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NUMERICAL STUDY OF TRANSIENT ONE DIMENSIONAL BLOWDOWN PROBLEM

Part 1: Technique Development

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ABSTRACT

In this paper, a numerical technique for integrating the transient one-dimensional conservation equations of mass, momentum, and energy is presented. The technique is based on an implicit finite difference formulation (sometimes called Brsching's method). A computer program is developed and used to study the thermo-fluid transients of blowdown problem (i.e. sudden depressurization of a straight pipe of a given initial pressure condition and sudden openning of one end). The developed program is simple, flexible, and could be extended to analyse the sudden depressurization of two phase flow cases.

NOMENCLATURE

A	Flow path area	m^2
$\tilde{C}$	Matrix defined in equation (1)	
D	Diameter	m
e	Specific internal energy	J/kg
$\tilde{F}$	Matrix defined in equation (1)	
f	Coefficient of friction	
$\tilde{G}$	Matrix defined in equation (8)	
G	Mass Flux	$kg/m^2.s$
g	Gravitational Acceleration	m/s^2
H	Height	m
l	Length	m
M	Mass	kg
P	Pressure	Pa
Q	Wall heat flux	W/m^2
t	Time	s
u	Velocity	m/s
$\tilde{U}$	Matrix defined in equation (1)	
U	Total energy	J
V	Volume	m^3
W	Flow rate	kg/s
x	Flow direction coordinate	
$\tilde{Y}$	Matrix defined in equation (5)	

Greek Letters

γ	Specific heat ratio	
ϱ	Density	kg/m^3
τ	Wall shear stress	N/m^2

INTRODUCTION

When the back pressure P_b is reduced below an upstream pressure P_o in a flow system, flow begins and a gradient is established in the connecting channel between P_o and a pressure P_e at the exit of the channel. This flow increases as P_b is reduced further and the exit pressure P_e equals P_b up to the critical (choked) condition at which the exit velocity equals the speed of sound. This phenomenon occurs in both single and two-phase flows. Although it has long been observed in boiler and turbine systems, flow of refrigerants and rocket propellent, serious theoretical and experimental studies of the transient behaviour of the phenomenon have been made only in recent years. This phenomenon is of utmost importance in safety considerations of nuclear reactors. A break in a primary coolant loop, for example, results in a rapid loss of coolant which exposes the reactor core to a steam environment which may lead to a core melting. An evaluation of the transient rate of flow out from the broken channel is therefore of importance for the design of emergency core cooling and for the determination of the extend of damage in accidents, see reference [1] to [5].

Most current programs solve the conservation equations of mass, momentum, and energy using explicit numerical integration methods. The nature of the nonlinearity of governing differential equations associated with these systems require severe time-step restrictions in an explicit numerical integration methods. An implicit method that utilizes block inversion techniques will be presented in the present study. The developed program is given with a case study of the depressurization problem of a single phase transient in straight pipe.

DERIVATION OF CONSERVATION EQUATIONS

The derivation of the conservation equations in the present technique is based on the node-flow path concept in which control volume, denoted as nodes, are connected to other control volume via a flow area, denoted as flow path. In such node-flow path arrangement, Figure (1), the equations of conservation of mass and energy are solved in the nodes and the momentum equation is solved in the flow path. By solving these three equations in this manner, one value of the total mass and one value of total energy are known for each node and the average flow is known in a flow path. To obtain these average values, the conservation of mass and energy must be spatially integrated from X_i to X_{i+1} and the conservation of momentum equation integrated from $X_{i-\frac{1}{2}}$ to $X_{i+\frac{1}{2}}$, see Figure (1). Detailed derivation of these conservation equations is now presented.

The basic equations can be written in the vectorial form as follows:

$$\frac{\partial \tilde{U}}{\partial t} + \frac{\partial \tilde{F}}{\partial x} = \tilde{C} \qquad (1)$$

Where:

$$u = \begin{bmatrix} \rho \\ \rho u \\ \rho e \end{bmatrix} , \quad F = \begin{bmatrix} G \\ G^2/\rho + P \\ G(e + \frac{P}{\rho}) \end{bmatrix} , \quad C = \begin{bmatrix} 0 \\ -(\Upsilon + \rho g \frac{dH}{dz}) \\ Q \end{bmatrix}$$

Integrating these equations we get ,

Mass

$$\dot{M} = \frac{dM_i}{dt} = W_{i-1} - W_i \qquad (2)$$

Momentum

$$\dot{W} = \frac{dW_i}{dt} = C_2 \left(\frac{W^2_{i-1}}{\rho_{i-1} A^2_i} - \frac{W^2_i}{\rho_i A^2_{i+1}} \right) + C_3 (P_i - P_{i+1})$$

$$- fC_2C_3 \left(\frac{W^2_i}{\rho_i A^2_{i+1}} \right) - g (\rho_i H_{i+1} - \rho_{i-1} H_i)$$

Energy

$$\dot{U} = \frac{dU_i}{dt} = \left(\frac{U + PV}{M} W \right)_{i-1} - \left(\frac{U + P V}{M} W \right)_i + Q$$

Where; $C2 = \dfrac{2}{(L_i/A_i + L_{i+1}/A_{i+1})}$, $C3 = \left(\dfrac{L_i}{D_i} + \dfrac{L_{i+1}}{D_{i+1}} \right)$

From these equations, it is clear that the unkown variables are M_i, W_i, and U_i for all nodes. Let these variables put in a vector form $\tilde{Y}$ where;

$$\tilde{Y} = \begin{bmatrix} M_1 \\ W_1 \\ U_1 \\ M_2 \\ W_2 \\ U_2 \\ \cdot \\ \cdot \\ \cdot \\ M_n \\ W_n \\ U_n \end{bmatrix} \tag{5}$$

The conservation equations (2) to (4) can then be written as :

$$\dot{Y} = \frac{d\tilde{Y}}{dt} = F(Y) \tag{6}$$

Using an implicit finite difference representation, equation (6) can be written as:

$$\frac{\tilde{Y}_i^{\,n+1} - \tilde{Y}^n_{\,i}}{\Delta t} = F(Y_i^{\,n+1}) \tag{7}$$

$$\hat{Y}_i^{n+1} - \tilde{Y}_i^n - \Delta t \cdot F\,(\tilde{Y}_i^{n+1}) = 0 = G\,(\tilde{Y}_i^{n+1}) \quad (8)$$

Using Newton method to get the roots of equation (8):

$$Y_{k+1}^{n+1} = Y_k^{n+1} - \frac{\tilde{G}(Y_k^{n+1})}{\dfrac{\partial \tilde{G}(Y_k^{n+1})}{\partial Y_k^{n+1}}} \qquad (9)$$

$$Y_{k+1}^{n+1} = Y_k^{n+1} - \frac{\partial \tilde{G}(Y_k^{n+1})}{\partial Y_k^{n+1}} = - \tilde{G}(Y_k^{n+1}) \qquad (10)$$

Where:

$$\frac{\partial G(Y_k^{n+1})}{\partial Y_k^{n+1}} = I - \Delta t \cdot \frac{\partial F(Y_k^{n+1})}{\partial Y_k^{n+1}} = \text{Jacobian Matrix}$$

Therefore equation (10) will be :

$$(Y_{k+1}^{n+1} - Y_k^{n+1})(I - \Delta t. \frac{\partial F(Y_k^{n+1})}{\partial Y_k^{n+1}}) = Y_k^{n+1} - Y_k^n - \Delta t. F(Y_k^{n+1})$$

or

$$[DY]. \ [RJ] = [BB] \qquad (11)$$

where ;

$$[DY] = Y_{k+1}^{n+1} - Y_k^{n+1}$$

$$[RJ] = \text{Jacobian Matrix} = I - \Delta t \cdot \frac{\partial F(Y_k^{n+1})}{\partial Y_k^{n+1}}$$

$$[BB] = Y_k^{n+1} - Y_k^n - \Delta t. F(Y_k^{n+1})$$

Equation (11) is 3N simultanious linear equations and could be solved for Dy where:

$$Y = \begin{bmatrix} M_1 \\ W_1 \\ U_1 \\ \\ M_N \\ W_N \\ U_N \end{bmatrix} , \quad F(Y) = Y = \begin{bmatrix} M_1 \\ W_1 \\ U_1 \\ \\ M_N \\ W_N \\ U_N \end{bmatrix} = \begin{bmatrix} F_1(M_1, W_1, U_1, \ldots U_N) \\ F_2(M_1, W_1, U_1, \ldots U_N) \\ F_3(M_1 \ldots\ldots\ldots U_N) \\ \\ \\ F_N(M_1, W_1, U_1, \ldots U_N) \end{bmatrix}$$

$$\frac{\partial F_i}{\partial Y_j} = \begin{bmatrix} \frac{\partial \dot{M}_1}{\partial M_1} & \frac{\partial \dot{M}_1}{\partial W_1} & \frac{\partial \dot{M}_1}{\partial U_1} & \frac{\partial \dot{M}_1}{\partial M_2} & \frac{\partial \dot{M}_1}{\partial W_2} & & \frac{\partial M_1}{\partial U_N} \\ \frac{\partial W_1}{\partial M_1} & \frac{\partial \dot{W}_1}{\partial W_1} & \frac{\partial \dot{W}_1}{\partial U_1} & \frac{\partial \dot{W}_1}{\partial M_2} & \frac{\partial \dot{W}_1}{\partial W_2} & & \frac{\partial \dot{W}_1}{\partial U_N} \\ \ldots & \ldots & \ldots & \ldots & \ldots & & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & & \ldots \\ \frac{\partial U_N}{\partial M_1} & \frac{\partial U_N}{\partial W_1} & \frac{\partial U_N}{\partial U_1} & \ldots & \ldots & & \frac{\partial U_N}{\partial U_N} \end{bmatrix}$$

The Jacobian matrix will end as a block diagonal matrix as:

$$[RJ] = \begin{bmatrix}
x\ x\ x\ x\ x\ x & & & & 0 \\
x\ x\ x\ x\ x\ x & & & & \\
x\ x\ x\ x\ x\ x & & & & \\
x\ x\ x\ x\ x\ x\ x\ x\ x & & & & \\
x\ x\ x\ x\ x\ x\ x\ x\ x & & & & \\
x\ x\ x\ x\ x\ x\ x\ x\ x & & & & \\
\qquad\quad x\ x\ x\ x\ x\ x\ x\ x\ x & & & & \\
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\qquad\quad x\ x\ x\ x\ x\ x\ x\ x\ x & & & & \\
\qquad\quad \cdots\cdots\cdots\cdots & & & & \\
\qquad\quad \cdots\cdots\cdots\cdots & & & & \\
 & & \qquad x\ x\ x\ x\ x\ x\ x\ x\ x & & \\
0 & & \qquad x\ x\ x\ x\ x\ x\ x\ x\ x & & \\
 & & \qquad x\ x\ x\ x\ x\ x\ x\ x\ x & & \\
 & & \qquad\qquad\quad x\ x\ x\ x\ x\ x & & \\
 & & \qquad\qquad\quad x\ x\ x\ x\ x\ x & & \\
 & & \qquad\qquad\quad x\ x\ x\ x\ x\ x & &
\end{bmatrix}$$

The derivatives of the momentum equation (3) are ;

$$\frac{d\dot{W}}{dM_i} = C2 \left(\frac{W_i^2 V}{M_i^2 A_i^2} \right) + C3 \frac{\partial P_i}{\partial M_i} - f\, C2\, C3 \left(\frac{W_i^2 V}{M_i^2 A_{i+1}^2} \right)$$

$$\frac{d\dot{W}}{dW_i} = C_2 \left(\frac{-2 W_i}{\rho A_{i+1}^2} \right) + C_3 \frac{\partial P_i}{\partial W_i} - f\, C_2\, C_3 \left(\frac{2W_i}{\rho A_{i+1}^2} \right)$$

$$\frac{d\dot{W}}{dU_i} = C_3 \frac{\partial P_i}{\partial U_i}$$

Other terms are calculated in a similar way and are omitted here.

Computation Steps

The compuation steps to be followed can be summarized as, shown in Figure (2),

1. Read the required data such as nodes number, volumes, flow area, length, and time step.

2. supply the initial conditions for each node, as, mass, flow rate, and energy (Subroutine IC).

3. Supply the problem boundary conditions and flow properties, equation of state (Subroutine PROP).

4. Calculate the matrices [RJ] and [BB] and then solve equation (11) for the matrix [DY] iteratively.

5. Use these results as initial values for the new time step and print the results if necessary.

6. Advance the time by an incremental time step dt, and repeat from step 3 above.

7. When the computation time specified is over, or steady state is reached, stop calculations.

CASE STUDY

The case study that will be presented herein is for ideal gas. The ideal gas state equation can be written as a function of M_i, W_i, and U_i as follows:

$$PV = MRT \quad = M\frac{R}{c_v}(c_v . T)$$

But; the specific internal energy $e = c_v . T$, and the specific heat $c_v = R/(\gamma - 1)$, therefore:

$$PV = M(\gamma - 1)\, e.$$

The total energy U is equal to internal and kinitic energies so that; $U = M(e + 0.5\, u^2)$, therefore:

$$e = \frac{U}{M} - 0.5\, u^2$$

$$PV = M(\gamma - 1)\left(\frac{U}{M} - 0.5\, u^2\right)$$

From the continuity equation, the flow velocity u can be given as : $u = W/\rho A = V.W/(M.A)$, therefore:

$$P_i = \frac{(\gamma - 1)}{V}\left(U_i - \frac{V^2 W_i^2}{2.A_i^2 M^2}\right)$$

and :
$$\frac{\partial P_i}{\partial M_i} = ((\gamma - 1).\ V.W_i^2 / (2.A_i^2 . M_i^2))$$

$$\frac{\partial P_i}{\partial W_i} = (-V(\gamma - 1)W_i / A_i^2 M_i)$$

$$\frac{\partial P_i}{\partial U_i} = (\gamma - 1)/V$$

Initial and Boundary Conditions

Figure (1) presents the schematic of the blowdown case study system. It represents a 50 m long pipe of 0.4 m^2 flow area. The pipe is divided into 10 equal control volumes of 2 m^3 each. The initial and boundary conditions are given in table (1) below; where:

$$\rho = P/RT, \quad M = \rho V, \quad W = \rho.u.A$$

$$U = M(e + 1/2.\ u^2) = M(c_v T + 1/2.\ u^2)$$

Table (1) Initial and Boundary Conditions

P(0) ata		2	5	10
I.C.				
$P(0,x) \times 10^{-5}$	N/m^2	2.026	5.06	10.135
W(0,x)	kg/s	0	0	0
$\rho(0,x)$	kg/m^3	2.35	5.875	11.75
M(0,x)	kg	4.7	11.75	23.5
$U(0,x) \times 10^6$	J	1.01	2.525	5.05
B.C.	$P(t, IMAX + 1) = 1.01 \times 10^5 \ N/m^2$			

Figure (3) shows the transient developent of the system parameters; pressure, flow rate, velocity, total energy, and enthalpy. These results are shown for two different nodes (control volumes) No.7 and No. 10. The results indicate that after a period of time (dependent upon the initial and boundary conditions as well as flow areas), the outlet velocity and flow rate reach their maximum values after which they remain constant. This condition is known as critical or choked conditions during which the outlet velocity reaches the sonic speed at the local and transient conditions. Increasing the system initial pressure to 5 ata and 10 ata changes the depressurization rate and choking conditions, Figure (4).

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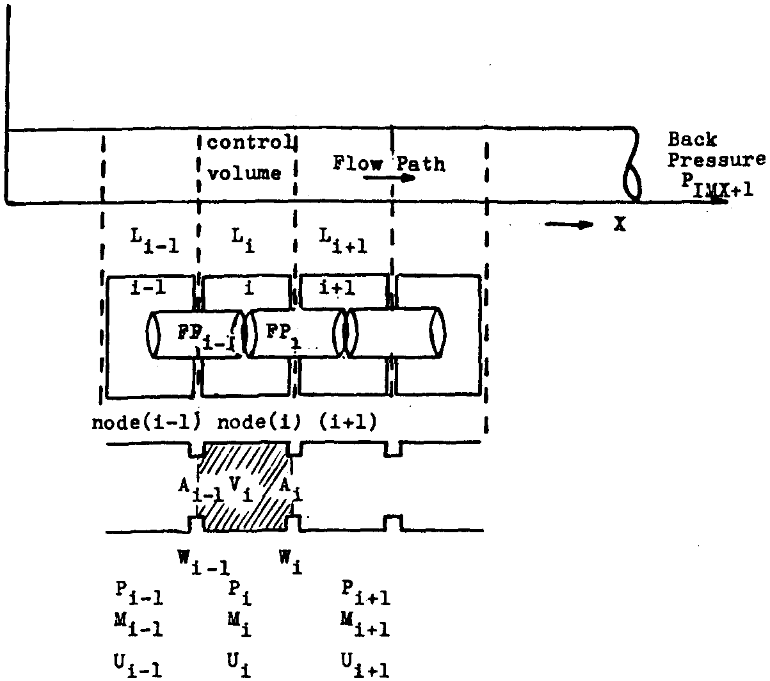

Figure (1) The node (control volume) - flow path representation of a region .

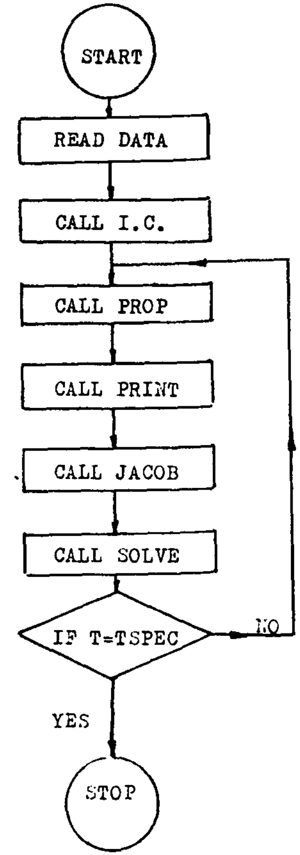

Figure (2) Flow Chart of The Computation Steps

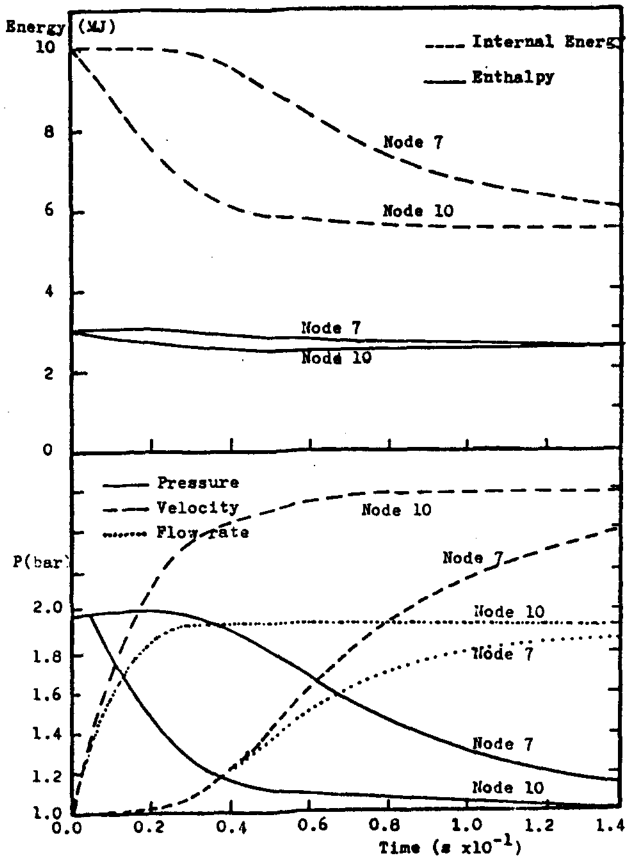

Figure (3) Blowdown Transient Results

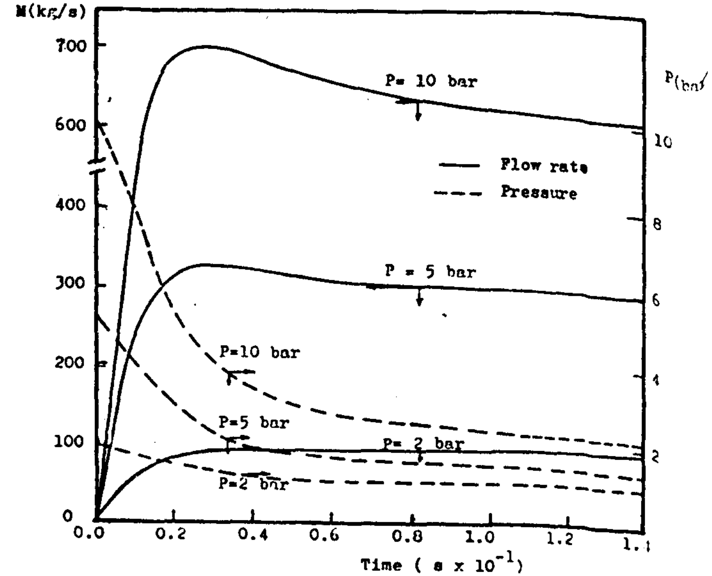

Figure (4) Effect of Initial Pressure on Blowdow Results