Heat transfer for laminar flow in partially heated tubes

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Fully developed laminar convective heat transfer in a partially heated pipe was investigated. The tube was assumed to be heated along a peripheral angle θ_h while the rest of its surface was assumed to be adiabatic. Two different heating conditions were investigated; uniform temperature and uniform heat flux along the heated surface. The governing equations have been solved numerically using finite volume technique to obtain the temperature distribution and heat transfer coefficient represented by Nusselt number. Effect of the angle of the heated surface on the heat transfer process was investigated. The values of Nusselt number for various heating angles were obtained. The analysis shows a decrease of Nusselt number as the angle of the heated portion of the tube surface increases. As it is expected Nusselt number approaches the known values of completely heated tubes for uniform wall-temperature and uniform wall-heat flux.

يعرض البحث دراسة عددية لانتقال الحرارة بالحمل القسري لسريان طبقي كامل التطور داخل ماسورة معرضة لتسخين جزئي على سطحها الجانبي ، لحالتين هما عندما يكون هذا الجزء ذو درجة حرارة ثابتة و عندما يكون هذا الجزء معرض لفيض حراري منتظم . تم استخدام طريقة الحجم المحدد في هذه الدراسة ، و بذلك تم دراسة تأثير قيم زاوية المحيط المعرضة للتسخين على توزيع درجات حرارة خلال المائع و معامل انتقال الحرارة ممثلا في رقم نسلت ، تم تحديد رقم نسلت لزوايا تسخين مختلفة ، أوضحت الدراسة أن رقم نسلت يقل بزيادة زاوية التسخين حتى يصل إلى رقم نسلت المعروف في حالة المواسير المعرضة كليا للتسخين أي ٣,٦٦ و ٤,٣٦ في حالتي التسخين بسطح ذي درجة حرارة ثابتة أو التسخين بفيض حراري منتظم على الترتيب ، كما بينت الدراسة أن رقم نسلت في حالة التسخين بفيض حراري منتظم أعلى منه في حالة التسخين بسطح ذي درجة حرارة ثابتة .

Keywords: Laminar, Forced convection, Tube, Partially heated

1. Introduction

Circular duct is widely used geometry in fluid flow and heat transfer applications. Accordingly, its fluid flow and heat transfer characteristics have been analyzed extensively for various boundary conditions. Circular channel flow is encountered in many applications including heat exchangers, boilers and solar energy applications, ... etc. Harnet [1], Shah and Bhatti [2] and Shah and London [3] reviewed laminar flow in circular duct. Steady fully developed incompressible laminar flow through a circular duct is referred to as Hagen-Poiseuille flow. Laminar convection in circular channel was deeply investigated for channels subjected to homogeneous heating conditions. Graetz in 1883 and Nusselt in 1910 as reported by [3] investigated heat transfer of laminar flow in tubes of constant wall-temperature. Michelsen and Villadsen [4] included axial conduction for tube of uniform wall-temperature. Ou and Cheng [5] took in consideration viscous dissipation effect. Arici [6] investigated effect of axial conduction in tube wall on heat transfer process of fully developed laminar flow in the tubes under constant wall-temperature, constant heat flux and convection heat transfer boundary conditions at the pipe outer surface. Claser as reported by [3], Tyagi [7], Tao [8] and Lee and Howel [9] investigated laminar flow in tubes subjected to uniform heat flux. Reynold [10, 11] investigated laminar flow in tubes subjected to circumferential variable heat flux.

Shah and London [3], Sparrow and Patankar [12], Clements and Lee [13],

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Quaresma and Cotta [14] and Srivastava [15] investigated Laminar flow in tubes subjected to axially varied wall heat flux.

In many applications such as the water walls of boilers and solar collectors, the heating surfaces are only partially heated along the circumference of the pipe while the rest of tube circumference is thermally insulated. This study represents a numerical analysis of forced convection heat transfer of fully developed laminar flow in a circular tube heated along a peripheral angle θ_h of its circumference while the rest of the circumference is adiabatic. The analysis was carried out for two different heating conditions; constant wall-temperature and uniform heat flux along the heated portion of the tube surface.

The laminar forced convection through Newtonian fluids, in case of pipe flows, was well investigated. The velocity profile of the fully developed laminar flow inside tube is well known, Kays and Crawford [16]:

$$u/u_m = 2(1-(r/r_o)^2), \tag{1}$$

where u, u_m, r and r_o are the velocity at general point, the mean velocity, the radial position and the tube radius, respectively. The mean velocity is given by:

$$u_m = -\frac{1}{\mu}\frac{dP}{dz}r_o^2, \tag{2}$$

where the dynamic viscosity is denoted by μ and dP/dz is the derivative of the pressure with respect to the axial coordinate z. In dimensionless form, the velocity profile is:

$$U = 2(1-R^2), \tag{3}$$

where U is the dimensionless velocity, which defined as u/u_m and R is the dimensionless radial coordinate ($R = r/r_o$).

It is well known that the forced convection process of laminar flow in channels is very sensitive to thermal boundary conditions and to channel geometry. Nusselt number for hydrodynamically and thermally fully developed flow in circular tube as reported by [2], in case of uniform wall-temperature, is:

$$Nu_T = 3.65679, \tag{4}$$

and for uniform wall-heat flux is:

$$Nu_H = 4.3636. \tag{5}$$

In the present investigation, the heat transfer process in case of partially heated tube was investigated.

2. Analysis

Fig. 1 shows the flow domain of the partially heated pipe studied in the current study. The pipe has a radius r_o. The pipe is heated along a peripheral angle θ_h of its circumference. The flow is assumed to be steady, laminar and fully developed. Moreover, it is assumed that the fluid is Newtonian and has uniform properties and the viscous dissipation within the fluid is neglected. Due to the geometrical symmetry of the flow domain, the energy equation was solved for half of the tube shown in fig. 1. For a fully developed temperature profile and negligible axial conduction, the energy equation can be reduced to:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \left(\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) = \frac{\rho C_p}{k}u\frac{\partial T}{\partial z}. \tag{6}$$

This equation coupled with its appropriate boundary conditions and the known velocity profile inside tube given by eq. (1) was solved numerically to determine the heat transfer characteristics of partially heated tube. Two heat transfer cases were investigated, heating using isothermal wall and uniform wall heat flux. In the case of uniform wall-temperature along the heated portion of the tube surface, the boundary conditions are:

$$\begin{aligned}
&\text{at } r = r_o, && 0 \le \theta \le \theta_h/2: && T = T_w,\\
&\text{at } r = r_o, && \theta_h/2 < \theta \le \pi: && q = 0,\\
&\text{at } \theta = 0, && 0 \le r \le r_o: && \partial T/\partial\theta = 0,\\
&\text{at } \theta = \pi, && 0 \le r \le r_o: && \partial T/\partial\theta = 0.
\end{aligned} \tag{7}$$

In the case of uniform wall-heat flux along the heated tube portion, the boundary conditions are:

$$\begin{array}{llll} \text{at } r = r_o, & 0 \le \theta \le \theta_h/2: & q & = q_w, \\ \text{at } r = r_o, & \theta_h/2 < \theta \le \pi: & q & = 0, \\ \text{at } \theta = 0, & 0 \le r \le r_o: & \partial T/\partial\theta & = 0, \\ \text{at } \theta = \pi, & 0 \le r \le r_o: & \partial T/\partial\theta & = 0. \end{array} \quad (8)$$

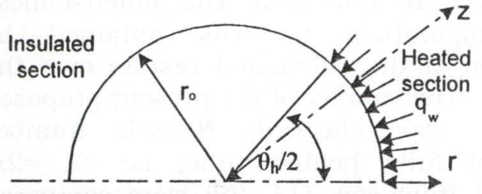

Fig. 1. Physical description and coordinate system of the present problem.

2.1. Constant temperature along heated peripheral angle

In this section, the heating portion of tube wall is assumed to have uniform temperature along angular and axial directions, T_w, and the rest of tube circumference is assumed to be adiabatic. In dimensionless form, the axially invariant fully developed temperature profile can be written in the following form:

$$\varphi(r,\theta) = \frac{T(z,r,\theta) - T_w}{q_w(z)\, r_o / k}, \quad (9)$$

where q_w is the average wall heat flux along the heated peripheral angle. The heat balance at the heating tube surface may be given by the following equation:

$$h r_o \theta_h (T_w - T_b) = q_w r_o \theta_h = \dot{m} C_p \frac{dT_b}{dz}, \quad (10)$$

where θ_h is the heated peripheral angle of the pipe circumference and h is the average heat transfer coefficient along the heated peripheral angle. Using the dimensionless variable given by eq. (9) and invoking eq. (10), the energy equation can be written in the following dimensionless form:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\varphi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\varphi}{\partial\theta^2} = \frac{\theta_h}{\pi} U \frac{\varphi}{\varphi_b}. \quad (11)$$

The appropriate boundary conditions for the above equation are:

$$\begin{array}{llll} \text{at } R = 1, & 0 \le \theta \le \theta_h/2: & \varphi & = 0, \\ \text{at } R = 1, & \theta_h/2 < \theta \le \pi: & \partial\varphi/\partial R & = 0, \\ \text{at } \theta = 0, & 0 \le R \le 1: & \partial\varphi/\partial\theta & = 0, \\ \text{at } \theta = \pi, & 0 \le R \le 1: & \partial\varphi/\partial\theta & = 0. \end{array} \quad (12)$$

The Nusselt number, which represents the heat transfer along the heated circumference of the tube surface, is defined by:

$$Nu_T = \frac{h(2\, r_o)}{k}. \quad (13)$$

From which and with the aid of eq. (10), it follows:

$$Nu_T = \frac{2\, r_o\, q_w}{k\,(T_w - T_b)}, \quad (14)$$

and in dimensionless form:

$$Nu_T = \frac{-2}{\varphi_b}. \quad (15)$$

The value of Nusselt Number given by the above equation should approach the value of Nusselt number of the uniformly heated tube, i.e. $Nu_T = 3.66$.

2.2. Uniform wall heat flux along heated peripheral angle

In this case, the tube wall is partially heated by a uniform heat flux in the axial direction. Assuming that the wall temperature is uniform along the peripheral angle of heating. In dimensionless form, the axially invariant fully developed temperature profile can be written in the following form:

$$\psi(r,\theta) = \frac{T(z,r,\theta) - T_w(z)}{T_b(z) - T_w(z)}. \quad (16)$$

Using this dimensionless variable and the heat balance at the heated portion of the tube surface, eq. (10), the energy equation can be written in the following dimensionless form:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\psi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\psi}{\partial\theta^2} = -\frac{\theta_h}{\pi}\frac{Nu_H}{2} U, \quad (17)$$

where Nu_H is the Nusselt number. Using a new dimensionless variable $\xi = \psi/Nu_H$, the above equation can be reduced to:

$$\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial \xi}{\partial R}) + \frac{1}{R^2}\frac{\partial^2 \xi}{\partial \theta^2} = \frac{-\theta_h}{2\pi}U \quad (18)$$

The appropriate boundary conditions for the above equation are:

At $R = 1$, $0 \leq \theta \leq \theta_h/2$: $\xi = 0$,
at $R = 1$, $\theta_h/2 < \theta \leq \pi$: $\partial\xi/\partial R = 0$,
at $\theta = 0$, $0 \leq R \leq 1$: $\partial\xi/\partial\theta = 0$,
at $\theta = \pi$, $0 \leq R \leq 1$: $\partial\xi/\partial\theta = 0$. (19)

The Nusselt number can be calculated from $\xi_b = \psi_b/Nu_H$. For the present case, $\psi_b=1$ and consequently Nusselt number can be reduced to:

$$Nu_H = \frac{1}{\xi_b}. \quad (20)$$

The value of Nusselt number given by the above equation should approach the value of Nusselt number of the tube uniformly heated with uniform heat flux, i.e. $Nu_H = 4.36$.

3. Computational procedure

The differential eqs. (11, 18) subjected to the boundary conditions given by eqs. (12, 19), respectively, were solved numerically using finite volume method. Details of this method are given in Patankar [17]. The velocity and temperature profiles in the channel are symmetric, hence only half of the channel was simulated. The computational domain is defined in fig. 1. The domain was divided into a finite number of cells. Discrete equations for the nodal points were derived. A grid system of 51 nodes in radial and 63 nodes in tangential directions was employed through out the computation. The grid spacing in r-direction was chosen to be finer near the inner tube wall. Exploratory computations of finer grids were conducted and the resulting changes in results were too small compared with the increase of computation time. The discretized equations were obtained by integrating eqs. (11, 18) over the domain of control volumes. The velocity profile given by eq. (3) was used as input to the eqs. (11, 18). These equations were solved by iteration until a converged solution was obtained where the change in temperature from eqs. (11, 18) at any point of the domain was less than 10^{-7} of its value of the previous iteration step. After each iteration φ_b, needed in eq. (11), was calculated by integrating the results over the flow area. The dimensionless bulk temperature ξ_b was obtained by integration of the calculated results over the flow area. The validity of the present proposed technique was checked. Nusselt number results of fully heated tube, i.e. $\theta_h = 2\pi$, calculated from eqs. (14, 20) were compared with the well known results of the fully developed laminar flow in tube with isothermal surface and uniform heat flux. The predicted Nusselt numbers, after present technique, for these two cases are 3.659 and 4.366 respectively. The differences in both cases are 0.06 % and 0.055%, respectively.

4. Results and discussions

4.1. Constant temperature along heated peripheral angle

Samples of the isotherms (constant dimensionless temperature, $(T - T_w)/(T_b - T_w)$, lines) over the whole domain are shown in fig. 2 for different heating peripheral angles. As shown in the figure, part of the tube surface has uniform dimensionless temperature of zero while the rest is adiabatic. As shown in the figure, the heat is transferred in the radial and the tangential directions from the heated surface. The heat dissipated in the tangential and the radial directions based on the heating angle. The results in fig. 2 show that the heat is transferred faster in the tangential direction for small heating angle, θ_h. To clarify that, the flow can be divided into two domains; heated domain where $(T - T_w)/(T_b - T_w) < 1$ and unheated domain where $(T - T_w)/(T_b - T_w) > 1$. The heated fluid layer along the adiabatic wall is longer for small circumference heating angles. As the heating angle increases, the heat diffusion in the angular direction decreases and the radial diffusion becomes the dominant as the problem changes from two into one dimensional problem at $\theta_h = 360^o$.

In this case, Nusselt number was calculated from eq. (15). The results of Nusselt number obtained for heating portion of the

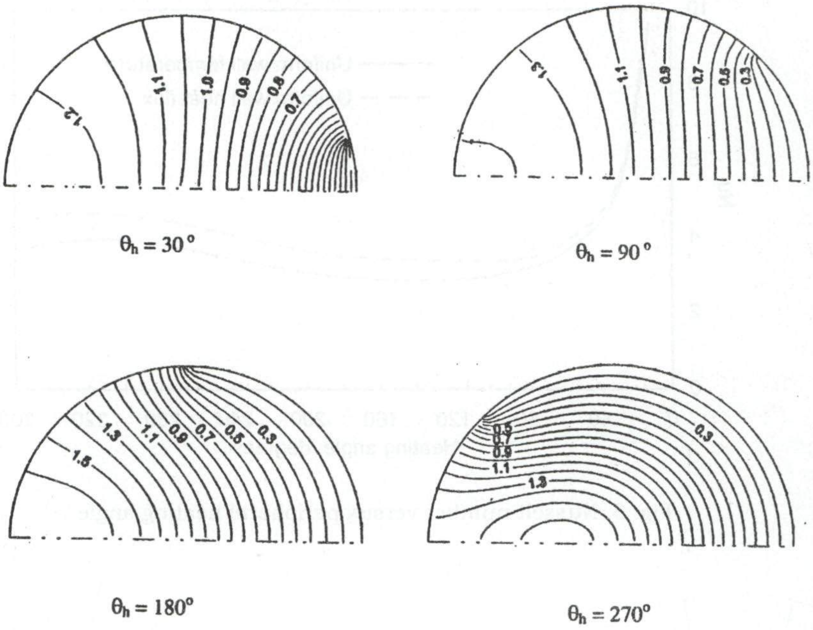

Fig. 2. Isothermal lines, $(T - T_w)/(T_b - T_w)$, for partially heated tube with uniform wall-temperature.

tube wall of angle ranging from 12 to 360° are shown in fig. 3. As seen from the figure, the Nusselt number is higher for small heating peripheral angles and decreases generally as the heating peripheral angle increases. The Nusselt number reaches the well-known value, Nu_T = 3.6579, at heating angle of θ_h = 360°, i.e. completely heating of the tube surface.

4.2. Uniform wall heat flux along peripheral heated angle

Typical isotherms (constant dimensionless temperature, $(T - T_w)/(T_b - T_w)$, lines) over the flow domain of fully developed laminar flow inside tube partially heated by uniform heat flux are shown in fig. 4 for different heating circumference angles. In this case, it was assumed that the heated portion of tube surface had uniform temperature along the angular direction. Fig. 4 shows that part of the tube surface has uniform temperature along the heated circumference while the rest is insulated. This figure shows similar trend to the previous case. The heat is dissipated in radial and tangential directions from heated circumference. The results in fig. 4 show that the heat is dissipated faster in the tangential direction for small heating angle, θ_h, of the tube circumference. As the heating angle increases, the heat diffusion in the tangential direction decreases as the problem changes from two into one dimensional problem at θ_h = 360°.

In this case, Nusselt number was calculated from eq. (20). Nusselt number data obtained for different heating angles in the range from 12 to 360° are also shown in fig. 3. The Nusselt number of uniform heat flux case is higher than that of the uniform wall temperature case. Similar to the previous case, the Nusselt number is higher for small heating peripheral angles and decreases generally as the heating peripheral angle increases The Nusselt number reaches the well-known value Nu = 4.3636 at a heating angle of θ_h = 360°, i.e. completely heating surface of the tube.

5. Conclusions

Fully developed laminar convective heat transfer in a pipe partially heated was solved numerically for two different heating conditions; uniform wall-temperature and uniform wall heat flux. Effects of the angle of the heated circumference on the heat transfer

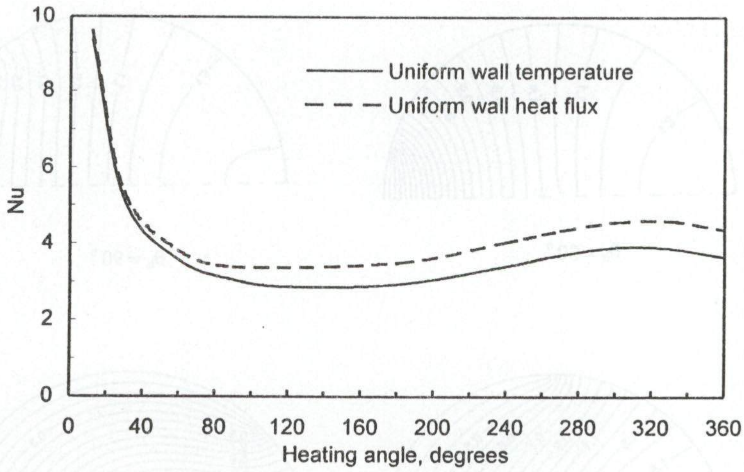

Fig. 3. Nusselt number versus peripheral heating angle.

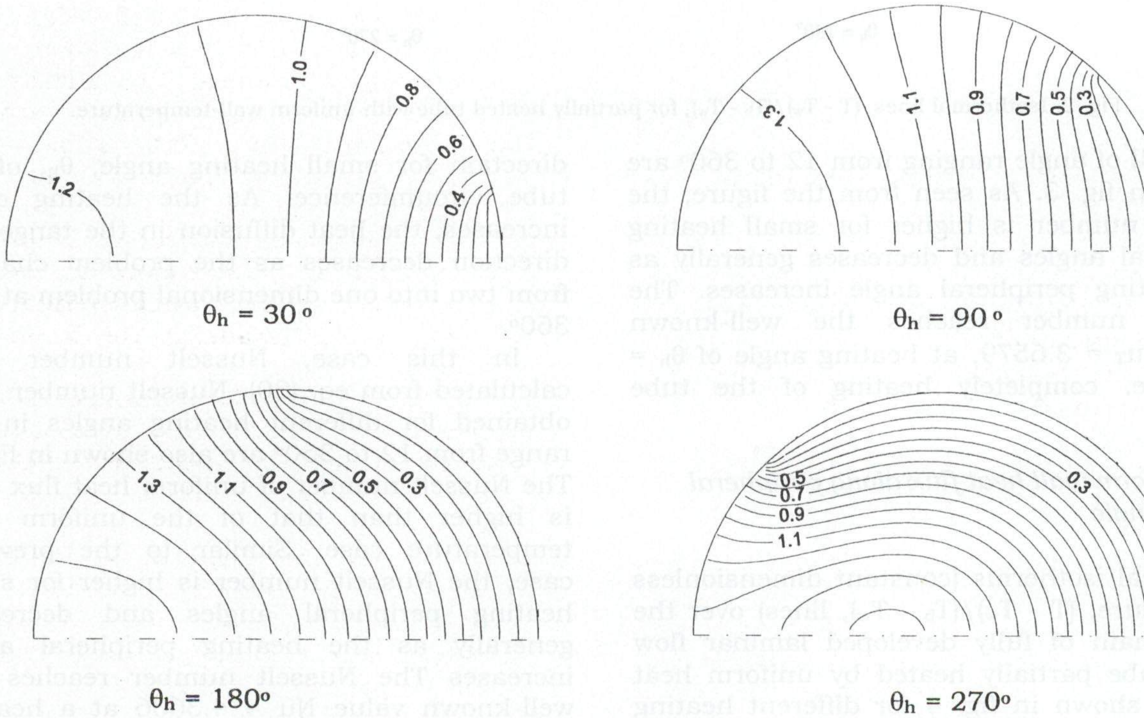

Fig. 4. Isothermal lines, $(T - T_w)/(T_b - T_w)$, for partially heated tube with uniform wall-heat flux.

process were investigated. Data for Nusselt number for various heating angle were represented versus peripheral heating angle. The data show a decrease in Nusselt number as the peripheral angle of heating increases. The Nusselt number data of uniform wall heat flux are higher than that of the uniform wall temperature heating boundary condition.

Nomenclature

C_p	specific heat at constant pressure,
h	average heat transfer coefficient along heated circumference, W/m^{2}K,
k	thermal conductivity of fluid, W/mK,
$\dot{m}$	mass flow rate, kg/s,
Nu	Nusselt number, h 2r_o/k,

Nu_H Nusselt number for uniform wall heat flux condition,
Nu_T Nusselt number for uniform wall temperature,
P pressure, Pa,
q heat flux, W/m^2,
q_w heat flux along heated portion of tube surface, W/m^2,
r radial coordinate, m,
R dimensionless radial coordinate r/ro,
Re Reynolds number $u_m 2r_o \rho/ \mu$,
r_o pipe inner radius, m,
T temperature, K,
T_b bulk temperature, K,
u axial velocity, m/s,
U dimensionless velocity,
u_m mean axial velocity, m/s, and
z axial coordinate, m.

Greek symbols

φ dimensionless temperature, $(T - T_w)/(q_w r_o/k)$,
φ_b dimensionless bulk temperature, $(T_b - T_w)/(q_w r_o/k)$,
μ dynamic viscosity, Pa s,
θ tangential coordinate ,
θ_h peripheral angle subjected to heating,
ρ density, kg/m^3 ,
ξ dimensionless temperature, ψ/Nu_H ,
ξ_b dimensionless bulk temperature, ψ_b/Nu_H,
ψ dimensionless temperature, $(T - T_w)/(T_b - T_w)$.

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