THE IMPEDANCE OF A FRACTURE

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ABSTRACT

The existence of a fracture on the hole surface changes the wave impedance in the hole and it causes a reflection of the wave at the location of the fracture. Thus, the dimension of the fracture may be estimated by analyzing the reflected waves . The fracture impedance of the fundamental mode (Stoneley wave) is obtained by analyzing the wave motion of a fluid layer laying between two elastic half space. A numerical investigation of the fracture impedance for different typical rock materials is presented.

Keywords: Waves motion, Stoneley Waves, Fracture Impedance.

1. INTRODUCTION

It is known that the existence of a fracture intersecting a borehole will change the characteristic of the wave motion in the borehole, many parameters have been introduced in the literature to represent fracture intersecting a borehole, e.g. the fracture dynamic conductivity (Tang and Cheng 1989, the fracture stiffness (Chouet (1986) , the hydraulic impedance of a fracture (Holzhouzen 1985, Paige and et al. 1992), and the fracture impedance (Mathieu 1984, Ashour 1994). The wave motion in a fluid-filled fracture was studied by Tang et al. (1989) and by Hornaby et al. (1989) for acoustic logging applications. In their studies, they assumed the formation is rigid and modeled the fracture as a parallel-surface channel of thickness 2h. Also, the wave motion in a fluid laying between two elastic medium have been investigated by V. Ferrazzini and K. Aki (1987) as a model for tremor and for long-period events observed at volcanoes. Paillet and White (1982) used the wave motion in sandwich fluid layer between two elastic half space as a simple model of wave motion in a fluid-filled borehole. Tang and Cheng (1988) carried out an experimental study to investigate the mode trapping characteristics of a fluid filled fracture between two elastic half space. In this study, the wave motion in a fluid-filled a horizontal fracture intersecting a borehole is considered using cylindrical coordinates. The formation is assumed elastic and isotropic. Then, the characteristic equation (dispersion relation) of the wave motion in the fluid is determined from which the phase and the group velocities of the waves in the fluid filled a fracture can be determined. Finally, an expression of the fracture impedance is obtained.

II. FORMULATION OF THE PROBLEM

Consider a horizontal fracture intersecting a borehole. Let us consider a fluid layer of thickness 2h and infinite extent laying between two infinite elastic half space. Define cylindrical coordinates (r, ϕ, z) , where r is measured from the borehole axis, ϕ is the polar angle, z is the distance in the direction of the borehole axis, z = 0 at the center of the fracture. The medium in the borehole and the fracture is fluid with density ρ_f and velocity v_f. Here, R_w is the radius of the borehole.

1- The acoustic field in the fracture

For a non-viscous fluid, the acoustic velocity is given by

$$\bar{v} = \bar{\nabla}\phi, \tag{1}$$

where ϕ is the acoustic wave potential. For time harmonic wave ($e^{-i\omega t}$), the equation of the wave motion in the fluid can be written as

$$\nabla^2\phi + \frac{\omega^2}{v_f^2}\phi = 0, \tag{2}$$

Applying the method of separation of variables, the solution of the above equation after considerable algebraic manipulations, let $(\omega^2/v_f^2) > k_z^2$, can be obtained as

$$\phi = \left[H_o^{(1)}(k_r r)\right]\left[A\cos(k_z z) + B\sin(k_z z)\right], \quad (3)$$

where $H_o^{(1)}(K_r r)$ is Hankel functions of first kind and zero order and $k_z^2 = \omega^2/v_f^2 - k_r^2$ and for the case $(\omega^2/v_f^2) < k_z^2$,

$$\phi = \left[H_o^{(1)}(k_r r)\right]\left[A\cosh(k_z Z) + B\sinh(k_z Z)\right], \quad (4)$$

where, $k_z^2 = k_r^2 - \omega^2/v_f^2$ and k_r is the wave number to be determined from the characteristic equation of the wave motion in the fluid layer. In Eqns (3) and (4), note that the function $H_o^{(1)}(k_r r)$ represents outgoing wave.

Since the fluid motion is axisymmetric, the fluid particle velocities have only two components, u_f and w_f in the r and z directions, respectively. The fluid particle velocities in terms of the acoustic wave potential ϕ is given by

$$u_f = \frac{\partial\phi}{\partial r}, \quad (5)$$

$$w_f = \frac{\partial\phi}{\partial z}. \quad (6)$$

Since the wave propagation in the fracture is symmetric with respect to the fracture axis, the radial component of the velocity u_f is even with respect to z; i.e., $u_f(z) = u_f(-z)$. From Eqn. (5), this require that ϕ is symmetric with respect to z, i.e., $\phi(z) = \phi(-z)$. These can be satisfied by setting B = 0 in Eqns. (3), (4). The substitution of Eqn. (4) in Eqns (5) and (6) and using the following recursion relation (Hildbrand 1976),

$$\frac{dH_o^{(1)}(r)}{dr} = -H_1^{(1)}(r) \quad (7)$$

yields

$$u_f = -\left\{H_1^{(1)}(k_r r)\right\} x \left\{A\, k_r \cosh(k_z Z)\right\}, \quad (8)$$

$$w_f = \left\{H_o^{(1)}(k_r r)\right\} x \left\{A\, k_z \sinh(k_z Z)\right\}, \quad (9)$$

where $(\omega^2/v_f^2) < k_z^2$ and the pressure p can be obtained from the relation

$$p = -\rho_f \frac{\partial\phi}{\partial t} = i\omega\rho_f\phi,$$

$$\therefore\ p = i\,\omega\,\rho_f\,A_1\,H_o^{(1)}(k_r r)\cosh k_z z. \quad (10)$$

2- The elastic fields in the formation

Next, the wave motion in the elastic formation is considered, the displacement $\bar{u}$ can be expressed in terms of a scalar potential ϕ and vector potential $\bar{\psi}$ as:

$$\bar{u} = \bar{\nabla}\phi + \bar{\nabla} x \bar{\psi} \quad (11)$$

For vertical polarization wave [SV], the potential $\bar{\psi} = \psi\hat{e}_\phi$ where $\hat{e}_\phi$ is the unit vector along the increasing in ϕ direction. Thus, the displacement has only two components

$$u_r = \frac{\partial\phi}{\partial r} - \frac{\partial\psi}{\partial z} \quad (12)$$

$$u_z = \frac{\partial\phi}{\partial z} + \frac{1}{r}\frac{\partial(r\psi)}{\partial r} \quad (13)$$

For time harmonics wave, it can be shown that ϕ and Ψ satisfy the following wave equations:

$$\nabla^2\phi + \frac{\omega^2}{v_c^2}\phi = 0 \quad (14)$$

$$\nabla^2\psi - \frac{\psi}{r_2} + \frac{\omega^2}{v_s^2}\psi = 0 \quad (15)$$

where, v_c and v_s are the velocities of the compressive and shear waves respectively,

$$v_c = \sqrt{\frac{(\lambda + 2\mu)}{\rho_s}} \quad \& \quad v_s = \sqrt{\frac{\mu}{\rho_s}}$$

where, λ and μ are the Lamé elastic constants and ρ is the density of the formation. The components of the stresses tensor in terms of the potential ϕ and potential Ψ can be written as:

$$\sigma_r = \lambda \nabla^2\phi + 2\mu \frac{\partial}{\partial r}\left[\frac{\partial \phi}{\partial r} - \frac{\partial \psi}{\partial r}\right] \quad (16)$$

$$\sigma_{rz} = \mu\left[\frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial r} - \frac{\partial \psi}{\partial z}\right) + \frac{\partial}{\partial r}\left(\frac{\partial \phi}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(r\psi)\right)\right] \quad (17)$$

$$\sigma_z = \lambda \nabla^2\phi + 2\mu \frac{\partial}{\partial r}\left[\frac{\partial \phi}{\partial z} + \frac{1}{r}\frac{\partial r \psi}{\partial r}\right] \quad (18)$$

From the condition that the displacement and stresses, therefore the potentials vanish at ($z \to \infty$), the solution of the wave equations can be written as

$$\phi = A_2 H_o^1(k_r r) e^{-k_c z} \qquad z>0 \quad (19)$$

$$\psi = B_2 H_1^1(k_r r) e^{-k_s z} \qquad z>0 \quad (20)$$

where $H_o^{(1)}$ and $H_1^{(1)}$ are Hankel functions of zero and first order of the first kind respectively, and

$$k_c^2 = k_r^2 - \frac{\omega^2}{v_c^2} \quad (21)$$

$$k_s^2 = k_r^2 - \frac{\omega^2}{v_s^2} \quad (22)$$

Substitution from Eqns. (19), (20) into Eqns (15), (17) and (18), the normal stress σ_z and the shear stress σ_{rz} can be written as

$$\sigma_z = \left\{A_2\left[(\lambda + 2\mu)k_c^2 - \lambda k_r^2\right]e^{-k_c z} - 2\mu k_r k_s B_2 e^{-k_s z}\right\} H_o^1(k_r r) \quad (23)$$

$$\sigma_{rz} = \mu\left\{2k_r k_c A_2 e^{-k_c z} - B_2\left[k_s^2 + k_r^2\right]e^{-k_s z}\right\} H_1^1(k_r r) \quad (24)$$

and the normal displacement u_z is

$$u_z = [A_2 k_c e^{-k_c z} + B_2 k_r e^{-k_s z}]\, H_o^1(k_r r) \quad (25)$$

3- Boundary conditions and the dispersion relations:

From the continuity conditions at the fluid-solid interface one can write the following three boundary conditions at $z = h$:

1- The continuity of the normal stress:

$$p = -\sigma_z \quad (26)$$

2- The shear stress must vanish.

$$\sigma_{rz} = 0 \quad (27)$$

3- The continuity of the normal velocities.

$$\frac{du_z}{dt} = w_f \quad (28)$$

Substitution of Eqns (12), (13), (23), (24) and (25) into Eqns (26), (27) and (28) , the following set of equations in the three unknown A_1, A_2 and B_2 can be obtained for the case $(\omega^2/v_f)^2) < k_z^2$

$$\begin{bmatrix} i\omega\rho_f \cosh k_z h & k_c^2(\lambda + 2\mu) - \lambda k_r^2 & -2\mu k_r k_s \\ 0 & 2k_r k_c & -(k_s^2 + k_r^2) \\ k_z \sinh k_z h & -i\omega k_c & i\omega k_r \end{bmatrix}$$

$$\chi \begin{bmatrix} A_1 \\ A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (29)$$

For non trivial solution, the determined should vanish, one can obtain the following dispersion relation of the wave motion in the fluid layer after some algebraic manipulation:

Applying the method of separation of variables, the solution of the above equation after considerable algebraic manipulations, let $(\omega^2/v_f^2) > k_z^2$, can be obtained as

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and the normal displacement u_z is

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$$\chi \begin{bmatrix} A_1 \\ A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \chi = \begin{bmatrix} i\omega\rho_f \cosh k_z h & k_c^2(\lambda + 2\mu) - \lambda k_r^2 & -2\mu k_r k_s \\ 0 & 2k_r k_c & -(k_s^2 + k_r^2) \\ k_z \sinh k_z h & -i\omega k_c & i\omega k_r \end{bmatrix} \tag{29}$$

For non trivial solution, the determined should vanish, one can obtain the following dispersion relation of the wave motion in the fluid layer after some algebraic manipulation:

$$\omega^2 \rho_f k_c (k_r^2 - k_s^2) +$$

$$\rho_s v_s^2 k_z\{(k_s^2+k_r^2)^2 - 4k_r^2 k_c k_s\} \tanh(k_z h) = 0, \quad (30)$$

which is the dispersion relation of the fundamental mode (Stoneley mode) and for the case $(\omega^2/v_f^2) > k_z^2$, the dispersion relation becomes

$$\omega^2 \rho_f k_c (k_r^2 - k_s^2) - \rho_s v_s^2$$

$$\times\ k_z \{(k_s^2+k_r^2)^2 - 4k_r^2 k_c k_s\} \tan(k_z h) = 0, \quad (31)$$

which is the dispersion relation of the normal modes. The above dispersion relations are relationships between the phase velocity and the frequency, with the elastic properties of the fluid-solid system as parameters. This dispersion relations are nonlinear equations in k_r, which can be solved numerically. The phase and group velocities can be obtained from the relations:

$$v_{ph} = \frac{\omega}{k_r} \quad \& \quad v_g = v_{ph} + k_r \frac{dv_{ph}}{dk_r}.$$

4- The fracture impedance

The fracture impedance of the fundamental mode is of particular interest in this section. The fracture impedance Z_f is defined as

$$Z_f = \frac{<p>}{<u>}\Big|_{r=r_w}, \quad (32)$$

where < > implies the average value over the cross-sectional area of the fracture opening. Substitution of Eqns. (25) and (26) into Eqn.(32) gives

$$Z_f = \left(\frac{-iv_f^2 \rho_f k_r}{A_f \omega}\right)\left\{\frac{H_o^{(1)}(k_r R_w)}{H_1^{(1)}(k_r R_w)}\right\}, \quad (33)$$

where $A_f = 4\pi R_w h$ is the fracture opening area. Note that for rigid formation, i.e., $k_r = \omega/v_f$, the fracture impedance reduces to

$$Z_f = \left(\frac{-iv_f \rho_f}{A_f}\right)\left\{\frac{H_o^{(1)}(k_r R_w)}{H_1^{(1)}(k_r R_w)}\right\}, \quad (34)$$

which is the acoustic impedance of a uniform pulsating cylindrical fills the space between two parallel rigid plates.

At high frequency, the phase velocity approaches Stoneley waves and therefore $k_r = \omega/v_{St}$. Using the asymptotic expressions of Hankel functions $H_o^{(1)}(k_r r)$ and $H_1^{(1)}(k_r r)$ at high frequencies ($k_r r \to \infty$), (Hildbrand 1976)

$$H_m^{(1)}(z) \approx \sqrt{\frac{2}{2\pi}} e^{j[z-(m+1/2)\frac{\pi}{2}]},$$

one can obtain,

$$Z_f = \left(\frac{v_f^2 \rho_f}{A_f V_{st}}\right).$$

Also, for hard formation $v_{St} \approx v_f$ and therefore, the fracture impedance reduces to the characteristic impedance of the wave motion in the fracture $Z_f = (\frac{v_f \rho_f}{A_f})$ (Mathieu and Toksoz (1982).

III. RESULTS AND DISCUSSIONS

In this section a study of the different types of waves which can propagate in the fluid layer is presented for both hard formation ($v_s > v_f$) and soft formation ($v_s < v_f$). First, we examine the fundamental mode, in this case $v_{ph} < v_f$, and the dispersion equation is given by Eqn. (30). The amplitude of this wave decay exponentially away from the interface in both media and has most of its energy in the fluid (B. A. Auld 1985) and the waves in this case is called Stoneley waves. This waves exist for all frequency (has no cutoff frequency). At low frequency as $f \to 0$, the phase velocity approaches zero (Aki 1990). At high frequency, $k_z W_f$ is large and Eqn (30) is reduced to

$$\omega^2 \rho_f k_c (k_r^2 - k_s^2) + \rho_s v_s^2 k_z$$

$$\times \{(k_s^2 + k_r^2)^2 - 4k_r^2 k_c k_s\} = 0, \quad (35)$$

the roots of this equation gives the Stoneley waves velocity (Ewing 1960), which exists in the fluid-elastic half-space boundary. The fluid layer in this case is equivalent to fluid half-space (i.e. the surface wave is damped out before it reach to the other boundary). Figure (1) shows the dispersion curves for Stoneley waves for different rocks materials as listed in Table (1).

Table 1. Seismic velocities and physical parameters for test cases

Case	Formation	ρ_f	ρ_s (gm/cm^3)	Poisson's (ratio)	V_c (m/sec)	V_s (m/sec)	V_f (m/sec)
1	Granite	1000	2600	0.25	5900	3450	1500
2	Sandstone	1000	2300	0.27	3950	2200	1500
3	Shale (1)	1380	2300	0.3	2000	1100	1500
4	Shale (2)	1380	2300	0.35	2000	1000	1500
5	Shale (3)	1380	2300	0.45	2000	600	1500

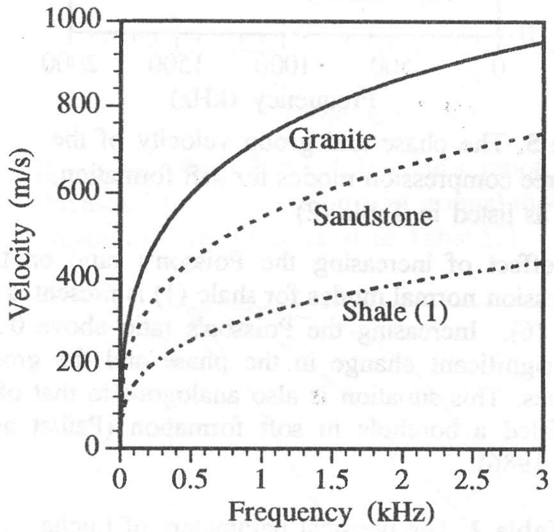

Figure 1. The phase velocity of Stoneley mode for different rocks materials as listed in Table (1).

In the hard formation, the velocity of Stoneley wave is very close to the wave velocity in the fluid, e.g. $V_{St.} = .997\ v_f$ for granite and $V_{St.} = .992\ v_f$ for sandstone. On the other hand, in the case of soft formation ($v_s < v_f$) shale (1), the Stoneley wave velocity is much less than the wave velocity in the fluid $V_{St.} = .67\ v_f$. The effect of fracture width on the fundamental mode is illustrated in Figure (2). The effect of decreasing the fracture width is to decrease the Stoneley wave velocity in the fracture.

Next, the wave motion in the hard formation is considered $v_f < v_s < v_c$ and $v_f < v_{ph} < v_s$. In this case a finite number of normal modes exist , each mode has a cutoff frequency and starts from the shear velocity at the cutoff frequency. Both the phase and the group velocities approach the fluid velocity as $k_r h \rightarrow \infty$ (see Paillet and White, 1982; and Tang and Cheng 1988).

Figure (3) shows the phase and the group velocities of the first three normal modes of sandstone formation as listed in Table (1).

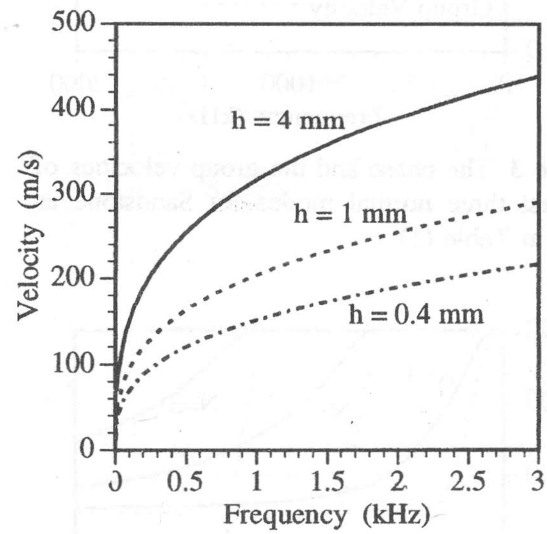

Figure 2. The phase velocity of Stoneley mode for different fracture widths of shale (1) as listed in Table (1).

In the case of soft formation, ($v_s < v_f < v_c$) in which a number of Leaky-P modes exist each has different cutoff frequency. The phase and the group velocities start at the compression velocity of solid at the cutoff frequency and approach the fluid velocity at high frequency (Tang and Cheng 1988). The phase and group velocity as function of frequency is illustrated in Figure (4). Shale (1) of Table (1) is used for this calculation. Another interesting case of soft formation (Lucite) used by Tang and Cheng (1988) in their experimental apparatus is studied. The physical parameters for this case study is given in Table (2). Figure (5) shows the phase and the group velocity for this case. It can be noticed that the group velocity of the fundamental mode in this case has distinguish stationary point .

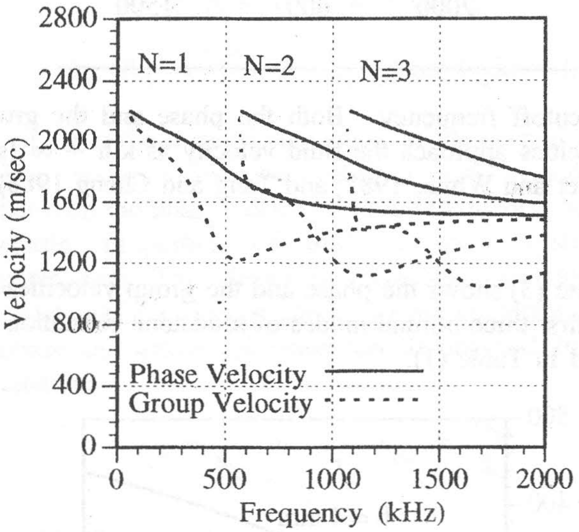

Figure 3. The phase and the group velocities of the first three normal modes for Sandstone as listed in Table (1).

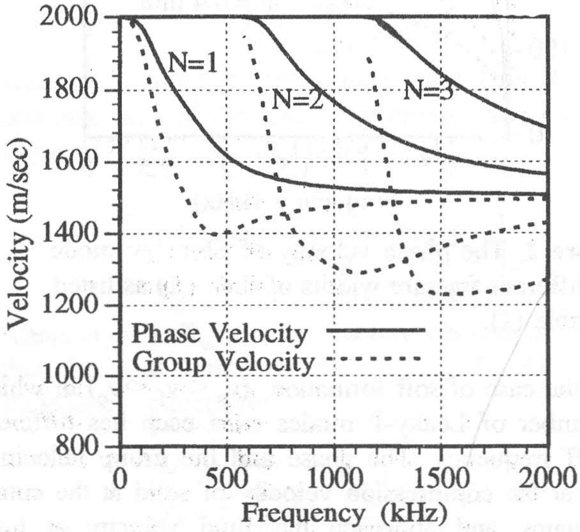

Figure 4. The phase and the group velocities of the first three compression modes for soft formation, Shale (1) as listed in Table (1).

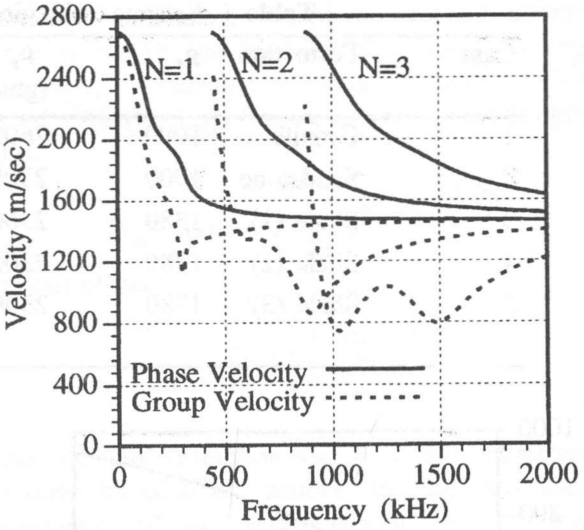

Figure 5. The phase and group velocity of the first three compression modes for soft formation, Lucite as listed in Table (2).

The effect of increasing the Poisson's ratio on the compression normal modes for shale (1) is presented in Figure (6). Increasing the Poisson's ratio above 0.35 has a significant change in the phase and the group velocities. This situation is also analogous to that of a fluid filled a borehole in soft formation (Paillet and Cheng 1986).

Table 2. The physical parameters of Lucite

ρ_s(g/cm^3)	v_s (m/sec)	v_c(m/sec)
1.2	1300	2700

The real and imaginary parts of the normalized fracture impedance (The fracture resistance and reactance) versus frequency are plotted in Figure (7a, 7b) for different rock materials as listed in Table (1). They are normalized by the characteristic impedance of the fracture ($Z_f = (v_f \rho_f)/A_f$). Note that for soft formation, Shale (1), has higher resistance and reactance than the hard formation, Granite and Sandstone. At high frequency the fracture resistance reduces to the characteristic impedance of the fracture while the fracture reactance approaches zero as explained at the end of section IV.

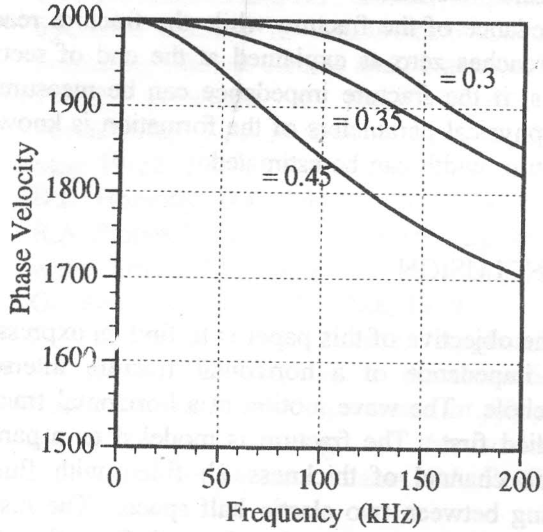

Figure 6a. The effect of Poisson's ration on phase velocity of the compression first normal mode for soft formation (Shale 1) as listed in Table (1).

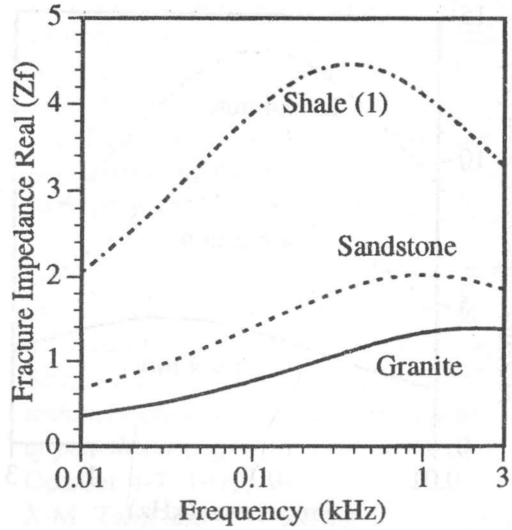

Figure 7a. The real part of the normalized fracture impedance versus frequency for different rocks materials as listed in Table (1).

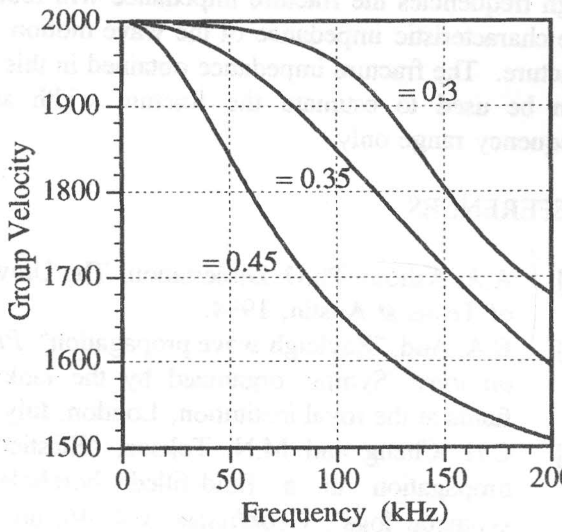

Figure 6b. The effect of Poisson's ration on group velocity of the compression first normal mode for soft formation (Shale 1) as listed in Table (1).

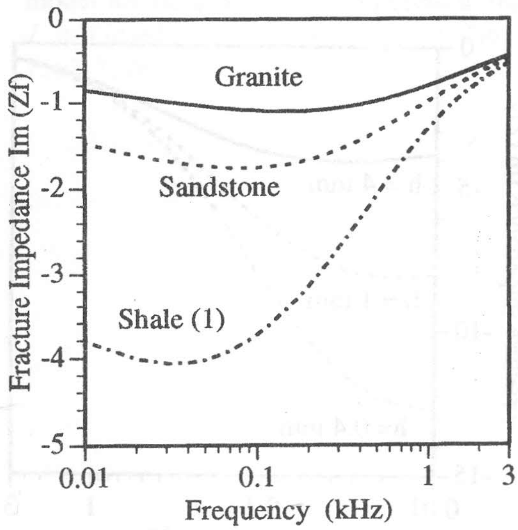

Figure 7b. The imaginary part of the normalized fracture impedance versus frequency for different rocks materials as listed in Table (1).

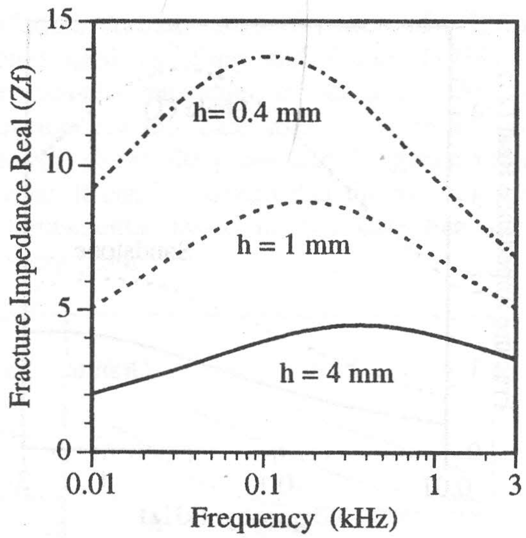

Figure 8a The real part of the normalized fracture impedance (The Fracture Resistance) of Shale (1) of Table (1) versus frequency for different fracture thicknesses.

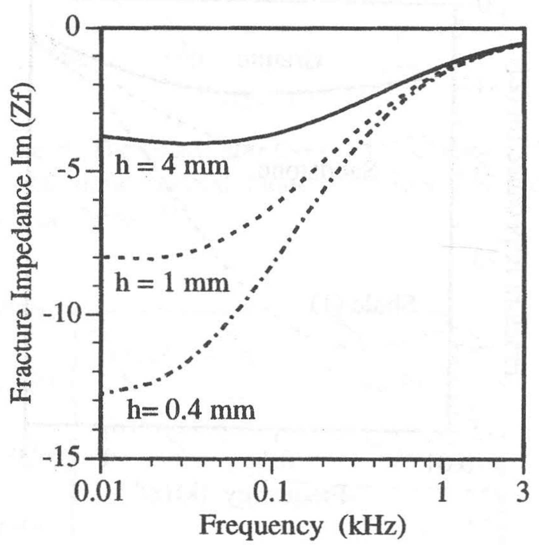

Figure (8b). The Imaginary part of the normalized fracture impedance (The Fracture Reactance) of Shale (1) of Table (1) versus frequency for different fracture thicknesses.

The calculated real (resistive) and imaginary (reactive) parts of the normalized fracture impedance versus frequency for different fracture width 2h is shown in Figure (8a, b). It is clear that increasing the fracture width will cause the fracture resistance and the fracture reactance to decrease. Also, at high frequency the fracture resistance reduces to the characteristic impedance of the fracture while the fracture reactance approaches zero as explained at the end of section ii. Thus, if the fracture impedance can be measured and the physical parameters of the formation is known, the fracture width can be estimated

CONCLUSION

The objective of this paper is to find an expression of the impedance of a horizontal fracture intersects a borehole. The wave motion in a horizontal fracture is studied first. The fracture is modeled as a paralleled plane channel of thickness 2h filled with fluid and laying between two elastic half space. The results of this study demonstrate that the soft formation (Shale) has higher resistance and reactance than the harder formation (Sandstone and Granite) Also, fracture with small width has higher resistance and reactance. At high frequencies the fracture impedance will reduce to the characteristic impedance of the wave motion in the fracture. The fracture impedance obtained in this study can be used to estimate the fracture width at low frequency range only.

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