Tensioned flexible membrane breakwater for obliquely incident waves

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This paper introduces an analytical method to study the effect of a tensioned vertical flexible membrane wave-absorber. Three configurations of the membrane are considered. The membrane may be partially immersed from the water surface to a given depth, submerged from a given depth and extends to the bottom or submerged from a given depth but does not extend to the bottom. The method is based on using the shape modes of the membrane and the eigenfunction expansion of the flow potential. The model takes into account the wave direction. It was found that the performance of the membrane depends on, the tension, the wave direction, the membrane length and location.

يقدم هذا البحث طريقة تحليلية لدراسة استخدام الأغشية المرنة المشدودة كحواجز أمواج. الطريقة المقدمة صالحة للتطبيـــق علـــى أوضاع مختلفة للأغشية سواء كانت هذه الأغشية ممتدة من السطح إلى القاع أو تشغل حيز محدود من كامل الارتفـــاع. و يخلـــص البحث إلى أهمية كل من مقدار الشد، طول الغشاء بالإضافة إلى وضع الغشاء في عامود الماء على فاعلية الحـــاجز.

Keywords: Membrane breakwater, Wave absorber, Tensioned membrane, Wave transmission

1. Introduction

A variety of mathematical methods have been used to study the water wave diffraction problem by fixed vertical thin barriers within the framework of the linearized theory of water waves. Among the different methods are; the complex variable technique, the integral equations procedures based on Havelock expansion and Green integral theorem. In this paper we treat the problem of using a flexible tensioned vertical membrane as a breakwater.

Different concepts for using flexible membrane, as a wave absorber have been introduced for many coastal and ocean applications. The improvement in longevity, durability and robust behavior of the fabric materials, increased their importance as a construction material for coastal and ocean work. Some of the applications that may make use of the flexible membranes are breakwaters, military deployment, protecting coastal and offshore structures and submerged oil storage tanks.

In this paper we study the behavior of a vertical flexible membrane as a wave absorber. The tension, T, in the membrane is much larger then the dynamic tension resulting from the waves, hence the tension will be assumed constant. Six different configurations of the membrane are considered. It may be

I) extended from the water surface to the bed,
II) deployed from the water surface to a given depth,
III) submerged from a given depth z_t to depth z_b less than h, the water depth,
IV) submerged from a given depth and extend to the bottom, and finally, and
V) extending from the water surface to the bed, with one or several horizontal gaps, fig. 1.

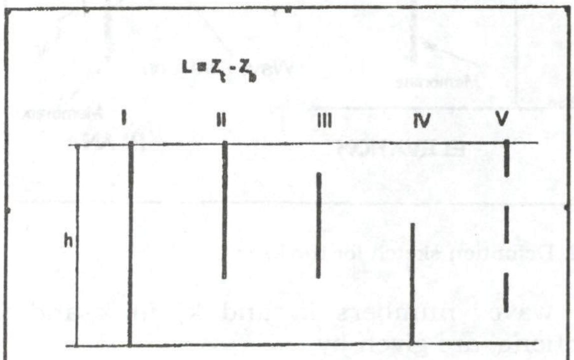

Fig. 1. Definition sketch for the different membrane configurations.

From practical point of view the last configuration is hard to construct and deploy. The first and second types are located in the region with the highest wave energy. They are expected to absorb energy more than the third and fourth configurations with the same tension and material. They may be used to trap oil spill. However, the third and fourth configuration may be preferred for aesthetic reasons. The third configuration has the advantage that it does not affect the movement of the bed load, sand transport. In all cases the mathematical formulation is the same, except for the limits of the integration, as will be seen in the paper.

2. Mathematical formulation

The wave crest makes an angle 2 with the membrane, which extends along the y-axis. The x axis is normal to the membrane and point to the east. The vertical axis z is directed upward and the origin of coordinates is located on the still water surface. The membrane has a length $l = z_t - z_b$ in the z direction, fig. 2.

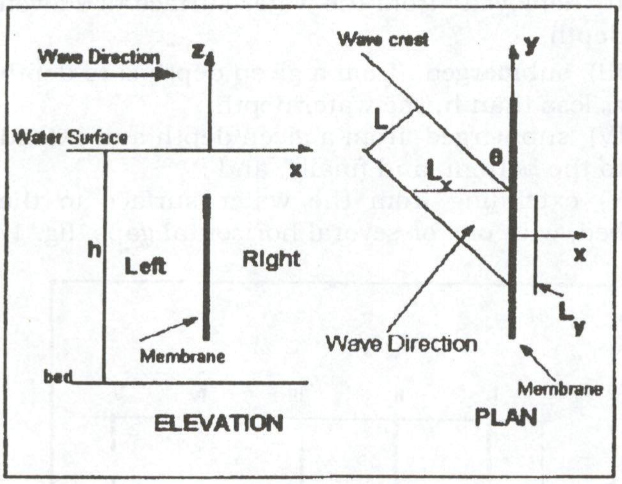

Fig. 2. Definition sketch for the axes

The wave numbers k_x and k_y in x and y directions, are given by,

$$k_x = \frac{2\pi}{L_x} = \frac{2\pi\cos\theta}{L} = k\cos\theta , \tag{1}$$

$$k_y = \frac{2\pi}{L_y} = \frac{2\pi\sin s\,\theta}{L} = k\sin\theta , \tag{2}$$

where, k and L are the wave number and length in the direction of the wave propagation. The potential N_L to the left of the membrane (x < 0) is given by Sharaki [1],

$$\Phi_L = \phi_L \exp(-i\omega t), \tag{3-a}$$

$$\phi_L = A_o f_o(z)\left|\exp(i k_x x) + R_C \exp(-i k_x x)\right| + \sum_{n=1}^{\infty} f_n(z) A_n \exp(\lambda_n x) . \tag{3-b}$$

The potential ϕ_R to the right of the membrane (x > 0) is given by,

$$\Phi_R = \phi_R \exp(-i\omega t), \tag{4-a}$$

$$\phi_R = A_o f_o(z) T_C \exp(i k_x x) + \sum_{n=1}^{\infty} f_n(z) B_n \exp(-\lambda_n x) , \tag{4-b}$$

where,

$$f_o(z) = \frac{\cosh\left[k_{o_o}(z+h)\right]\sqrt{2}}{\left[h + \frac{g\sinh^2(k_o h)}{\omega^2}\right]^{0.5}} , \tag{5-a}$$

$$f_n(z) = \frac{\cos\left[k_n(z+h)\right]\sqrt{2}}{\left[h - \frac{g\sin^2(k_n h)}{\omega^2}\right]^{0.5}} , \tag{5-b}$$

are normalized orthogonal functions, that is

$$\int_{-h}^{0} f_i(z) f_j(z)\,dz = \delta_{ij} , \tag{6}$$

where,

δ Kronecker delta function ($\delta_{ij} = 1$ i = j , $\delta_{ij} = 0$ i ≠ j),

g the gravity acceleration,

A_n the coefficients of the evansent modes to the left of the membrane n = 1,2,...,

B_n the coefficients of the evansent modes to the right of the membrane n = 1,2,...,
T_C the transmission coefficient,
R_C the reflection coefficient,
ω the radian frequency of the incident wave, and ,

$$A_o = \frac{-i\,Am_i\,g}{\omega\,f_o(0)}\,, \qquad (7)$$

is obtained by using the definition of the surface elevation and Am the amplitude of the incident wave

The quantities T, k_o and k_n are connected by the relations,

$$k_o \tanh(k_o h) = \frac{\omega^2}{g}\,, \qquad (8\text{-a})$$

$$k_n \tan(k_n h) = -\frac{\omega^2}{g} \quad (n-0.5)\pi < k_n h < n\pi\,, \qquad (8\text{-b})$$

k_x and 8_n are given by Das et al. [2],

$$k_x^2 = k_o^2 - k_y^2\,, \qquad (9\text{-a})$$

$$\lambda_n^2 = k_n^2 + k_y^2\,. \qquad (9\text{-b})$$

If we replace k_y by zero, (2 = 0) , we get the case of normal incident wave.

The horizontal velocity on both sides of the membrane (x = 0) are equal, so one must have

$$\frac{\partial \Phi_L}{\partial x} = \frac{\partial \Phi_R}{\partial x}\,. \qquad (10)$$

Using eqs. (3-b and 4-b), multiplying by f_n and integrating along the water depth, one gets,

$$A_o(1 - R_C) = A_o\,T_C\,, \qquad (11\text{-a})$$

$$A_n + B_n = 0 \qquad n = 1,2,.... \qquad (11\text{-b})$$

The equation of motion of the membrane is given by Graff [3] and Hartog [4],

$$\rho_m \frac{\partial^2 \xi}{\partial t^2} - T\frac{\partial^2 \xi}{\partial x^2} = \Delta P\,,$$

where,
T the tension
ρ_m the mass per unit length of the membrane
ξ the displacement of the membrane in x direction
ΔP the pressure difference along the two faces of the membrane.

For harmonic motion, one can write, using eqs. (3-b and 4-b)

$$(\phi_L - \phi_R) = \frac{i\,T}{\omega\,\rho_w}\left(\frac{\partial^2 \xi}{\partial x^2} + \alpha^2 \xi\right) \quad z_b \le z \le z_t\,, \qquad (12\text{-a})$$

$$\phi_L - \phi_R = 0 \text{ otherwise,} \qquad (12\text{-b})$$

where ρ_w the mass density of the water, and $\alpha = T\,(\rho_m / T)^{0.5}$.

Due to the surrounding water, Δ_m must be increased to take into account the effect of the added mass. The added mass for a rigid plate is given by, $m_a = \pi\rho_w / 4$, Sarpkaya [5]. In principle, the added mass depends on various modes and wave frequency.

The solution of eq. (12-a) may be given as the sum of the homogenous and particular solutions. The constants of the solution can be found by fulfilling the boundary conditions. To be consistent with the expressions introduced for the flow potential, a sum of the eignfunctions is used as the solution of eq. (12-a), Kim and Kee[6].

$$\xi(z,t) = \sum_{m=1}^{\infty} \frac{C_m}{\alpha^2 - \alpha_m^2} \sin\alpha_m (z + z_d)\exp(-i\omega t)\,, \qquad (13)$$

with $\alpha_m = m\pi / l$. C_m is constant for the m^{th} membrane mode.

The natural frequency, ωT_m of the m^{th} membrane mode is given by,

$$\omega_m = (m\pi / l)\,(T / \rho_m)^{0.5}\,,$$

Equating the velocity along the two sides of the membrane, one gets,

$$\frac{\partial \Phi_L}{\partial x} = \frac{\partial \Phi_R}{\partial x} = \frac{\partial \xi(z,t)}{\partial t} = -i\,\omega\,\xi\,(z,t)\,. \qquad (14)$$

Substituting the serial form for both the membrane and flow potential in eq. (14), multiplying by sin α_n $(z + z_b)$ and integrating from $z = - z_b$ to $z = - z_t$. We get M equations, where M is the number of the eigen shapes of the membrane to be used.

$$i k_x A_o (1 - R_C) I_{o\,m} + \sum_{n=1}^{N} \lambda_n A_n I_{n\,m} = \frac{-i\omega l}{2(\alpha^2 - \alpha_m^2)} C_m \quad m = 1,2,..,M , \tag{15}$$

where N is the number of the evanescent modes used to represent the flow potential , and

$$I_{n\,m} = \int_{-y_b}^{-y_t} f_n(z) \sin \alpha_m (z + z_b)\, dz \quad n = 0,1,2,.... \tag{16}$$

Working the same way with eqs. (12), but using the orthogonality conditions of f_i and integrating along the total depth h, we get ,

$$R_C = \frac{i\,T}{2\omega\rho_w} \sum_{m=1}^{M} C_m I_{o\,m} , \tag{17-a}$$

$$A_n = \frac{i\,T}{2\omega\rho_w} \sum_{m=1}^{M} C_m I_{n\,m} \quad n = 1,2, \cdots, N . \tag{17-b}$$

Eqs, (15 and 17) form a set of M+N+1 equations in N+M+1 unknowns. The unknowns are the reflection coefficient, R_C, the coefficients of the evanescent modes, (A_n n=1,2...N) of the waves and the amplitudes of the shape modes of the membrane (C_m m = 1,2...M). We can write the system of equations in a matrix form as follow,

$$[M]\{X\} = \{F\} , \tag{18}$$

where,

$$\{X\} = \{(R_c A_o)\ A_1\ A_2 \cdots A_N C_1 \cdots C_M\}^T ,$$

$$\{F\} = \{0\ 0 \cdots 0\ f_1\ f_2 \cdots f_M\}^T ,$$

$$f_j = -i\,k_x\,A_o\,I_{o\,j} ,$$

$$[M] = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} ,$$

$$[M11]_{(N+1).(N+1)} = \mathrm{diag}\,(1) ,$$

$$[M12]_{(N+1)(M)} = \frac{-i\,T}{2\omega\rho_w} \left[I_{n,m} \right] \quad 0 \le n \le N ,\ 1 \le m \le M ,$$

$$[M22]_{M\,M} = 0.5\, i\omega L\, \mathrm{diag}\, ((\alpha^2 - \alpha_n^2)^{-1}) ,$$

$$[M21]_{(M)(M)} = \left[\overline{M}_{21} \mid \overline{\overline{M}}_{21} \right] ,$$

$$\overline{M}_{21} = -i\,k_x \{I_{o1}\ I_{o2} \cdots I_{oM}\}^T , \text{ and}$$

$$\overline{\overline{M}}_{22} = [\lambda_n I_{n\,m}]_{(M)(N)} .$$

Since both M_{11} and M_{22} are diagonal matrices, then we do not need to inverse **M**. In addition taking into account the fact that, only the last M elements of **F** are non zero, one can use simple matrix algebra to find the reflection coefficient as follow. If the inverse of M is given by IM and we partite it in the same way as **M**, then,

$$IM_{12} = M_{12} M_{22}^{-1} \left[M_{12} M_{22}^{-1} M_{21} - I \right]^{-1} . \tag{19}$$

Where IM_{12} is (N+1) x M matrix, and the Reflection coefficient R_C is given by,

$$R_c = IM_{12}(1,1:M) * F_2 / A_o . \tag{20}$$

The dimensional analysis of eq. (20) shows that the reflection coefficient is directly proportional to both the membrane tension and length and inversely proportional to the wave frequency. This means that, increasing the tension and the membrane length decreases the transmission coefficient.

3. Results and discussion

In what follows, we use normalized tension T_n and normalized membrane mass per unit area ρ_n, where $T_n = T/g\,\rho_w h^2$ and $\rho_n = \rho_m / \rho_w h$.

Fig. 3 shows the transmission coefficient for a membrane deployed from the surface with length equal one half the water depth, for the case of normal incident wave. The values used for the normalized membrane mass and tension are $\rho_n = 0.01$ and T_n = .05, 0.1, 0.2 and 0.4, respectively. While fig. 4, shows the results for a rigid plate immersed to one half the water depth. The rigid membrane is simulated using very high tension, T_n = 400. Edmond [7], calculated the transmission coefficients using the same values for T_n and ρ_n. Edmond results are shown in figs. 3 and 4. From the figures, it is clear that the present method agrees well with that of Edmond. In the present method the number of evanescent modes used for the potential are 60 and the number of the mode shapes used for the membrane range between 20 and 40. Edmond [7] used 200 terms to represent the membrane and 80 terms to represent the potential. This means the proposed theory converges faster. The transmission coefficient for the membrane is less than that for the rigid barrier, for long waves (small kh). This may be explained as follows. The membrane acts as a wave maker, where the waves generated by its motion cancel or reduce the incident waves. To simulate the rigid barrier, very large tension is used.

Next we consider the case for a membrane covering the whole depth. Fig. 5 shows the transmission coefficient, given by both the present theory and that of Kim and Kee [6], for a membrane extending over the full depth. The normalized tension T_n is 0.064, 0.127, 0.255 and 0.51. The mass of the membrane is 5kg/m^2. The figure shows that, there is very close agreement between the proposed method and that of Kim and Kee [6]. Edmond used eigen function expansion, which is similar to the one introduced, While Kim and Kee used a numerical approach based on Green function.

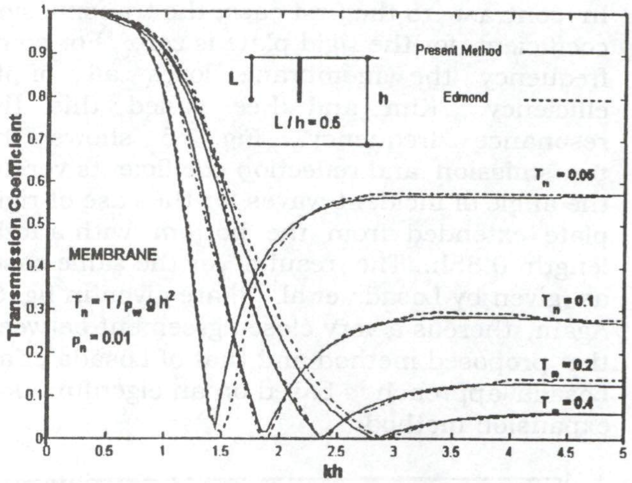

Fig. 3. Transmission coefficient for tensioned membrane.

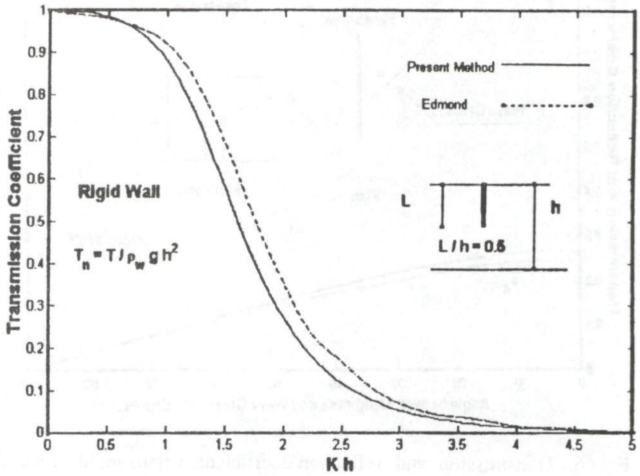

Fig. 4. Transmission coefficient for rigid plate.

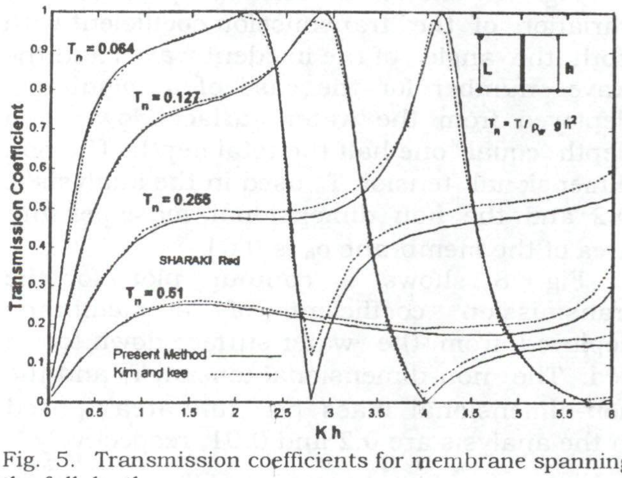

Fig. 5. Transmission coefficients for membrane spanning the full depth.

In contrast to the first case, the transmission coefficient for the rigid plate is zero. For some frequency the membrane loses all of its efficiency. Kim and Kee called this the resonance frequency. Fig. 6 shows the transmission and reflection coefficients versus the angle of incident waves for the case of rigid plate extended from the bottom with a total length 0.85h. The results for the same case, as given by Losada et al. [8] are given in fig. 6. Again, there is a very close agreement between the proposed method and that of Losada et al. Losada approach is based on an eigenfunction expansion method.

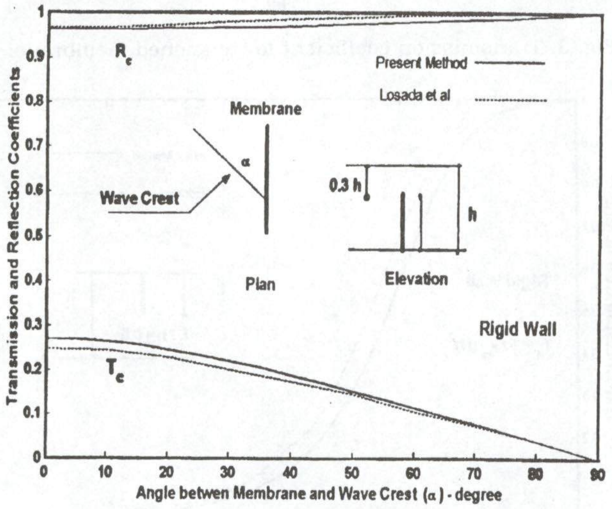

Fig. 6. Transmission and reflection coefficients versus incident wave angle.

Fig. 7 shows a contour plot for the variation of the transmission coefficient with both the angle of the incident waves and the wave number for the case of a membrane deployed from the water surface down to a depth equal one half the total depth. The non dimensional tension T_n used in the analysis is 0.2 and the non dimensional mass per unit area of the membrane ρ_n is 0.01

Fig. 8 shows a contour plot for the transmission coefficient of a membrane deployed from the water surface down to the bed. The non-dimensional tension T_n and the non-dimensional mass per unit area ρ_n used in the analysis are 0.2 and 0.01, respectively.

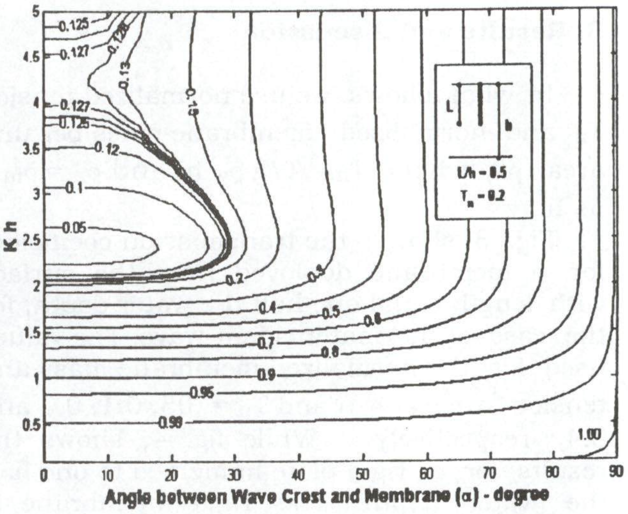

Fig. 7. contour plot of the transmission coefficients versus incident wave angle and wave number- - L / h = 0.5 T_n = 0.2.

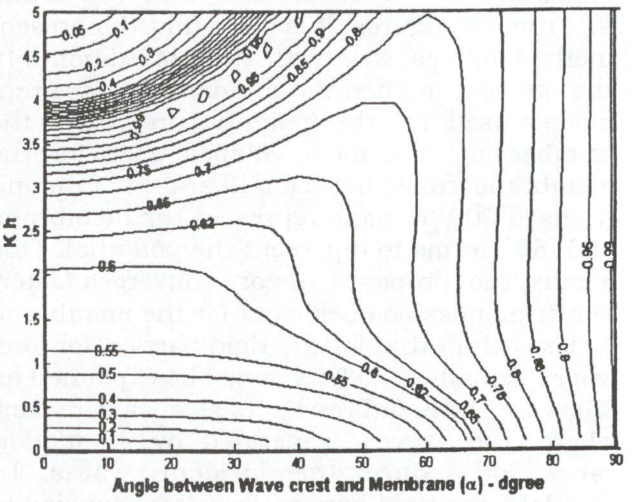

Fig. 8. Contour plot of the transmission coefficients versus incident wave angle and wave number - L/h = 1.0 T_n = 0.2.

4. Conclusions

An analytical method is introduced to predict the performance of a flexible tensioned membrane as a breakwater within the frame of linear wave theory. The method can be extended to cover the case of a rigid wall. Different configurations of the membrane can be handled using the present method. The results of the present method agree with the different methods for both flexible membrane and rigid wall. The membrane length, tension and location affect the performance of the membrane to a great extent. The mass of the membrane has small effect on its performance. The transmitted waves result from both the motion of the membrane and the diffraction from the gaps. In some cases they cancel one another. In this case the performance of the membrane outweighs that of a rigid plate. To achieve this result the length and tension of the membrane must be tuned with the wave frequency.

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