RELIABILITY OF ISUM MODELLED UNIAXIALLY LOADED PLATES

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ABSTRACT

The reliability of plates modelled with the Idealized Structural Unit Method (ISUM) and considered uniaxially loaded is studied. ISUM is used to assess the strength while a First Order Reliability Method (FORM) is selected to calculate the safety index of the plates for both failure modes, namely, yielding and buckling. Before the reliability analysis is carried out, the random variables involved are delineated and suitable probabilistic models are selected. The present study combines nonlinear strength analysis with an approximate method to evaluate the safety index of the plates under study with consideration of the effect of parameters such as plate thickness and statistical load variation on the safety index. Results for an example plate are presented in the form of charts, that show the target safety index to the required design load for different plate thicknesses, and that clearly show the influence of load variability on the safety of the plates.

Keywords: ISUM, Yielding, Buckling, Probability of Failure, Reliability, FORM.

Nomenclature

Note: The terms not defined here are uniquely defined in the sections in which they are used.

B	Strain-displacement matrix
D	Stress-strain matrix
E	Young's modulus of elasticity
H	Strain hardening stiffness matrix
K^B	Post-buckling stiffness matrix
$K_{im}{}^B$	Post-buckling stiffness matrix of the imaginary plate
K^e	Elastic failure-free stiffness matrix
K^P	Elastic-plastic stiffness matrix
M	Safety margin
R	Vector of nodal forces
R_i	Vector of nodal forces at point i
U	Vector of total nodal displacement
U_i	Vector of total nodal displacement at point i
u_i	Displacement in x direction at point i
v_i	Displacement in y direction at point i
p_f	Probability of failure
β	Safety index
Γ	Gamma function
Γ_B	Buckling function
Γ_Y	Yielding function
Γ_{yi}	Yielding function at checking point i
ϵ	Strain vector
Φ_i	$\{\partial\Gamma/\partial R\}_i$
υ	Poisson's ratio
σ	Stress vector
σ_o	Material yield stress
σ_x	Normal stress directed along the x axis
σ_{xav}	Maximum allowable average stress in x direction
σ_{xcr}	Critical buckling stress in x direction

1. INTRODUCTION

Plates, both stiffened and unstiffened, largely compose plated structures used in ship and offshore structures. They constitute the major structural component in ships and the main members in deck and accomodation structures of both fixed and floating platforms. The local response of these plates mainly influences the response of the major structural components. Usually, they are designed using special codes or rules. Such an approach while relatively straight forward and simple, does not yield the most efficient design. It is well recognised that structural problems are undeterministic, due to the random nature of the parameters involved. Clearly, the design of structures under different loads should be based on methods of nonlinear strength analysis and structural reliability analysis.

Ship plates are generally subjected to combined biaxial stresses, shear stress and lateral pressure, mainly due to still water bending moment and low frequency wave induced bending moment. It is important to study their reliability [1,2]. In this study, the strength analysis and the reliability analyis of rectangular plate panels subjected to uniaxial loads are combined and investigated.

Firstly, the Idealized Structural Unit Method (ISUM) is adopted to analyze the non-linear behavior of plate panels until their ultimate strength state. Thus, the response of the considered plate panels under uniaxial compression is evaluated at any loading condition.

The effect of initial imperfections suh as, initial deflection and residual stresses is not included but will be studied in a forthcoming paper by the same authors.

Secondly, a reliability analysis of the plate panels is performed using a First Order Reliability Method (FORM). The study of the reliability of plate panels includes statistical modeling of dimensions, material properties and loading. Also, possible formulations of strength requirements and failure criteria are included. Several examples of square plates with different thicknesses subjected to uniaxial load are carried out and the results are presented.

The aim of this study is to develop an efficient approach to assess the reliability, i.e. calculate the safety index of uniaxially loaded square plates in yielding and in buckling. Results are presented in the form of charts, that show the target safety index to the required design load for different plate thicknesses. To allow for inclusion of the influence of uncertainty in the load estimation, the effect of changing its coefficient of variation is included.

2. STRENGTH ANALYSIS

The hull of a ship is fundamentally regarded as a thin-walled box girder whose major portion is usually composed of stiffened plate panels. Under various combined loads the hull is subjected to longitudinal bending, shear and torsion. Locally, each member is subjected to lateral loads, axial forces, bending moments and shearing forces. The response of the hull girder is mainly affected by the response of the individual members, such as plate panels. In order to take such effects into account, and to evaluate stresses acting on each individual member, a nonlinear analysis of the hull girder should be performed, taking into account geometric and material nonlinearities. Different methods are available to analyze the behavior of structures until their ultimate strength is reached. The nonlinear Finite Element Method (FEM) is the most powerful method for analyzing complicated behavior until ultimate strength and it has a wide application in various fields. However, the resources required by this method put a practical limit on the size of the problem to be handled. In this paper, the Idealized Structural Unit Method (ISUM) [3] is adopted to analyze the behavior of a plate panel until and after its ultimate strength state .

2.1 *General Behavior of a Rectangular Plate Panel*

The behavior of a rectangular plate panel when subjected to an increasing proportional load composed of in-plane biaxial compressive forces, in-plane bending moment and in-plane shearing forces may be summarized as follows.

Before any failure has taken place, stresses in the middle plane of the plate are linearly distributed. Displacements are linearly proportional to the applied load. As the applied forces increase, the plate buckles when the acting forces satisfy the buckling criterion. When buckling occurs, lateral deflection is induced in the plate. As the load increases the lateral deflection becomes significant. This results in a decrease of the stiffness of the plate and stresses of nonlinear distribution are developed in the middle plane of the plate. The plate panel may continue to carry further load as long as its edges continue to be effectively supported. However, the large axial compressive stress, in combination with the bending stress may cause yielding to start and spread over a considerable area of the plate. This leads to further decrease in stiffness and causes the plate to reach its ultimate strength. As compressive displacement continues to increase after the ultimate strength is reached, the plate exhibits a reduction of its carrying capacity. If the characteristics of the plate are such that buckling does not occur, the plate may continue to carry further load until it reaches its fully plastic strength. Lateral load is disregarded when the inplane behavior of ship plates is considered. Consequently, lateral loads are not considered in this work.

2.2 *Description and Behavior of ISUM Rectangular Plate Element*

Ueda and Rashed [3] developed an effective method analysis of nonlinear behavior of large structures. In this method, the structure is divided into the biggest possible structural units (components), whose geometric and material nonlinear behaviors are idealized. In the middle eighties [4,5], a rectangular plate element and a rectangular stiffened plate element were developed. The developed elements predict the behavior until the ultimate strength is reached. An improved ISUM rectangular plate element was developed to evaluate the reduction of the post-ultimate strength [6].

The ISUM element is a rectangular plate as shown in Figure (1a). Its edges are assumed to remain straight after deformation. The plate element has four nodal points, one at each corner. Bending stiffness of the element is assumed to be negligible in comparison with the bending stiffness of the whole structure. Therefore the element is treated as a membrane. Each nodal point has two translatory degrees of freedom. The nodal displacement and the nodal force vectors are represented as follows:

$$U = [U_1\ U_2\ U_3\ U_4]^T,\ U_i = [u_i, v_i]^T \quad (1)$$

$$\mathbf{R} = [\mathbf{R}_1\ \mathbf{R}_2\ \mathbf{R}_3\ \mathbf{R}_4]^T,\ \mathbf{R}_i = [R_{xi}, R_{yi}]^T \quad (2)$$

where a suffix T indicates the transposed matrix.

The plate is simply supported at its edges. In-plane biaxial compressive forces, in-plane bending moments and in-plane shearing forces are applied as shown in Figure (1b).

The behavior of the plate is investigated based on fundamental theories, refined theoretical analysis such as the finite element method and experimental results. The behavior is then idealized and conditions are formulated for the possible or expected failures in the plate such as buckling and yielding. Stiffness matrices are derived in each respective state, i.e. before any failure and after different combinations of failure. The incremental method is applied and the response of the element is evaluated at each loading step. The idealized behavior of the rectangular plate element when subjected to an increasing load is illustrated in Figure (2) and may be summarized as follows.

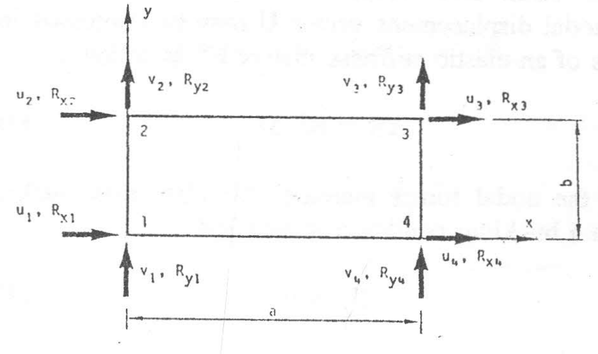

Figure 1a. ISUM rectangular plate element.

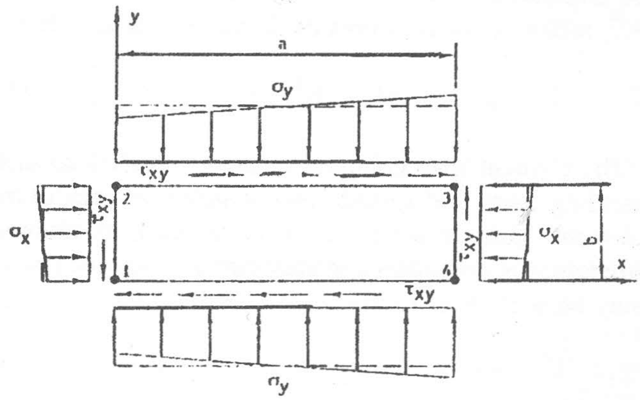

Figure 1b. Rectangular plate panel and applied loads.

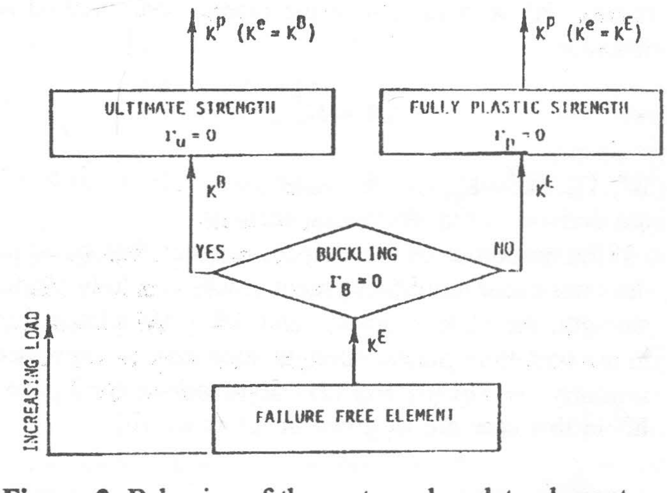

Figure 2. Behavior of the rectangular plate element.

The relation between the nodal force vector **R** and the nodal displacement vector **U** may be conveniently expressed in the incremental form. Before any failure has taken place, the relation between an increment **ΔR** of the nodal force vector **R** and an increment **ΔU** of the nodal displacement vector **U** may be expressed in terms of an elastic stiffness matrix $\mathbf{K}^e$ as follows,

$$\Delta \mathbf{R} = \mathbf{K}^e \, \Delta \mathbf{U} \qquad (3)$$

As the nodal forces increase, the plate may buckle when a buckling condition is satisfied.

$$\Gamma_B = 0 \qquad (4)$$

where Γ_B is a buckling function.

After buckling, the relation between **ΔR** and **ΔU** may be expressed in terms of a tangential stiffness matrix $\mathbf{K}^B$, taking account of post-buckling effects, as follows,

$$\Delta \mathbf{R} = \mathbf{K}^B \, \Delta \mathbf{U} \qquad (5)$$

The element may continue to carry further load until yielding starts and spreads over a sufficient area of the element. This causes the element to reach its ultimate strength. A condition for yielding, Γ_{yi} at any point i may be written as follows:

$$\Gamma_{yi} = 0 \qquad (6)$$

After yielding starts, the relation between **ΔR** and **ΔU** may be expressed in terms of an elastic-plastic stiffness matrix $\mathbf{K}^P$ with the aid of the plastic node method as follows:

$$\Delta \mathbf{R} = \mathbf{K}^P \, \Delta \mathbf{U} \qquad (7)$$

$\mathbf{K}^e$, Γ_B, $\mathbf{K}^B$, Γ_{yi} and $\mathbf{K}^P$ appearing in Eqs. (3) to (7) are derived in the following sections.

If the properties of the element are such that buckling does not occur until the element reaches its fully plastic strength, the yield condition and **ΔR** - **ΔU** relationship in the post-fully-plastic strength state may be expressed similarly by Eqs.(6) and (7). Expressions for Γ_{yi} and $\mathbf{K}^P$ in this case are as given in reference [6].

2.2.1 *Failure-Free Stiffness Matrix*

Before any local failure, such as buckling, of the plate element occurs, membrane strains are assumed to be linearly distributed, which is reasonable for a plate panel in a large structure. The displacement functions satisfying the conditions of linearly varying boundary displacement and constant shear strain along the plate sides are assumed as follows:

$$u_{xy} = a_1 + a_2 x + a_3 y + a_4 xy + b_4 (b^2 - y^2)/2$$

$$v_{xy} = b_1 + b_2 x + b_3 y + b_4 xy + a_4 (a^2 - x^2)/2 \qquad (8)$$

where u_{xy} and v_{xy} are the displacements in x and y directions at a point (x,y), a_i and b_i are coefficients, and a and b are the length and breadth of the element.

The relation between $\Delta\epsilon$, an increment of the strain vector ϵ, and **ΔU**, an increment of the nodal displacement vector **U**, may be derived as follows:

$$\Delta \epsilon = \mathbf{B} \, \Delta \mathbf{U} \qquad (9)$$

where,

$$\Delta \epsilon = [\Delta\epsilon_x \; \Delta\epsilon_y \; \Delta\gamma_{xy}]^T \text{ and}$$

B is the strain-displacement matrix.

The relation between $\Delta\sigma$ an increment of the stress vector σ and $\Delta\epsilon$ may be written as

$$\Delta \sigma = \mathbf{D}^e \, \Delta \epsilon \qquad (10)$$

where $\Delta\sigma = [\Delta\sigma_x \; \Delta\sigma_y \; \Delta\tau_{xy}]^T$ and

$\mathbf{D}^e$ is the stress-strain matrix in the elastic range.

The elastic failure free stiffness matrix $\mathbf{K}^e$ may then be derived as follows:

$$\mathbf{K}_e = \int \mathbf{B}^T \, \mathbf{D}_e \, \mathbf{B} \, dV \qquad (11)$$

where V is the volume of the element.

The stress in the element may be expressed as,

$$\sigma = \mathbf{D}^e \, \epsilon = \mathbf{D}^e \, \mathbf{B}\mathbf{U} \qquad (12)$$

2.2.2 *Buckling Condition and Post-Buckling Stiffness Matrix*

The buckling condition Γ_B of the rectangular plate element may then be written in terms of the average normal stresses σ_{xav} in x direction as:

$$\Gamma_B = \sigma_{xav}/\sigma_{xcr} - 1 \tag{13}$$

where σ_{xcr} is the buckling critical stress.

After the plate element has buckled, out-of-plane deflection is induced and the stress distribution in the middle plane of the element (membrane stress) becomes nonlinear. In order to continue to use the same displacement functions as in Eq.(8) in the post buckling range, an imaginary flat plate with linear stress distribution is considered. The material properties of this imaginary plate are determined such that it shows overall deformation equal to that of the buckled plate under the same load (same stiffness K_{im}^B). Then the buckled plate is replaced by an imaginary flat plate of homogeneous material, linear stress distribution and exhibits displacement similar to that of the original plate when subjected to the same load. That is, it has the same stiffness [6]:

$$K^B = K_{im}^B = \int B^T D^B B \, dV \tag{14}$$

2.2.3 *Ultimate Strength Condition and Elastic-Plastic Stiffness Matrix*

In the case of a simply supported rectangular plate which has buckled under uniaxial stress, normal stresses along a half buckling wave become as shown in Figure (3). This stress distribution is developed repeatedly along each half buckling wave length of the plate. Under this kind of stress distribution, yielding starts at any location where the induced stresses satisfy the yield condition applicable to the material of the plate. Yielding is assumed to start at any of the checking points where the von Mises yield condition is satisfied, that is:

$$\Gamma_y = \sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 - \sigma_o^2 = 0 \tag{15}$$

Ultimate strength will be reached after yielding has occurred at a sufficient number of locations. To evaluate the post-ultimate strength elastic-plastic stiffness matrix, it is first necessary to evaluate the elastic-plastic stress-strain relationship. Then an equivalent inplane elastic-plastic stiffness is evaluated taking account of plasticity and out-of-plane deflection. Finally the stiffness matrix is evaluated such that.

$$K^p = K^B - K^B \Phi_i \Phi_i^T K^B / S_{pi} \tag{16}$$

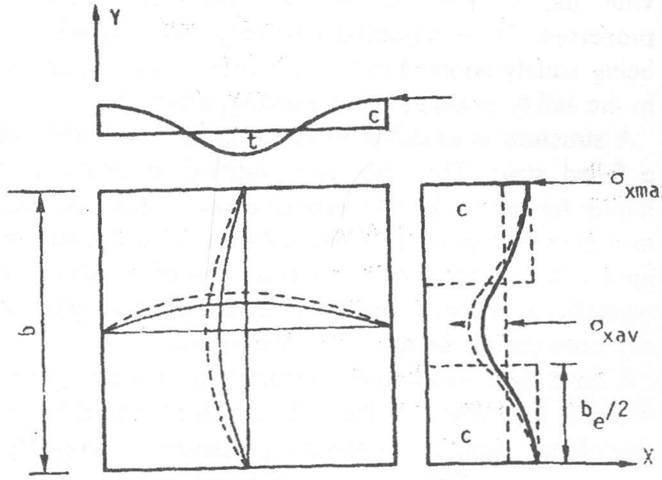

Figure 3. Stress distribution in a buckled plate panel under uniaxial compression.

When yielding occurs at m nodes, K^P may similarly be derived as follows,

$$K^p = K^B - K^B \Phi S_p^{-1} \Phi^T K^B \tag{17}$$

where, $\Phi = [\Phi_1, \Phi_2, ..., \Phi_m], S_p = \Phi^T(K^B + K^o)\Phi$

$K^o = \int B^T H B \, dV$ and H is an equivalent strain hardening matrix ,see Ref.[6].

Thus, σ_{xav} , the response of the considered plate under uniaxial compression load can be evaluated by calculating the considered stiffness matrices K^e, K^B, and K^P at any loading condition . That means, the response of the plate is calculated at the pre-buckling stage (linear stage), buckling point, post-buckling stage (nonlinear stage) and yielding point.

3. RELIABILITY ANALYSIS

3.1 *Introduction to Reliability Theory*

A deterministic approach was traditionally adopted in the analysis of structures in general and of ship structures in particular. Safety factors were established by means of engineering judgment to account for any uncertainties.

Fluctuations in loads, variability of material properties and uncertainties in the adopted analytical models have lead to the development of methods capable of dealing with the random nature of loads and material properties. These so-called reliability methods are now being widely adopted in the structural design stage and in the safety evaluation of existing structures.

A structure is assumed to be either in a safe state or a failed state. The state is quantified in terms of a failure function. The structure changes from a safe state to a failure state in different failure modes through its limit states. A limit state is a condition of the structure whereby it becomes unfit for its intended purpose at any time during its specified design life.

A limit state function or performance function g(**x**)is defined in terms of the set of basic variables **x**, describing loads, material properties, geometry, scantlings, etc. The limit state function satisfies:

$$g(\mathbf{x}) \begin{cases} <0 & \mathbf{x} \text{ in failure set} \\ =0 & \mathbf{x} \text{ on limit state surface} \\ >0 & \mathbf{x} \text{ in safe set} \end{cases} \tag{18}$$

The "reliability" of a structure is its ability to fulfill its design purposes for some specified time. Or, it is the probability that the structure will not reach any of its limit states during a specified time. It can be defined as:

$$R = 1 - p_f \tag{19}$$

The "probability of failure", p_f, is generally calculated as follows:

$$p_f = P\,(M \leq 0) = P\,(g(\mathbf{x}) \leq 0) = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{x}}(\mathbf{x}).d\mathbf{x} \tag{20}$$

where the safety margin, M, is defined as: $M = C - D = g\,(\mathbf{x})$ and $f_{\mathbf{x}}(\mathbf{x})$ is the joint probability density function (PDF) of the random vector **X**. The above multi-dimensional integral is practically impossible to calculate. Due to the previous fact, attention has focused on developing methods (first- and second-order methods) as well as methods based o efficient simulation techniques. Reliability theory and the methods mentioned above are comprehensively reviewed in [7] and [8].

3.2 *Modeling of Uncertainties*

In structural reliability, the calculated load effects (demand) and the strength of the structure (capacity) must be modeled to enable the assessment of its safety, see Figure (4).

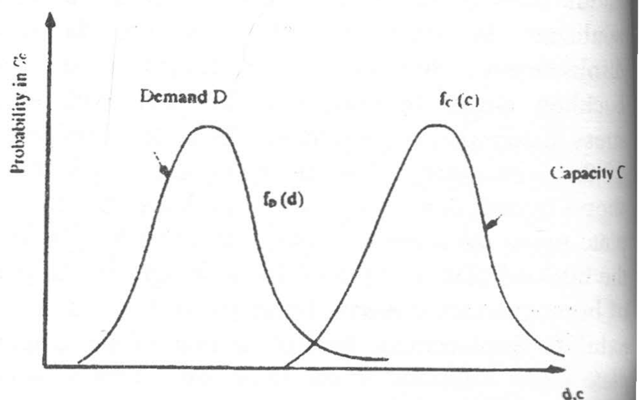

Figure 4. Typical distribution function of demand and capacity.

Uncertainties are involved in all steps. Thes uncertainties are the result of the stochastic nature c the loads, the variability in material properties an dimensions and the simplifications and assumption used in the different adopted models.

Uncertainties are classified into two categories [9] random (natural) and modelling. The former are due t the random nature of the loading environment and th resulting loads and to variability in material propertie and structure dimensions. The latter are due to lack o data on various phenomena and simplifications an idealizations in the analysis procedures. Modellin uncertainties will not be considered in this study.

In order to quantify the uncertainties, the basi random variables such as loads, material properties dimensions, etc. are defined and denoted as previousl stated by the vector **X**, and their outcomes denoted b the corresponding lower case vector, **x**. A listing of th variables affecting the reliability of uniaxially loade plates and their statistical variability, presented in [1 and [2], is given in Table (1).

Table 1. Statistical modelling of variables affecting plate reliability

Variable	Symbol	Description	Distribution	cov %
X(1)	t	Plate thickness	Log-normal	0.5-4
X(2)	σ_x	Normal stress in x-direction	Normal	15-40
X(3)	E	Young's modulus	Normal	1-2
X(4)	σ_o	Yield stress	Normal	4-10

3.3 *Reliability Analysis Method*

A reliability method is a method to decide if a structure is acceptably reliable. In this investigation, a First Order Reliability Method (**FORM**) is adopted whereby the limit state function is linearized.

The first step in the analysis is to transform the basic correlated random variables **X** to standard normal correlated variables **Z**, by using the Nataf model, see [10]. Uncorrelated variables **Y** are then obtained by multiplying **Z** with the triangularized covariance matrix. The limit state or performance function than becomes:

$$G_Y(y) = g_x(x) = 0 \qquad (21)$$

The idea in **FORM** is to approximate the limit state surface at the point **y*** closest to the origin 0 by its tangent hyperplane, refer to Figure (5). $\beta = |Y^*|$ is the Hasofer-Lind [11] safety index. To determine **y*** an optimization problem must be solved and a suitable algorithm [8] chosen.

The approximation to the failure probability is:

$$p_f \approx \Phi(-\beta) \Leftrightarrow \beta \approx -\Phi^{-1}(p_f) \qquad (22)$$

where Φ is the standard normal distribution function. The safety index β has no absolute significance but is a relative measure assessing the safety of the structure.

3.4 *Performance Functions for the ISUM Plate Element*

A design criterion is a condition that a structural element does not reach any of its limit states. A limit state is a state in which the structural element loses one of its intended functions. Structures must be designed with limits or constraints against different modes of failure. The two types of failures considered in this study for plate elements are:

- Buckling
- Yielding

Each failure mode is represented by a corresponding performance function in terms of the basic design variables.

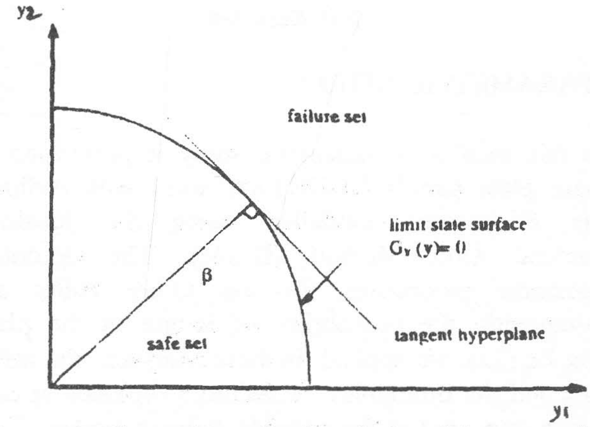

Figure 5. Illustration of safety index β in uncorrelated standard normal space.

3.4.1 *Performance Function for Design against Buckling*

Buckling occurs when the plate carries in-plane compressive loads. Here only loading in the x direction is considered. The buckling condition Γ_B representing the performance function of the rectangular plate element under consideration may be written in terms of the average normal stress σ_{xav} in the x direction, as given in Eq. (13), from which the following is deduced:

$$M = 1 - \frac{\sigma_{xav}}{\sigma_{xcr}} \begin{cases} <0 & \text{Failure set} \\ =0 & \text{Failure surface} \\ >0 & \text{Safe set} \end{cases} \qquad (23)$$

3.4.2 *Performance Function for Design against Yielding*

When the axial compressive load is small or the plate element is relatively thick and short, the plate will not buckle. When the maximum equivalent stress reaches the material yield stress, a substantial loss of stiffness occurs and the plate becomes unable to carry further load.

The yielding condition Γ_Y of the rectangular plate element is the von Mises criterion, which may be written as in Eq.(15).

The safety margin, M, then becomes:

$$M=\sigma_o^2-\sigma_x^2+\sigma_x\ \sigma_y-\sigma_y^2 \begin{cases} < 0 \text{ Failure set} \\ =0 \text{ Failure surface} \\ >0 \text{ Safe set} \end{cases} \quad (24)$$

4. PARAMETRIC STUDY

In this section, a parametric study is performed on square plate panels (1000x1000 mm.) with different plate thicknesses modelled using the Idealized Structural Unit Method (ISUM). The described calculation procedures for the safety index and consequently the probability of failure of the plate, using Eq.(22), are applied. In these analyses, the safety index and the probability of failure are plotted at each loading step and at the possible failures modes.

Fixed deterministic values are assumed for Poisson's ratio and for the length and width of the plate. Normal distributions are assumed for Young's modulus with a mean value of 21,000 kg/mm^2 and yield stress with a mean value of 28 kg/mm^2 . The coefficients of variation are taken to be 2% and 10%, respectively, based on values given in references [1] and [2].. As has been noted previously, modelling uncertainties have not been included. The results of the parametric study are presented in the following subsections with two important parameters considered, namely: the mean of the plate thickness and the coefficient of variation (cov) of the average stress in the x-direction σ_{xav}. The results are all plotted against the mean of normalized load , the normalized load defined as:

$$\text{Normalized load} = \sigma_{xav} / \sigma_0$$

4.1 *Effect of Plate Thickness*

The plate thickness is modelled by a log-normal distribution with a 2% coefficient of variation. σ_x is assumed to have a normal distribution with a 15% cov. The safety index for three typical values of plate thickness; namely, t=10mm, 16mm, and 24mm is calculated and plotted against normal load. The results are shown in Figures (6), (7) and (8) for both buckling and yielding modes of failure. Figures (9), (10) and (11) show the relationships of probability of failure to normalized load for the same plate thicknesses. Figures. (12) to (15) depict similar relationships for a wider range of plate thicknesses.

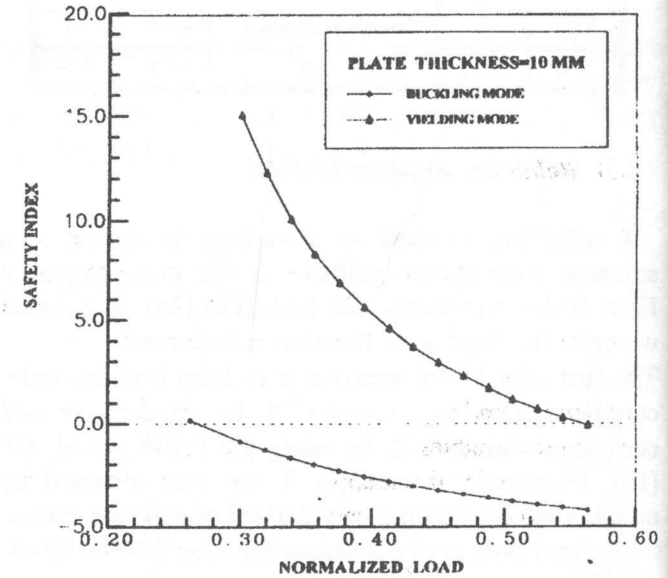

Figure 6. Relationships of safety index to normalized load for thin plate, t=10 mm.

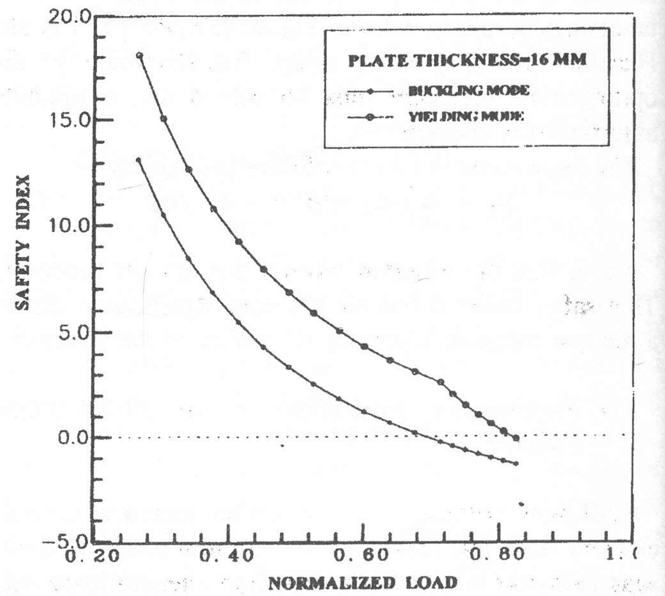

Figure 7. Relationships of safety index to normalized load for medium plate, t=16 mm.

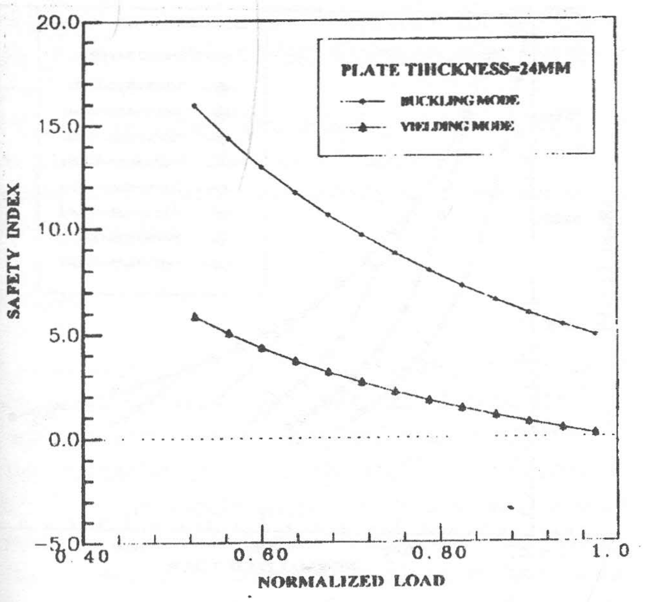

Figure 8. Relationships of safety index to normalized load for thick plate, t=24 mm.

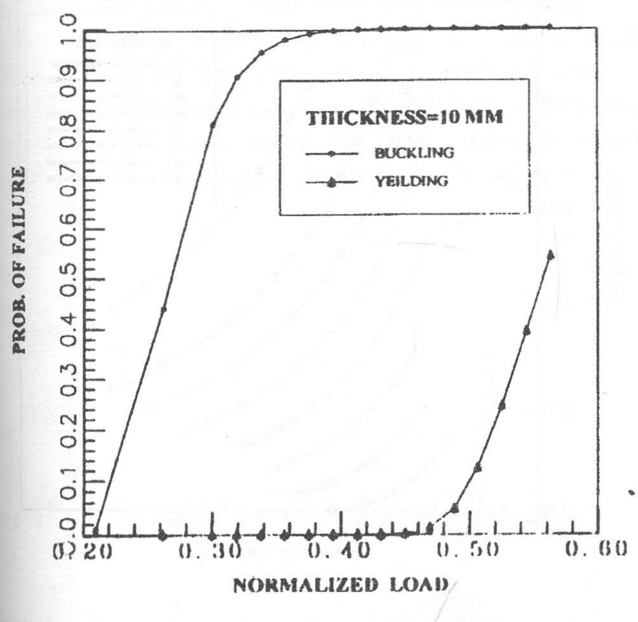

Figure 9. Relationships of probability of failure to normalized load for thin plate, t=10 mm.

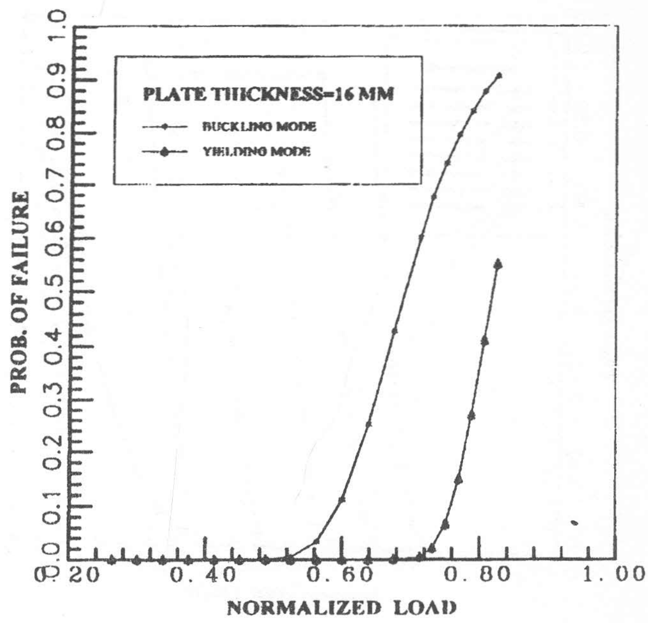

Figure 10. Relationships of probability of failure to normalized load for medium plate, t=16 mm.

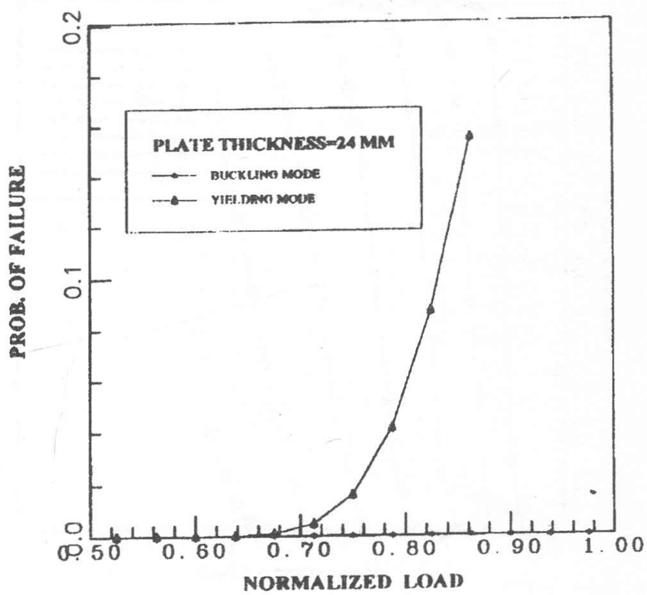

Figure 11. Relationships of probability of failure to normalized load for thick plate, t=24 mm.

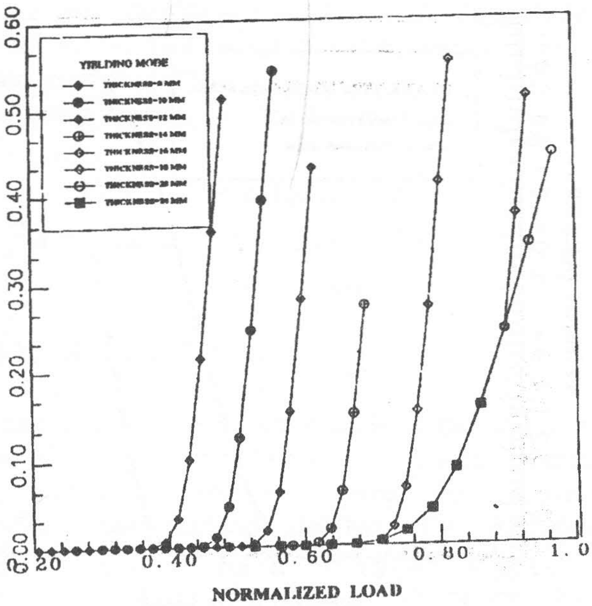

Figure 12. Effect of plate thickness on the probability of failure, yielding mode.

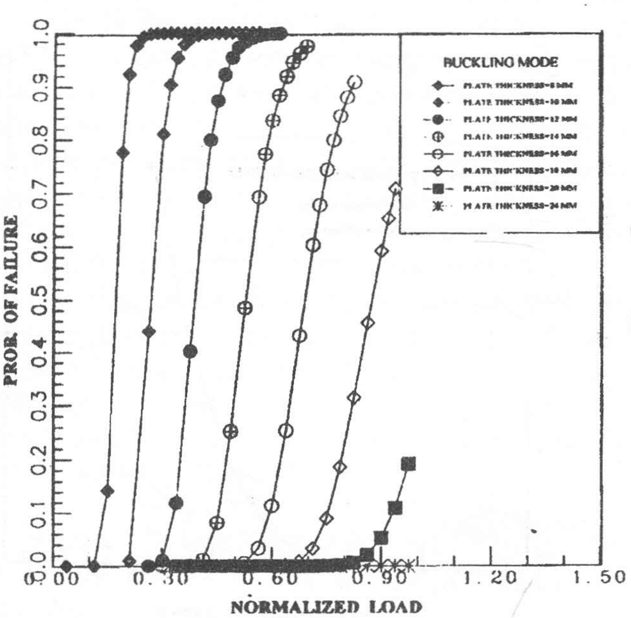

Figure 13. Effect of plate thickness on the probability of failure, buckling mode.

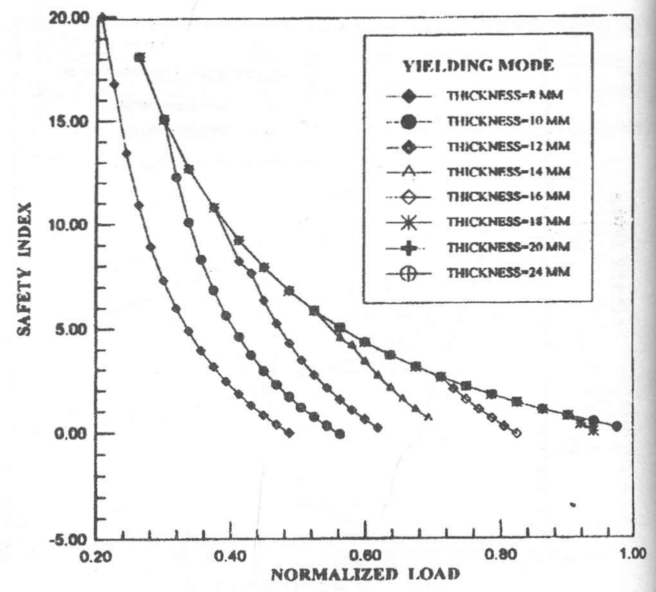

Figure 14. Effect of plate thickness on the safety index, yielding mode.

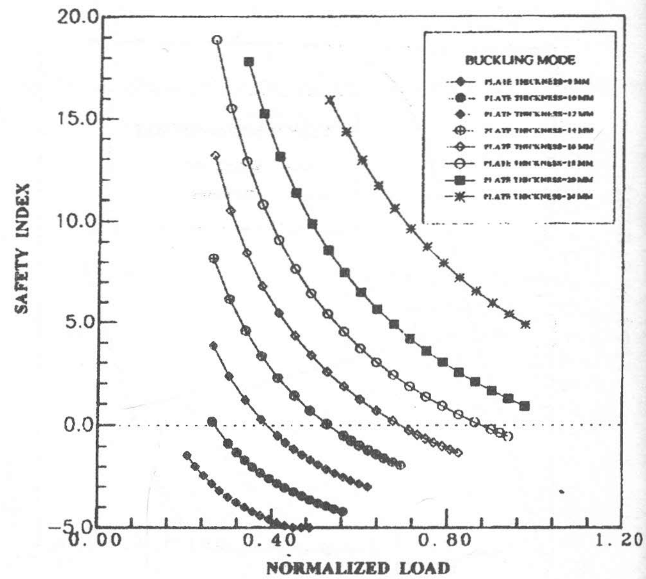

Figure 15. Effect of plate thickness on the safety index, yielding mode.

It is shown from the obtained results that the safety index, corresponding to the buckling failure mode, is the smaller for thin plates; also the safety index, corresponding to the yielding failure mode, is the smaller for thick plates for all loading steps. And hence buckling is the suitable design criterion for thin plates while yielding is the suitable design criterion for thick plates.

It should be noted that a safety index equal to 0 corresponds to a probability of failure equal to 0.5. A negative safety index corresponds to a probability of failure larger than 0.5.

4.2 *Effect of COV of Load*

A most important source of uncertainty in the safety assessment of plate panels is that related to the loading. To study its effect on the safety index and consequently on the probability of failure, the coefficient of variation (cov) of σ_x is varied from 0.1 to 0.4 with a 0.05 increment. Figures (16) and (17) show the relationships of the probability of failure with the normalized load, with varying cov of load, in the yielding and buckling failure modes, respectively. Figures (18) and (19) show the relationships of the safety index with the normalized load , with varying cov of load, in the yielding and buckling failure modes, respectively.

The figures show that the larger the cov of the load, the smaller the safety index for the yielding mode of failure. For the buckling mode of failure, the curves intersect at the point where the safety index becomes equal to 0, or the probability of failure equal to 0.5.

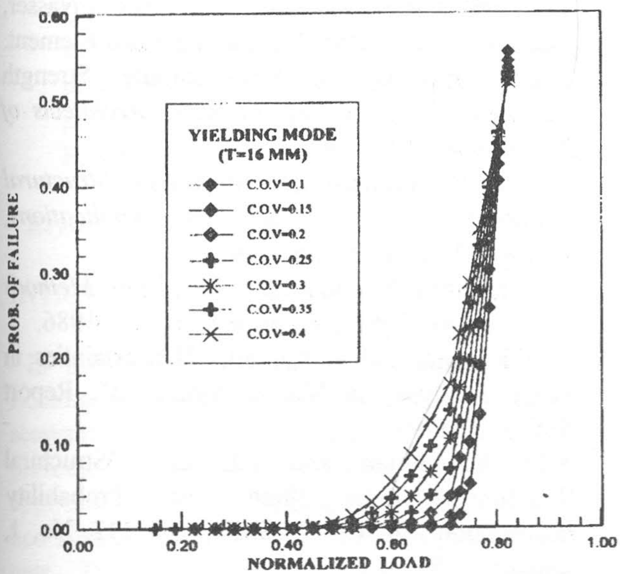

Figure 16. Effect of the coefficient of variation of the load on the probability of failure in yielding.

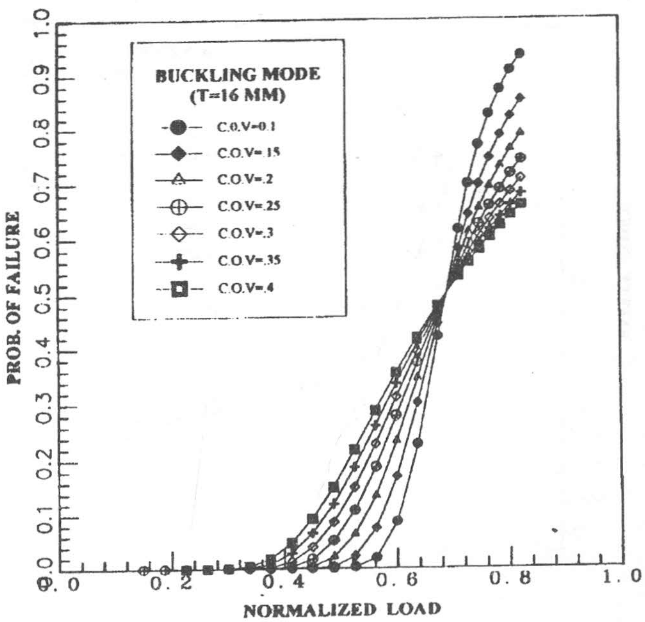

Figure 17. Effect of the coefficient of variation of the load on the probability of failure in buckling.

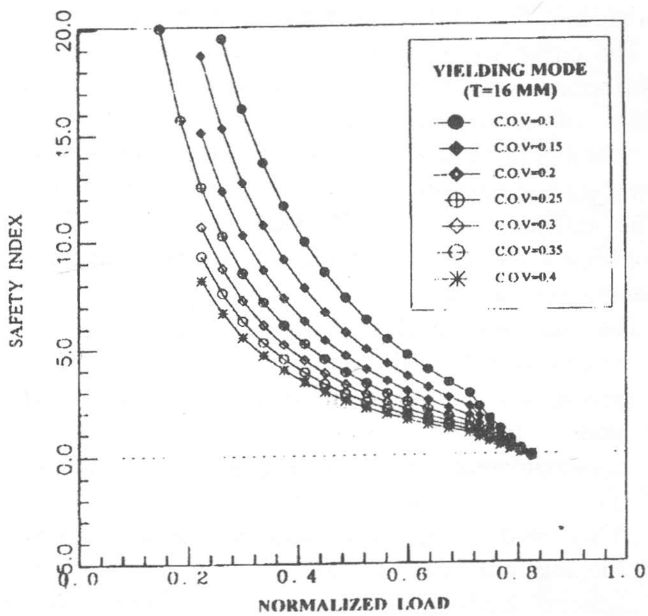

Figure 18. Effect of the coefficient of variation of the load on the safety index in yielding.

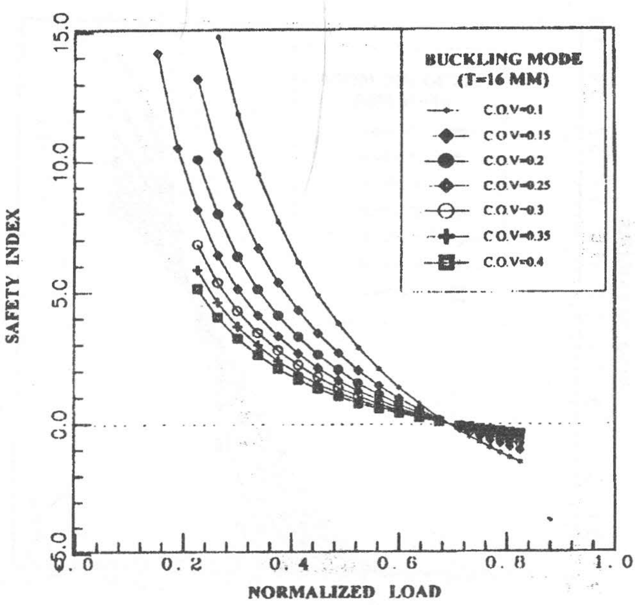

Figure 19. Effect of the coefficient of variation of the load on the safety index in buckling.

5. CONCLUSIONS

This paper has demonstrated an efficient method for predicting the safety of rectangular plate panels.

The paper has presented the results of the analysis performed on a 1000x1000mm square plate panel in the form of charts showing the safety index or the probability of failure of the plate panel considering both yielding and buckling, as a function of normalized load. The influence of plate thickness and that of load coefficient of variation, being the two most important parameters, are shown.

The following conclusions may be deduced:

1. ISUM and FORM were combined to assess the probability of failure of rectangular plate panels under uniaxial loading. Both methods are approximate methods yielding good results suitable for design purposes.
2. This study was performed for single uniaxial loading but may be extended to include more general loading on the plate: combined biaxial stresses, shear stress and lateral pressure.
3. The methods described can handle different distribution types for the design variables involved.
4. The procedure may be extended to include modelling uncertainties.
5. The results of the following study may be extended for different size plates and used for rule calibration and to help in the choice of safety factors for uniaxially loaded plates.

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