THE EFFECT OF CONDUCTING AND NON-CONDUCTING SURROUNDINGS ON THE EARTH SURFACE POTENTIAL OF BURIED GROUNDING SYSTEMS

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ABSTRACT

In this paper, a moment method-variational approach which has been used in field theory of guided waves is applied, after modifications, in the analysis of complex grounding systems. The effect of surroundings, such as foundations, rocks, conducting tanks, etc., on the earth surface potential at a buried grounding system(s) is studied. The study deals with the calculation of the reduction or the increase in the stored electrostatic energy due to the surroundings. The interaction between different conducting structures of multiple grounding systems is analyzed. The results obtained in the present paper are compared with the accurate but very time consuming point matching moment method results and the agreement is good.

Keywords: Grounding systems, Touch voltage, Grounding resistance, Grounding grid, Grounding rodbed.

1. INTRODUCTION

Solving complicated engineering field and circuit problems by the simplest possible techniques, without sacrificing accuracy, depends on the selection of the related Green's function which is used in the moment method. In the field of antenna theory, digital circuits, microstripline, and electrooptics the transformation of infinite number of images into a fewer complex images so that the impedance matrix has few elements has led to the reduction in the required calculation time by several order of magnitudes [1].

Following similar goal (i.e. reduce computation time and keeping the accuracy high), in previous papers, different theories and techniques are used to derive simple formulas for the resistance of different grounding systems buried in two-layer earth. These theories and techniques such as the analytical use of the moment method, a novel approach of synthetic asymptote, dual concept between resistance and capacitance of the system, a semi-Pathygreon theory, linear interpolation technique, and the theory of multiple images have been applied to achieve a high accuracy in the resistance calculation (average error < 8%) [2-8]. These formulas are ready to use in the field by engineers, however, such techniques cannot be used to derive formulas to calculate the surface potentials on the earth (a very important requirement in safety grounding system analysis). The earth surface potential calculation requires knowing the accurate current distribution outward from each conductor in the system. Employing the moment method in field calculation to get a high accurate current distributions requires a very fine segmentation of each conductor which will lead to a huge matrix size (at least 2000 X 2000) for a typical grounding system. Such a technique is simple but requires a large computer resources. In previous papers [9-14], we have proposed a different approach which depends on using an error reduction property technique, namely the Galerkin's moment method - variational principle. This technique enabled us to assume a constant current density on each conductor, then using matrix equations to solve for these current. The variational principle (averaging the voltage on each conductor) enabled us to reduce the resistance matrix down to 30X30 while reserving the high accuracy (resistance error < 3%). Despite the high accuracy in the total current outward from the grounding system, the current distribution along each conductor differs from the exact one especially at the conductor's tips. This error in the current distribution resulted in boundary condition violation at the conductor's tips (i.e. the assumed constant voltage). To satisfy the boundary conditions at the conductor's tips a new technique is proposed where a sphere at each conductor end and at each conductors intersection is placed. The current of the spheres is determined so that the boundary conditions are satisfied on the spheres. This has led to the increase in the size of the resistance matrix, however, the size is still too small as compared with the conventional point matching moment method matrix size. The error in calculated earth surface potential using the Galerkin's moment method, the variational principle, and the concept of matching spheres is < 5%.

The high speed of calculations combined with the high accuracy was the power of the proposed technique. This powerful tool enabled us to perform a parametric analysis on grounding system to have a problem insight and to tackle more complicated problems such as the study of general grounding system buried near foundations and other conducting structures (connected or disconnected from the active grounding system). In this paper, the powerful Galerkin's moment method - variational principle - matched spheres technique is used to study and analyze multiple grounding systems buried near foundations and other current obstacles. The concept used in calculation is based on calculating the reduction or the increase in the stored electrostatic energy so that the variation in the grounding resistance is calculated. The new current distributions due to the obstacles and the nearby conductors are calculated to determine the variation in the earth surface potential. The present results are compared with the output of the very time consuming point matching moment method computer program and the agreement is good.

2. VARIATIONAL PRINCIPLE IN DIFFERENTIAL FORM : STORED ELECTROSTATIC ENERGY "W_e"

For a volume V_o which contains an active grounding structure (see Figure 1).

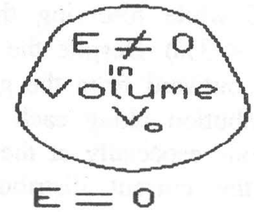

Figure 1. A non zero electric field volume.

the stored electrostatic energy W_e can be written as

$$W_e = \frac{\epsilon}{2}\iiint_{V_o} \nabla\phi_o \cdot \nabla\phi_o \, dv \tag{1}$$

where

ϕ_o true potential function

V_o volume in which $\nabla\phi_o$ not equal to 0

and $E = -\nabla\phi$

Let ϕ_o be the exact solution and $\phi = \phi_o + \delta\phi$ be an approximate solution in the same volume V_o. We shall then show that $\delta\phi$ would cause only a second order change in W_e.

$$\delta W_e = \frac{\epsilon}{2}\iiint_{V_o} \nabla(\phi+\delta\phi)\cdot\nabla(\phi+\delta\phi)dv - \frac{\epsilon}{2}\iiint_{V_o}\nabla\phi\cdot\nabla\phi dv$$

$$= \frac{\epsilon}{2}\iiint_{V_o} 2\nabla\phi\cdot\nabla\delta\phi dv + \frac{\epsilon}{2}\iiint_{V_o}\nabla\delta\phi\cdot\nabla\delta\phi dv \tag{2}$$

The last term of Eqn. (2) is very small and can be neglected. By using divergence theorem on function (U+V)

$$\iiint_{V_o} \nabla U\cdot\nabla V \, dv = \iiint_{V_o} (\nabla U\cdot\nabla V + U\nabla^2 V) \, dv$$

$$= \iint_{S_o} U\nabla V\cdot dS = \iint_{S_o} U\frac{\partial V}{\partial n} dS \tag{3}$$

Let $\phi = U$ and $\delta\phi = V$

$$\therefore \iiint_{V_o} \nabla\phi\cdot\nabla\delta\phi dv = \iint_{S_o} \delta\phi\frac{\partial\phi}{\partial n} dS \tag{4}$$

Now as $r \to \infty$, $\phi \alpha 1/r$, and $\partial\phi/\partial n \alpha 1/r^2$ and $\int\int dS = S_o \alpha r2$, $\alpha \equiv$ is proportional to

$$\therefore \iint_{S_o} \phi\frac{\partial\phi}{\partial n} dS = 0 \text{ as } r \to \infty \tag{5}$$

This means that

$$\delta W_e = \frac{\epsilon}{2}\iiint_{V_o} \nabla\delta\phi_o\cdot\nabla\delta\phi_o \, dv \tag{6}$$

Thus a first order change $\delta\phi$ from the exact solution ϕ_o causes only a second order change in W_e.

For the case of lossy earth instead of dielectric medium, we may express the power dissipated as

$$P_o = \frac{1}{\rho}\iiint_{v_o} \nabla\phi_o \cdot \nabla\phi_o \, dv \tag{7}$$

and

$$\delta P = \frac{1}{\rho}\iiint_{v_o} \nabla\delta\phi \cdot \nabla\delta\phi \, dv \tag{8}$$

where ρ is the resistivity of the medium.

3. GROUNDING STRUCTURE NEARBY HOUSING FOUNDATION

Figure (2) shows a grounding system (in the form of grid and rodbed) nearby a buried foundation.

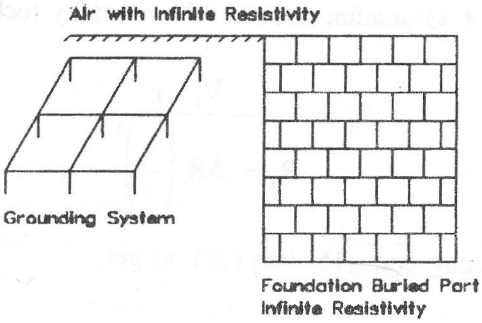

Figure 2. A buried foundation nearby the grounding system.

In foundation $\rho_f = \infty$, therefore from variational principle

$$P = \frac{1}{\rho_1}\iint_{v=v_f} \nabla\phi_o \cdot \nabla\phi_o \, dv \tag{9}$$

where v_f is the volume of the buried part of the foundation and ρ_1 is the earth resistivity. ϕ_o is the original potential distribution without foundation. With the foundation ϕ_o is an approximation. The error in ϕ from the true solution causes only a second order change in P and may be neglected. The resistance, R_f, of the system with foundation can be written as

$$R_f = \frac{V^2}{P} = \frac{1}{G} \tag{10}$$

where P is given by Eqn. (9), Eqn. (8) can be written as

$$p = p_O - \frac{1}{\rho_1}\iiint_{v_f} \nabla\phi_o \cdot \nabla\phi_o \, dv \tag{11}$$

$$G = G_o - \frac{1}{\rho_1 V^2}\iiint_{v_f} \nabla\phi_o \cdot \nabla\phi_o dv \tag{12}$$

$$= G_o - \delta G \tag{13}$$

and

$$R_f = \frac{1}{G_o - \delta G} \tag{14}$$

The integration of Eqn. (12) can be done numerically as follows

$$-\nabla\phi = \overline{E} = -\overline{a}_x\frac{\partial\phi}{\partial x} - \overline{a}_y\frac{\partial\phi}{\partial y} - \overline{a}_z\frac{\partial\phi}{\partial z}$$

then

$$\nabla\phi\cdot\nabla\phi = \left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2$$

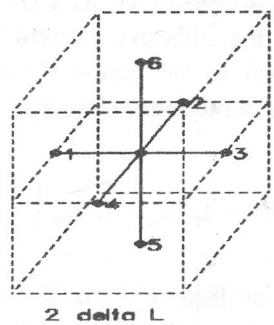

Figure 3. Coordinates for electric field calculations.

$$\nabla\phi\cdot\nabla\phi = \left(\frac{1}{2\Delta\ell}\right)^2[(\phi_3-\phi_1)^2+(\phi_4-\phi_2)^2+(\phi_6-\phi 5)^2]$$

$$(\nabla\phi\cdot\nabla\phi)\Delta V = (2\Delta\ell)\,[(\phi_3-\phi_1)^2+(\phi_4-\phi_2)^2+(\phi_6-\phi_5)^2]$$

Therefore the foundation can be divided into small cubes, the electric field at each cube center is calculated and the integral of Eqn (12) is computed as a summation. The following form is used to calculate the integral

$$\iiint_{V_f} \nabla\phi_o \cdot \nabla\phi_o \, dv = 2\Delta\ell \sum_{V_f} [(\phi_{x(n+1)} - \phi_{x(n-1)})^2$$

$$+ (\phi_{y(m+1)} - \phi_{y(m-1)})^2 + (\phi_{z(p+1)} - \phi_{z(p-1)})^2]$$

4. DERIVATION OF THE TOUCH VOLTAGE

Figure (4) shows a grounding system and a nearby rock. If the equipotential on the grounding system is V_1 and the current outward from this structure is I_1, then the resistance, R_1 of the structure without the rock can be calculated from

$$V_1 = I_1 R_1 \tag{15}$$

where R_1 is determined from the Galerkin's moment - method - variational principle technique. With the rock, the modified current outward from the grounding system is I_2 and the modified resistance is R_2. Since the voltage is the same as V_1, one can write

$$V_2 = V_1 = I_2 R_2 \tag{16}$$

R_2 is known from the differential variational principle of Eqn. (14). I_2 is considered as a constant and has to be adjusted for the effective current due to the rock which is assumed to be remote from the grounding system. We may write Eqn. (16) as

$$V_1 = I_2 R_1 + I_2 R_m \left(\frac{r_o}{r}\right)^2 \tag{17}$$

The first term of Eqn. (17) is the self term of the system where I_2 has to be calculated. The second term is the induced term of grounding system due to the rock, this mutual term is due to the induced dipole on the rock. R_m is the induced resistance which is the change in the resistance due to the existence of the rock. In Eqn. (17) we have taken $1/r^2$ variation due to the induced dipole. R_m can be written as

$$R_m = (R_2 - R_1) = \delta R \tag{18}$$

The touch voltage with the rock, V_{Tf} is related to the touch voltage without the rock, V_T through the following relation :

$$V_{Tf} = V_T \left(\frac{I_2}{I_1}\right) \tag{19}$$

i.e. the rock does not introduce touch voltage directly, only the grounding system does. At location r of grounding system close to the rock I_2 is smaller due to more contribution of voltage from rock and at location r farther from rock I_2 is larger. This variation is now introduced to the touch voltage equation (19). Substitution of Eqn. (18) into (17), gives

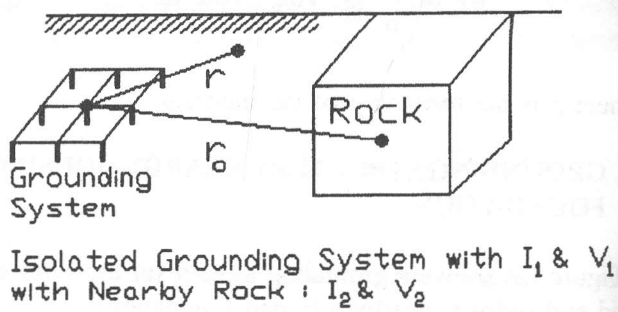

Figure 4. Grounding system with a nearby rock.

$$I_2 = \frac{V_1}{R_1 + \delta R \left(\frac{r_o}{r}\right)^2} \tag{20}$$

Substituting eqn. (15) into (20), to get

$$\frac{I_2}{I_1} = \frac{1}{1 + \frac{\delta R}{R_1}\left(\frac{r_o}{r}\right)^2} \tag{21}$$

Now from eqns. (21) and (19), to get

$$V_{Tf} = V_T \frac{1}{1 + \frac{\delta R}{R_1}\left(\frac{r_o}{r}\right)^2} \tag{22}$$

5. MULTIPLE GROUNDING SYSTEMS

Figure (5) shows a multiple grounding system, each grounding system consists of grid and rodbed.

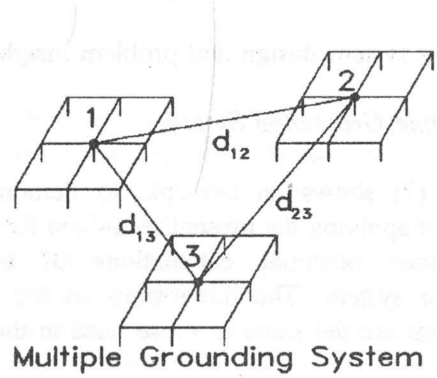

Figure 5. Multiple Grounding System.

Let all grounding systems be linked with unearthed wires (in this case $V_1 = V_2 = ... = V_N = V$). The resistance matrix of the combined grounding structure is constructed as

$$\begin{pmatrix} V \\ V \\ . \\ . \\ V \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & .. & .. & R_{1N} \\ R_{21} & R_{22} & .. & .. & R_{2N} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ R_{N1} & R_{N2} & .. & .. & R_{NN} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ . \\ . \\ I_N \end{pmatrix} \tag{23}$$

where R_{nn} are known (i.e. the grounding resistance of each grounding system in isolation which is calculated using the Galerkin's - moment method and n is 1 to N. N is the total number of grounding systems. R_{mn} is the mutual resistance between grounding systems m and n. The distance between different grounding systems is, typically, greater than the dimensions of the grounding systems, therefore, Coulomb's law can be applied as

$$R_{mn} = \frac{\rho(d_{mn})}{2 \pi d_{mn}} \tag{24}$$

where d_{mn} is the distance between the centroids of the grounding systems m and n and $\rho(d_{mn})$ is the apparent resistivity of reference [15] and is rewritten as

$$\rho(d_{mn}) = \rho_{upper} + 4\rho_{upper} \sum_{p=1}^{\infty} \frac{K^p}{\sqrt{1+\left(\frac{2ph}{d}\right)^2}} - 4\rho_{upper} \sum_{p=1}^{\infty} \frac{K^p}{\sqrt{4+\left(\frac{2ph}{d}\right)^2}} \tag{25}$$

where h is the height of the upper layer in the two-layer model earth, d is the distance between the two electrodes (which is used in resistivity measurement) or the horizontal distance between the two grounding systems center to center.

Since all grounding systems are connected with unearthed wire they will be at the same potential (Vo say). The current-voltage matrix can be written as

$$[V_o] = [R_{total}]\,[I_i] \tag{26}$$

where I_1, I_2, ...,I_N (the current outward from grounding systems number 1, 2, ..., N) can be determined by solving the above matrix equation. The total resistance of the multiple grounding system is given by

$$R_{total} = \frac{V_o}{\sum_{n=1}^{N} I_n} \tag{27}$$

The touch voltage above the active grounding system number n is given by

$$V_{T_{n,multi}} = V_{T_{n,isolated}} \frac{I_{n,multi}}{I_{n,isolated}} \tag{28}$$

6. RESULTS AND DISCUSSION

6.1 *Effects of Foundations*

The amount of stored electrostatic energy in the system is decreased or increased depending on the resistivity of the obstacle (rock, conducting bank, another soil, etc.) with respect to the resistivity of the soil surrounding the active grounding system (see Figure (2)). For values of ρ_f other than ∞ Eqn. (20) is modified as follows

$$\delta R_{modified} = \delta R\, \rho_1 \left(\frac{1}{\rho_1} - \frac{1}{\rho_f}\right) \tag{28}$$

Figure (6) shows an example of a grounding system of the following parameters :

- Earth resistivity 100 Ω.m
- Sunken depth 0.5 m

- Grid size 2X2 — 20m side length and conductor diameter of 0.01m
- 9 Driven rods — 10m length and 0.01m radius
- Obstacle — Buried depth = 5m and resistivity = 1000 Ω.m

Due to assumption violation, reliable calculations are limited within an area equal to 1.2 times the grounding system area as shown by the dotted box of Figure (6).

Table (I) demonstrates the touch voltage above the grounding system for a number of selected points (1 to 4 shown in Figure 6). The present results are compared with a time consuming point matching moment method computer program.

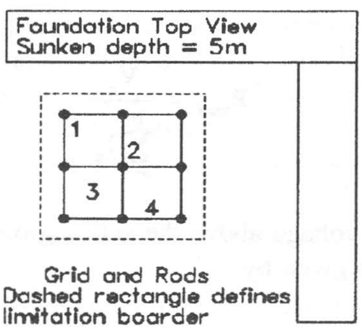

Figure 6. The layout of the grounding system and the obstacle of the example.

Table I. A comparison between the present results and the point matching moment method results.

point Number	Present V_{Tf} (%)	P.M.M.M V_{Tf} (%)
1	19.2	20.0
2	20.9	20.3
3	23.0	21.0
4	21.0	20.5

Note : P.M.M.M = point matching moment method.

As shown in Table (I) the agreement is satisfactory. Such an agreement is acceptable since the present technique is faster than the point matching moment method by several orders of magnitudes. In the mean time, the present technique is developed, mainly, to give the field engineer a proper tool for a preliminary grounding system design and problem insight.

6.2 *Multiple Grounding System*

Figure (7) shows an example to demonstrate the validity of applying the present technique for resistance and surface potential calculations of a multiple grounding system. The dimensions of the grids and driven rods are the same as those used in the previous example.

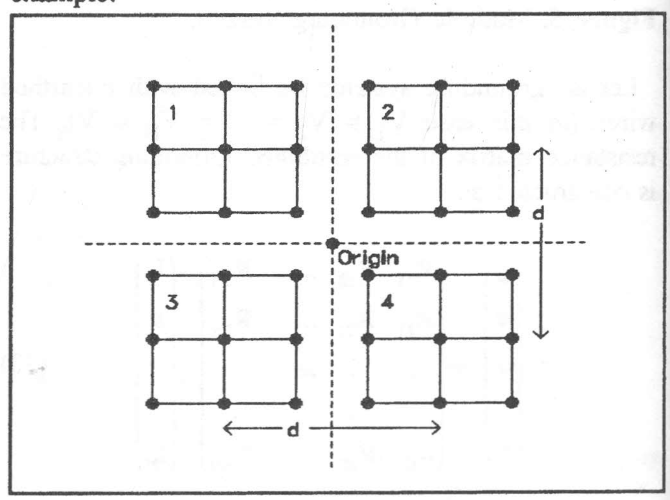

Figure 7. A multiple grounding system.

Table II. Variation of the multiple grounding system resistance with the distance between its components.

Resistance (Ω)	d (m)
1.5	30
1.1	50
0.77	100
0.66	150
0.58	250
$R_{isolated, present}$ = 1.79 Ω and 1.81 Ω [16]	

The four grounding systems are selected to be identical to prove the validity of the present technique. It is shown in Table (II) that the mutual resistances are neglected if the distance between the centroids of the different grounding system becomes very large. In this case the resistance of the four grounding systems is the resistance of the isolated grounding system divided by 4.

7. CONCLUSIONS

The Galerkin's moment method with the variational principle has been used to calculate the grounding resistance and the surface potential of complex grounding system structures. The present paper combined with the previous papers represent a powerful tool for gr⌐..nding system analysis and performing parametric studies. Such studies were not possible due to the large computer and calculation time requirements. Despite the many approximations in the present derivations, the results gave a good problem insight so that the variation trends (of the grounding resistance and the surface potential) can be determined rapidly with a good accuracy.

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