A Theoretical study for seepage through an earth dam founded on an inclined impervious base

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A theoretical study for seepage flow through an earth dam, with a cut off wall based on an inclined impervious base, is prepared in this work. Four equations are deduced represent drops in free water surface and seepage discharge. Effect of penetration depth of cut off wall and its distance, measured from the heel of the dam on both seepage discharge and loss of head due to cut off wall, are studied. A comparison between the theoretical work and similar experimental one is also done.

يهدف هذا البحث لدراسة تسرب المياه خلال جسم سد ترابى يحتوى بداخله على ستارة رأسية غير منفذة. ويرتكز جسم هذا السد على طبقة مائلة غير منفذة. فى هذا البحث تم استنتاج أربعة معادلات نظرية لايجاد الهبوط الحادث فى شكل سطح المياه الحر داخل جسم السد و كذلك ايجاد التصرف المتسرب داخله. كذلك تم أيضا فى هذا البحث دراسة تأثير العمق المتغلغل للستارة الرأسية الغير منفذة وكذلك دراسة تأثير المسافة بين موقع هذه الستارة الرأسية و حافة السد فى الأمام على كل من التصرف المتسرب داخل جسم السد و على الفواقد فى ضاغط المياه نتيجة وجود هذه الستارة الرأسية الغير منفذة. كذلك تم عمل مقارنة نتائج الدراسة النظرية التى تم التوصل اليها فى هذا البحث مع نتائج دراسة معملية مماثلة.

Keywords: Seepage, Cut off wall, Earth dam

1. Introduction

The problem of seepage through an earth dam, with cut off wall based on an impervious inclined base, was studied experimentally by Abd El-Razek and Rabiea [1]. Different theoretical approaches for earth dams with horizontal impervious base were treated by Rozanov [2]. An approximate solution for seepage through earth dam founded on layer of finite depth with cut off wall was first given by Polubarinova-Kochina [3] based on the solution given earlier by Voshchinin [4]. The same problem was also studied by Mkhitarian [5] and Nelson-Shornyakov [6]. Shornyakov [6] gave an approximate solution for the seepage characteristics through a homogeneous earth dam. Also, he solved the problem of earth structure with cutoff wall at toe mathematically.

The present work aims to find a theoretical solution for an earth dam with a cut off wall based on an inclined impervious base. Four equations are established to solve this problem, and to find effect of penetration depth of cut off wall and its distance on both head loss due to cut off wall and seepage discharge. A Comparison between results of the recommended theoretical solution and these given by the experiments given by Abd El-Razek and Rabiea [1] is studied. The experimental study given by Abd El-Razek and Rabiea was carried out on a Hele-shaw model representing an earth dam with cutoff wall based on inclined impervious base.

2. Theoretical study

A theoretical study is prepared, in case of earth dam with cut off wall based on an inclined impervious base, fig. 1, to calculate the seepage discharge through the dam and to find the loss of head due to constructing cut off wall. The solution mainly based on dividing the dam body into four zones, each one is treated separately. Four equations are deduced for the above-mentioned four zones. Drops in free water surface and seepage discharge are the parameters of these equations.

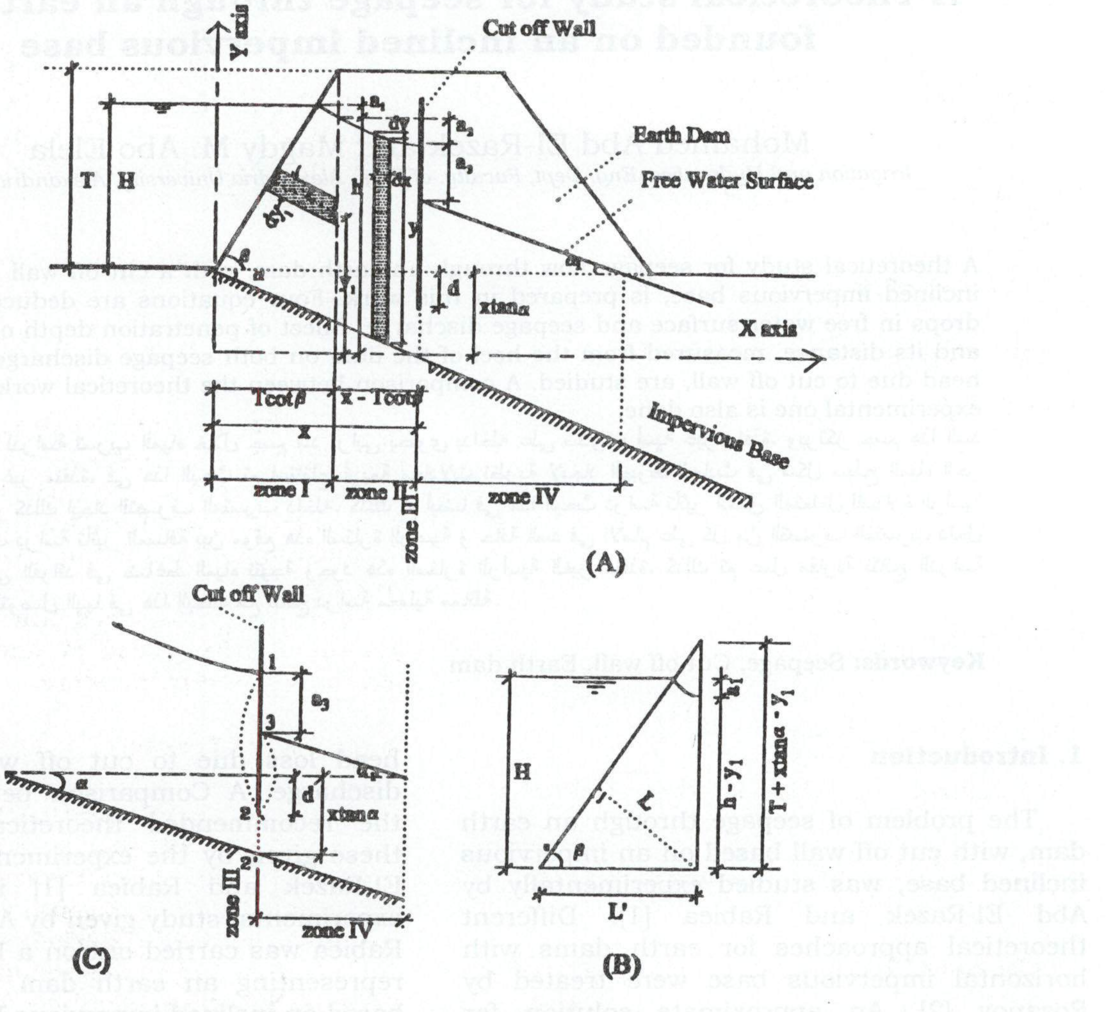

Fig. 1. Cross section of the earth dam under study.

2.1. Solution of zone I

Applying Darcy's law at zone I, (see fig. 1-b):

$$q = -k \cdot i \cdot A,$$

$$dq = -k \cdot \frac{a_1}{L} \cdot dy_1,$$

$$q = -k \cdot a_1 \int_{\tan\alpha(x - T\cot\beta)}^{h} \frac{dy_1}{\cos\beta(T + x \cdot \tan\alpha - y_1)},$$

$$q = -k \cdot a_1 \times \int_{\tan\alpha(x - T\cot\beta)}^{h} \frac{dy_1}{\cos\beta(T + x \cdot \tan\alpha) - \cos\beta \cdot y_1},$$

$$q = -k \cdot a_1 \times \left[\frac{-1}{\cos\beta} \cdot \ln[\cos\beta(T + x \cdot \tan\alpha) - \cos\beta \cdot y_1] \right]_{\tan\alpha(x - T\cot\beta)}^{h},$$

$$q = -k \cdot a_1 \times \left[\frac{-1}{\cos\beta} \cdot \ln[\cos\beta(T + x \cdot \tan\alpha) - h \cdot \cos\beta] + \frac{1}{\cos\beta} \cdot \ln[\cos\beta(T + x \cdot \tan\alpha) - \cos\beta \cdot \tan\alpha(x - T \cdot \cot\beta)] \right]$$

$$q = \frac{-k \cdot a_1}{\cos\beta} \times \left[\ln[\cos\beta(T + x \cdot \tan\alpha) - \cos\beta \cdot \tan\alpha(x - T \cdot \cot\beta)] - \ln[\cos\beta(T + x \cdot \tan\alpha) - h \cdot \cos\beta] \right],$$

rearranging:

$$q = \frac{-k \cdot a_1}{\cos\beta}\left[\ln\frac{T(1+\tan\alpha\cdot\cot\beta)}{(T-H)+a_1}\right]. \qquad (1)$$

The above equation eq. (1) is a function of a_1, i.e. $q = f(a_1)$.

2.2. Solution of zone II

Applying Darcy's law at zone II:

$$q = -k\cdot i\cdot A = -k\cdot\frac{dy}{dx}(y\cdot 1),$$

$$q\cdot dx = -k\cdot y\cdot dy,$$

$$q\int_{T\cot\beta}^{x} dx = -k\int_{h}^{h-a_2} y\cdot dy,$$

$$q = \frac{-k}{(x-T\cdot\cot\beta)}\left[\frac{y^2}{2}\right]_h^{h-a_2},$$

$$= \frac{-k}{2(x-T\cdot\cot\beta)}\left[(h-a_2)^2-h^2\right]$$

rearranging and putting $h = H + x\cdot\tan\alpha - a_1$

$$q = \frac{k\cdot a_2}{2(x-T\cdot\cot\beta)}\left[2H+2x\cdot\tan\alpha-2a_1-a_2\right]. \qquad (2)$$

The above eq. (2) is a function of a_1 & a_2, i.e. $q = f(a_1, a_2)$.

2.3. Solution of zone III

Applying Darcy's law at zone III:

$$q = -k\cdot i\cdot A = -k\cdot\frac{a_3}{l'}(x\cdot\tan\alpha-d),$$

where l' is the path length of the stream line = path length 123, (see fig. 1-c),

$$l' = 2H - 2a_1 - 2a_2 - a_3 + 2d.$$

Substituting l' in the formula of q,

$$q = \frac{-k\cdot a_3(x\cdot\tan\alpha-d)}{(2H-2a_1-2a_2-a_3+2d)}. \qquad (3)$$

The above eq. (3) is a function of a_1, a_2 and a_3, i.e. $q = f(a_1, a_2, a_3)$.

2.4. Solution of zone IV

In this zone, the seepage line is assumed to be linear and its slope is α_1 based on the experimental results, which was done by Mohamed Abd El-Razek and Rabiea Nasr [1]. Applying Darcy's law:

$$q = -k\cdot i\cdot A = -k\cdot\tan\alpha\cdot\tan\alpha_1 x \left[x+\frac{H-a_1-a_2-a_3}{\tan\alpha_1}\right]. \qquad (4)$$

The above eq. (4) is a function of a_1, a_2 and a_3, i.e. $q = f(a_1, a_2, a_3)$.

It is assumed that the slope of seepage Line D.S. the cutoff wall (α_1) equals the angle of inclination of the impervious base of the dam (α).

Eqs. (1-4), which are functions in a_1, a_2, a_3, and q, can be solved to find the mentioned four unknowns parameters.

3. Analysis of results

The problem of seepage through an earth dam, with cut off wall based on an inclined impervious base, is mathematically treated in this paper. Four equations are established to find the total head loss due to cut off wall and the seepage discharge through the dam.

Figs. 2 to 4 are plotted to show the relation between theoretical head loss due to cut off (a_3/x) versus the retained water head $H/(x\cdot\tan\alpha-d)$ for $\alpha = 10°$, $20°$ & $30°$ respectivel y. It is noticed that, the relations are linear and the value of (a_3/x) increases with increasing $H/(x\cdot\tan\alpha-d)$. This means that increasing the penetration depth of cut off wall (d) decreases the path length of discharge which known by the term $(x\cdot\tan\alpha-d)$, consequently the drop (a_3) due to cut off wall increases. The figures are put in the form of design charts for different values of (d/x).

Fig. 5 shows the relation between (a_3/x) and $H/(x\cdot\tan\alpha-d)$ for the experimental data given by Abd El-Razek and Rabiea [1]. It is noticed that, the relations shown in figs. 2-4 have the same trend of the experimental one, which indicate the validity of the theoretical solution.

A comparison between theoretical solution given in this work and the experimental one given by Abd El-Razek and Rabiea [1] for the head loss due to cut off wall (a_3/x) for $\alpha = 10°$, is shown in by fig. 6 and recorded in table 1. It is also noticed that, for $H/(x \cdot \tan\alpha - d)$ equal one, the value of (a_3/x) is not changed as (d/x) increases for both theoretical and experimental results, as shown in fig. 6 and table 1.

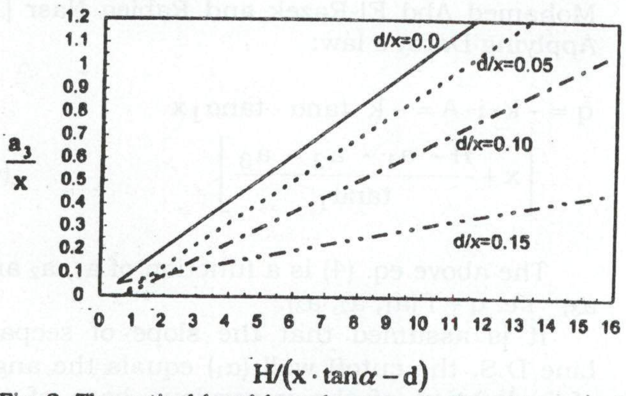

Fig. 2. Theoretical head loss due to cut off wall (a_3/x) versus $H/(x \cdot \tan\alpha - d)$ for α = 10°.

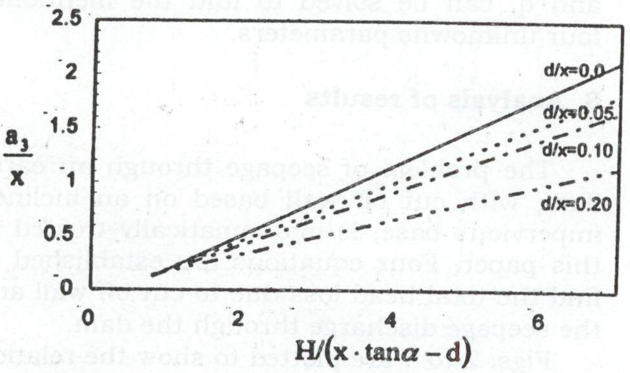

Fig. 3. Theoretical head loss due to cut off wall (a_3/x) versus $H/(x \cdot \tan\alpha - d)$ for α = 20°.

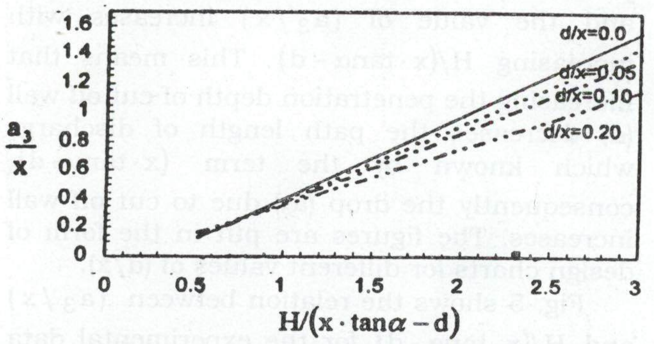

Fig. 4. Theoretical head loss due to cut off wall (a_3/x) versus $H/(x \cdot \tan\alpha - d)$ for α = 30°.

From the tabulated results, it is clear that for $H/(x \cdot \tan\alpha - d) > 1.0$, the theoretical values of (a_3/x) are bigger than the experimental values given by Abd El-Razek and Rabiea [1] for d/x = 0.0, 0.10 and 0.15. Also, it is clear that the difference between the theoretical and experimental values of (a_3/x) reduces with increasing the value of d/x. The average percentage difference between theoretical results and experimental one is about 50% as calculated in table 1. The difference between these results related to the assumptions which given in the theoretical solution.

Discharges in the two mentioned cases are compared according to the fig. 7 and the results are recorded in table 2. It is evident from the table that theoretical values of (q/k.x) are less than experimental values given by Abd El-Razek and Rabiea [1] for d/x = 0.0, 0.10 and 0.15. Also, it is clear that the difference between the theoretical and experimental values of (q/k.x) reduces with increasing the value of d/x. The average percentage difference between theoretical results and experimental one is about 75%.

A sample of the free water surface through earth dam with cut off wall is presented in figs. 8 to 11 for both theoretical solution and experimental one given by Abd El-Razek and Rabiea [1] for different values of H/T & $d/(x \cdot \tan\alpha)$.

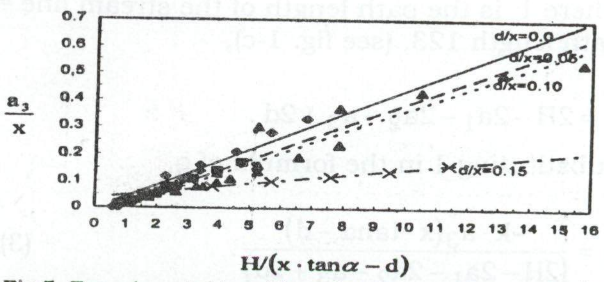

Fig.5. Experimental head loss due to cut off wall (a_3/X) versus $H/(x \cdot \tan\alpha - d)$ for α = 10°.

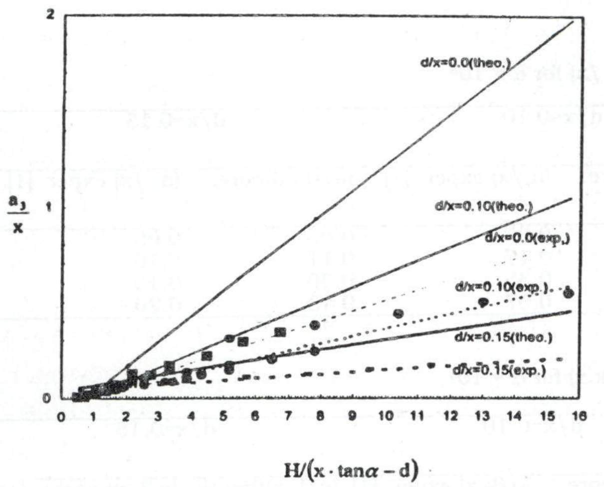

Fig.6. Comparison between theoretical and experimental results of (a_3 / X) for α = 10°.

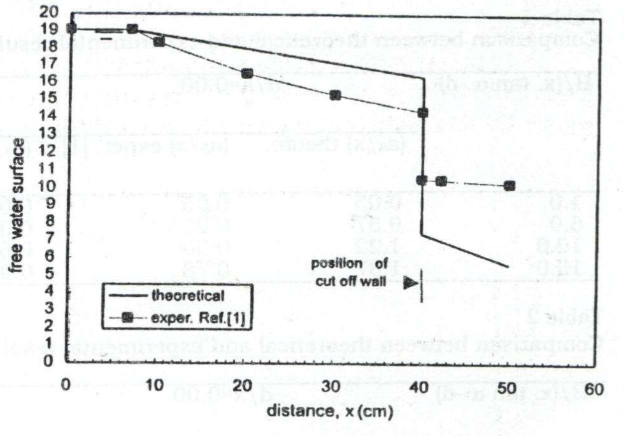

Fig. 9. Theoretical and experimental free water surface for α = 10° (H/T = 0.92 & $d/(x \cdot \tan\alpha)$ = 0.57).

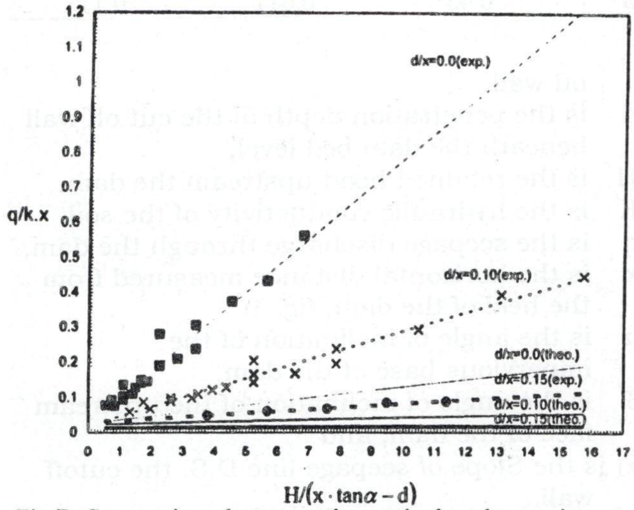

Fig.7. Comparison between theoretical and experimental results of (q/k.x) for α = 10°.

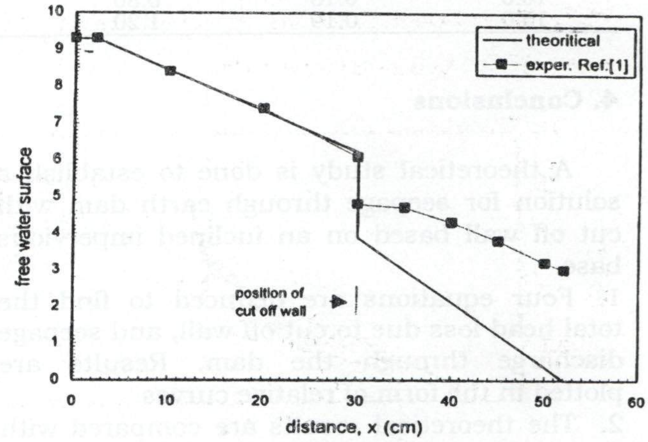

Fig. 10. Theoretical and experimental free water surface for α = 10° (H/T = 0.92 & $d/(x \cdot \tan\alpha)$ = 0.0).

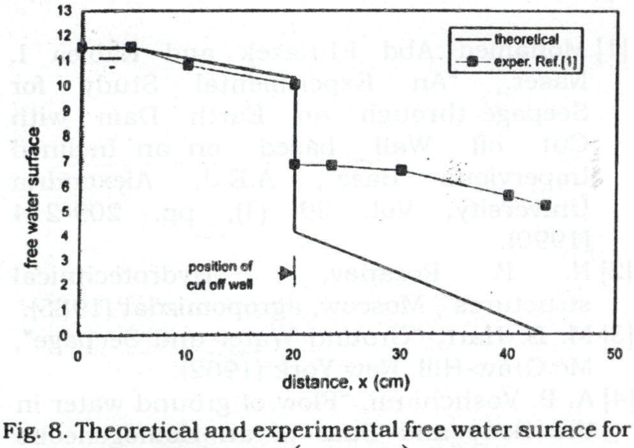

Fig. 8. Theoretical and experimental free water surface for α = 10° (H/T = 0.62 & $d/(x \cdot \tan\alpha)$ = 0.57).

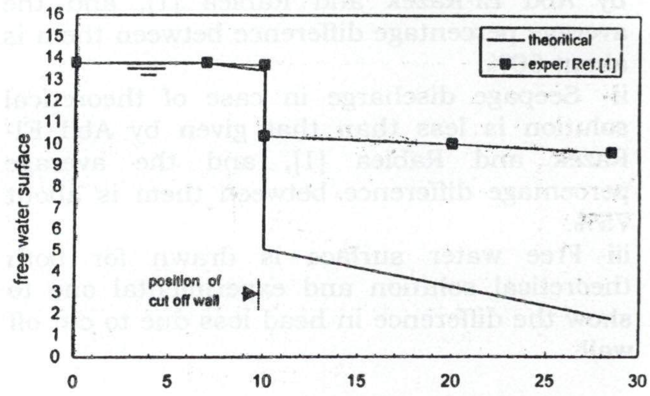

Fig. 11. Theoretical and experimental free water surface for α = 10° (H/T = 0.31 & $d/(x \cdot \tan\alpha)$ = 0.57).

Table 1
Comparison between theoretical and experimental results of (a_3/x) for $\alpha = 10°$

H/(x. tan α -d)	d/x=0.00		d/x=0.10		d/x=0.15	
	(a_3/x) theore.	(a_3/x) exper. [1]	(a_3/x) theore.	(a_3/x) exper. [1]	(a_3/x) theore.	(a_3/x) exper. [1]
1.0	0.05	0.05	0.05	0.05	0.05	0.05
5.0	0.57	0.23	0.30	0.17	0.17	0.10
10.0	1.22	0.50	0.65	0.35	0.30	0.15
15.0	1.87	0.78	0.98	0.57	0.43	0.20

Table 2
Comparison between theoretical and experimental results of $(q/k.x)$ for $\alpha = 10°$

H/(x. tan α -d)	d/x=0.00		d/x=0.10		d/x=0.15	
	$(q/k.x)$ theore.	$(q/k.x)$ exper.[1]	$(q/k.x)$ theore.	$(q/k.x)$ exper. [1]	$(q/k.x)$theore.	$(q/k.x)$ exper. [1]
1.0	0.03	0.10	0.022	0.04	0.017	0.03
5.0	0.08	0.41	0.033	0.16	0.017	0.06
10.0	0.13	0.80	0.044	0.30	0.017	0.09
15.0	0.19	1.20	0.055	0.43	0.017	0.11

4. Conclusions

A theoretical study is done to establish a solution for seepage through earth dam with cut off wall based on an inclined impervious base.

1. Four equations are deduced to find the total head loss due to cut off wall, and seepage discharge through the dam. Results are plotted in the form of relative curves.
2. The theoretical results are compared with the experimental one given by Abd El-Razek and Rabiea [1], and the following are found:

i- The head loss due to cut off wall in case of theoretical solution is bigger than that given by Abd El-Razek and Rabiea [1], and the average percentage difference between them is about 50%.

ii- Seepage discharge in case of theoretical solution is less than that given by Abd El-Razek and Rabiea [1], and the average percentage difference between them is about 75%.

iii- Free water surface is drawn for both theoretical solution and experimental one to show the difference in head loss due to cut off wall.

Nomenclature

a_3 is the drop in free water surface due to cut off wall,
d is the penetration depth of the cut off wall beneath the dam bed level,
H is the retained head upstream the dam,
K is the hydraulic conductivity of the soil,
q is the seepage discharge through the dam,
x is the horizontal distance measured from the heel of the dam, fig. 1,
α is the angle of inclination of the impervious base of the dam,
β is the angle of inclination of the upstream face of the dam, and
α_1 is the Slope of seepage line D.S. the cutoff wall.

References

[1] Mohamed Abd El-Razek and Rabiea I. Naser., "An Experimental Study for Seepage through an Earth Dam with Cut off Wall based on an Inclined Impervious Base", A.E.J, Alexandria University, Vol. 29 (4), pp. 209-214 (1990).
[2] N. P. Rozanov, "Hydrotechnical structures", Moscow, agropomizdat (1985).
[3] M. E. Harr, "Ground Water and Seepage", Mc Graw-Hill, New York (1962).
[4] A. P. Voshchinin, "Flow of ground water in the Body and Base of an Homogeneous Earth Dam with an Horizontal Underdrain

with a Finite Depth of Permeable Material", DAN, Moscow, Vol. 25 (9) (1939).

[5] A. M. Mkhitarian, "Computations for the Seepage through an Earth Dam with a Sheetpile and Drain", Inzhenerisbornik, Vol. 15 (1953).

[6] F. B. Nelson-Shornyakov, "Seepage in Homogeneous Media", Gosudarctvennoe Izd. Sovetskaya Nauka, Moscow (1949).

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