach profile.

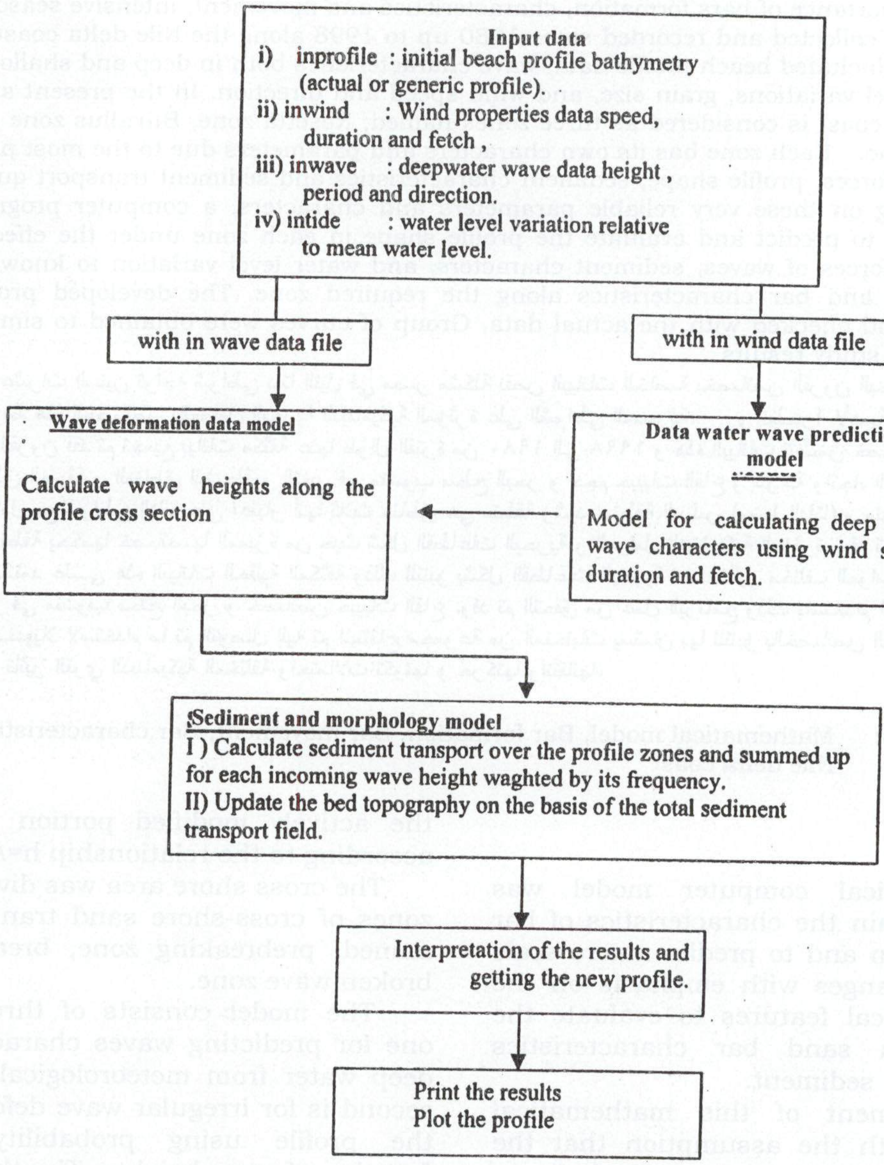

Fig. 1. General flow chart for the preset work.

3. Factors affecting beach profile morphology

The factors affecting the formation and migration of bars can be summarized as follows:

1. Existing beach profile morphology.
2. Characteristics of bed material.
3. Wave characteristics, height, length and period.
4. The tide characteristics in the study area.

All these factors were included in a new mathematical model depending on the most recommended and reliable formulae developed by the previous researches. These formulae were modified and new coefficients were developed to get a most applicable model for predicting bars formation, characteristics and migration for the Nile delta coast zones.

4. Boundary conditions

The cross-shore beach profile zones named; breaker zone, broken zone, and prebreaking zone were characterized by the following boundary water depth shown in fig. 2. The following reliable equations are used and represent the boundary conditions.

1. The limiting depth of erosion is given by [1]:

$$d_c = 2.28\, H_s - 68.5\, (H_s^2/gT_s^2). \quad (1)$$

2. The depth of wave breaking is given by [2]:

$$d_b = 1.28\, H_s. \quad (2)$$

3. The depth of plunging point is given by [3]:

$$d_p = 1.1\, H_s. \quad (3)$$

Where:
H_s = Significant wave height.
T_s = Significant wave period.
g = Acceleration due to gravity.

5. Parts of the model

5.1. Wind generated wave growth model

In this model, simple wave growth formula that follow is used and provide quick estimates for wind-wave growth in the deep and shallow water. The formulae sued in the presented work are recommended by shore protection manual [4] and Smith [5].

5.2. Boundary conditions and assumptions

The model has, the following major assumptions:

1. Winds prescribed at the 10-m depth (Z = 10m).
2. Relatively short fetch geometric (F < 100 km).
3. Fixed value of drag coefficient (cd = 0.001).
4. Natural stability conditions.

5.2. Wave deformation model

The second part of the model is to get waves characteristics along the cross shore beach profile using the wave characteristics in the deep water. This application yields cumulative probability distribution of wave heights as a field of irregular wave propagate from deep water throughout the profile.

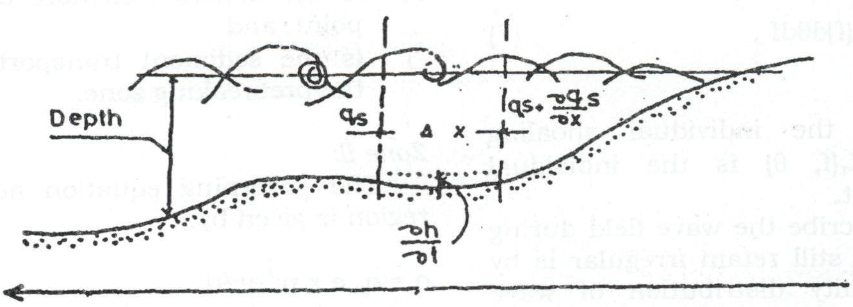

Fig. 2. Definition sketch for coastal profile model.

The model based on two random-wave theories given by [6] concerns transformation of random waves shoaling over a bottom and refraction procedure for random waves propagating over a shoaling bottom.

The process modeled includes the following wave criterion i.e. wave refraction, refraction shoaling and breaking. The spreading function G (f, θ) used in this application is that of Mitsuyasu [7]:

$$G(f, \theta) = G_o \cos^{2S}(\theta/2), \tag{4}$$

Where,

$$G_o = \frac{2^{2S-1}}{\Gamma}\frac{\Gamma^2(S+1)}{\Gamma(2S+1)}, \tag{5}$$

G is the angular deviation from principle direction,
S is the parameter representing directional energy concentration around a peak,
S max = peak value of S,
= 10 for wind waves,
= 25 for steep swell,
= 75 for flat swell,
Γ is the gamma function,
f is the frequency, and
θ is the angle of spreading.

The effective refraction coefficient ($k_{r\ eff}$) for the spectrum is defined as the average refraction coefficient for the entire spectrum and given by [7]:

$$k_{r\ eff} = \left[\frac{1}{m}\int_0^{\infty}\int_{\theta\min}^{\theta\max} S(f,\theta)k_S^2(f)k_r^2(f,\theta)d\theta df\right]^{1/2}. \tag{6}$$

Where;

$$m = \int_0^{\infty}\int_{\theta\min}^{\theta\max} S(f,\theta)k_S^2(f)d\theta df, \tag{7}$$

where $K_s(f)$ is the individual shoaling coefficient, and $K_r(f, \theta)$ is the individual refraction coefficient.

One way to describe the wave field during transformation but still retain irregular is by using the probability distribution of wave heights.

Assuming a Rayligh distribution of wave height, a probability density function of wave height normalized by Ho is given by [6]:

$$P_o(x) = 2a^2x\left(e^{(-a/x^2)}\right). \tag{8}$$

Where; $P_o(x)$ = probability density function,

$X = \frac{H}{H_o}$ = normalized wave height,

$a = \frac{1.416}{K_s}$, and

Ks = shoaling coefficient.

6. Sediment transport morphology model

The surf zone is considered as three parts, illustrated in fig. 3, Zone I extends from the depth of closure to the beginning of the bar (breaker point), zone II is the bar zone (breaker zone), and zone III starts from the end of the bar up to the shore.

Each zone is governed by a certain equation as follows, given by [8]:

Zone I:

Sediment transport in this zone is governed by the equation,

$$q = q_b \exp^{-\lambda_1(x-x_b)}. \tag{9}$$

Where:
q_b is the mass transport rate regime in zone I,
X is the distance offshore at any point within the zone $x > x_b$,
x_b is the distance offshore of the breaking point, and
λ_1 is the sediment transport parameter in the prebreaking zone.

Zone II:

The governing equation adopted in this region is given by,

$$q = q_p\, e\, x\, p^{-\lambda_2(x-x_p)}. \tag{10}$$

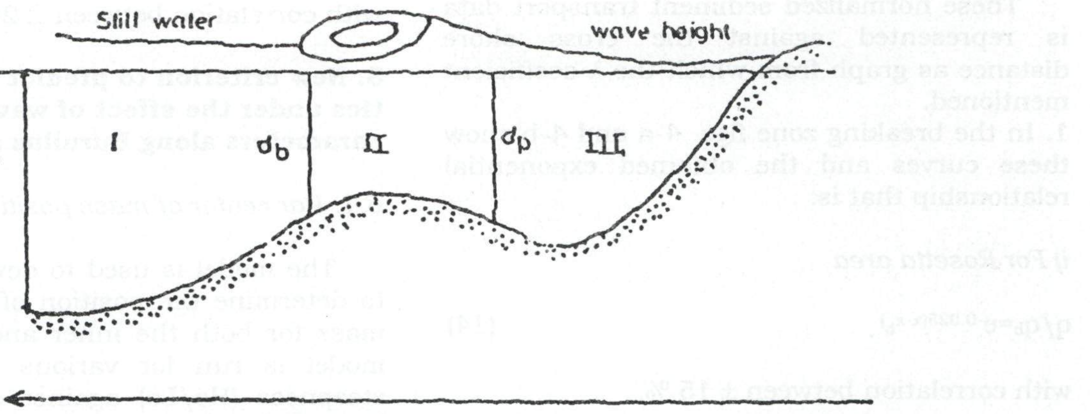

Fig. 3. Beach protile zones

Where:

q_p is the mass transport rate regime in zone II,

X is the distance offshore at any point within the zone $x_b \geq x > x_p$,

x_p is the distance offshore of the plunging point, and

λ_2 is the sediment transport parameter in the breaking zone.

Zone III:

The governing equation for sediment transport within the surf zone is given by,

$$q = K (E - E_{eq}). \quad (11)$$

Where;

K is the empirically sand transport rate coefficient.,

E,Eeq are the actual and equilibrium energy dissipation per unit volume of water under given local wave and water level conditions.

To calculate the development of the coastal beach profile under the effect of waves and tides, the general continuity equation for sediment is given by,

$$\frac{\partial h}{\partial t} + \frac{1}{1-\nu}\left(\frac{\partial q}{\partial x} + \frac{\partial q}{\partial y}\right) = 0. \quad (12)$$

If the alongshore conditions are stable i.e., isopaths are parallel and there is no alongshore current, then $\frac{\partial q}{\partial y} = 0$ and the profile conservation can be expressed as,

$$\frac{\partial h}{\partial t} = \frac{1}{1-\nu}\frac{\partial q}{\partial x}. \quad (13)$$

Where:

ν is the bed sediment porosity,

h is the bed level, and

q is the sediment rate.

7. Sediment transport equations verification

The cross shore profile area is divided into three parts as mentioned. Transport equations are developed for two coastal profile areas; one illustrates the mean features of Rosetta area profiles and the other for Burullus and Ras El Bar area profiles.

For each system the cross shore area of the average profile calculated for each survey line between 1980 to 1998 is divided into the three zones named, prebreaking, breaker transition and broken wave zone. Comparing each average profile with the corresponding modified equilibrium profile curve, the normalized sediment transport rate for both the prebreaking and breaker transition zone is calculated along the cross shore distance from the breaking and plunging points, respectively.

These normalized sediment transport data is represented against the cross shore distance as graph from which the λ coefficient mentioned.

1. In the breaking zone figs. 4-a and 4-b show these curves and the obtained exponential relationship that is:

i) For Rosetta area

$$q/q_b=e^{-0.025(x-x_b)}, \quad (14)$$

with correlation between ± 15 %.

ii) For Ras El Bar and Burullus areas

$$q/q_b=e^{-0.03(x-x_b)}, \quad (15)$$

with correlation between ± 18 %.

2. In the breaker transition zone figs. 4-c and 4-d, yielded:

i) For Rosseta area

$$q/q_p=e^{-0.05(x-x_p)}, \quad (16)$$

with correlation between ± 18 %

ii) For Ras El Bar and Burullus areas

$$q/q_p=e^{-0.07(x-x_p)}, \quad (17)$$

with correlation between ± 18 %.

On the other hand, the average profile is compared with the Modified Equilibrium Profile (MEP) curve for each area profile to determine the transport rate governing the broken zone. The values are presented in curve in figs. 5-a and 5-b from which the governing transport rate equation are obtained which are:

i) For Rosetta area

$$q=0.0015(E-E_{eq}), \quad (18)$$

with correlation between ± 18 % .

ii) For Ras El Bar and Burullus areas

$$q=0.0010(E-E_{eq}), \quad (19)$$

with correlation between ± 20 %.

8. New criterion to predict bar characteristics under the effect of waves and sediment parameters along Burullus and Ras El Bar

8.1. Bar center of mass position

The model is used to develop a new graph to determine the position of the bar center of mass for both the inner and outer bars. The model is run for various deep water wave steepness (Ho/Lo) against offshore slope for determining the position of bar center of mass for the outer bar, while breaking wave steepness (Hb/Lo) versus beach (near-shore) slope data are used to obtain the inner bar center of mass position.

So, by knowing the breaker or deep water wave steepness and the wave length, the position of the inner or outer bar center of mass can be obtained easily.

A group of curves are obtained and presented in figs. 6 and 7 for the area of Burullus and Ras El Bar coasts.

It is clear from these figures that the distance of the bar center of mass is increasing by increasing the deep water wave length, decreasing of deep or breaker wave steepness and decreasing in beach or offshore slope.

8.2. Bar crest height

By using several cases of deep water wave steepness (Ho/Lo) with beach slope for the evaluation of outer bar crest height, and breaker wave steepness (Hb/Lo) with beach slope. The outer and inner bar crest heights are predicted, respectively.

Figs 8 and 9 show the simple criterion developed to predict the inner or outer bar crest height under the prevailing wave and sediment characteristics for systems of coastal areas that representing Burullus and Ras El Bar area.

It is concluded from these curves that the bar crest height (hc) is increasing in the breaker zone by increasing in breaker wave steepness (Hb/Lo), decreasing in beach slope

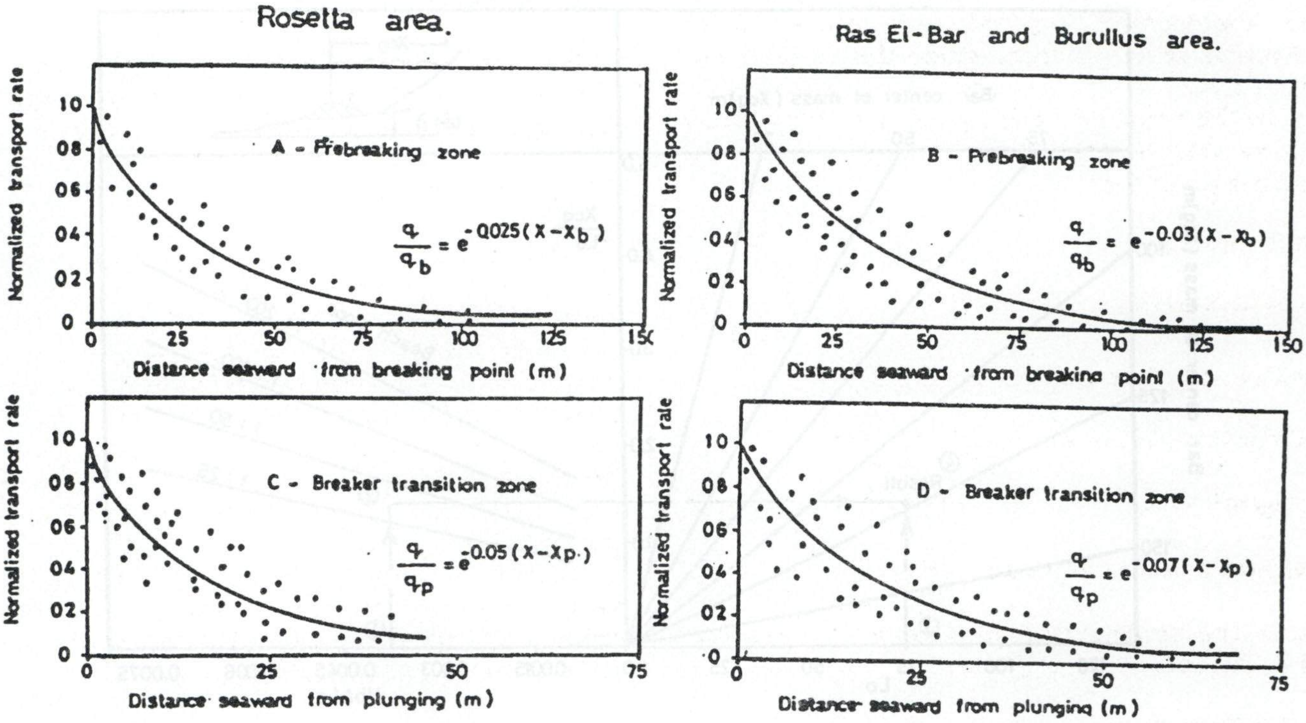

Fig. 4. Net cross-shore sand transport rates.

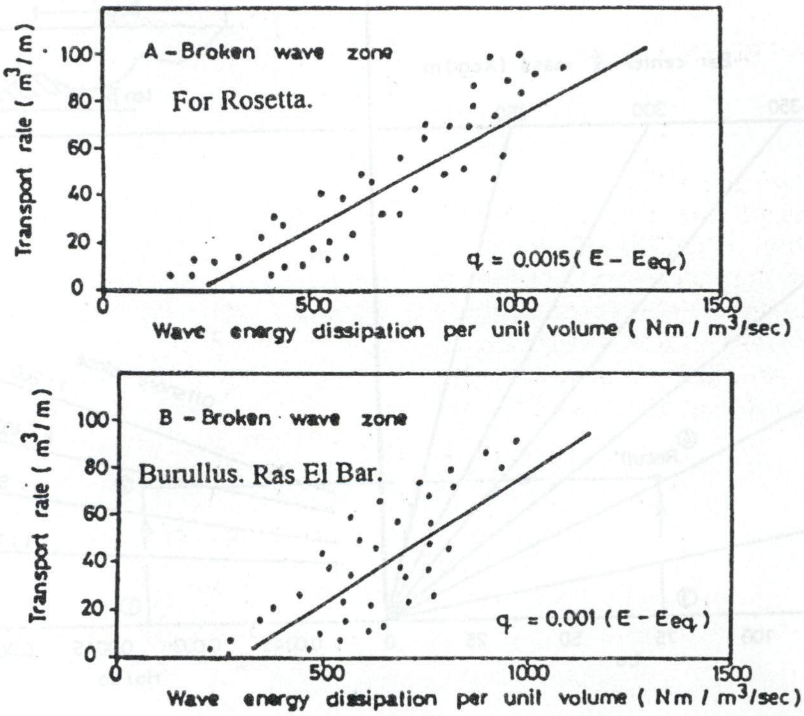

Fig. 5. Net cross-shore sand transport rates for problem wave zone.

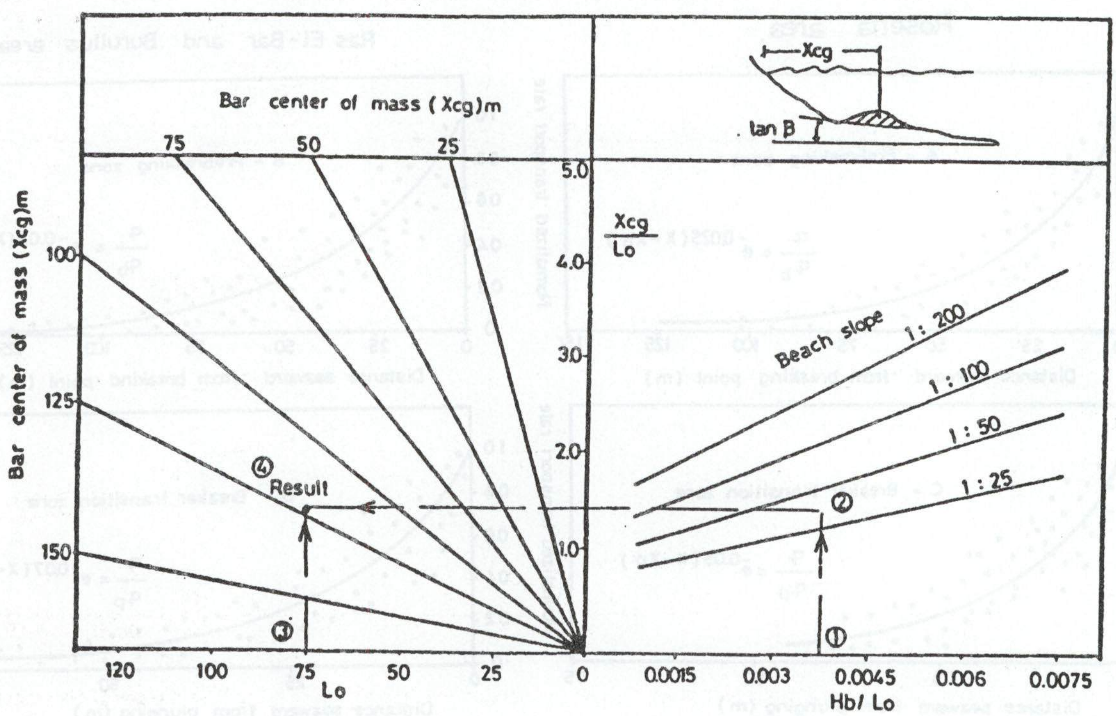

Fig. 6. Graph to determine bar center of mass knowing wave steepness (H_b/L_o) and beach slope for Burullus and Ras El-Bar areas inner bar.

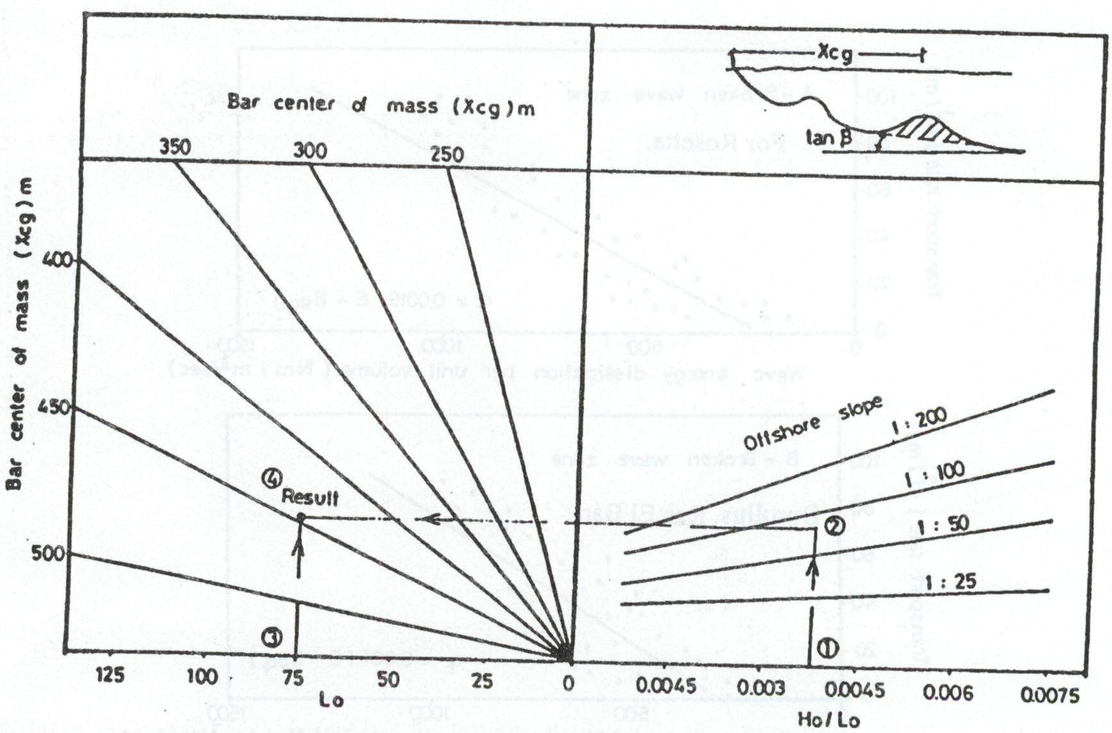

Fig. 7. Graph to determine bar center of mass knowing deep water wave steepness (H_b/L_o) and offshore slope Burullus and Ras El-Bar areas inner bar.

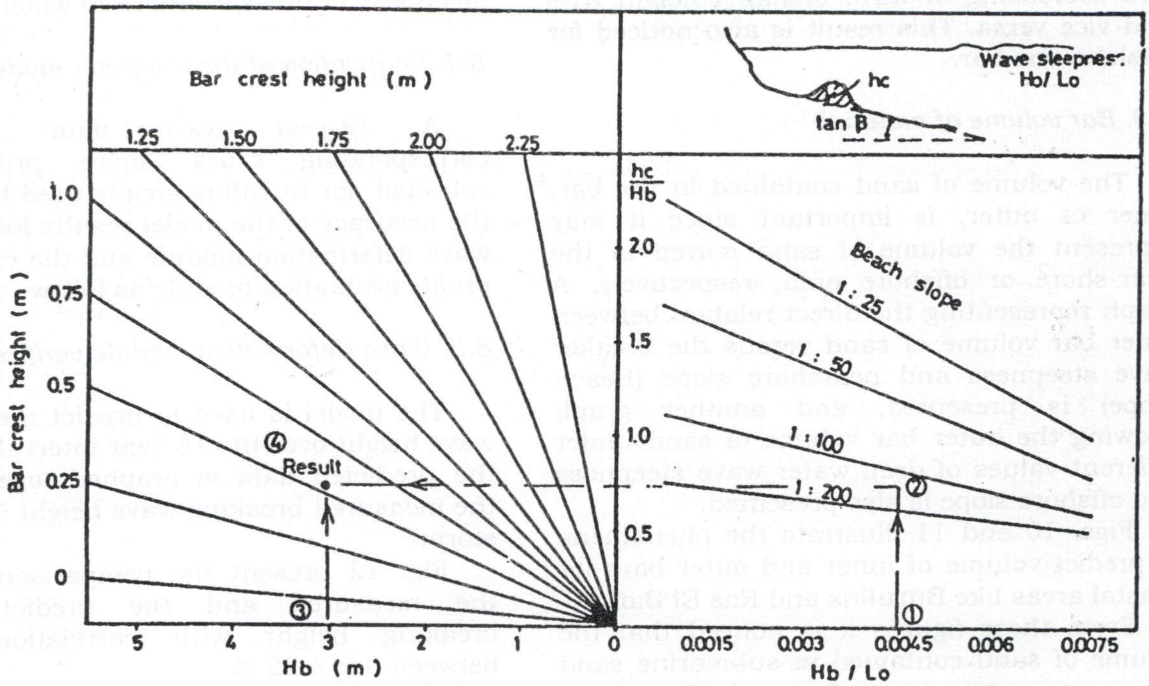

Fig. 8. Graph to determine bar crest height (h_c) knowing wave steepness (Hb/Lo) and beach slope for Burullus and Ras El-Bar areas inner bar.

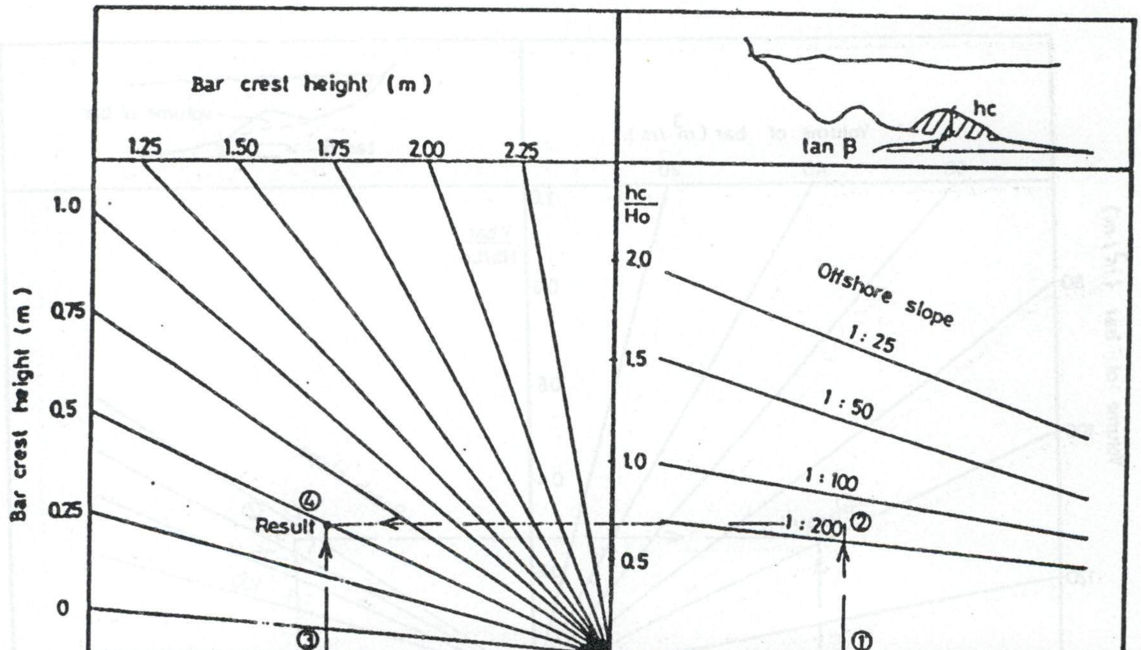

Fig. 9. Graph to determine bar crest height (h_c) knowing deep water wave steepness (Ho/Lo) and offshore slope for Burullus and Ras El-Bar areas outer bar.

and decreasing in wave breaking height (Hb) and vice versa. This result is also noticed for offshore one bar.

8.3. Bar volume of material

The volume of sand contained in the bar, inner or outer, is important since it may represent the volume of sand moved in the near-shore or offshore area, respectively. A graph representing the direct relation between inner bar volume of sand versus the breaker wave steepness and nearshore slope (Beach Slope) is presented, and another graph showing the outer bar volume of sand under different values of deep water wave steepness and offshore slope is also presented.

Figs. 10 and 11 illustrate the phenomena to predict volume of inner and outer bars for coastal areas like Burullus and Ras El Bar.

From these figures it is noticed that the volume of sand contained in submarine sand bars are strongly influenced by wave steepness in breaker and deep water zones.

8.4. Verification of the computer model

A 18-year wave data and the corresponding cross shore profile data collected for Burullus area is used to measure the accuracy of the model results for both, the wave deformation module and the cross shore profile evaluation module as follow:

8.5. Wave deformation module verification

The model is used to predict the breaking wave height over the 18-year interval and then the predicted data is graphed together with the measured breaking wave height during the storm.

Fig. 12 present the comparison between the measured and the predicted wave breaking height with correlation ranges between 0 to ± 22 %.

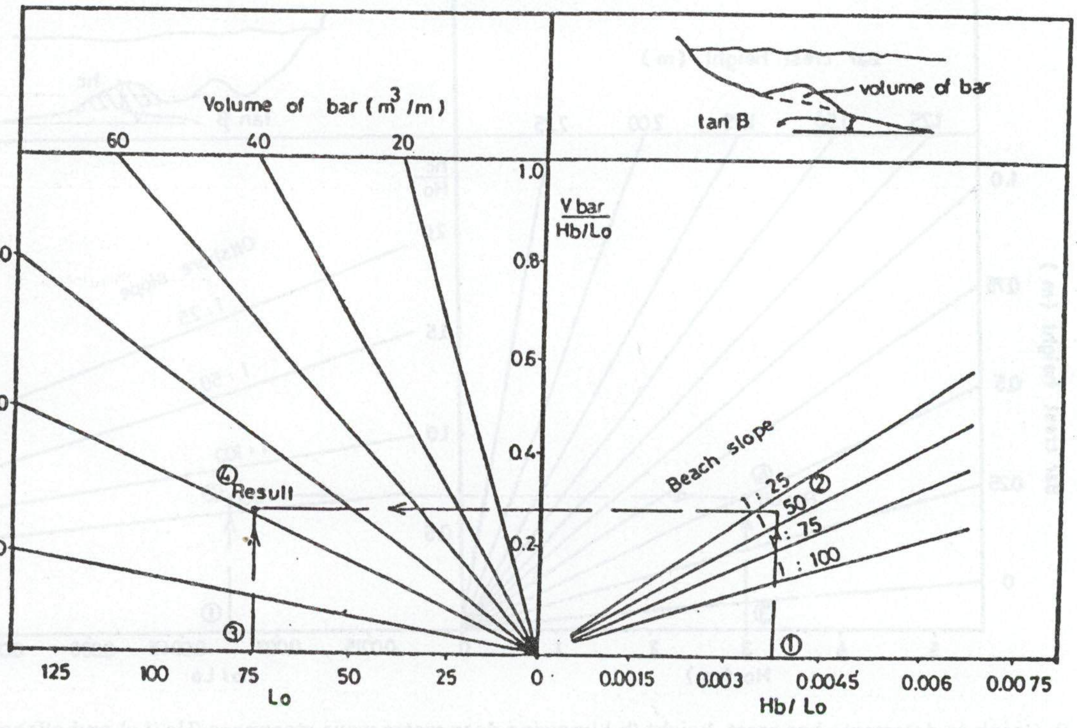

Fig. 10. Graph to determine volume of bar knowing wave braker steepness and beach slope for Burullus Ras El-Bar areas inner bar.

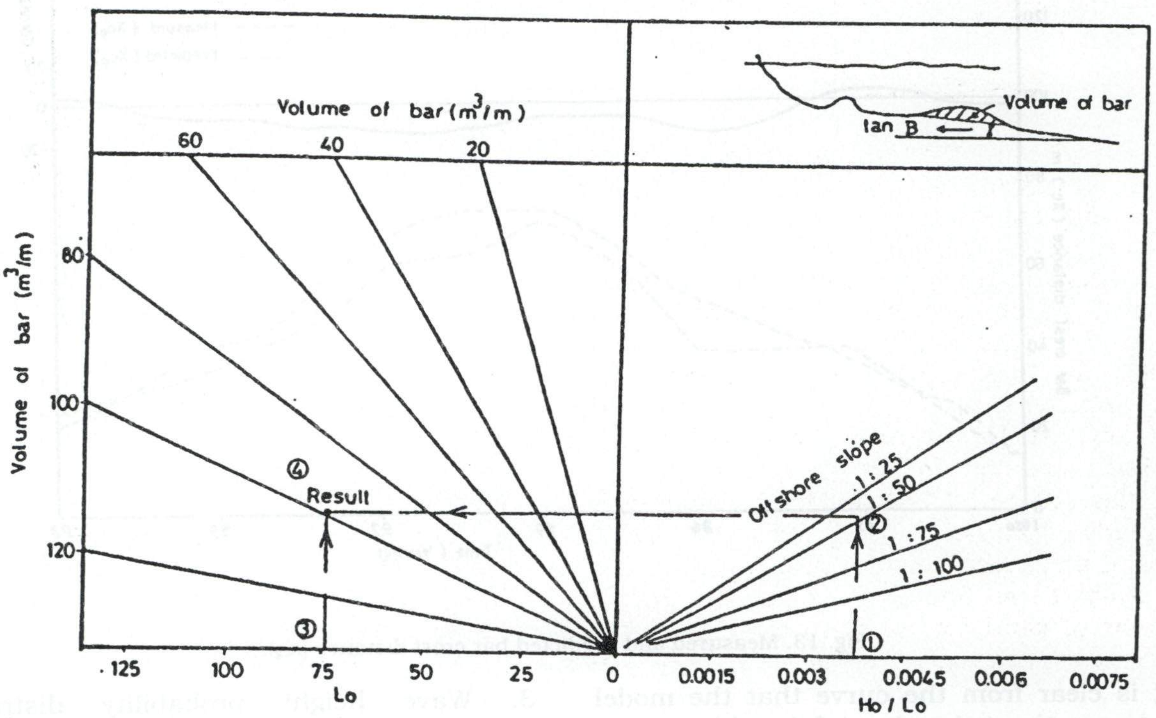

Fig. 11. Graph to determine volume of bar knowing wave steepness and offshore beach slope for Burullus and Ras El-Bar areas outer bar.

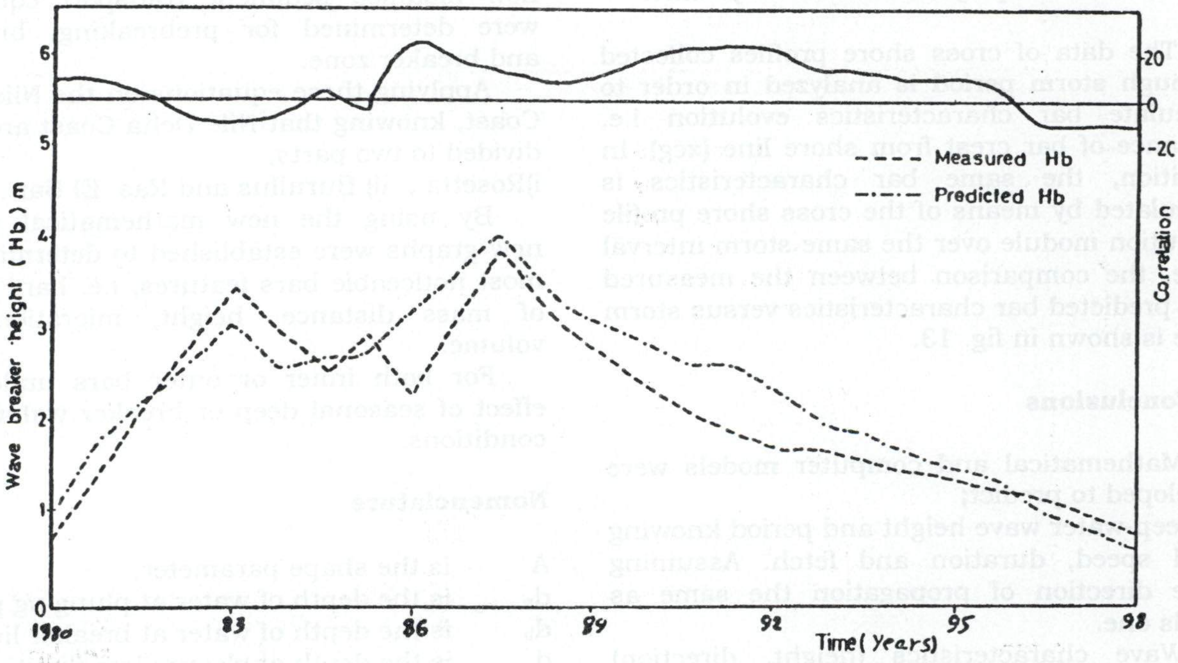

Fig. 12. Measured and predicted variation wave breaker depth.

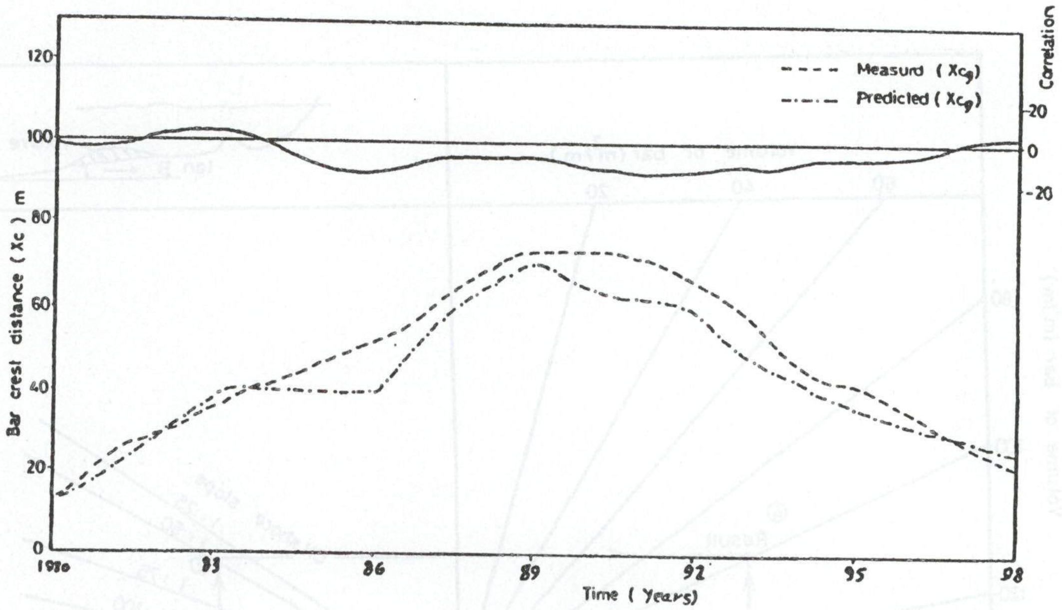

Fig. 13. Measured and predicted bar crest distance (x_{cg}).

It is clear from the curve that the model gives an estimated value of breaking wave heights during the break period and the storm decay period within the range of (0 to 22%) which is reasonable.

8.5. Cross shore profile evolution verification

The data of cross shore profiles collected through storm period is analyzed in order to calculate bar characteristics evolution i.e. distance of bar crest from shore line (xcg). In addition, the same bar characteristics is calculated by means of the cross shore profile evolution module over the same storm interval time, the comparison between the measured and predicted bar characteristics versus storm time is shown in fig. 13.

9. Conclusions

Mathematical and computer models were developed to predict;

1. Deep-water wave height and period knowing wind speed, duration and fetch. Assuming wave direction of propagation the same as winds one.
2. Wave characteristics (height, direction) variation along the cross-shore profile.
3. Wave height probability distribution function for each depth and direction of propagation.
4. Cross-shore beach profile evolution under the dynamic effect of waves and tides with emphasis on bars formation and migration. A new modified sediment transport equations were determined for prebreaking, breaking and breaker zone.

Applying these equations on the Nile Delta Coast, knowing that Nile Delta Coast area was divided to two parts,
i)Rosetta , ii) Burullus and Ras El Bar .

By using the new mathematical model, new graphs were established to determine the most noticeable bars features, i.e. bars center of mass distance, height, migration and volume.

For both inner or outer bars under the effect of seasonal deep or breaker water wave conditions.

Nomenclature

A	is the shape parameter,
d_p	is the depth of water at plunging point,
d_b	is the depth of water at breaker line,
d_c	is the depth at closure line (limiting depth of erosion),

D_{50} is the mean grain diameter,
E,Eq are the actual and equilibrium time – dependent energy dissipation unit volume of water,
f is the Frequency,
G is the acceleration due to gravity,
G is the angular determination from principle direction,
G(f,θ) is the spreading friction,
h is the depth of water positive below the still water level,
h_c is the bar crest depth,
H_o is the deep water wave height,
H_b is the break water wave height,
H_s is the significant wave height,
K is the transport rate parameter,
Kr_{eff} is the effective refraction coefficient,
K_s is the individual shoaling coefficient,
$K_r(f,\theta)$ is the individual refraction coefficient,
L_o is the deep water wave length,
L_b Length of bar across the shore,
$P_o(x)$ is the probability density function,
q is the sand transport rate,
q_b is the sand transport rate at breaking point,
q_p is the sand transport at the plunging point,
S is the parameter representing directional energy concentration around a peak
S_{max} is the peak value of S
= 10 for steep waves
= 25 for steep swell
= 75 for flat swell,
T is the wave period,
$\tan\beta_i$ is the beach bed slope,
$\tan\beta_o$ is the offshore bed slope,
t is the time,
T_s is the significant wave period,
V is the bed sediment porosity,
V_{bar} is the volume of material contained in bar pen unit width and
W is the fall speed of sediment material,
X is the distance to the offshore,
X_b is the distance offshore of the breaking point,
X_p is the distance offshore of the plunging point,
X_{cg} is the distance of bar center of mass,
Γ is the Gamma function,
θ is the Angle of spreading,
λ_1 is the sediment transport parameter in the pre-breaking zone,
λ_2 is the sediment transport parameter in the breaking zone,
π is the constant 22/7.

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