# RELIABILITY ANALYSIS OF TUNNELS

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## ABSTRACT

The increasing world-wide activity in the construction of tunnels necessitates the investigation of the reliability of tunnels under different construction conditions. Those reliability studies should primarily serve to assess practical design rules, for example, partial safety factors. The reliability analysis of such complex structures requires approximate techniques in the mechanical and/or probabilistic models. In this paper a response surface method is used to link the finite element analysis phase with the reliability calculations. The response surface is evaluated using nonlinear regression technique. The use of FORM/SORM requires that the response surface to be twice differentiable. Hence, an efficient simulation-based reliability method is used in conjunction with the response surface method in order to evaluate the component reliability of cavity system. One type of limit state function is used; serviceability limit state at cavity crown. A parametric study is performed in order to investigate the effect of the statistics of the system parameters on the cavity reliability and the partial factor of safety.

***Keywords***: Reliability, Partial Safety Factors, Tunnels, Serviceability, FORM/SORM

## INTRODUCTION

It is an important fact that tunnels play a vital role in our life. On the other hand, for many years, tunneling has been carried out at small depth. In recent years and as a result of a moving reliable design schemes and high technological of construction machines, tunnels are carried out at deep depths.

Indeed, there are two categories of methods which can handle the reliability of complex nonlinear structures; namely, direct and indirect methods. In the first group lie FORM/SORM in conjunction with FEM and the adaptive conditional expectation method (ACEM) in conjunction with FEM [1], while in the second group lies any efficient reliability method like FORM/SORM in conjunction with FEM via a response surface method. The use of FORM/SORM in conjunction with FEM requires full access to the finite element code in order to efficiently evaluate the gradients of the response quantities with respect to the basic random variables. Unfortunately, the evaluation of these gradients may be very difficult or impossible especially in the vicinity of the critical point. A random interface element is introduced between the tunnel material and surrounding soil domain.

A high risk in tunnel design appears due to the relatively high variation in soil properties; cohesion, angle of internal friction and unit weight along the tunnel axis. One of the main objectives of this paper is to determine the probability of failure of a chosen tunnel due to the change of coefficient of variation of soil properties around the tunnel during construction (cavity case).

In addition, the probability of failure is determined for both material and geometrically nonlinear analysis techniques and also for both the Cap and Drucker-Prager soil models. From a reliability point of view, a response surface technique was used to determine FORM and SORM reliability index of the deformation at tunnel crown. Also, the design point (critical point) and associated partial factor of safety of each parameter is determined according to EUROCODE [2].

## LIMIT STATE FUNCTION

In estimating reliability or failure probability of a structural system, the failure criterion for the system should be expressed as a so-called limit state function with respect to random design variables involved in the system, Figure 1.

In general, a limit state function is defined in the following form:

$$g(x_i)\begin{cases} >0: \text{safe} \\ \leq 0: \text{failure} \end{cases} \tag{1}$$

where

$x^T=(x_1, x_2, .., x_n)$ : vector of basic variables.

$N$ : number of basic variables.

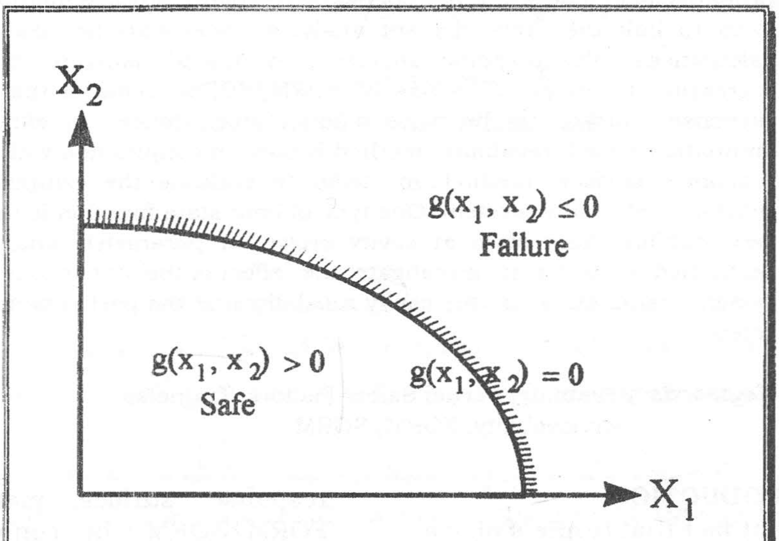



***Figure 1*** Schematic Sketch of Limit State Function.

## RESPONSE SURFACE

Response surface can be determined by two methods, Taylor Series Expansion and Interpolation surfaces. If one selects an initial point $x_o$, e.g., the means of x, the second order form of Taylor series expansion around $x_o$ is [3]:

$$\bar{g}(x) \cong g(x_o) + \sum_{i=1}^{n} \frac{\partial g(x)}{\partial x_i}\left(x_i - x_{i_o}\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 g(x)}{\partial x_i x_j}\left(x_i - x_{i_o}\right)\left(x_j - x_{j_o}\right) \tag{2}$$

On the other hand, interpolation surface states that

$$\bar{g}(x) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \tag{3}$$

is suggested, in which $x_i$, i=1,......., n are the basic random variables and the parameters a, $b_i$, $c_i$, i=1,....., n, have to be determined. It is seen that Equation 3 does not contain mixed terms $x_i$, $x_k$, hence, the function $\bar{g}(x)$ basically represents the original function g(x) along the coordinate axes $x_i$. Since the number of free parameters in Equation 3 is rather low, i.e., 2n + 1, only few numerical experiments (calculations of the actual g(x) are required to obtain a unique response surface $\bar{g}(x)$.

## RELIABILITY INDEX AND DESIGN POINT

The reliability index β is the shortest distance from the origin O to the failure surface in the normalized coordinate system. The point $X_D$ shown in Figure 2 is called the design point, and it is on the failure surface. It is also called the check point for the safety of the structure. Hence, in this method also, the important relation is expressed as,

$$\beta = -\phi_o^{-1}(P_f) \Leftrightarrow P_f = \phi_o(-\beta) \tag{4}$$

The above relation can be used, provided that the failure function is a linear function of the normally distributed basic variables.

From the above discussions, it is obvious that the first order reliability index, β can be obtained for a nonlinear function by expanding the function as a tangent hyper-plane about the design point $X_D$ [4-5].

Then, this tangent hyper-plane may be used to approximate the value of β by numerical integration.

In the second order reliability method (SORM), the probability of failure is approximated by replacing the integration boundary g(x) = 0 by a parabolic extended around the design point XD in a transferred normal space [6] as shown in Figure 2. So, the first step in SORM is to determine the design point.

In fact, the flatter the response surface, the closer β FORM to β SORM. On the other hand, if the surface is nonflat, which occurs when the performance function is nonlinear, β SORM can be obtained [7-8], by,

$$P_{f2} \cong \phi_o(-\beta)\prod_{i=1}^{n}(1+\beta\varsigma_i)^{\frac{-1}{2}} \tag{5}$$

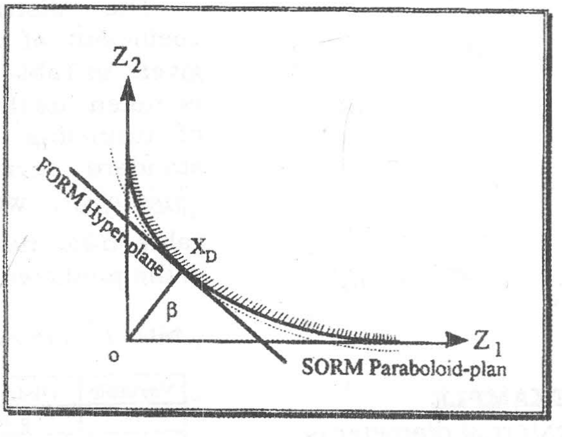



***Figure 2*** Reliability Index, β and design point $X_D$.

## NON-NORMAL BASIC VARIABLES

To overcome the problem that the definition space of the set of basic variables $x_1,\dots,x_n$ is not the normal space, one can use one-to-one transformations [9]. The transformation is chosen in such a way that the values of the original density function $f_{x_i}$, and the original distribution function $F_{x_i}$ for the random variables $x_i$ are equal to the corresponding values of the density function and distribution function for the normal distribution variable at the design point $X_D$.

$$F_{x_i}\left(x_i^*\right) = \phi\left(\frac{x_i^* - \mu'_{x_i}}{\sigma'_{x_i}}\right) \qquad (6)$$

$$f_{x_i}\left(x_i^*\right) = \frac{1}{\sigma_{x_i}}\varphi\left(\frac{x_i^* - \mu'_{x_i}}{\sigma'_{x_i}}\right) \qquad (7)$$

where, $X_D = \left(x_1^*,\dots.x_i^*,\dots.x_n^*\right)$ the design point and $\mu'_{x_i}$ and $\sigma'_{x_i}$ are the unknown mean and standard deviation of the approximate normal distribution. Solving Equations 6 and 7 for $\mu'_{x_i}$ and $\sigma'_{x_i}$ the following can be obtained.

$$\sigma'_{x_i} = \frac{\varphi\left(\phi^{-1}\left(F_{x_i}\left(x_i^*\right)\right)\right)}{f_{x_i}\left(x_i^*\right)} \qquad (8)$$

$$\mu'_{x_i} = x_i^* - \phi^{-1}\left(F_{x_i}\left(x_i^*\right)\right)\sigma'_{x_i} \qquad (9)$$

## APPLICATION EXAMPLE

A cavity with 9.00 m external diameter is chosen. The depth of crown point is 27 m below ground surface. The geometry, finite element mesh and boundary conditions of the problem are shown in Figure 3.

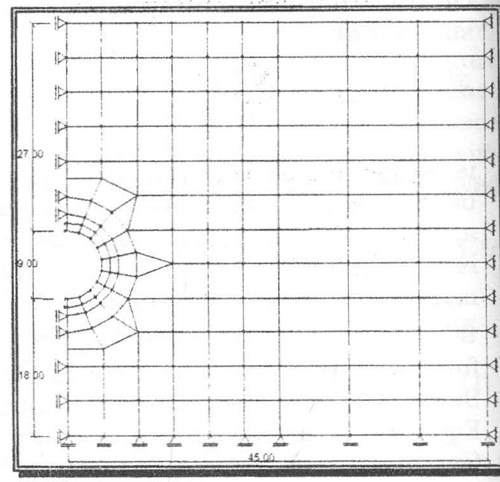



***Figure 3*** Geometry and finite element mesh of cavity.

The section of the cavity is assumed to be far from inlet and outlet, hence, the plane strain condition can be adopted. The analysis is carried out for geometric nonlinearity, with employing the Cap and Drucker-Prager plasticity models. In addition, the death technique is used to simulate the excavation of the tunnel problem. A parametric study is performed to evaluate the effect of the variability of different variables; cohesion, angle of internal friction and unit weight on the tunnel reliability and probability of failure.

### Stochastic Model

The distribution, mean value and coefficient of variation of all variables are given in Table 1. The limit value in this case is taken as the clearance between the head of tunneling machine and the soil. The standard deviation of loss value equals $\sqrt{(OBS-PRE)^2}$, where, OBS is the observed value from numerical experiments and PRE is the predicted value.

***Table 1*** Stochastic Model

| Variable | Distribution | Mean Value | C.O.V. |
|---|---|---|---|
| C | Log-Normal | 3.00 t/m² | 15 % |
| ϕ | Log-Normal | 20 ° | 15 % |
| γ | Log-Normal | 1.85 t/m³ | 5 % |
| LOSS | Normal | 0.00 | to be calculated |
| LIMIT | Constant | -20 cm ↓ | -------- |

**Limit State Functions**

The limit state functions for all cases of analysis are summarized in Table 2.

**Comparison Between Form and Sorm Analysis**

Results for FORM (MD-F, GD-F, MC-F and GC-F) and SORM (MD-S, GD-S, MC-S and GC-S) analysis [10] are summarized in Table 3. Also, Figures 4 to 6 show the effect of C.O.V. of C, $\phi$ and $\gamma$ on the reliability index and the probability of failure.

*Table 2* Limit State Functions for all Cases

| Case | Limit State Function |
|---|---|
| MC | $FLIM = -18.6805 + 5.689101\sqrt{C} - \frac{0.002746}{[\tan(\phi)]^5} - 1.68488\gamma^2 + LOSS - LIMIT$ |
| MD | $FLIM = -17.4601 + 7.048041\sqrt{C} - \frac{0.237321}{[\tan(\phi)]^5} - 2.0166\gamma^2 + LOSS - LIMIT$ |
| GC | $FLIM = -20.0429 + 6.482682\sqrt{C} - \frac{0.003256}{[\tan(\phi)]^5} - 1.86873\gamma^2 + LOSS - LIMIT$ |
| GD | $FLIM = -20.4872 + 8.171346\sqrt{C} - \frac{0.002991}{[\tan(\phi)]^5} - 2.25831\gamma^2 + LOSS - LIMIT$ |

*Table 3* FORM and SORM Results

| PT | Case | $\beta$ | $\beta$ | $P_f$ | $P_f$ | Design point | | | Specified point | | | Partial Factor of Safety | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Form | Sorm | Form | Sorm | C | $\phi$ | $\gamma$ | C | $\phi$ | $\gamma$ | $F_C$ | $F_\phi$ | $F_\gamma$ |
| Crown | MD | 3.156 | 3.121 | $10^{-4}\times7.9984$ | $10^{-4}\times9.019$ | 2.685 | 12.61 | 1.90 | | | | 1.117 | 1.586 | 1.027 |
| | GD | 2.745 | 2.677 | $10^{-3}\times3.0213$ | $10^{-3}\times3.7149$ | 2.635 | 13.60 | 1.91 | 3 | 20 | 1.85 | 1.14 | 1.47 | 1.032 |
| | MC | 2.818 | 2.784 | $10^{-3}\times2.4167$ | $10^{-3}\times2.6828$ | 2.528 | 13.45 | 1.896 | | | | 1.187 | 1.487 | 1.025 |
| | GC | 2.37 | 2.313 | $8.8953\times10^{-3}$ | $1.0356\times10^{-2}$ | 2.695 | 14.25 | 1.899 | | | | 1.113 | 1.404 | 1.026 |

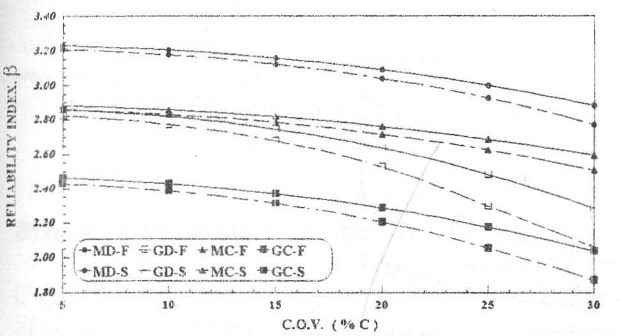



*Figure 4* Effect of change of C.O.V. of C on reliability index $\beta$.

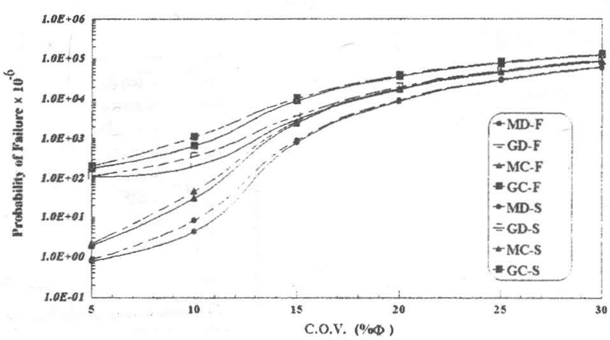



*Figure 5* Effect of change of C.O.V. of $\phi$ on probability of failure $P_f$.

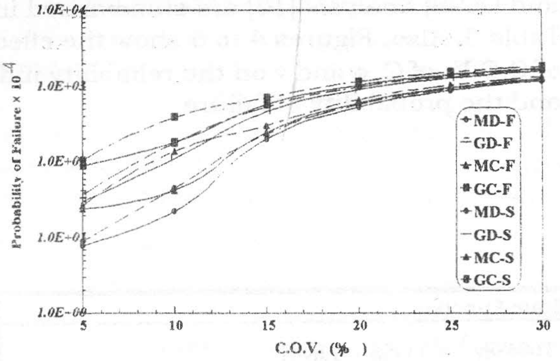


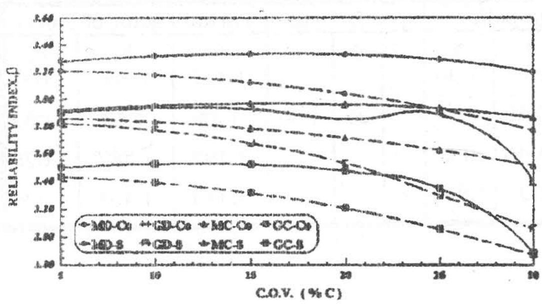

*Figure 6* Effect of change of C.O.V. of $\gamma$ on probability of failure $P_f$.

### Form and Sorm Results For Correlation Between C and $\phi$ Equals TO -0.30

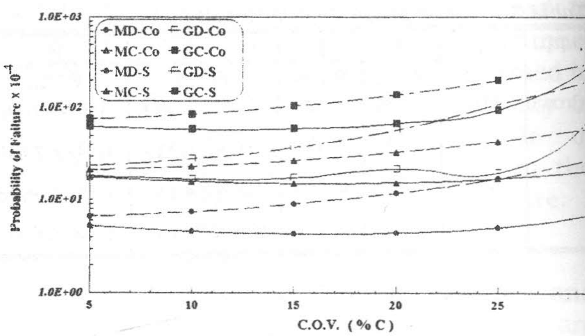
In the sequel, the correlation coefficient between C and $\phi$ is assumed to be (-0.3). The SORM results for correlated and uncorrelated cases (MD-Co, GD-Co, MC-Co and GC-Co), (MD-S, GD-S, MC-S and GC-S) respectively, are given in Table 4. Figures 7-9 show the effect of C.O.V. of C, $\phi$ and $\gamma$ on reliability index, probability of failure and partial safety factors for each parameter.

*Table 4* Results for Correlated Case.

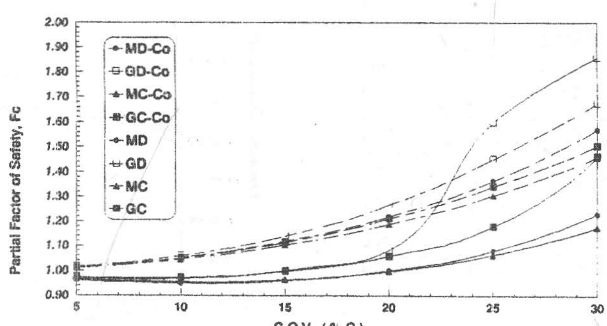
| PT | Case | $\beta$ Form | $\beta$ Sorm | $P_f$ Form | $P_f$ Sorm | Design point C | Design point $\phi$ | Design point $\gamma$ | Specified point C | Specified point $\phi$ | Specified point $\gamma$ | Partial Factor of Safety $F_c$ | $F_\phi$ | $F_\gamma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Crown | MD | 3.363 | 3.332 | $3.857\times10^{-4}$ | $4.3187\times10^{-4}$ | 3.125 | 12.20 | 1.899 | | | | 0.96 | 1.64 | 1.026 |
| | GD | 2.987 | 2.924 | $1.409\times10^{-3}$ | $1.7285\times10^{-3}$ | 3.00 | 13.08 | 1.91 | 3 | 20 | 1.85 | 1.00 | 1.529 | 1.032 |
| | MC | 2.997 | 2.966 | $1.364\times10^{-3}$ | $1.5107\times10^{-3}$ | 3.114 | 12.86 | 1.894 | | | | 0.963 | 1.555 | 1.024 |
| | GC | 2.572 | 2.516 | $5.062\times10^{-3}$ | $5.9267\times10^{-3}$ | 3.01 | 13.82 | 1.90 | | | | 0.997 | 1.447 | 1.027 |




(a)




(b)




(c)

*Figure 7* Effect of change of C.O.V. of C on:
(a) SORM reliability index $\beta$,
(b) SORM probability of failure $P_f$, and
(c) partial factor of safety, $F_c$.

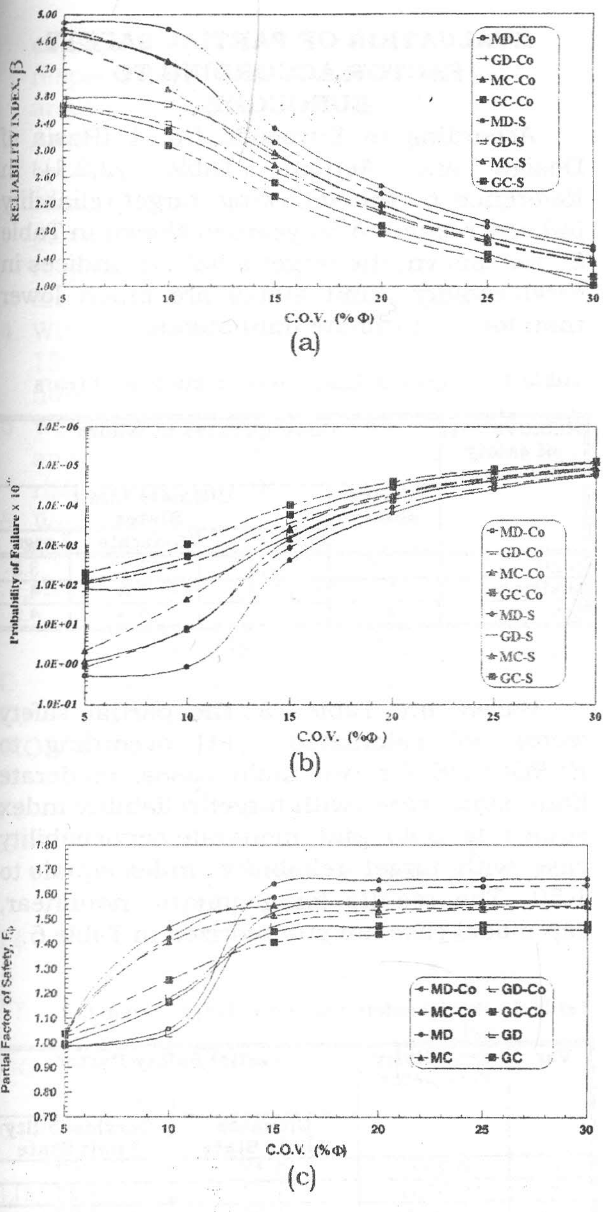



***Figure 8*** Effect of Change of C.O.V. of $\phi$ on:
(a) SORM reliability index $\beta$,
(b) SORM probability of failure Pf, and
(c) partial factor of safety, Fc.

## Effect of Variable Distribution Types

For the case of geometric nonlinearity-Cap Model, five types of distributions are chosen, Lognormal (LN) (original distribution), Normal (N), Exponential (E), Gamma (G) and Rayleigh (R) distributions to study the effect of distribution type on reliability indices. Figures 10-11 show the effect of C.O.V. of C and $\phi$ on SORM reliability index, SORM probability of failure and partial safety factors for each parameter with different distributions types.

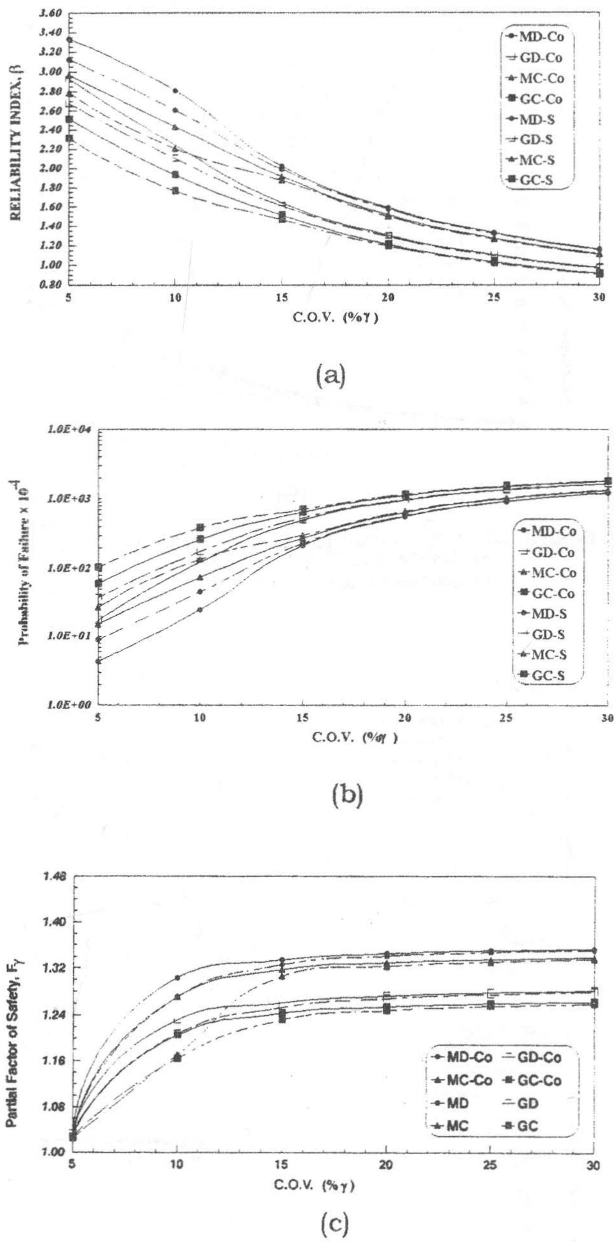



***Figure 9*** Effect of Change of C.O.V. of $\gamma$ on:
(a) SORM reliability index $\beta$,
(b) SORM probability of failure Pf, and
(c) partial factor of safety, Fc.

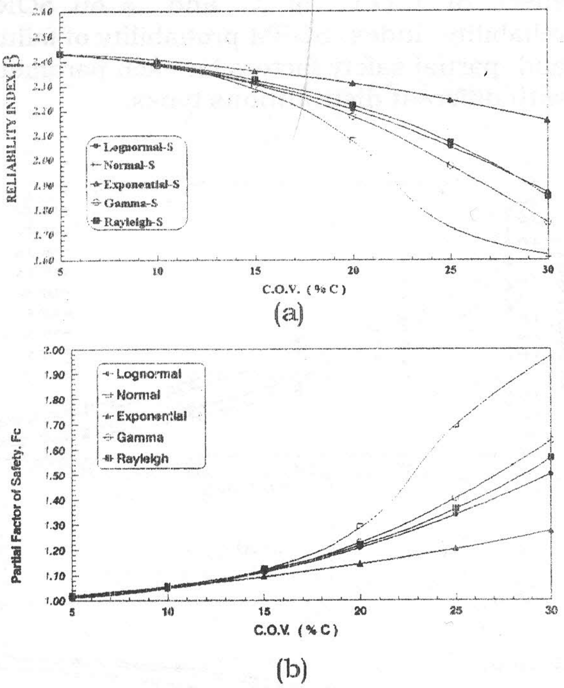



***Figure 10*** Effect of change of C.O.V. of C on:
(a) SORM reliability index β,
(b) partial factor of safety, Fc.

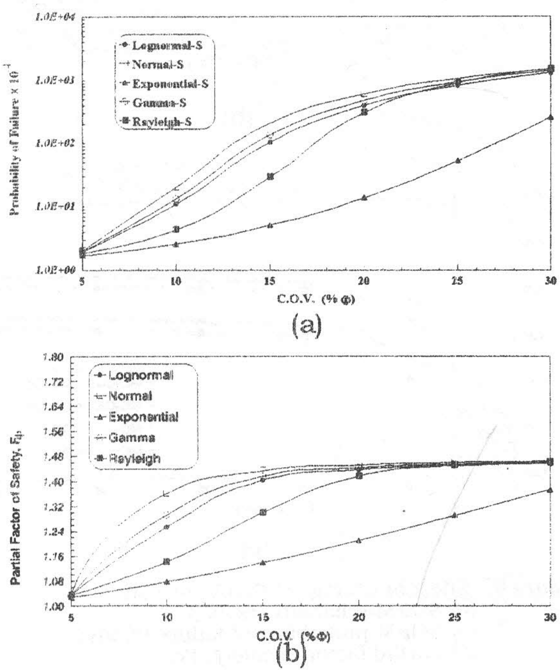



***Figure 11*** Effect of change of C.O.V. of ϕ on:
(a) SORM reliability index β,
(a) partial factor of safety, Fc.

## EVALUATION OF PARTIAL SAFETY FACTOR ACCORDING TO EUROCODE

According to Eurocode No. 1 (Basis of Design and Actions) Table (2.2.1) in Reference 6, the following target reliability indices related to 50 years is shown in Table 5. As shown, the target reliability indices in serviceability limit states are much lower than for the ultimate limit states.

***Table 5*** Target reliability indices related to 50 years

| Relative cost of safety measures | Consequences of failure | | | |
|---|---|---|---|---|
| | Service-ability | Ultimate Limit States | | |
| | | Low | Moderate | Large |
| High | 1.0 | 2.8 | 3.3 | 3.8 |
| Moderate | 1.5 | 3.3 | 3.8 | 4.3 |
| Low | 2.0 | 3.8 | 4.3 | 4.8 |

Based on Table 5, the partial safety factor is calculated [11] according to *EUROCODE* for two main cases, moderate limit state case with target reliability index equals to 3.80 and moderate serviceability case with target reliability index equals to 1.50. The results for geometric nonlinear, cap model case are summarized in Table 6.

**Table 6** Partial safety Factor for Target Reliability index.

| Var. | Sensitivity Parameter, $\alpha_I$ | Partial Safety Factor | |
|---|---|---|---|
| | | Ultimate Limit State | Serviceability Limit State |
| C | 0.2591 | 1.17 | 1.07 |
| ϕ | 0.9351 | 1.72 | 1.25 |
| γ | -0.2187 | 1.04 | 1.02 |

## CONCLUSIONS

1. Analysis and Design on material nonlinearity basis gives a risk of about (11-15)%, (16-22)% and (16-20)% with the change of C.O.V. of C, ϕ and γ respectively, in comparison with the analysis and design on geometric nonlinearity basis.
2. Analysis and Design on Drucker-Prager Model basis gives a risk of about (10-15)%, (10-15)% and (4-9)% with the

change of C.O.V. of C, $\phi$ and $\gamma$ respectively, in comparison with the analysis and design on Cap Model basis.

3. Analysis and Design on no Correlation between C and $\phi$ basis gives a risk of about (15-20)%, (2-5)% and (2-3)% on partial safety factors, $F_C$, $F_\phi$ and $F_\gamma$ respectively, in comparison with the analysis and design on Correlation basis.
4. With the change of C.O.V. of $\phi$ and $\gamma$ (over 15%), geometry nonlinearity and material nonlinearity give the same risk.
5. With the change of C.O.V. of C, Gamma and Rayleigh distributions give the same risk as Lognormal distribution.
6. With the change of C.O.V. of $\phi$ and $\gamma$, Normal and Gamma distributions have the same risk as Lognormal Distribution.

## NOMENCLATURE

C Cohesion of soil in $t/m^2$.
Co Correlation case.
C.O.V. Coefficient of variation of a variable.
F FORM results.
Fc Partial factor of safety of cohesion.
$F\delta$ Partial factor of safety of angle of internal friction between soil and concrete lining.
$F\phi$ Partial factor of safety of angle of internal friction of soil Particles.
$F\gamma$ Partial factor of safety of soil unit weight.
GC Geometric nonlinearity analysis, Cap model case.
GD Geometric nonlinearity analysis, Drucker-Prager case.
LIMIT Allowable values.
LOSS Loss values from regression results.
MC Material nonlinearity analysis, Cap model case.
MD Material nonlinearity analysis, Drucker-Prager case.
Pf Probability of failure.
-S SORM results
$\beta$ Reliability index.
$\alpha i$ Normalized sensitivity parameter.
$\delta c$ Vertical displacement at crown, cm.
$\phi$ Angle of internal friction of soil particles..
$\gamma$ Unit weight of soil in t/m3.
$\varsigma$ The main curvature of limit state surface at the design point.
$P_{f2}$ The second order probability failure.
$\phi_o$ The cummulative distribution function of the standard normal distribution.

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# تحليل وثوقي للأنفاق

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## ملخص البحث

ان الزيادة المطردة فى تنفيذ الأنفاق عالميا تتطلب بحث قيم الوثقية لتشييد الأنفاق تحت ظروف التنفيذ المختلفة حيث أن عملية الأنفاق قد تتعرض لمخاطر عديدة نظرا لتواجد جسم النفق فى مجموعة من المتغيرات الكثيرة على سبيل المثال قيمة التماسك وزاوية الإحتكام بين حبيبات التربة بعضها البعض ووزن وحدة الحجوم للتربة. لذا فمن أهداف هذا البحث هو توحيد مدى إحتمال إنهيار النفق نتيجة لمجموعة المتغيرات السابقة وذلك أثناء التنفيذ، بالاضافة الى تعيين قيم الوثوقية سواء من الدرجة الأولى **(FORM)** أو من الدرجة الثانية **(SORM)** وذلك بإستخدام طريقة سطح الإنفعال **(Response Surface)** كأداة لربط النتائج التجريبية وقيم الوثوقية.

يتناول هذا البحث أيضا عمل مقارنة بين طرق التحليل المختلفة ومنا الطريقة الغير خطية للمـــواد المكونـــة للعنصـــر الإنشائى والتربة المحيطة **(Material Nonlinearity)** وطريقـــة التشكـــلات الكبـــيرة **(Geometric Nonlinearity)** ، وكذلك المقارنة بين بعض النماذج الميكانيكية للدونة مثل **(cap, Drucker-Prager Models)**، وفى النهاية تم دراسة تأثـــير هذه المقارنت على قيم الوثوقية للنفق.

ونظرا لصعوبة تحديد معامل أمان للنفق كوحده واحده، فإن من أهم أهداف هذا البحث هو تعيين معـــاملات الأمـــان الجزئية للمتغيرات المختلفة والمؤثره على جسم النفق طبقا للكود الأوروبى ولك لكل من مفهوم الحدود القصـــوى **(Ultimate Strength State)** ومفهوم حدود الإستخدام **(serviceability Limit State)** ودراسة مدى تطبيـــق ذلـــك بـــالكود المصرى.