Free lateral vibrations of moderately thick trapezoidal plates

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In the present paper, the free vibration of moderately thick trapezoidal plates has been studied. The analysis based on the Mindlin shear deformation theory. The solutions are determined using the pb-2 Rayleigh-Ritz method. The transverse displacement and the rotations of the plate are approximated by Ritz functions defined as two dimensional polynomials of the trapezoidal domain variables and a basic function that satisfies the essential boundary conditions. Three different arrangements of boundary conditions; the cantilevered, the simply supported and the clamped edge conditions are considered. The effects of both the transverse shear and the rotary inertia are accounted for. Convergence of the solutions is verified by considering polynomials of several subsequent degrees till the results converge. The present results are compared with those available in the open literature. Comparisons indicate good agreement between the present results and those previously published. A set of tabulated results for a wide range of variation of both thickness to root width (H/a) and the trapezoid angle θ for each of the three different cases of boundary conditions are presented.

يختص هذا البحث بدراسة الاهتزازات الحرة للألواح ذات الشكل الشبه المنحرف و متوسطة السمك و التي لها ثلاث حالات مختلفة من طرق التثبيت و هي التثبيت التام و الارتكاز البسيط و الكابولى. و يستهل البحث بعرض موجز يقتصر على ماتم إنجازه في مجال الاهتزازات الحرة للألواح شبه المنحرفة فقط ثم بتكوين النموذج الرياضي من معادلات الحركة و طريقة الحل و هي طريقة رايلى – ريتز التي تعتمد على دالة أساسية تحقق الشروط الاطارية و كثيرة حدود ثنائية. و قد تمت دراسة تقارب الحلول و مقارنتها بما هو متاح من نتائج لباحثين سابقين حيث ظهر التقارب الكبير بين النتائج الحالية و السابقة مما يبرهن على دقة الحل. وقد خلص البحث إلى عدة نتائج هامة منها أن الترددات الطبيعية للوح تزيد بزيادة زاوية شبه المنحرف و تقل بزيادة كل من سمك اللوح و نسبة الطول للعرض.

Keywords: Free vibrations, Transverse shear, Rotary inertia, Trapezoidal plates

1. Introduction

Many aircraft wings can be modeled as either trapezoidal or quadrilateral plates. The free vibration analysis of such models is a necessary prerequisite to design them to operate under different loading conditions. Based on classical thin plate theory and several different approximate mathematical methods, there exists a reasonable amount of work related to the vibration of thin trapezoidal plates of constant thickness. In [1,2], Chopra and Durvasula have investigated the free oscillatory motion of simply supported symmetric and asymmetric trapezoidal plates by applying Galerkin`s method. Orris and Petyt [3] used the finite element method to study the free vibration of simply supported and clamped triangular and trapezoidal plates. In [4], Nagaya applied the integral equation technique to investigate the free vibration of plates of arbitrary shapes that have free and simply supported mixed boundary conditions. Results for thin trapezoidal plates were presented as numerical examples of such arbitrary plates. Srinivasan and Babu [5] used the integral equation method to study the free vibration of cantilevered quadrilateral and trapezoidal plates. In [6], Narita et al. presented the results of the experimental study of the free transverse vibration of clamped trapezoidal plates. Bert and Malik [7] applied the differential quadrature method to study the free lateral oscillations of plates of irregular domains. They presented the solutions for simply supported and clamped trapezoidal plates as worked examples.

For trapezoidal plates that have variable thickness, there is a little amount of work related to the free vibration analysis of such plates. Laura et al. [8] applied the Rayleigh-

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Ritz method to investigate the free vibration of tapered cantilevered trapezoidal plates. In [9], the author has investigated the problem of transverse vibration of plates which have span-wise quadratic thickness variation. The finite element method was applied and the results for cantilevered trapezoidal plates were presented. In [10], three different cases , which are the linear, the quadratic and the exponential thickness variations were considered. In each case, Galerkin`s method was applied to solve the problem of free vibration of clamped trapezoidal plate.

In the present study, the effects of both the transverse shear deformation and the rotary inertia on the free vibration characteristics of plates are accounted for. The Mindlin plate theory [11] is employed. The pb-2 (Two dimensional polynomial and a basic function) Rayleigh- Ritz method is applied. Three different cases of edge conditions that are the cantilevered, the simply supported and the fully clamped are considered. Convergence of the present solutions is demonstrated through using polynomials of several subsequent degrees. The results for moderately thick trapezoidal plates are not available in the open literature. So, the results for isosceles triangular Mindlin plates are obtained as special cases from trapezoidal plates and then compared with those presented by other researchers. Also, the results for thin trapezoidal plates that are obtained as special solutions from those concerning thick plates are found to be in good agreement with the previously published results. The effects of variation of the thickness to root width ratio (h/a), the trapezoid angle(θ) and the aspect ratio of the plate (γ) on the frequency coefficients are studied.

2. Mathematical formulation

A thick isotropic symmetric trapezoidal plate of uniform thickness H , length b and root width a is considered. The geometry of the plate is shown in fig. 1. Following Karunasena et al. [12], the energy functional Π for a Mindlin plate can be written in terms of the maximum strain energy U_{max} and the maximum kinetic energy T_{max} as:

$$\Pi = U_{max} - T_{max} , \tag{1}$$

where:

$$U_{max} = \frac{1}{2}\int_A \left(D \left\{ \left(\frac{\partial \theta_X}{\partial X} + \frac{\partial \theta_Y}{\partial Y} \right)^2 - 2(1-\upsilon)\left[\frac{\partial \theta_X}{\partial X}\frac{\partial \theta_Y}{\partial Y} - \frac{1}{4}\left(\frac{\partial \theta_X}{\partial Y} + \frac{\partial \theta_Y}{\partial X} \right)^2 \right] \right\} + kGH\left[\left(\theta_X + \frac{\partial W}{\partial X} \right)^2 + \left(\theta_Y + \frac{\partial W}{\partial Y} \right)^2 \right] \right) dA. \tag{2}$$

$$T_{max} = \frac{1}{2}\omega^2 \int_A \left[\rho H W^2 + \frac{1}{12}\rho H^3 \left(\theta_X^2 + \theta_Y^2 \right) \right] dA, \tag{3}$$

in which W is the transverse displacement, θ_X is the rotation about the Y- axis, θ_Y is the rotation about the X- axis,ω is the natural frequency of the plate, ν is the Poisson`s ratio, ρ is the density of the plate, k is the shear correction factor, D is the flexural rigidity of the plate [$D = E H^3/12(1 - \nu^2)$ where E is the modulus of elasticity] , G is the modulus of rigidity [$G = E / 2(1 + \nu)$] and A is the plate surface area.

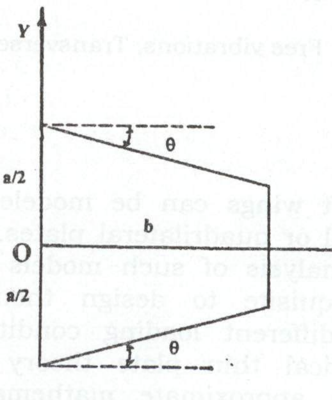

Fig.1. The geometry of the plate.

In the following formulation, the X,Y coordinates, the thickness H and the transverse displacement of the plate middle surface W are normalized by a characteristic length which is the plate root width a. ($x = X/a$, $y = Y/a$, $h = H/a$ and $w = W/a$).The lateral displacement and the rotations will be approximated by a set of pb-2 Ritz functions in the x-y plane as follows:

$$w(x,y)=\sum_{i=1}^{n}c_i\Phi_i(x,y)=\{c\}^T\{\Phi\}, \quad (4\text{-a})$$

$$\theta_x(x,y)=\sum_{i=1}^{n}d_i\Psi_{xi}(x,y)=\{d\}^T\{\Psi_x\}, \quad (4\text{-b})$$

$$\theta_y(x,y)=\sum_{i=1}^{n}e_i\Psi_{yi}(x,y)=\{e\}^T\{\Psi y\}, \quad (4\text{-c})$$

where {c}, {d}, {e} are the unknown coefficients vectors containing c_i, d_i, e_i , which are the unknown coefficients of the Ritz functions, as respective elements, {Φ}, $\{\Psi_x\}$, $\{\Psi_y\}$ are the Ritz functions vectors associated to w, θ_x, θ_y respectively, whose respective elements are Φ_i, ψ_{xi}, ψ_{yi} and T denotes the transpose of a matrix or a vector. The Ritz functions are defined over the domain of the plate by the products of basic functions, that must satisfy the geometric boundary conditions, and complete two dimensional polynomials, which will be assumed here to have the same degrees, as follows:

$$\{\Phi\}=P(x,y)\{f\} \quad , \quad (5\text{-a})$$

$$\{\Psi_x\}=Q(x,y)\{f\}, \quad (5\text{-b})$$

$$\{\Psi_y\}=R(x,y)\{f\}, \quad (5\text{-c})$$

where P(x,y), Q(x,y), R(x,y) are the basic functions that satisfy the essential boundary conditions associated to w,θ_x, θ_y, respectively, they are chosen according to the plate edge conditions as follows:

For a plate which is cantilevered along the y-axis:

P (x, y) = Q (x, y) = R (x, y) = x.

For a simply supported plate:

P (x, y) = x(x-γ)(y-cx+0.5)(y + cx-0.5),

Q (x, y) = 1,

R (x, y) = x (x-γ).

For a fully clamped plate:

P (x, y) = Q(x, y) = R(x , y)
= x(x-γ)(y − cx + 0.5)(y + cx - 0.5),

in which γ is the plate aspect ratio (γ = b/a) and c = tan θ, where θ is the trapezoid angle. The elements of the vector {f} are those of complete two dimensional polynomials of x, y that may have variable degrees. As an example, for a polynomial of degree p = 4, the components of {f} are given by:

$$\{f\}^T=\{1 \quad x \quad y \quad x^2 \quad xy \quad y^2 \quad x^3 \quad x^2y \quad xy^2 \quad y^3 \quad x^4 \quad x^3y \quad x^2y^2 \quad xy^3 \quad y^4\}.$$

For a two dimensional polynomial of degree = p , the total number of elements n of the vector {f} is given by : n = (p+1)(p+2)/2

Substituting from eq. (4) into eqs. (1-3) the energy functional Π can be written as:

$$\Pi=\frac{1}{2}\{q\}^T\left([K]-\lambda^2[M]\right)\{q\}, \quad (6)$$

in which $\lambda = \omega a^2\sqrt{\rho H/D}$ is the normalized natural frequency coefficient and

$$\{q\}^T=\langle \{c\}^T \ \{d\}^T \ \{e\}^T \rangle, \quad (7)$$

$$[K]=\begin{bmatrix} K_{cc} & K_{cd} & K_{ce} \\ & K_{dd} & K_{de} \\ \text{symmetric} & & K_{ee} \end{bmatrix}, \quad (8)$$

$$[M]=\begin{bmatrix} M_{cc} & M_{cd} & M_{ce} \\ & M_{dd} & M_{de} \\ \text{symmetric} & & M_{ee} \end{bmatrix}, \quad (9)$$

where the sub matrices inside [K] and [M] are defined by:

$$K_{cc}=\frac{6k(1-\upsilon)}{h^2}\int_A\left(\frac{\partial\Phi}{\partial x}\frac{\partial\Phi}{\partial x}^T+\frac{\partial\Phi}{\partial y}\frac{\partial\Phi}{\partial y}^T\right)d\bar{A}, \quad (10\text{-a})$$

$$K_{cd}=\frac{6k(1-\upsilon)}{h^2}\int_A\frac{\partial\Phi}{\partial x}\Psi_x^T d\bar{A} \quad , \quad (10\text{-b})$$

$$K_{ce}=\frac{6k(1-\upsilon)}{h^2}\int_A\frac{\partial\Phi}{\partial y}\Psi_y^T d\bar{A} \ , \quad (10\text{-c})$$

$$K_{dd} = \int_A \left[\frac{\partial \Psi_x}{\partial x} \frac{\partial \Psi_x^T}{\partial x} + \frac{(1-\upsilon)}{2} \frac{\partial \Psi_x}{\partial y} \frac{\partial \Psi_x^T}{\partial Y} + \frac{6k(1-\upsilon)}{h^2} \Psi_x \Psi_x^T \right] d\bar{A}, \quad (10\text{-d})$$

$$K_{de} = \int_A \left[\upsilon \frac{\partial \Psi_x}{\partial x} \frac{\partial \Psi_y^T}{\partial y} + \frac{(1-\upsilon)}{2} \frac{\partial \Psi_x}{\partial y} \frac{\partial \Psi_y^T}{\partial x} \right] d\bar{A}, \quad (10\text{-e})$$

$$K_{ee} = \int_A \left[\frac{\partial \Psi_y}{\partial y} \frac{\partial \Psi_y^T}{\partial y} + \frac{(1-\upsilon)}{2} \frac{\partial \Psi_y}{\partial x} \frac{\partial \Psi_y^T}{\partial x} + \frac{6k(1-\upsilon)}{h^2} \Psi_y \Psi_y^T \right] d\bar{A}, \quad (10\text{-f})$$

$$M_{cc} = \rho h \int_A \Phi \Phi^T d\bar{A}, \quad (11\text{-a})$$

$$M_{cd} = 0, \quad (11\text{-b})$$

$$M_{ce} = 0, \quad (11\text{-c})$$

$$M_{dd} = \frac{1}{12} \rho h^3 \int_A \Psi_x \Psi_x^T d\bar{A}, \quad (11\text{-d})$$

$$M_{de} = 0, \quad (11\text{-e})$$

$$M_{ee} = \frac{1}{12} \int_A \Psi_y \Psi_y^T d\bar{A}, \quad (11\text{-f})$$

where $\bar{A}$ is the nondimensional area ($d\bar{A}$ = dxdy).

Substituting from eq. (5) into eqs. (10, 11), for each of the three different cases of boundary conditions and carrying out the associated integration over the domain of the trapezoidal plate, the elements of both the overall stiffness and the mass matrices are evaluated. Setting the first variation of the energy functional in eq. (6) to zero results in the following eigenvalue problem.

$$\left([K] - \lambda^2 [M] \right) \{q\} = \{0\} \quad . \quad (12)$$

It must be mentioned that each of the overall stiffness and mass matrices in eq. (12) are symmetric and of order 3n × 3n. Using EISPACK routines [13], eqn. (12) is solved for the natural frequency coefficient λ for the considered plates.

3. Numerical work and discussion

Three different cases of boundary conditions, which are, the cantilevered, the simply supported and the fully clamped plates, will be considered. In each case, the convergence of the solutions is demonstrated through using polynomials of successive different degrees till the convergence is achieved. The accuracy of results is checked by comparisons with the previously published solutions which are available for both the isosceles triangular Mindlin plates and the thin trapezoidal plates. Finally, a series of tabulated results , for thick trapezoidal plates ,are given. To execute correct comparisons, the Poisson`s ratio is taken to be 0.3 and the shear correction factor is considered to be k = 5/6 = 0.833 in all the following calculations, which are the same values that were considered in [12, 14, 15].

In table 1, the first four natural frequency coefficients for both thin square plates (H/a = 0.001, θ = 0, γ =1) and moderately thick trapezoidal plates (H/a = 0.1, θ = 10, γ = 1) are presented. For each case of boundary conditions, the results that correspond to four successive values of polynomial degrees are computed. From the expressions of the basic functions which satisfy the essential boundary conditions, one can expect that, for simply supported and clamped plates, two dimensional polynomials of degrees (p) whose values are less than those considered for the cantilevered plates will result in efficiently accurate solutions. So, for cantilevered plates, the results that correspond to values of p , starting from p = 7 up to p = 10 are given, while, for both simply supported and clamped plates, the values of p are taken to be in the range from p=6 up to p = 9. As shown in table 1, the results for both thin square plates and thick trapezoidal plates are monotonically converged to their accurate values as the degrees of the polynomials increase.

The present results for thin square plates are found to be in good agreement with those obtained by thin plate theory solutions. For moderately thick trapezoidal plates, the

Table 1
Convergence of results for square and trapezoidal plates

Boundary condition	H/a	θ	P	λ_1	λ_2	λ_3	λ_4
CFFF*	0.001	0	7	3.481	8.522	21.317	27.213
			8	3.477	8.519	21.311	27.202
			9	3.475	8.517	21.305	27.201
			10	3.474	8.514	21.301	27.200
			Ref.[14]	3.475	8.513	21.301	27.205
	0.1	10	7	3.862	11.454	20.990	33.968
			8	3.861	11.449	20.985	33.886
			9	3.861	11.443	20.981	33.877
			10	3.861	11.443	20.979	33.861
SSSS+	0.001	0	6	19.745	49.510	49.512	92.554
			7	19.739	49.509	49.511	79.398
			8	19.739	49.348	49.348	79.398
			9	19.739	49.348	49.348	79.317
			Exact	19.739	49.374	49.374	78.957
	0.1	10	6	23.125	49.501	59.728	88.023
			7	23.109	49.217	59.638	85.672
			8	23.065	49.081	59.614	85.170
			9	23.063	49.074	59.602	85.097
CCCC++	0.001	0	6	35.999	74.286	74.286	108.587
			7	35.999	73.431	73.431	108.586
			8	35.988	73.431	73.431	108.262
			9	35.988	73.412	73.412	108.262
			Exact	35.99	73.80	73.80	108.27
	0.1	10	6	40.202	68.093	80.853	106.879
			7	40.199	68.073	80.845	106.720
			8	40.198	68.066	80.841	106.680
			9	40.197	68.064	80.840	106.674

* One edge is clamped and the other three ones are free.
\+ The four edges are simply supported.
++ The four edges are clamped.

present results are new in literature, but the good convergence of such results demonstrates their accuracy.

For certain values of the trapezoid angle θ and its aspect ratio γ, the symmetric trapezoidal shape becomes an isosceles triangular one. To demonstrate the accuracy of the present solutions for thick plates, the problem of the isosceles triangular Mindlin plates is solved. In table 2, the first four natural frequency coefficients for isosceles triangular plates ($\theta = 30°$, $\gamma = \sqrt{3}/2$) are determined. The present results are compared with those available in [14, 15]. As can be shown, they agree well with both of the two sets of the available results and the maximum percentage difference between the three sets of different solutions does not exceed 0.5 %.

In table 3, the results for both thin and moderately thick cantilevered trapezoidal plates, which correspond to three different values of aspect ratios are given. For each aspect ratio, three different values of H/a are considered and for each value of H/a, the solutions are determined for three different values of θ. The present results, for thin plate ($H/a = 0.001$, $\gamma = 1$) are found to be in good agreement with those obtained in [16] by thin plate theory solution. The variation of the natural frequency coefficients λ against the three varying parameters γ, H/a, θ is found to be as follows: For certain values of γ and H/a, the increase of θ leads to a corresponding increases of λ. The increase of the value of H/a tends to decrease the value of λ, while the values of both θ and γ remain stationary. The increase of γ, for stationary values of both θ and H/a tends to the decrease of λ. Such behavior of λ against the three varying parameters may be explained as follows: For a single degree of freedom vibrating system, the natural frequency is given by $\omega^2 = K/M$, where K is the stiffness and M is the mass. For the same value of K, ω^2 is inversely proportional to M. For a freely vibrating plate, the stiffness is mainly dependant on its boundary conditions , while the mass of the plate is depending on the plate thickness and the plate surface area. As θ increases, the area, and hence, the mass of the plate decreases. The increase of both γ and H leads to an increase of the mass of the plate. In tables 4, 5, the results for simply supported and clamped Mindlin trapezoidal plates are presented. The variation of the natural frequency coefficient λ against each of the three varying parameters is found to be nearly the same as that of the case of the cantilevered plates.

Table 2
Comparison of results for isosceles triangular Mindlin plates

Boundary condition	H/a	Ref.	λ_1	λ_2	λ_3	λ_4
CFF	0.001	Present	8.920	35.089	38.482	89.603
		[15]	8.922	35.092	38.485	89.598
		[12]	8.922	35.086	38.482	89.606
	0.1	Present	8.647	31.435	34.836	75.522
		[15]	8.646	31.410	34.809	75.398
		[12]	8.646	31.435	34.839	75.522
	0.15	Present	8.414	28.445	31.877	65.116
		[15]	8.413	28.457	31.831	64.987
SSS	0.001	Present	52.637	123.081	123.081	210.941
		[15]	52.634	122.836	122.836	210.550
	0.15	Present	42.332	85.873	85.873	128.943
		[15]	42.311	85.326	85.326	128.365
CCC	0.001	Present	99.031	189.025	189.025	295.403
		[15]	99.023	188.998	188.998	295.247
	0.15	Present	64.696	104.841	104.841	145.984
		[15]	64.591	104.741	104.741	145.330

Table 3
Results for cantilevered trapezoidal Mindlin plates

γ	H/a	θ	λ_1	λ_2	λ_3	λ_4
1	0.001	5	3.663	10.072	21.770	33.823
		10	3.909	12.207	22.216	37.641
		15	4.261	15.297	22.790	43.255
	Ref.[9]	5	3.663	10.070	21.768	33.813
		10	3.910	12.207	22.217	37.640
		15	4.262	15.300	22.793	43.266
	0.2	5	3.520	8.575	17.978	25.759
		10	3.750	10.211	18.352	28.173
		15	4.071	12.486	18.794	31.547
0.5	0.001	5	14.314	23.160	45.367	85.278
		10	14.672	25.161	50.852	89.143
		15	15.075	27.551	57.475	90.277
	0.1	5	13.799	21.177	39.919	71.757
		10	14.136	22.899	44.530	73.455
		15	14.512	24.943	50.062	74.304
	0.2	5	12.638	18.145	32.507	51.976
		10	12.929	19.475	35.913	52.542
		15	13.249	21.032	39.421	53.173
2.0	0.001	0	0.8607	3.703	5.363	12.053
		5	0.9740	5.2952	5.569	14.958
		10	1.197	5.970	8.377	15.624
	0.1	0	0.8558	3.548	5.274	11.444
		5	0.9688	5.027	5.476	14.045
		10	1.190	5.861	7.841	15.042
	0.2	0	0.8480	3.318	5.058	10.455
		5	0.9596	4.620	5.252	12.605
		10	1.177	5.614	6.980	13.761

Table 4
Results for simply supported trapezoidal Mindlin plates

γ	H/a	θ	λ_1	λ_2	λ_3	λ_4
1.0	0.001	5	21.777	51.445	57.208	87.456
		10	24.684	54.761	67.093	100.96
		15	28.966	60.836	78.677	110.82
	0.1	5	20.480	46.561	51.503	74.994
		10	23.060	49.073	59.595	85.083
		15	26.860	53.662	68.801	92.159
	0.2	5	18.293	38.587	42.168	58.030
		10	20.389	40.322	47.851	64.558
		15	23.441	43.440	54.084	69.270
0.5	0.001	5	50.304	82.755	136.77	168.79
		10	51.503	87.438	146.78	170.31
		15	53.064	93.377	158.07	173.33
	0.1	5	45.651	71.409	111.04	133.82
		10	46.567	74.907	117.88	134.67
		15	47.720	79.296	125.58	136.33
	0.2	5	37.975	55.698	80.862	94.798
		10	38.612	58.010	84.978	95.291
		15	39.421	60.882	89.562	96.240
2.0	0.001	0	12.337	19.739	32.082	41.946
		5	16.643	25.100	37.509	55.037
		10	22.098	36.865	54.950	66.811
	0.1	0	11.902	18.690	29.886	38.926
		5	15.949	23.480	34.453	49.676
		10	20.939	33.977	48.463	59.433
	0.2	0	11.083	16.812	25.946	33.167
		5	14.612	20.744	29.341	40.583
		10	18.783	29.050	39.615	47.767

Table 5
Results for clamped trapezoidal Mindlin plates

γ	H/a	θ	λ_1	λ_2	λ_3	λ_4
1.0	0.001	5	39.813	76.539	85.303	120.05
		10	45.555	81.829	100.52	139.08
		15	54.241	92.378	118.45	148.87
	0.1	5	35.643	64.316	70.517	94.686
		10	40.198	68.066	80.841	106.68
		15	46.820	75.010	92.369	113.04
	0.2	5	28.669	47.754	51.347	66.614
		10	31.674	50.072	57.313	73.219
		15	35.853	54.169	63.851	77.150
0.5	0.001	5	99.416	131.96	189.72	257.07
		10	100.85	137.86	202.73	258.50
		15	102.80	145.69	219.02	261.05
	0.1	5	77.734	99.837	135.95	166.57
		10	78.737	103.61	143.18	167.38
		15	80.086	108.45	151.75	168.70
	0.2	5	53.744	67.981	89.514	102.02
		10	54.421	70.190	93.424	102.55
		15	55.312	72.940	97.841	103.49
2.0	0.001	0	24.579	31.829	44.820	63.986
		5	33.380	42.763	46.846	75.667
		10	43.068	62.587	83.135	100.41
	0.1	0	22.971	29.132	40.182	54.982
		5	30.228	38.236	48.315	62.233
		10	38.083	53.245	69.030	80.644
	0.2	0	19.240	24.205	32.473	41.499
		5	24.548	30.281	37.618	47.311
		10	29.956	40.023	50.126	57.106

4. Conclusions

The free lateral vibration of thin and moderately thick trapezoidal plates that have three different arrangements of boundary conditions are analyzed by applying the pb-2 Rayleigh-Ritz method. From the preceding analysis, it is possible to conclude the following remarks:

1. The convergence of the present solutions is achieved through using some successive values of the degrees of the two dimensional polynomials contained in the Ritz functions. The accuracy of the present results is demonstrated by comparisons with most of those available in the open literature.
2. The effects of variations of the plate aspect ratio, the trapezoid angle and the thickness to root width ratio on the frequency coefficients has been investigated. The results indicate that, the frequency coefficients increase as the trapezoid angle increases while they decrease against the increase of both the thickness and the aspect ratio.

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