Analysis and prediction of the monthly electric load in Jeddah city

Abdullah O. Ba-Fail

Industrial Eng. Dept., King Abdulaziz University, PO Box 80204, Jeddah 21489, Saudi Arabia

Energy usage in Jeddah and its surrounding cities in Saudi Arabia exhibits a strong seasonal pattern due to the higher demand in summer season and the additional load during pilgrimage months. Six long range and four short range forecasting models are developed to analyze energy demand in Jeddah city. Three different types of trigonometric time series, a multiplicative seasonal model and multiple linear regression model using dummy variables are considered. In addition, a composite nonlinear model is proposed which combines the seasonality of the multiple linear regression model and the peak type pattern to smooth out the peaks. To determine the parameters of this model, least squares fit by parabolic expansion of a chi-square function is proposed. Moreover, smoothing base techniques such as moving average, double exponential smoothing, Winters and a multiplicative seasonal model with the properties of additive trend are investigated. Extensive computational results and statistical tests are used to test the validity of the nonlinear regression model. It is observed that the proposed nonlinear model is superior to others that investigated the electric demand prediction in Jeddah city.

يتطلب استخدام الكهرباء فى مدينة جدة والمدن المجاورة أحمالا عالية فى اشهر الصيف وموسم الحج، وقد تم تطوير ستة نماذج للتنبؤ طويل المدى وأربعة نماذج للتنبؤ قصير المدى لمتطلبات أحمال الكهرباء بجدة، وقد استخدمت ثلاثة أنواع من المتتالية الثلاثية الزمنية ونموذج تفصيلي متضاعف ونموذج خطى ترددي متضاعف بالإضافة الى اقتراح نموذج مركب بين النموذج الخطى الترددي المتضاعف مع تمهيد الذروة. ولتحديد مكونات النموذج تم استخدام نموذج مقترح يلائم اقل التربيعات باستخدام امتداد الدالة المكافئة بالإضافة إلى استخدام تقنية حقل الأساس. وقد استخدمت نتائج حسابات مستفيضة بالحاسب واختيارات إحصائية لتقويم النموذج الغير خطى الترددي المتضاعف وتلاحظ ان هذا النموذج يفوق جميع النماذج الأخرى التي تم اختيارها للتنبؤ بأحمال الكهرباء المتطلبة في مدينة جدة.

Keywords: Electric load, Forecasting, Time series, Multiple linear and nonlinear regression, Parabolic extrapolation

1. Introduction

Power is supplied to the western region of Saudi Arabia through Saudi Consolidated Electric company in the western region (SCECO-west). Models of peak electric load demand forecast for SCECO-west, have been proposed in Ahmed and Al-Jiffry [1], Moreb and BaFail [2], Ahmed, Aljifrry and Ghulman [3]. Most of these forecast models are of short and long range types.

The analysis of the monthly demand of electric load in Jeddah is important for maintenance scheduling, coordination of power sharing arrangements and for establishing fixed capacity. The monthly demand of electric load for Jeddah city in mega-watt-hour in 12 months cycles starting from June 1992 ,to Dec 1998, is shown in fig. 1. The figure shows that there is a strong seasonality and a slight upward trend. The seasonal pattern is due to the extreme weather conditions of the long summer months, mainly May through October followed by a gradual decrease for electric load demand in other months. The winter months in Jeddah are moderate and do not require room heating. The electric load demand during winter months is minimum for domestic consumption. Moreover, the pilgrimage months have an extra peak demand apart from the average daily peak demand. The objective of this paper is to analyze the monthly demand of electric load in Jeddah city and to predict the load for a few months ahead. Ten different time series models for the electric load were developed.

© Faculty of Engineering Alexandria University, Egypt.

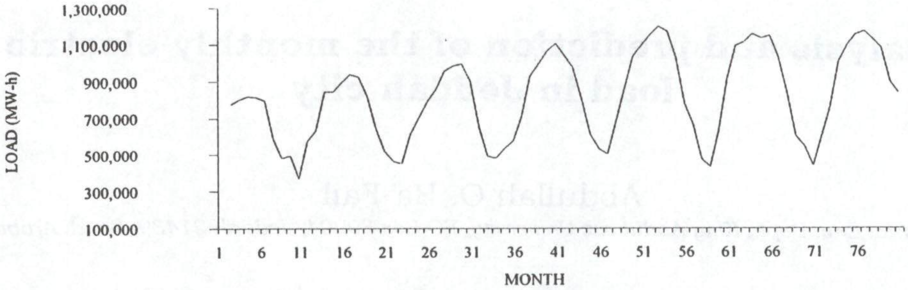

Fig. 1. Actual monthly electric load for Jeddah city: June 1992- Dec 1998.

The first five models: 1, 2, 3, 4, and 5 are regression based type. Models 6, 7 and 8 are of smoothing based types. Model 9 is a multiplicative decomposition method, while model 10 is a proposed non-linear regression type model. All these models are explained in details in section 2. Also a new heuristic procedure for estimating the parameters of the non-linear model is proposed and explained in section 3. The analysis of the results is explained in section 4 and finally conclusion is stated in section 5.

2. Developments of time series models

Many different time series characteristics may be modeled using trigonometric functions involving sine and cosine terms. This can be achieved by controlling the three characteristics of the sine wave, the amplitude, phase angle, and period. Thus, $x = \beta \sin \omega t$ defines a sine wave with amplitude β and origin at $t = 0$. A 12-period season is obtained by setting $\omega = 2\pi/12$, while ω is in radians. In some time series we may find that the amplitude of the seasonal cycle is proportional to the trend. Model 1 as shown in table 1 capture this characteristic. A seasonal pattern that exhibits more irregular behavior may be modeled by including a second sine-cosine pair to introduce a higher-frequency harmonic into the theoretical model. This process is represented by Model 2. A model containing four harmonic frequencies which can describe almost any periodic process is represented by Model 3, with irregular 12-point periodic function. The reader may refer to Montgomery et al. [4] and Mandenlhall and Sincich (1989) [5] for more information.

Model 4 is a multiple linear regression model. The seasonality is modeled by introducing (0,1) dummy variables for the 12 month periods. Model 5 is a multiplicative model based on Model 4. A simple moving average method is expressed by Model 6, while Model 7 is a double exponential smoothing method. Winters method is described by Model 8. The multiplicative decomposition method is represented by Model 9. This model first fits a straight line on the early seasonal data and then corrects it with a seasonal ratio. Model 10 is a nonlinear time series, where a Gaussian function is embedded with model 4 in order to account for the peak load with quadratic background for more accurate data modeling. Table 1 lists the mathematical formula for all these models. Parameters estimation for Model 1 through 9 is straightforward. The methods of estimating the parameters can be found in Mandenlhall and Sinicich [5]. Since some of the parameters of Model 10 are nonlinear, the parameters are estimated by methods proposed in the next section.

2.1. Models with nonlinear parameters

Model 10 as shown in table 1 is a nonlinear regression model with a set of m parameters α_j for $j = 1,\ldots,m$. In the proposed model α_j are δ_0, δ_1, $\beta_1, \beta_2, \ldots\ldots, \beta_{11}$, Ω_1, Ω_2 and Ω_3. One way to estimate the parameters is to adopt goodness of fit criterion. The measure of goodness of fit can be defined by the following chi-square (χ^2) expression.

Table 1
Time series models

MODEL:1	$X_t = \delta_0 + \delta_1 t + [\beta_1 + \beta_2 t]\ SIN\left(\frac{2\Pi t}{12}\right) + [\gamma_1 + \gamma_2 t]\ COS\left(\frac{2\Pi t}{12}\right) + \varepsilon_t$
MODEL:2	$X_t = \delta_0 + \delta_1 t + \sum_{j=1}^{2}\left[\beta_j\ SIN\left(\frac{2\Pi jt}{12}\right) + \gamma_j\ COS\left(\frac{2\Pi jt}{12}\right)\right] + \varepsilon_t$
MODEL:3	$X_t = \delta_0 + \delta_1 t + \sum_{j=1}^{4}\left[\beta_j\ SIN\left(\frac{2\Pi jt}{12}\right) + \gamma_j\ COS\left(\frac{2\Pi jt}{12}\right)\right] + \varepsilon_t$
MODEL:4	$X_t = \delta_0 + \delta_1 t + \sum_{j=1}^{12-1}\beta_j M_j + \varepsilon_t$; ε_t = Error term at time t ; X_t = predicted value at time t Where, $M_1 = \begin{cases} 1 & If\_Month\_1 \\ 0 & If\_Month\_2,3,4,...,12 \end{cases}$ $M_2 = \begin{cases} 1 & If\_Month\_2 \\ 0 & If\_Month\_1,3,4,...,12 \end{cases}$ .... $M_{11} = \begin{cases} 1 & If\_Month\_11 \\ 0 & If\_Month\_1,2,....,10,12 \end{cases}$; (Consider a season of 12 months)
MODEL:5	$X_t = e^{\left(\delta_0 + \delta_1 t + \sum_{j=1}^{12-1}\beta_j M_j\right)} + \varepsilon_t$
MODEL:6	$X_t = \sum_{i=1}^{nprd}\frac{Y_{t-i}}{nprd} + \varepsilon_t$; Where, nprd = n-Point moving average; Y_{t-i} = Response at time t-i
MODEL:7	$X_t = \nu Y_t + (1-\nu)(X_t + T_{t-1}) + \varepsilon_t$ $T_t = \lambda(X_t - X_{t-1}) + (1-\lambda) T_{t-1}$ Where, Y_t = Response at time t ; X_t = Predicted value at time t T_t = Trend component ; $1 \geq \nu, \lambda \geq 0$
MODEL:8	$X_t = (\Phi_0 + \Phi_1 t)\psi_t + \varepsilon_t$ Where, Φ_0 = Base signal; Φ_1 = Linear trend component; ψ_t = Multiplicative seasonal factor
MODEL:9	$X_t = \left[\frac{\delta 0 + \delta 1t}{12}\right] \dfrac{\left[\left[\sum_{j=1}^{nyrs} Y_{jk}\right] / nyrs\right]}{\left[\sum_{j=1}^{12}\left[\left[\sum_{i=1}^{nyrs} Y_{ij}\right] / nyrs\right] / 12\right]} + \varepsilon_t$; k=1,2,3,........,12
MODEL:10	$X_t = \delta_0 + \delta_1 t + \sum_{j=1}^{12-1}\beta_j M_j + \left[\Omega 3 . e^{-\frac{1}{2}\left(\frac{t-\Omega 1}{\Omega 2}\right)^2}\right] + \varepsilon_t$ Where, $M_1 = \begin{cases} 1 & If\_Month\_1 \\ 0 & If\_Month\_2,3,4,...,12 \end{cases}$ (*Regression model with gaussian peak*) $M_2 = \begin{cases} 1 & If\_Month\_2 \\ 0 & If\_Month\_1,3,4,...,12 \end{cases}$ (ε_t = Error term at time t) .... (X_t = predicted value at time t) $M_{11} = \begin{cases} 1 & If\_Month\_11 \\ 0 & If\_Month\_1,2,....,10,12 \end{cases}$; (Consider a season of 12 months)
	Where, (δ_0 & δ_1) , (Ω_1, Ω_2, ..., Ω_k) (β_0 , β_1 , β_2 ,........, β_L), (γ_1 , γ_2 , γ_3 ,......., γ_M), are coefficients and the values of K, L and M are dependent on the model. nyrs = Data for 'nyrs' number of years, (t=1,2,3,..., nyrs*12).

$$\chi^2 = \sum_{t=1}^{n} \frac{1}{\sigma^2}\left(X_t - \sum_{j=1}^{m} \alpha_j T_j(t) \right)^2$$

where σ^2 = uncertainties in the data point X_t at time t,
where t = 1,2,3,..., n, and n = number of data points.
There are three sources of errors that contribute to the values of χ^2,

(i) the fluctuations of χ^2 about the expected value of χ_i which may be statistically greater than or less than the expected theoretical uncertainty α_j ;
(ii) χ^2 is a continuous function with all parameters of α_j ; and
(iii) the choice of the approximate function as a true analytical function.
The contribution of errors can be minimized in two steps. First the parameters α_j can be estimated by the least squares method by minimizing χ^2. Second, an iterative procedure can be used to test several different functions with arbitrary choice of α_j such that a most suitable function is determined.

A modified form of Gauss Newton method was suggested by Hartley [6] to solve such a problem. For several years it was the only alternative to steepest descent method. The method behaves unsatisfactorily when some of the residuals are large. Nazareth [7], and McKeown [8] explain the demerits of this method. Marquaradt [9] put forth an elegant method related to an earlier suggestion of Levenberg [10] for varying smoothly between the extremes of the inverse Hessian method and the steepest descent method. Marquardt's first observation about Hessian matrix was that it provides no precise information, but gives information about the order of magnitude of the scale of the problems. The second observation was the selection of step size so that χ^2 function is minimized. This method is somewhat more dependable than the modified Gauss Newton method, but frequently fails to converge, as claimed by Dennis [11] and Meyer [12]. χ^2 function is quasi convex type. It is shown in Kennedy [13] and Press et al. [14] that the function contains local minimal. The author computational experience reveals that gradient based search process faces difficulty due to the ill condition of the Hessian matrix. It is claimed in Ralston [15] that this type of problem would not perform better with an algorithm that requires derivative information. He suggested a derivative free algorithm. Code implementing Newton type method can be found in Dennis et al. [16]. Another method which classifies derivative free algorithm is the simplex method given by Nelder and Mead [17]. In this paper a new algorithm was developed to estimate the parameters of the non-linear model type 10. This algorithm approximates the χ^2 function and uses gradient information by numerical approximation for derivatives. The algorithm was called Parabolic Approximation of χ^2 hypersurface (PAXS) and is explained in more details in the following section.

2.2. Searching parameters in m-dimensional space

One of the most difficult computational step in searching parameters α_j by minimizing χ^2 function is that there are more than one local minimum within the reasonable range of α_j. χ^2 can be considered a continuous function of m parameters α_j, describing a hypersurface in m-dimensional space. By expanding the χ^2 function to a second order Taylor series in parameters α_j, χ^2 surface can be approximated with a parabolic hypersurface. So, the parameters can be searched on the approximate function with initial estimates made at the beginning of the computation. If the estimates are close enough to the minimum point, the parabolic approximation of the χ^2 hypersurface will be close to the original function. Hence, it would provide an accurate estimate of α_j.

2.3 Algorithm (parabolic approximation of χ^2 hypersurface: PAXS)

The first step in estimating the parameters α_j for j =1,...,m, is to define a χ^2 merit function and determine the best fit parameters by its minimization through an iterative procedure. By giving trial values for the parameters an algorithm was developed that improves the solution. Fig. 2 shows the flow chart of the steps of the algorithm (PAXS). The algorithm performs well with user supplied initial estimates of the parameters. The proposed PAXS algorithm builds up information during its search procedure and needs good computational facility. This algorithm is currently implemented on an IBM Pentium II.

3. Response of time series models

The data available were divided into two sets. The first set contains the data from June 1992 till May 1998 and is called initialization or fitting data. The second set contains the data from June 1998 to December 1998 and is called test data. The first set of data is used to find the best fitted model(s) based on statistical criteria such as F-test, adjusted R-square ($\bar{R}^2$), Mean absolute percentage error (MAPE), D-W test. The second set of data are used to test the forecasts made by the models. MAPE, relative percentage error and absolute error criteria are used for checking the validity of the forecasts.

Table 2 contains the values of the coefficients of Models 1 through 5 and Models 9 and 10 using initialization data. Parameters of the smoothing based methods are determined by minimizing the sum of errors by an iterative procedure. The Winters method (Model 8) has the parameters α,β and γ with values 0.02, 0.3 and 0.02, respectively. The double exponential method (Model 7) selects both the parameters as (0.089). Comprehensive statistical analysis of the models is presented in table 3. Figs. 3 through 7 show the performance of the ten models for the initialization data set, i.e., the period from June 1992 till May 1998.

Table 2
Values of coefficients

Coeff.	Model 1	Model 2	Model 3	Model 4	Model 5	Model 9	Model 10
δ_0	653424	652058	652759	829188	13.61	7592003	833220
δ_1	3918.9	38819	3800	3781	0.0045	544486	3803
β_1	184161	286269	286198	54251	0.0555		48601
β_2	51138	-38250	-38284	76455	0.0773		70474
β_3			-1440	58724	0.0596		53709
β_4			-5679	-33975	-.0340		-38534
β_5				-244087	-.2934		-249408
β_6				-404555	-.5419		-410620
β_7				-474041	-.6644		-478891
β_8				-521017	-.7646		-525920
β_9				-365494	-.4642		-371182
β_{10}				-218137	-.2550		-223984
β_{11}				-36816	-.0414		-41421
Ω_1							-3168
Ω_2							48.17
Ω_3							149.82
γ_1	2864	74992	75011				
γ_2	709	33062	33082				
γ_3			17180				
γ_4			-5661				

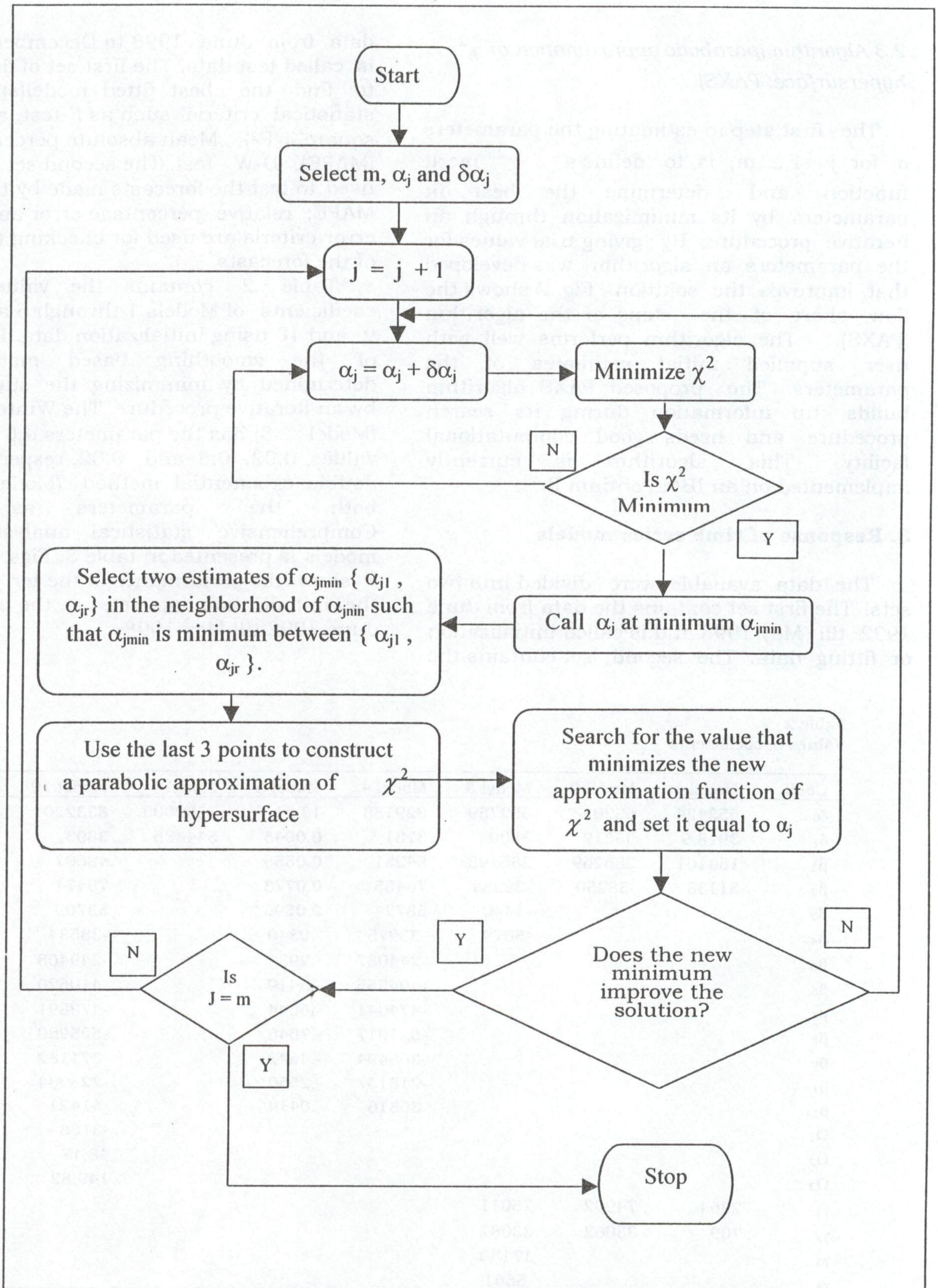

Fig. 2. Flow chart of the algorithm of the Parabolic Approximation of χ^2 hypersurface (PAXS).

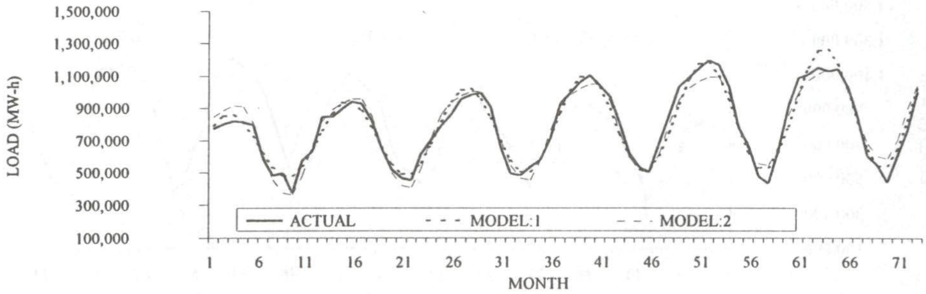

Fig. 3. Electric load forecast for initialization period: June 1992 – May 1998.

Table 3
Statistical parameters

	Regression Models					Model	Smoothing Models			
Statistics	1	2	3	4	5	10	6	7	8	9
MSE x 10^{10}	0.4337	0.408	0.414	0.412	0.244	0.346				
SSE x 10^{10}	24.29	26.94	25.64	24.28	17.38	22.81	325.0	115.1	16.1	15.95
σ^2 x 10^{10}	0.396	0.410	0.414	0.412	.25	0.346				
σ_ε x 10^5	0.63	0.639	0.643	0.642	0.5	0.588	2.170	1.264	0.473	0.471
R^2	0.938	.9350	.9311	.9168	.92	.9387				
Adjusted $\bar{R}^2$	0.922	.9211	.9211	.9220	.8990	.9341				
F-TEST	56.83	169.2	93.12	70.44	54.20	202.2				
DW-TEST	0.636	0.878	0.814	0.640	0.804	1.263	0.394	0.733	0.916	0.869
A.E. x 10^5	0.002	0	0	0	.021	0	0.015	-.032	-.058	0
Rel % ε	-0.396	-.501	-.493	-.435	0.343	0.243	-5.46	1.38	-1.09	-.413
MAE	0.473	0.501	0.483	0.473	0.405	0.351	1.866	1.018	0.378	0.384
MAPE	6.79	7.20	6.90	6.78	5.56	5.20	26.74	14.43	5.40	5.36

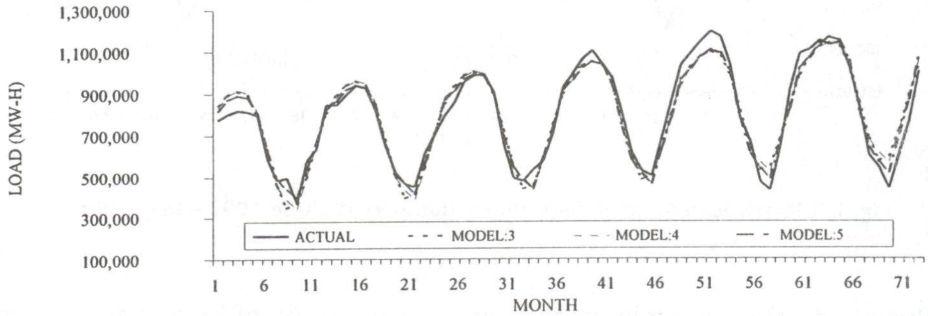

Fig. 4. Eelectric load forecast for initialization period: June 1992 – May 1998.

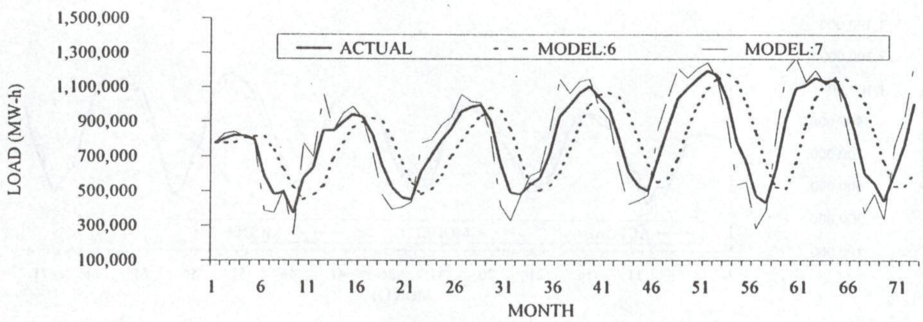

Fig. 5. Electric load forecast for initialization period: June 1992 – May 1998.

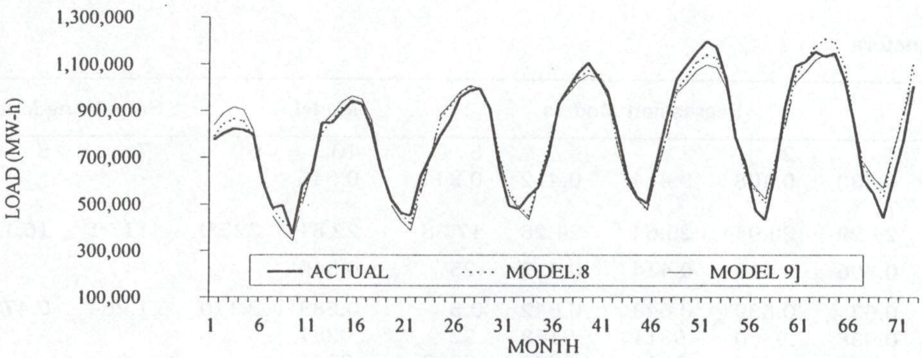

Fig. 6. Electric load forecast for initialization period: June 1992 – May 1998.

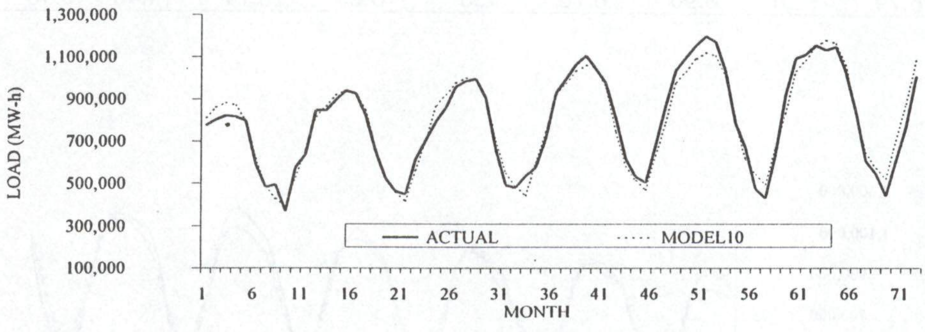

Fig. 7. Electric load forecast for initialization period: June 1992 – May 1998.

The analysis of the Models based on statistical criteria could be categorized into two groups. Models 1, 2, 3, 4, 5 and 10 are regression-based and, hence, have the capability of long range forecast, while Models 6, 7, 8 and 9 are smoothing-based, and therefore are good for short range forecast.

Different test statistics, as shown in table 3, are used to rank the models.

F-test indicates that Models 1 through 5 and 10 are highly significant. The regressors explain significant amount of variability - more than 90% - in the change of monthly electric load for Jeddah city. Adjusted R^2 values are also highly significant in all the models. Model 10 appears to be best explained by the R^2 statistics with a value of 0.9387. Model 1 has a close value with $R^2 = 0.9380$. It should, however, be noted that R^2 alone does not increase the appropriateness of the model, [Montgomery et al. [6]. Root mean square error of 58821 for model 10 implies that one can expect to predict, 95% of the time, monthly peak demand accurate to within 117642 mega-watt-hour of its true value. It is slightly higher for other series.

The statistics based on the values of mean absolute percentage error and mean absolute error suggest that Model 10 is most suitable for demand forecast followed by Model 5. D-W statistics generally provides information whether or not the model has been correctly fitted. A value of 2 would imply that the fitting errors are random. All the models have values of D-W less than 2. It appears that there exists a positive autocorrelation and, hence, a relatively smooth pattern cannot be ruled out. Model 10 provides a D-W value of 1.263 and, therefore, it is preferred to Models 1, 2, 3, 4, and 5.

Among the smoothing based-models, Model 9 has the smallest mean absolute percentage error with a value of 5.36, followed by Model 8 with a value of 5.40. Mean absolute error for Model 8 is 37800 which is lower than that of Model 9 with a value 38400. In case of D-W statistics, Model 8 slightly outperforms Model 9 with values of 0.916 and 0.869 respectively.

Based on the above discussion, Models 1, 5 and 10 from regression-based models and Models 8 and 9 from smoothing based-models are acceptable candidates for forecasting. Based on the forecasting performance on the test data (i.e., the forecasts from June 1998 till December 1998), the forecasted values were compared with the actual values to identify the best model. Referring to table 4, Model 1 is unrealistic due to higher standard deviation of error. Using the lowest values of MAPE, relative percentage error and absolute error as criteria, it can be concluded that Model 10 performs best, followed by Models 5 and 9. Forecasts of data from June 1998 till December 1998 by models 1, 5, 8, 9 and 10 are shown in figs 8, 9 and 10. In conclusion, as shown in the above analysis, Model 10 seems superior to all models for predicting the electric demand in Jeddah city.

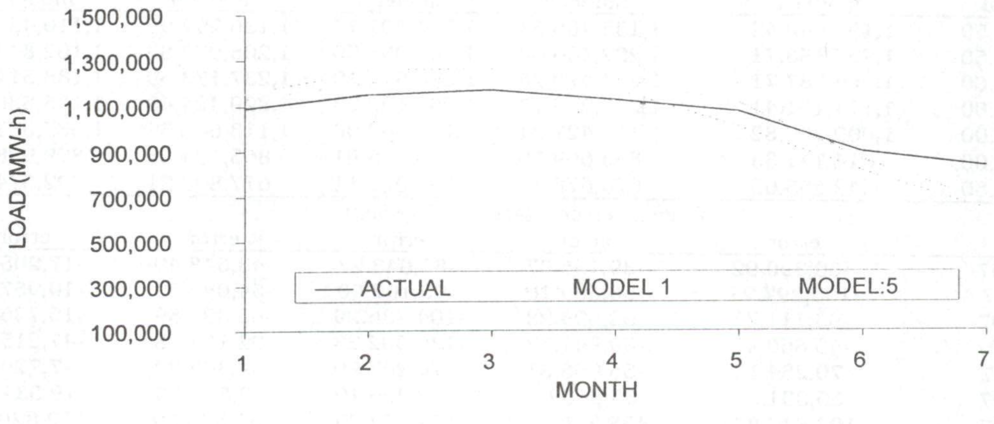

Fig. 8. Electric load forecast for test period: Jnue 1998 – Dec 1998.

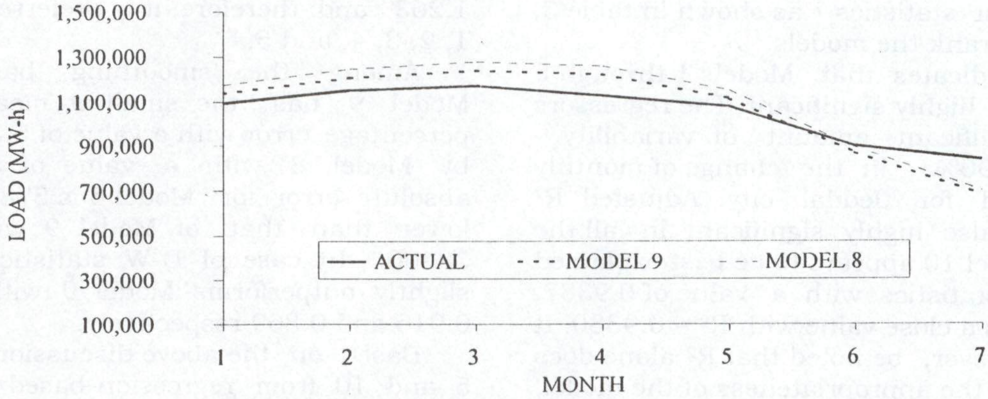

Fig. 9. Electric load forecast for test period: June 1998 – Dec 1998.

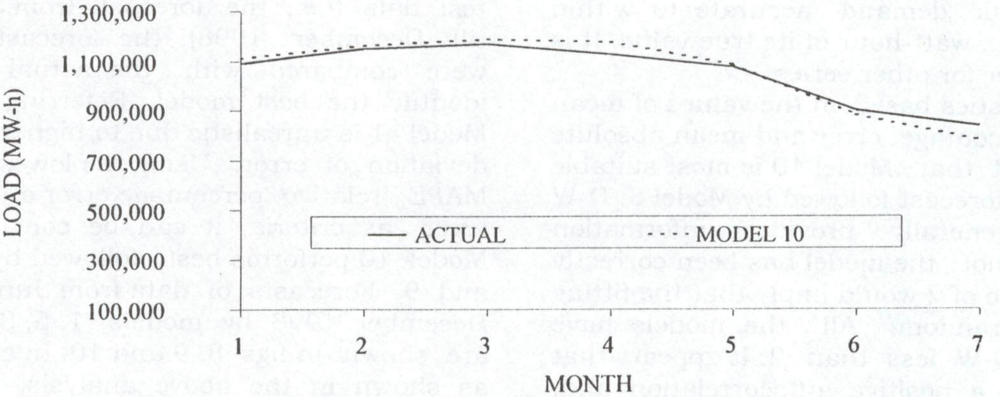

Fig. 10. Electric load forecast for test period: June 1998 – Dec 1998.

Table 4
Analysis of results in the test period

Actual data actual	Forecast model 1	Forecast model 5	Forecast model 8	Forecast model 9	Forecast model 10
1,093,249.50	1,153,640.42	1,132,405.57	1,174,292.77	1,136,297.99	1,110,457.53
1,151,891.50	1,257,188.71	1,202,400.60	1,244,998.50	1,205,973.82	1,162,848.76
1,171,776.00	1,264,887.71	1,234,470.28	1,276,512.29	1,237,173.59	1,188,512.26
1,128,321.00	1,174,181.41	1,218,262.73	1,258,623.23	1,220,424.48	1,175,536.60
1,079,362.00	1,009,107.82	1,114,427.31	1,149,569.90	1,113,665.32	1,087,082.62
899,533.00	814,141.33	863,669.35	890,436.81	865,934.35	879,998.16
835,394.50	642,555.65	676,678.37	697,022.11	677,809.21	722,574.45
		forecast error: (actual - forecast)			
	error	error	error	error	error
Jun 97	-60,390.92	-39,156.07	-81,043.27	-43,048.49	-17,208.03
Jul 97	-105,297.21	-50,509.10	-93,107.00	-54,082.32	-10,957.26
Aug 97	-93,111.71	-62,694.28	-104,736.29	-65,397.59	-16,736.26
Sept 97	-45,860.41	-89,941.73	-130,302.23	-92,103.48	-47,215.60
Oct 97	70,254.18	-35,065.31	-70,207.90	-34,303.32	-7,720.62
Nov 97	85,391.67	35,863.65	9,096.19	33,598.65	19,534.84
Dec 97	192,838.85	158,716.13	138,372.39	157,585.29	112,820.05
sum error	43,824.45	-82,786.71	-331,928.10	-97,751.25	32,517.12
Std. deviation	111,540.26	84,490.58	92,740.34	84,962.37	51,549.53
abs. error	653,144.95	471,946.27	626,865.28	480,119.13	232,192.66
mape	0.66	0.48	0.60	0.48	0.25
rel. % err	-0.12	0.02	0.25	0.03	-0.07

4. Conclusions

From the preceding discussion it is clear that Model 10 appears to be the most suitable model for forecasting the peak electric load demand for SCECO-west, followed by Models 5 and 9. However, Model 9 is simple in concept, economical to maintain and could be further improved for long term prediction. The regression based models are more reliable due to the fact that the prediction can be made with confidence interval. It should be noted that PAXS algorithm requires several runs to find the best estimates of statistical parameters. The reported results cannot be claimed as a global minimum. A simulation strategy may provide more reasonable estimate of the parameters. The reported results are found with initial estimates for all α_j = 100 and all $\delta\alpha_j$ = 0.01. The analysis was carried out by using the software developed by the author in the Department of Industrial Engineering, King Abdulaziz University, Jeddah.

Acknowledgement

Many thanks for Dr. Mohammed Sadik Al-jiffry for his valuable comments and contribution and special thanks for Mr. Shmmus El-deen Ahamed for his effort in running the algorithm (PAXS).

References

[1] S. Ahmed, M.S. Al-Jiffry,. Forecasting electric load demand in Jeddah city: A comparative study. The Fourth Saudi Engineering Conference, King Abdul Aziz University, Jeddah (1995).

[2] A. Moreb, and A. O BaFail, Dynamic Data Systems Models for Predicting the Load Demand.Computer and Industrial Engineering, Vol. 25 (1-4), pp. 291-294 (1993).

[3] Ahmed, S. Aljifiry & Gulman, A season-Hegriah Months - Sensitive Short term Electric Load Forecasting Model. Proc. of the Saudi Symposium on Energy, Vol. 2 (1990).

[4] D. C. Montgomery, L. A. Johnson, and J. S. Gardiner, Forecasting and Time Series Analysis. Mc. Graw Hill, NY, USA. Montgomery et al. (1990).

[5] W. Mendenhall and T.A. Sincich, Second Course in Business Statistics: Regression Analysis. Dellen Pub. Co., San Francisco, Calif. USA (1989).

[6] H. O. Hartley, The modified Gauss-Newton method for fitting nonlinear regression functions by least squares. Techno., Vol. 3, pp. 269-280 (1961).

[7] L. Nazareth, Some recent approach to solving large residual non-linear least squares problems. Proceedings of the ninth interface symposium on computer science and statistics, edited by D. Hoaglin and R. Welsch, Prindle, Weber and Schmidt, Boston, pp. 161-167 (1976).

[8] J.J. Mckeown, Specialized versus general purpose algorithms for minimizing functions that are sums of squared terms. Mathematical Programming 9, pp. 57-68 (1975).

[9] D. W. Marquardt, An algorithm for least squares estimation of nonlinear parameters. SIAM J. , Vol. 11, pp. 431-441 (1963).

[10] Levenberg, K. A method for the solution of certain nonlinear problems in least squares. Quart, Applied Math. 2, pp. 164-168 (1944)

[11] J. E. Jr Dennis, J. E., Jr., Some computational techniques for the nonlinear least squares problem. Numerical solutions of systems of nonlinear algebraic equations, edited by G. D. Bryne and C. A. Hall, Academic Press, NY, pp. 157-183 (1973).

[12] R. R. Meyer, Theoretical and computational aspects of nonlinear regression, in nonlinear programming, edited by J. Rosen, O. Mangasarian, and K. Ritter, Academic Press., NY, pp. 465 - 486 (1970).

[13] W. J. Jr. Kennedy and J.E. Gentle Statistical Computing. Marcel Dekker, Inc, NY (1981).

[14] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery. Numerical Recipes in FORTRAN The art of scientific computing. 2nd. ed. Cambridge University Press (1992).

[15] M.L. Ralston, DUD: A derivative-free algorithm for nonlinear regression. Ph.D. thesis, University of California, LA (1975).

[16] J.E. Dennis, D.M. Gay, D.M., and R.E. Welsch, Code NL2SOL . ACM Transaction on mathematical software, Vol. 7, pp. 348-368. (1981).

[17] J.A. Nelder, and R. Mead., A Simplex method for function minimization. Comp. J., Vol. 7, pp. 308-313 (1965).

Received Augusts 10, 2000
Accepted Augusts 19, 2001