cases, f_2 (t) = R/L and f_s (t) = (V/L) sin (wt + α) and then f_s (t) = o. Tables giving values of w/w_o, $R/w_o L$ and other intermediate parameters for steady state oscillation, with V = 0, are given, for various "k". Simple closed form characteristic relations between the parameters "k" and w_o^2/w^2 for periodic oscillations in the three cases are attained, (V = 0 = $f_2(t)$), with no need for using series expansion and continued fractions techniques.

1. INTRODUCTION

It is well-known that continuous linear time-varying systems' analysis is much more difficult than that of time invariant systems, since the response of the latter is described by simple exponential functions while the response of time-varying systems must be described by unkown complex function of two variables, namely the time "τ" at which the excitation (input) $v_s(\tau)$ is applied and the time "t" at which the response x(t) is measured and

$$x(t) = \int_{-\infty}^{\infty} h(t,\tau)v_s(\tau)d\tau \qquad (1)$$

This superposition integral is generally cumbersome to work with. If the system is time invariant, $h(t,\tau)$ depends only on $(t-\tau)$, and $x(t)=\int_{-\infty}^{\infty} h(t-\tau)v_s(\tau)\,d\tau$, this is the familiar convolution. In the frequency domain, and for periodically time-varying systems, (1) is given by [1]:

$$x(w) = \sum_k P_k(w - 2\pi k/T) . V_s(w - 2\pi k/T) \qquad (2)$$

where $p(t-\tau) = h(t,t-\tau)$; T is the period of the periodic variations of the system. For time invariant systems k=0 and $x(w) = H(w)\ V_s(w)$. It is clear from (2) that the spectral input-output relation of periodically time-varying systems is much more complicated and cumbersome to work than that of time invariant systems. If the input signal

$v_s(t) = V\ \cos\ w_s t$, equation (2) gives

$$x(w) = \pi V_s \sum_k P_k(w-2\pi k/T)[\delta(w-w_s-2\tau k/T)+\delta(w+w_s-2\pi k/T)]$$

and if $[(2\pi k/T)w_s]$ is a rational number, the response x(t) is a complex periodic wave of fundamental angular frequency "w_s".

The series $L\text{-}C_t\text{-}R$ circuit with a sinusoidal input signal of angular frequency $w/2\pi$ and with C_t periodically varying with time in a Fourier series form of fundamental angular frequency 2w, was analysed by the author, a long time ago, using the methods of superposition and successive approximations [2]. The same methods were also used by the author at that time for analysing $L\text{-}C_t$ and $L_t\text{-}C$ circuits with no input signal [3], [4], for the whole range of periodic variations, where $C_t=C_o/\ (1+k\ \cos 2wt)$ and $L_t=L_o(1+k\ \cos\ 2\ wt)$. In connection with numerical methods, spectral analysis of periodically time varying linear networks with a sinusoidal input signal appropriate for computer-aided design is given in [5], steady-state solution is only considered.

It is clear that there is still a need for a simple analytical method for analysing such systems. This is the purpose of the present paper. The method adopted is the distorted time scale method originated by the author [6], [7]. Although the method can be applied to periodically time varying systems of any order so long as the output response is represented by a complex periodic wave-yet second order types are analysed since they represent many of the applications.They are governed by the following equation:

$$f_1(t)dx/dt+f_2(t).x+f_3(t)\int x\ dt=f_s(t) \qquad (3)$$

$f_1(t)$, $f_2(t)$ and $f_3(t)$ are single-valued synchronized periodic functions of time represented in a Fourier series forms, and $f_s(t)$ is the driving source given in Fourier series form.

The distorted time scale method depends upon the fact that when the time scale of a periodically complex wave $x(t)$ is instantaneously distorted in a suitable manner, the resulting wave is a simple cosine in a distorted time parameter "θ", given by $x(t) = A \cos\theta$, and $\theta = w_s t + \gamma(t)$. The peak amplitude "A" is constant, and w_s is the fundamental angular frequency of the input signal. (w/w_s). $(=p)$ must be a rational number for the output $x(t)$ to be a complex periodic wave of fundamental angular frequency w_s.

Without loss of generality, the present analysis is limited to the steady-state solutions of waveform $x(t)$ expressed in odd harmonics which occur when $f_1(t)$, $f_2(t)$ and $f_3(t)$ are of even harmonics while $f_s(t)$ is an odd harmonic function.

In this case, $\gamma(t)$ is periodic of period π/w_s. Under the condition of having one maximum amplitude over a complete cycle, the parameter "θ" is steadily increasing with time and $dt/d\theta$ is positive and the θ-t relation can be inverted to $t = (1/w)\theta$ + a periodic function of "θ" of period π and

$$dt/d\theta = (1/w_s)[1 + \sum_{r=1} (h_r \cos 2r\theta + g_r \sin 2r\theta)] \tag{4}$$

Integrating $dt/d\theta$ gives,

$$w_s t = w_s t_s + \theta + \sum_{r=1} [h_r/2r)\sin 2r\theta - (g_r/2r)\cos 2r\theta] \tag{4.a}$$

"$w_s t_s$" is the constant of integration; thus "θ" is a function of "$t-t_s$". "t" is the time at which the excitation (input signal) occurs and "$t-t_s$" is the time at which the response $x(t)(=A' \cos\theta)$ occurs. "$w_s t_s$" has an important role in the solution.

In this case, equation (3) is given by :

$$(1+ \sum_{r=1} a_r \cos 2rwt)dx/dt + b_o[1+ \sum_{r=1} b_r \cos 2rwt]x +$$

$$d_o[1 + \sum_{r=1} d_r \cos 2rwt]\int x\,dt = \sum_{s=1} v_s \sin(2s-1)w_s t + \alpha] \tag{5}$$

where "α" is a phase angle. This phase angle "α" is used to control the exchange of energy between the driving electrical input source $f_s(s)$ and the driving synchronized mechanical (or otherwise) sources producing the periodic variations of the system. This control is enhanced if $w_s = w$ and thus, in the following analysis "w_s is teken equal to "w".

2. DETERMINATION OF THE RESPONSE X(t) OF NETWORKS REPRESENTED BY (5)

Equation (5) is put in the form:

$$\{1+\sum_{1} a_r \cos[2\, rw \int (dt/d\,\theta) d\theta\,]\}\ (dx/d\theta)/(dt/d\,\theta\,)+$$

$$b_o\ \{1+\sum_{1} b_r \cos\ [2\ r\ w \int (dt/d\ \theta) d\ \theta\ ]\}\ x\ +$$

$$d_o\ \ 1+\sum_{1} d_r\ \cos[2\ r\ w \int (dt/d\ \theta)\ d\ \theta\ ]\} \int x(dt/d\theta\,)d\,\theta$$

$$= \sum_{1} V_s\ \sin\ [(2s-1)w \int (dt/d\ \theta) d\theta\ + \alpha\,] \tag{6}$$

Substitution of (4-a) and putting $x=A \cos\theta$ in (6) gives an algebraic equation in the unknowns h_r , g_r , A and wt_s . Solution is attained by matching at several values of "θ" equal to the number of unknowns. The algebraic equation is a truncated Fourier series containing $\sin n\,\theta$, $\cos n\,\theta$, with n=1,3,5, etc. If the sampling (matching) points are equispaced and the number of samples were close to double the highest harmonic order in the series, the attained values of h_r , g_r , "A" and "wt_s" would give an almost optimum solution in the least square sense.

If the ($dt/d\,\theta$) series, given in (4), is truncated to one harmonic (r=1), the algebra is simplified but the solution is still satisfactory. The reason is that the response x(t) (=A $\cos\theta$) still gives infinite harmonic time series with infinite discrete spectrum approximating satisfactorily the actual response spectrum. This is completely different from the case of approximating the formal infinite terms' time Fourier series solution by a finite number of terms, since the spectrum would then be of finite discrete terms. Truncating the $dt/d\theta$ series to one harmonic (r=1), only four unknowns exist, namely h_1(=h), g_1 (=g), "A" and "wt_s". Putting $\theta = 0, \pi/4, \pi/2$ and $3\pi/4$ in the algebraic equation gives four equations from which the four unkowns are determined. The parameters "h" and "g" are measures of the deviation of the steady state solution x(t) from a pure sine wave.

Knowing h,g,A and wt_s, the closed form parametric solution is given by, $x(t)=A \cos\theta$,

$$\theta = [wt - wt_s - (h/2) \sin 2\theta + (g/2) \cos 2\theta]$$

If the driving electrical source is absent ($V_s = 0$), the unknowns are h, g, w/w_o and wt_s.

2.1 Analysis of a series R-L-C_t circuit with an applied signal V sint (wt + α), $C_t = C_o/(1+k \cos 2wt)$:

The differential ecuation of the current x(t) is given by,

$$dx/dt+(R/L)x+w_o^2(1+k \cos 2wt) \int x\, dt=(V/L)\sin(wt + \alpha) \qquad (5.a)$$

where $w_o^2 = 1/LC_o$. This is a special case of (5). The algebraic equation is :

$$[-(w^2/w_o^2)\sin\theta/(1+h \cos 2\theta+g \sin 2\theta)]+(w/w_o)(R/w_oL) \cos\theta$$
$$+ \{1+k \cos[(2\theta+h \sin 2\theta-g \cos 2\theta+2wt_s]\}$$
$$[(1+h/2)\sin\theta- (g/2) \cos\theta+(h/6)\sin 3\theta -(g/6)\cos 3\theta]$$
$$=(wV/w_o^2AL)\sin [\theta+wt_s+h/2 \sin 2\theta -g/2 \cos 2\theta + \alpha] \qquad (7)$$

Putting θ=0, π/4, π/2 and 3π/4 in (7) gives, respectively:

$$F_1=[(w/w_o)(R/w_oL)]-(2g/3)[1+k\cos(2wt_s-g)]-(wV/(w_o^2AL)) \qquad (8)$$
$$\sin(wt_s-g/2 +\alpha) = 0 \quad (8)$$

$$F_2=[(w^2/w_o^2)(1+g)-(w/w_o)(R/w_oL)-[1-k\ \sin(h+2w\ t_s)].$$

$$(1+2h/3-g/3)+(\sqrt{2wV}/(w_o^2A\ L)\sin\ (wt_s+h/2+\pi\ /4+\alpha\ ) = 0 \quad (9)$$

$$F_3=[(w^2/w_o^2)/(1-h)-[1-k\ \cos(g+2wt_s)](1+h/3)\ +$$

$$(wV/(w_o^2AL).\ \cos\ (wt_s+g/2\ +\ \alpha)= 0 \quad (10)$$

$$F_4=[(w^2/w_o^2)(1-g)]+(w/w_o)(R/w_oL)-[1-k\ \sin\ (h-2wt_s)].$$

$$(1+2h/3+g/3)+(\sqrt{2}\ wV\ /\ (w_o^2\ AL)\cos(wt_s-h/2\ +\ \pi\ \ /4+\alpha)=0 \quad (11)$$

(8)-(11) are four nonlinear algebraic equations in the four unknowns h,g,$wV/(w_o^2\ AL)$ and "wt ". (w/w_o), $(R/w_o\ L)$ and k are usually given. Aided with a microcomputer with a program using Newton's method and the Jacobian matrix, the four unkonwns are determined. Table 1 gives h,g,A/(V/R) and "wt_s" at different values of "α" ranging from 0° to 180°, for k=0.75, $w/w_o= 1$ and $R/w_oL = 1$. V/R is the value of the current x(t) for k = 0. It is clear from the table that A/(V/R) attains a maximum value of about 1.61 at $\alpha = 50^\circ$, and a minimum value of about 0.68 at $\alpha= 142.5^\circ$. It is clear from (7) that if "α" is replaced by ($\alpha + 180$) values of h, g and A will be the same, only "wt_s" will change to ($wt_s+ 180^\circ$).

Table 2 gives h,g,A/(V/R) and "wt_s" at different values of "k" ranging from 0 to more than 2.0, for $\alpha=0$, $w/w_o=1$ and $R/w_oL = 1$. It is clear from this table that "k" affects the response x(t) waveform indicated by varriations in "h" and "g" , but it has little effect on the amplitude "A", so long

Table 1 : R - L - C_t Circuit , Signal Applied

$k = 0.75$ $\quad \nu/\nu_0 = 1$ $\quad R/\nu_0 L = 1$

α (degree)	h	B	$\nu V/(\nu_0^2 LA)$	νt_s (radians)	A/(V/R)
0	0.4277	0.3972	0.9761	1.178	1.0245
10	0.4291	0.4378	0.8620	1.0568	1.1601
20	0.4124	0.4500	0.7655	0.9504	1.3064
30	0.3811	0.4471	0.6932	0.8547	1.443
40	0.3333	0.4340	0.6451	0.7646	1.5502
45	0.3010	0.4240	0.6297	0.7199	1.5880
50	0.2602	0.41135	0.6202	0.6741	1.6123 (max.)
60	0.1351	0.3752	0.6215	0.5749	1.6089
70	-0.0917	0.3185	0.6577	0.4641	1.5204
80	-0.3340	0.2591	0.7134	0.3666	1.4017
90	-0.4982	0.2123	0.7692	0.2749	1.3000
100	-0.6064	0.1731	0.8267	0.1704	1.2100
110	-0.6709	0.1353	0.8876	0.0395	1.1266
120	-0.6765	0.0838	0.9586	-0.1462	1.0431
125	-0.6167	0.0259	1.0176	-0.2980	0.9826
130	-0.2552	-0.2323	1.2535	-0.5912	0.7978
135	0.0154	-0.4035	1.4181	-0.7256	0.7052
140	0.1597	-0.4826	1.4727	-0.8306	0.6790
142.5	0.2300	-0.5073	1.4739	-0.8858	0.6785 (min.)
145	0.2499	-0.5247	1.4595	-0.9446	0.6852
147.5	0.2808	-0.5354	1.4300	-1.0084	0.6993
150	0.3036	-0.5288	1.3855	-1.0786	0.7218
155	0.3236	-0.5129	1.2516	-1.2485	0.7989
160	0.3004	-0.3103	1.0759	-1.5250	0.9295
165	0.3615	0.1591	1.1050	-1.7406	0.9050
170	0.3974	0.2883	1.0807	-1.8238	0.9254
175	0.4174	0.3564	1.0328	-1.8964	0.9683
177.5	0.4255	0.3792	1.0050	-1.9306	0.9950
180	0.4277	0.3972	0.9761	-1.9636	1.0245
	0.4277	0.3972	0.9761	1.1780 (=-1.9636+π)	

Table 2: R -L-C_t Circuit, Signal Applied $w/w_o = 1.0$ $R/w_oL = 1$ $\alpha = 0$

k	h	g	$wV/(w_o^2 LA)$	wt_s (radians)	A/(V/R)
0	0	0	1	$\pi/2$	1.0000
0.01	6.5445×10^{-3}	-2.4112×10^{-3}	1.0017	1.5664	0.9933
0.05	0.0321	-0.0109	1.0070	1.5484	0.9931
0.1	0.0626	-0.0182	1.0116	1.5249	0.9885
0.2	0.1194	-0.0178	1.0135	1.4734	0.9867(min)
0.3	0.1729	0.0114	1.0078	1.4142	0.9923
0.4	0.2283	0.0790	0.9997	1.3481	1.0003≃1.0
0.5	0.2889	0.1767	0.9943	1.2844	1.0057
0.6	0.3494	0.2763	0.9900	1.2328	1.0101
0.7	0.4036	0.3611	0.9821	1.1939	1.0182
0.75	0.4277	0.3972	0.9761	1.1780	1.0244
0.8	0.4499	0.2496	0.9685	1.1639	1.0325
0.9	0.4892	0.4848	0.9485	1.1393	1.0543
1.0	0.5226	0.5300	0.9218	1.1181	1.0843 (max)
1.1	0.5513	0.5674	0.8882	1.099	1.1259
1.2	0.5761	0.5988	0.8476	1.0808	1.1798
1.5	0.6335	0.6681	0.6757	1.027	1.4799
1.6	0.6483	0.6851	0.5971	1.007	1.6748
1.7	0.6609	0.6996	0.5031	0.9845	1.9877
1.8	0.6713	0.7116	0.3469	0.9576	2.5846
1.9	0.6785	0.7206	0.2319	0.9212	4.3122
1.95	0.6796	0.7231	0.1239	0.8943	8.071
1.975	0.6785	0.7232	0.0519	0.8752	19.267
1.98	0.6780	0.7230	0.0347	0.8705	28.818
1.985	0.6773	0.7227	0.0161	0.8652	62.111
1.99	0.6765	0.7223	0.0004≃0	0.8594	±∞
2.00	0.6738	0.7208	-0.054	0.8445	-18.518
2.01	0.6670	0.7165	-0.138	0.8170	-7.2411

as it is far from the value at which the energy of the mechanical source compensates completely the energy dissipated in the resistance, and $A = \pm \infty$. In the given example, table 2, this value of $k = 1.99$.

The power P given to the circuit by the electrical input source is given by :

$$P = (1/T) \int_{t=0}^{T} V \sin(wt+\alpha)x(t)\, dt, \qquad T = 2\pi/w ;$$

but $x(t) = A \cos\theta$, $wt = wt_s + \theta + (h/2)\sin 2\theta - (g/2)\cos 2\theta$

Thus $$P=(AV/2\pi)\int_{\theta=0}^{2\pi} \cos\theta \,.\, \sin[wt_s + \alpha + \theta + (h/2)\sin 2\theta - (g/2)\cos 2\theta]d\theta \qquad (12)$$

$= (AV/2\pi)\, Fn(h,g,wt\ \alpha)$, where $Fn(h,g,w\alpha)$ is the definite integral.

For $k=0$, $h=0=g$, $wt_s + \alpha = \pi/2$, table 2 , and $P = P_o = AV/2$,

giving $$P/P_o = (1/\pi)\, Fn(h,g,wt_s\, \alpha) \qquad (13)$$

$$Fn(h,g,wt_s, \alpha) = \sin(wt_s + \alpha)\{J_o(g/2)[J_o(h/2) - J_1(g/2)] + \sum_{r=1} (-1)^r J_{2r}(g/2)[J_{2r-1}(h/2) + 2J_{2r}(h/2)]\} + \pi \cos(wt_s + \alpha).$$

$$[J_o(h/2)J_1(g/2) - \sum_{m=1} (-1)^{m-1} J_{2m}(h/2)J_{2m-1}(g/2)]$$

where J (x) is the Bessel function of the first kind of order n and argument x.

It is clear from (12) and table 1that " α " controls the power P and hence controls the exchange of energy from mechanical (producing the capacitance variation) to electrical and vice versa.

Case the Parameter "K" is negative

If "wt_s " and " α" (for given K) are replaced by ($wt_s + \pi/2$) and ($\alpha - \pi/2$) respectively, equations (7)-(11) are unchanged but the sign of k is reversed. Therefore, for "k" negative, same parameters h, g and A, given in table 1 are used, only α and wt_s are to be replaced by ($\alpha - \pi/2$) and ($wt_s+\pi/2$), respectively.

2.2 Analysis of Series R-L_t-C circuit with an applied Signal V sin (wt+ α), and L_t=L (1+k cos 2wt), it is outside the differentiation sign.

The differential equation of the current x(t) is given by,

$$(1+k \cos 2wt)dx/dt + (R/L)x+w_o^2 \int x \, dt= (V/L)\sin(wt+ \alpha) \quad (5\text{-}b)$$

where $w_o^2 = 1/LC$. This is a special case of (5). Following the same analysis as in 2.1, the algebraic equation is:

$$-[1+k\cos(2\theta+h \sin2\theta -g \cos 2\theta +2 wt_s)](w^2/w_o^2)\sin \theta \;/$$

$$(1+h \cos 2\theta +g \sin 2\theta)+(w/w_o)(R/w_oL)\cos\theta +[(1+h/2).$$

$$\sin\theta -(g/2)\cos \theta +(h/6)\sin 3 \theta -(g/6) \cos 3 \theta] =$$

$$[wV/(w_o^2 AL)] \sin [\theta + wt_s + (h/2)\sin 2\theta - (g/2)\cos 2\theta + \alpha] \quad (14)$$

Putting $\theta = 0$, $\pi/4$, $\pi/2$ and $3\pi/4$ in (14) gives, respectively,

$$F_1 = (w/w_o)(R/w_o L) - 2g/3 - [wV/(w_o^2 AL)] \sin (wt_s - g/2 + \alpha) = 0 \quad (15)$$

$$F_2 = [1 - k\sin(h + 2wt_s)](w^2/w_o^2)/(1+g) - (w/w_o).(R/w_o L) -$$

$$(1 + 2h/3 - g/3) + [2\, wV/(w_o^2 AL)]\sin.(\pi/4 + wt_s + h/2 + \alpha) = 0 \quad (16)$$

$$F_3 = [1 - k\cos(g + 2wt_s)](w^2/w_o^2)/(1-h) - (1 + h/3) +$$

$$[mV(w_o^2 AL)] \cos (wt_s + g/2 + \alpha) = 0 \quad (17)$$

$$F_4 = [1 + k\sin(2wt_s - h)](w^2/w_o^2)/(1-g) + (w/w_o).$$

$$(R/w_o L) - [1 + 2h/3 + g/3] + [\sqrt{2}\, w V/(w_o^2 AL)] \cos(\pi/4 +$$

$$+ wt_s - h/2 + \alpha) \quad (18)$$

(15)-(18) are four nonlinear algebraic equations in four unknowns, namely, h,g. $[wV/(w_o^2 AL)]$ and wt_s. Aided with a microcomputer and using the same program used in section 2.1, the unknowns can be determined for given k, (w/w_o) and $R/w_o L$.

Case the parameter "k" is negative

Same as in section 2.1, if "wt" and "α" are replaced by $(wt + \pi/2)$ and $(\alpha - \pi/2)$, respectively, equation (5-b) is unchanged, but the sign of K is reversed. Therefore, for K negative, same parameters h, g and A, given in table 3, are used, only " α " and "wt_s " are to be replaced by($\alpha - \pi/2$) and ($wt_s + \pi/2$), respectively.

3. ANALYSIS OF SERIES R-L-C_t and R-L_t-C Circuits with no Applied Signal

These two cases give the theory and analysis of novel power generators as well as other applications.

3.1 R-L-C_t Circuit

The differential equation of the current x(t) is given by:

$$dx/dt+(R/L)x+w_o^2(1+k\cos 2wt)\int x\,dt= 0 \qquad (5.c)$$

The algebraic equation is given in (7) but with V=0. The unknowns here are h, g, w/w_o and "wt_s and are determined using the same program used in section 2. The four equations for the determination of these unknowns are given in equations (8) to (11) with V=0 . There are two basic solutions for (5-c).

The special case of R=0 ,can be simply analysed. Putting R= 0 = V in (8) gives g = 0. Then putting R = 0 = V = g in (9), (10) and (11) give, respectively:

$$w^2/w_o^2= [\,1 - k\sin(h+2wt_s)]\,(1+2h/3) \qquad (9.a)$$

$$w^2/w_o^2=(1-k\cos 2\,wt_s)(1-2h/3)-h^2/3) \qquad (10.a)$$

$$w^2/w_o^2=[1-k\sin(h-2wt_s)](1+2h/3) \qquad (11.a)$$

(9.a) and (11.a) give $2wt_2= 0$ or π , and $wt_s=0$ or $\pi/2$.

For $wt_s = 0$, (9-a) and (11-a) reduce to one equation, namely,

$$w^2/w_o^2 = (1-k \sin h)(1+2h/3) \qquad (9\text{-b})$$

while (10-a) gives : $w^2/w_o^2 = (1-k)(1-2h/3 - h^2/3)$

$$\text{Thus, } h = -1 + \sqrt{1 + 3\,[1-(w^2/w_o{}^2/)(1-k)]} \qquad (19)$$

Substituting (19) in (9-b) gives the relation between "w/w_o" and "k". This case corresponds to the well known se_1 Mathieu function [8].

The case $wt_s = \pi/2$, gives the same relation as $wt = 0$, only the sign of "k" is reversed, and it corresponds to the well known "Ce_1" Mathieu function.

In both cases, w^2/w_o^2 may be positive or negative.

In both cases, w^2/w_o^2 may be positive or negative.

Putting R=0 in (5-c) and replacing "x" by dy/dt gives:

$$d^2y/dt^2+w_o^2\,(1+k \cos 2\,wt)\,y=0 \qquad (20)$$

This is the well known Mathieu's equation; the relation between w_o^2/w^2 $(=x_1)$ and k is [8]

$$x_1=1+(x_1k/2)-(x_1k)^2/32-(x_1k)^3/512-(x_1k)^4/24576 + 0\,(x_1k)^5 \qquad (21)$$

Table 3 gives the relation between k and w_o^2/w^2 , for

$wt_s = 0$, calculated using (9-b) (aided with(19)) and calculated using equation(21), reference [8], for k ranging from $-\infty$ to $+\infty$. The parameter "h" is shown in the table.

The percentage error=

$$\{ [(w_o/w)^2[8]-(w_o/w)^2]/(w_o/w)^2[8] \} 100,$$

is also shown assuming the reasonable accuracy of our method.

For $wt_s = \pi/2$, corresponding to "Ce_1" Mathieu function, same parameters in Table 3 are used, only the sign of "k" is reversed.

The closed form parameteric solution of (20) is given by:

$$y = \int x(t)\, dt = \int A \cos\theta \;(dt/d\theta) d\theta$$

$$= (A/w) \int \cos\theta \;(1+h \cos 2\theta) d\theta$$

$$= (A/w)[1+h/2) \sin\theta + (h/6) \sin 3\theta]$$

$$\theta = wt-wt_s-(h/2)\sin 2\theta$$

There are two periodic solutions,one for $wt_s = 0$, correspon-ding to "se_1" Mathieu function and the other for $wt_s = \pi/2$, corresponding to the "Ce_1" Matheieu function.

The general case, $R \neq 0$

Using equations (8) to (11) with $V = 0$, the unknown

Table 3 : L - C_t Circuit , No Signal Applied

Our method $\begin{cases} 1/x_1=(1-k\ \sin\bar{h})(1+2h/3) \\ h=-1+\sqrt{1+3\left[1-1/x_1(1-k)\right]} \end{cases}$

k - x_1 relation ($\omega t_s=0$)

($x_1=\nu_0^2/\nu^2$)

Reference [8] $x_1=1+x_1k/2-(x_1k)^2/32-(x_1k)^3/512-(x_1k)^4/24576+0(x_1k)^5$

% error = $[x_1[8]-x_1]/x_1[8]\times100$

k	ν_0^2/ν^2	h	(ν_0^2/ν^2) [8]	Error %
0	1.0000	0.0000	1.0000	0.0000
-0.05	0.9755	0.0350	0.9755	0.0000
-0.1	0.9520	0.0655	0.9522	0.0210
-0.2	0.9074	0.1157	0.9084	0.1100
-0.4	0.8273	0.1874	0.8303	0.3613
-0.6	0.7581	0.2356	0.7641	0.7852
-0.8	0.6983	0.2702	0.7072	1.2585
-0.95	0.6586	0.2903	0.6697	1.6276
-1.0	0.6465	0.2961	0.6578	1.7178
-1.5	0.5439	0.3393	0.5595	2.7882
-2.5	0.4116	0.3839	0.4297	4.2122
-5.0	0.2542	0.4257	0.2709	6.1646
-7.0	0.1945	0.4393	0.2084	6.6700
-10.0	0.1438	0.4503	0.1553	7.4050
-100.0	0.0163	0.4753	0.0178	8.4270
± ∞	0.0000 / $kx_1=1.651$	0.4774	0.0000 / $kx_1=1.816$	9.0859
100.0	-.0168	0.4803	-0.0185	9.1892
10.0	-.1932	0.5081	-0.2178	11.2948
7.0	-.2973	0.5226	-0.3394	12.4042
5.0	-.4634	0.5432	-0.5392	14.0579
4.0	-.6417	0.5626	-0.7617	15.7542
3.0	-1.0373	0.5981	-1.2810	19.0242
2.5	-1.4892	0.6300	-1.9121	22.1169
2.0	-2.585	0.6551	-3.5992	28.1785
1.75	-3.988	0.7312		
1.5	-8.066	0.8045		
0.6	1.8756	-0.9643	1.3953	- 34.4227
0.5	1.5205	-0.7677	1.3147	- 15.6309
0.4	1.3142	-0.5580	1.2403	- 5.958
0.3	1.1923	-0.3633	1.1716	- 1.7668
0.2	1.1132	-0.2054	1.1097	- 0.3154
0.1	1.0525	-0.0873	1.0525	- 0.0000
0.05	1.0256	-0.0404	1.0256	- 0.0000
0	1.0000	0.0000	1.0000	- 0.0000

parameters h,g, w/w_o and wt_s are determined adopting the same program used in section 2, but modified to allow for using w/w_o as a variable instead of wV/w_o^2 AL).Tables 4 and 5 give h,g,w/w_o and wt_s for different values of "k" and R/w_oL Figure 2 . The closed form parametric solution of (5-c) is $x(t)=A\cos\theta$,

$$\theta = wt-wt_s-(h/2)\sin 2\theta+(g/2)\cos 2\theta ;$$

h,g,w/w_o and wt_s are given in tables 4 and 5.

IF "wt" is replaced by $(wt+\pi/2)$, (5.c) reduces to

$$dx/dt+(R/L)x+w_o^2(1-k\cos 2wt)\int x\,dt=0 \qquad (5.d)$$

Therefore, for solution of (5-d), same parameters h,g, w/w_o given in tables "4" and "5" are used, only "wt_s " is to be replaced by $(wt_s+\pi/2)$, and the sign of k is reversed.

Tables "4"and "5"are very useful in the design of parametric power generators as well as other similar applications.

3.2 $R-L_t-C$ Circuit

The differential equation of the current x(t) is given in equation (5-b) with V = 0 as follows:

$$(1+k\cos 2wt)\,dx/dt + (R/L)\,x + w_o^2\int x\,dt = 0 \qquad (22)$$

The algebraic equation is given in (14) but with V = 0 and

Table 4 R - L - C_t Circuit , No Signal Applied

(k positive)

k	R/w_oL	h	g	w/w_o	wt_s
0.1	0.01	-0.0851	0.0133	0.9752	0.0960
	0.02	-0.0783	0.0268	0.9769	0.1962
	0.03	-0.0665	0.0407	0.9798	0.3065
	0.05	-3.44×10^{-3}	0.0732	0.9974	0.7114
0.2	0.01	-0.2038	0.0119	0.9481	0.0474
	0.02	-0.1991	0.0238	0.9491	0.0952
	0.03	-0.1913	0.0358	0.9507	0.1437
	0.04	-0.1802	0.0481	0.9529	0.1935
	0.05	-0.1660	0.0607	0.9559	0.2452
	0.06	-0.1483	0.0736	0.9596	0.2999
	0.07	-0.1267	0.0872	0.9642	0.3591
	0.08	-0.1005	0.1015	0.9698	0.4263
	0.09	-0.0617	0.1174	0.9774	0.5089
	0.10	-0.0133	0.1377	0.9908	0.6490
0.3	0.01	-0.3615	0.0106	0.9161	0.0332
	0.025	-0.3535	0.0265	0.9178	0.0830
	0.055	-0.3177	0.0594	0.9253	0.1829
	0.07	-0.2906	0.0765	0.9308	0.2336
	0.085	-0.2579	0.0943	0.9372	0.2855
	0.1	-0.2197	0.1131	0.9446	0.3398
	0.115	-0.1755	0.1330	0.9531	0.3987
	0.13	-0.1238	0.1547	0.9630	0.4665
	0.145	-0.0581	0.1802	0.9761	0.5773
	0.15	-0.0282	0.1908	0.9826	0.6027
	0.155	-0.0277	0.2080	0.9960	0.7000

Table 4 (continued)

k	$\frac{R}{w_o L}$	h	g	ν/ν_o	νt_s
0.4	0.01	-0.5564	0.0093	0.8727	0.0282
	0.03	-0.5428	0.0282	0.8763	0.0842
	0.05	-0.5169	0.0476	0.8830	0.1389
	0.07	-0.4801	0.0678	0.8920	0.1916
	0.1	-0.4084	0.1	0.9083	0.2677
	0.12	-0.3518	0.1228	0.92	0.3174
	0.15	-0.255	0.1595	0.9384	0.3944
	0.175	-0.1622	0.1935	0.9545	0.4674
	0.2	-0.0466	0.2344	0.9745	0.5696
	0.21	0.0338	0.2609	0.9905	0.66
0.5	0.01	-0.7657	0.0081	0.8117	0.0278
	0.03	-0.7502	0.0246	0.8173	0.0824
	0.06	-0.7020	0.0506	0.8339	0.1584
	0.1	-0.6049	0.0887	0.8637	0.2464
	0.13	-0.5162	0.1200	0.8871	0.3043
	0.16	-0.4181	0.1534	0.9094	0.3583
	0.2	-0.2751	0.2013	0.9361	0.4305
	0.22	-0.1976	0.2271	0.9454	0.4704
	0.25	-0.066	0.2709	0.9676	0.5474
	0.27	0.0918	0.3215	0.9946	0.6814
0.6	0.01	-0.9618	0.0068	0.7314	0.0308
	0.05	-0.909	0.0359	0.7572	0.1433
	0.10	-0.7850	0.0784	0.8095	0.2495
	0.15	-0.6326	0.1275	0.8608	0.3282
	0.20	-0.4646	0.1814	0.9036	0.393
	0.25	-0.2843	0.2389	0.9366	0.4551
	0.30	-0.0847	0.3021	0.9624	0.5326
	0.325	0.0440	0.3428	0.9778	0.6022
	0.33	0.0843	0.3554	0.9836	0.6308

Table 5 : $R - L - C_t$ Circuit , No signal Applied

(k negative)

k	$\frac{R}{w_o L}$	h	g	w/w_o	wt_s
-0.1	0.01	0.0644	0.0170	1.0244	-0.1084
	0.02	0.0612	0.0336	1.0226	-0.2209
	0.03	0.0551	0.0495	1.0196	-0.3434
	0.04	0.0438	0.0640	1.0145	-0.49
	0.05	0.0099	0.0743	1.0017	-0.7717
-0.2	0.01	0.1154	0.0196	1.0494	-0.0595
	0.03	0.1132	0.0576	1.0468	-0.1797
	0.05	0.1078	0.0922	1.0416	-0.3039
	0.07	0.0968	0.1216	1.0333	-0.4392
	0.1	0.0382	0.1481	1.0064	-0.7588
	0.101	0.0282	0.1472	1.0032	-0.7929
	0.1015	0.0175	0.1455	0.9997	-0.8297
-0.4	0.01	0.1875	0.0273	1.0991	-0.0388
	0.03	0.1889	0.0797	1.0968	-0.1150
	0.05	0.1911	0.1262	1.0926	-0.1881
	0.07	0.1931	0.1656	1.087	-0.2577
	0.1	0.1942	0.2124	1.0764	-0.3573
	0.12	0.1926	0.2367	1.0682	-0.4227
	0.14	0.1882	0.2562	1.059	-0.4891
	0.16	0.1798	0.2712	1.0486	-0.5593
	0.18	0.1442	0.2813	1.0362	-0.6378
	0.2	0.1313	0.2838	1.0193	-0.7407
	0.21	0.0822	0.2747	1.0026	-0.8397
	0.2105	0.0739	0.2726	1.0003	-0.8533
	0.211	0.0684	0.2710	0.9987	-0.8627

Table 5 (continued)

k	$\frac{R}{w_o L}$	h	δ	ω/ω_o	ωt_s
-0.6	0.01	0.2363	0.0426	1.148	-0.0352
	0.05	0.2420	0.1780	1.1354	-0.1713
	0.1	0.2643	0.2744	1.1009	-0.2973
	0.15	0.2751	0.3311	1.1015	-0.4036
	0.2	0.2744	0.3670	1.0806	-0.5039
	0.25	0.2712	0.3889	1.0573	-0.6072
	0.3	0.2414	0.3962	1.029	-0.7296
	0.325	0.1950	0.3873	1.005	-0.8222
	0.330	0.1707	0.3814	1.0009	-0.8557
	0.332	0.1508	0.3757	0.996	-0.8791
-0.8	0.01	0.2729	0.0835	1.1194	-0.054
	0.05	0.2964	0.2526	1.1778	-0.1546
	0.1	0.3195	0.3414	1.1575	-0.2[illegible]4
	0.2	0.3486	0.427	1.1188	-0.4365
	0.3	0.3597	0.4681	1.0754	-0.5742
	0.4	0.3437	0.4834	1.0323	-0.7232
	0.45	0.3033	0.4764	1.0028	-0.824
	0.47	0.2332	0.4564	0.9820	-0.9085
-0.95	0.01	0.3013	0.1086	1.2244	-0.0926
	0.03	0.3106	0.2268	1.2177	-0.1307
	0.04	0.3252	0.2923	1.2069	-0.1819
	0.06	0.3369	0.3346	1.1976	-0.2207
	0.08	0.3470	0.3665	1.1890	-0.2541
	0.1	0.3557	0.3900	1.1808	-0.2843
	0.15	0.3741	0.4347	1.1614	-0.3513
	0.2	0.3885	0.4658	1.1425	-0.4116
	0.25	0.3997	0.4886	1;1237	-0.4684
	0.3	0.4084	0.5059	1.1045	-0.5233
	0.35	0.4137	0.5183	1.0843	-0.5776
	0.4	0.4150	0.5259	1.0643	-0.6326
	0.45	0.4135	0.5336	1.0428	-0.6900
	0.5	0.4043	0.5351	1.0195	-0.7521
	0.525	0.3950	0.5338	1.0070	-0.7868
	0.550	0.3393	0.5300	0.9933	-0.8266
	0.560	0.3679	0.5274	0.9573	-0.8451
	0.570	0.3517	0.5236	0.9407	-0.8664
	0.580	0.3355	0.5173	0.9229	-0.4938
	0.585	0.3454	0.5113	0.9677	-0.9145

the unknown parameters are h,g, w/w_o and "wt_s" and are determined using the same program used in section 2. The four equations are given in equations(15) to (18), but with V=0.

The special case of R = 0 can be simply analysed as follows: Putting R = 0 = V in (15) gives g = 0,. Then Putting R = 0 = V = g in (16), (17) and (18) give, respectively:

$$(w^2/w_o^2)\ [\ 1 - k \sin\ (h + 2wt_s)] = 1 + 2h/3 \tag{16-a}$$

$$(w^2/w_o^2)\ [\ 1 - k \cos 2\ wt_s] = 1 - 2h/3 - h^2/3 \tag{17-a}$$

$$(w^2/w_o^2)\ [1+k \sin\ (2wt_s - h)] = 1 + 2\ h/3 \tag{18-a}$$

(16-a) and (18-a) give $2wt_s=0$, or $2wt_s = \pi$, $wt_s=\pi/2$
For $wt_s = 0$, (16-a) or (18-a) reduce to one equation, namely,

$$(w^2/w_o^2)(1-k \sin h)= 1 + 2\ h/3 \tag{16-b}$$

while (17-a) gives, $w^2/w_o^2\ (1-k)=(1-2h/3-h^2/3)$

$$\text{Thus, } h = -1 + \sqrt{1 + 3\ [1 - (w^2/w_o^2)(1-k)}\tag{23}$$

Substituting (23) in (16-b) gives the characteristic relation between "w_o^2/w^2" and "k", Fig. 1.
For "wt_s"$= \pi/2$, same relations are used as "wt_s" = 0 , only the sign of "k" is reversed.

It is clear from (23) that h=1 at k=1.0, $w_o^2/w^2 = 0.09512$.

Table 6 gives w_o^2/w and h for different values of k ranging from -0.625 to 1.00^o.

The general case, $R \neq 0$:

Using equations (15) to (18) with V = 0, the unknown parameters h,g, w/w_o and "wt_s" can be determined adopting the same program used in section 2.1. The closed from parametric solution of (22) is $x(t) = A \cos \theta$, $\theta = wt - wt_s - (h/2) \sin 2\theta + (g/2) \cos 2\theta$.

If "wt" is replaced by $(wt + \pi/2)$, (22) reduces to

$$(1-k \cos 2wt)\, dx/dt + (R/L)\, x + w_o^2 \int x dt = 0 \qquad (22.a)$$

Therefore, for the solution of (22-a), same parameters h,g,w/w_o are used only, "wt_s" is to be replaced by $(wt_s + \pi/2)$, and the sign of k is reversed.

4. Analysis of a siries R-L_t-C Circuit, and the Inductance Varation [L_t=L(1+K cos 2wt)]is inside the Differentiation sign. [9].

Since the total flux linkage is the one to be differentiated in summing up the emf's around the circuit, the inductance variation should be inside the differentiation sign. The

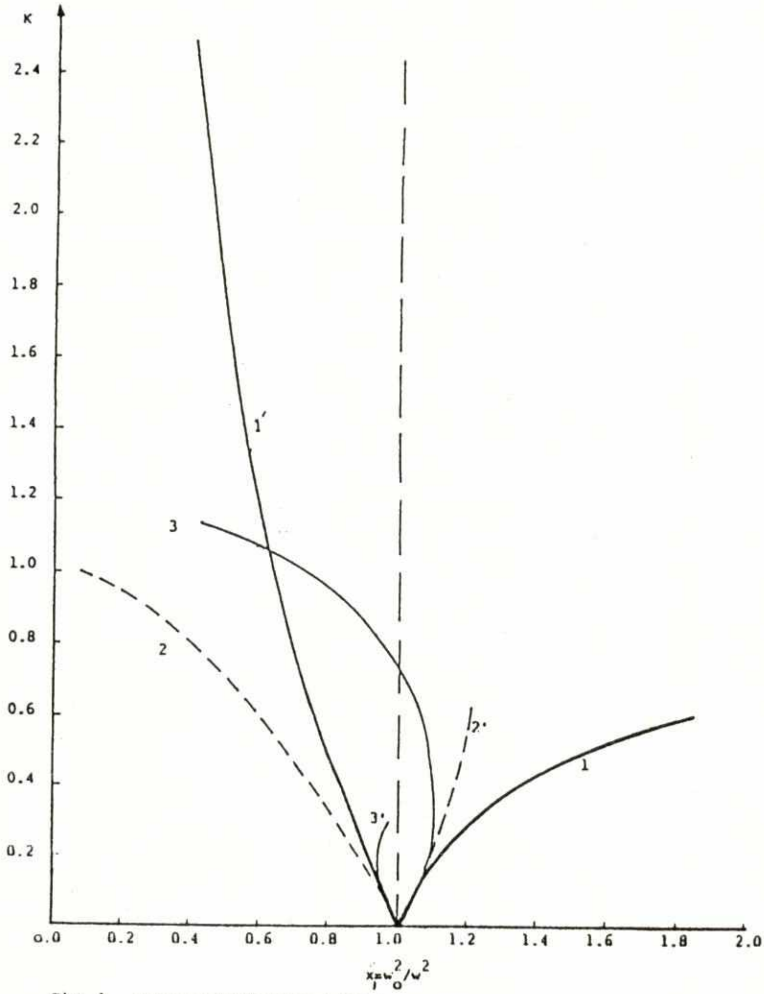

Fig. 1 Characteristic Curves for :

1. L-C_t circuits, curves "1" ($wt_s=0$), and "1′" ($wt_s=\pi/2$)
2. L_t-C circuits, curves "2" ($wt_s=0$), and "2′" ($wt_s=\pi/2$)
3. L_tC circuit, curves "3" ($wt_s=0$), and "3′" ($wt_s=\pi/2$)

Table 6 : L_t - C Circuit , No Signal Applied

"L_t" is outside the differentiation sign.

k - x_1 relation ($\omega t_s = 0$) : $x_1 = (1 - k\sin h) / (1 + 2h/3)$

($x_1 = \omega_o^2/\omega^2$) $h = -1 + \sqrt{1 + 3[1-(1-k)/x_1}$

k	w_o^2/w^2	h	k	w_o^2/w^2	h
0	1.0000	0.0000	0	1.0000	0.0000
0.05	0.9743	0.0368	-0.05	1.0243	-0.0384
0.1	0.9472	0.0723	-0.1	1.0472	-0.0788
0.2	0.8795	0.1402	-0.2	1.0885	-0.1679
0.3	0.8251	0.2062	-0.3	1.1235	-0.2728
0.4	0.7556	0.2719	-0.4	1.1527	-0.4028
0.5	0.6799	0.3392	-0.5	1.1789	-0.5721
0.6	0.5970	0.4107	-0.6	1.2132	-0.7912
0.7	0.5056	0.4898	-0.62	1.2229	-0.8397
0.8	0.4024	0.5839	-0.625	1.2255	-0.8515
0.9	0.2799	0.7112			
0.95	0.2041	0.8070			
0.97	0.1678	0.8611			
0.99	0.1239	0.9385			
0.995	0.1105	0.9658			
0.999	0.0984	0.9923			
0.9999	0.0955	0.9992			
1.000	0.0951	1.000			

differential equation of the current x(t) is therefore given by:

$$\frac{d}{dt}\ [(1+k\cos 2wt)x(t)]+(R/L)x(t)+w_o^2\int x(t)dt =$$

$$(V/L)\sin(wt+\alpha)\quad (24)\text{ , giving :}$$

$$(1+k\cos 2wt)\,dx/dt + w\,[(R/wL)-2k\sin 2wt]\,x +$$

$$w_o^2\int x\,dt = (V/L)\sin(wt+\alpha) \qquad (24\text{-a})$$

It is clear from (24-a) that the damping is periodically varying. Following the same analysis as in sections 2 and 3, the algebraic equation is :

$$-[1+k\cos(2\theta+h\sin 2\theta-g\cos 2\theta+2wt_s)](w^2/w_o^2)\sin\theta/$$

$$(1+h\cos 2\theta+g\sin 2\theta)+[(w/w_o)(R/w_oL)-2(w/w_o)^2 k\sin$$

$$(2\theta+h\sin 2\theta-g\cos 2\theta+2wt_s)\cos\theta+[(1+h/2)\sin\theta$$

$$-(g/2)\cos\theta+(h/6)\sin 3\theta-(g/6)\cos 3\theta] = (wV/w_o^2 AL)).$$

$$\sin[\theta+wt_s+(h/2)\sin 2\theta-(g/2)\cos 2\theta+\alpha]\quad (25)$$

Putting $\theta=0$, $,\pi/4$ $,\pi/2$ and $3\pi/4$ in (25) gives, respectively:

$$F_1 = [(w/w_o)(R/w_oL)-2(w/w_o)^2 k\sin(2wt_s-g)]-2g/3$$

$$-(wV/w_o^2 AL))\sin(wt_s-g/2+\alpha) = 0 \qquad (26)$$

$$F_2 = [1-k\sin(h+2wt_s)](w^2/w_o^2)/(1+g) - [(w/w_o)(R/w_o L)$$

$$-2(w/w_o)^2 k\cos(h+2wt_s)] - (1+2h/3-g/3)$$

$$+ (\sqrt{2}\, wV/w_o^2 AL)\sin(wt_s + h/2 + \pi/4 + \alpha) = 0 \quad (27)$$

$$F_3 = [1-k\cos(g+2wt_s)](w^2/w_o^2)/(1-h)$$

$$-(1+h/3) + (wV/w_o^2 AL)\cos(wt_s + g/2 + \alpha) = 0 \quad (28)$$

$$F_4 = [1+k\sin(2wt_s - h)](w^2/w_o^2)/(1-g) + [w/w_o)(R/w_o L) +$$

$$2(w^2/w_o^2)k\cos(2wt_s - h)] - (1+2h/3 + g/3 +$$

$$(\sqrt{2}\, wV/w_o^2 AL)\cos(wt_s - h/2 + \frac{\pi}{4} + 2) = 0 \quad (29)$$

The four unknown parameters h,g, $(wV/w_o^2 AL)$, and "wt_s" can be determined for given k, w/w_o and $R/w_o L$, using the four nonlinear algebraic equations (26) to (29).

The case with no applied signal is needed for parametric power generators' applications and the like, and will be analysed in details.

The special case R=0 is simply analysed as follows:

Putting R = 0 = V in (26) gives :

$$2(w/w_o)^2 K\sin(2wt_s - g) = -2g/3, \text{ giving}$$

$g = -\theta$, $2wts = \theta$ and "wt_s" $= \theta$; or $g = +\theta$, $2wt_s = \pi$

and "wt_s" = $\pi/2$

For $wt_s = 0$, (28) gives $(1-k)/x_1 = 1 - 2h/3 - h^2/3$,

and thus $h = -1 + \sqrt{1+3\,[1-(1-k)/x_1]}$ (30)

where $x_1 = w_o^2/w^2$

Equations (27) and (29) give the same relation, namely,

$(1-k \sin h + 2k \cos h)/(1+2h/3) = x_1$ (31) (31)

(31), aided with (30), gives the characteristic relation between k and x_1

It is clear from (30) that at k = 1 , h = 1 and w_o^2/w = 0.7435

For "wt " = $\pi/2$, same characteristic relation is used, only the sign of k is reversed. In this case, for h real, the maximum value of k is 0.2933, giving h=-0.88245, and w_o^2/w^2 = 0.97332.

Table 7 gives w_o^2/w^2 and h for different values of K ranging from -0.2933 to + 1.12, (for wt = 0), Fig. 1. For K > 1, h > 1.0 and $dt/d\theta$ is negative during a portion of the cycle. This could be allowed if the range of negative $dt/d\theta$ is very small.

The general case, R ≠ 0 ;

Using equations (26) to(29) with V=0, the unknown parameters h,g w/w_o and wt are determined adopting the same program used

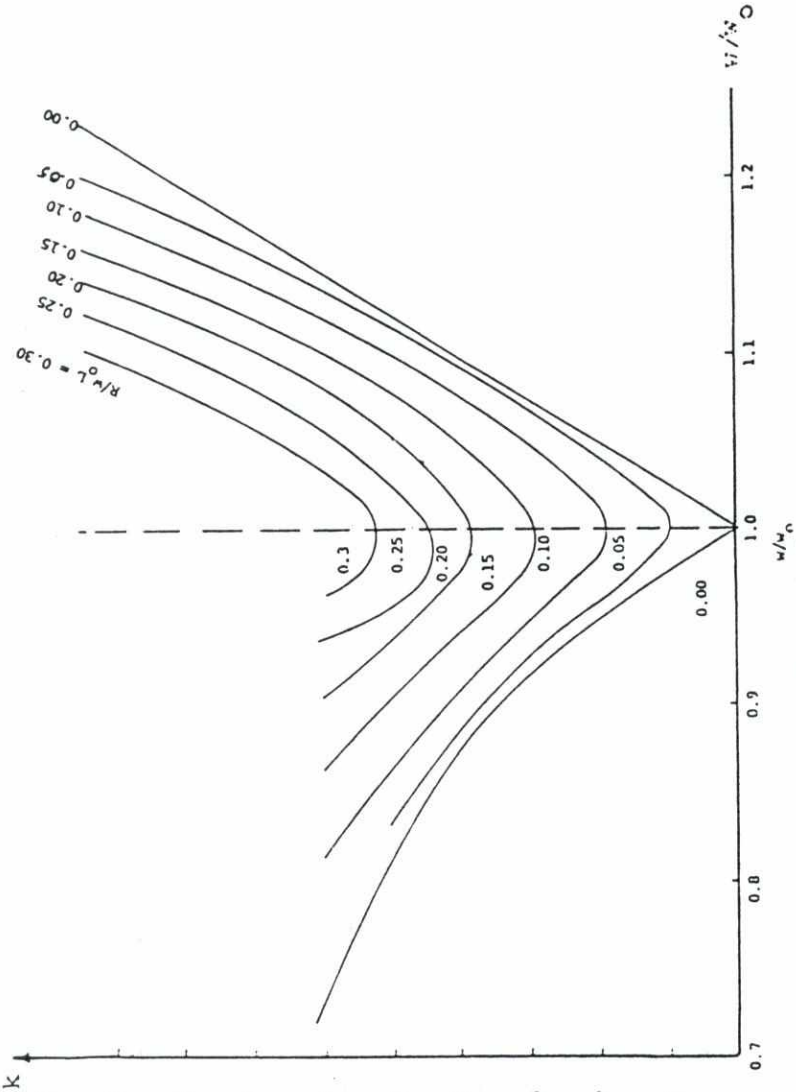

Fig. 2 Characteristic curves for R-L-C_t circuit

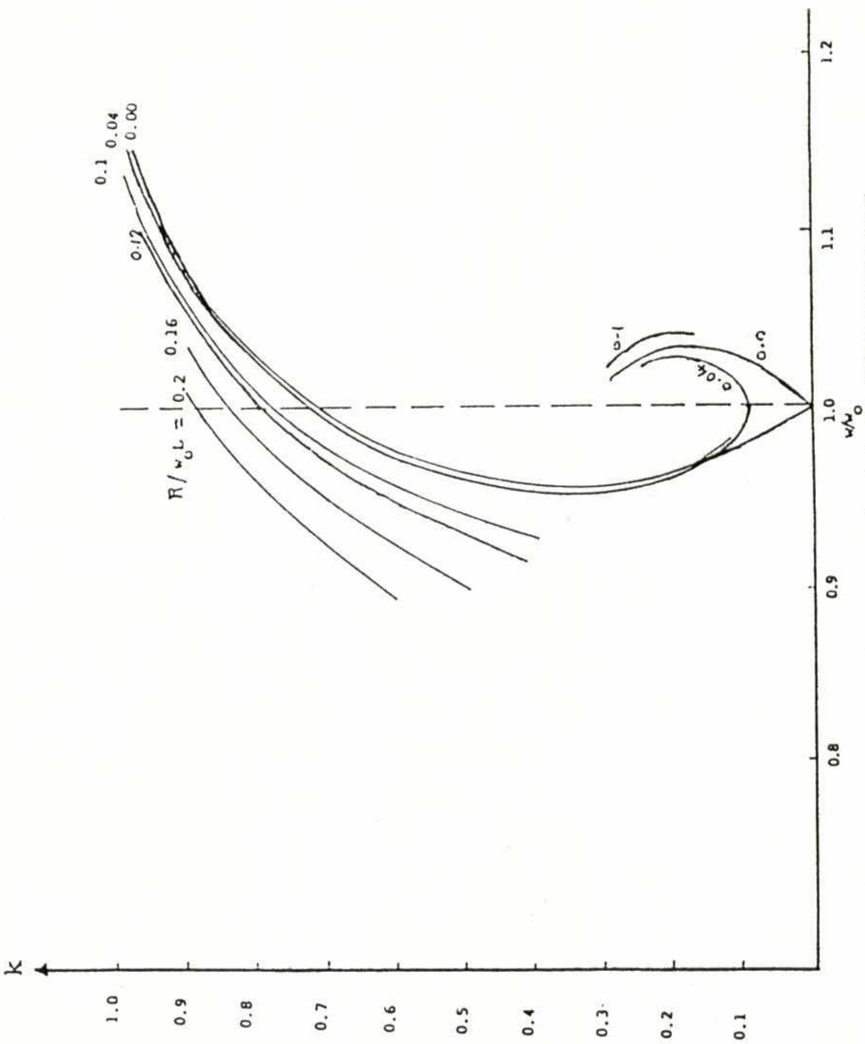

Fig. 3 Characteristic curves for $R-L_t-C$ circuit (L_t is inside differentiation sign).

Table 7. L_t-C circuit. No signal applied,

L_t is inside the differentiation sign.

k-x_1 relation (wt_s=0); $x_1 = w_o^2/w^2$

$x_1 = (1-k \sin h+2k \cosh)/(1+2h/3)$

$h = -1+\sqrt{1+3[1-(1-k)/x_1]}$

k	w_c^2/w^2	h	k	w_o^2/w^2	h
0	1	0	0	1	0
0.05	1.024	0.1030	−0.05	0.9748	−0.1233
0.1	1.045	0.1901	−0.1	0.9510	−0.2699
0.2	1.0763	0.3305	−0.15	0.9371	−0.4358
0.3	1.0926	0.4415	−0.20	0.9361	−0.6071
0.4	1.0944	0.53347	−0.25	0.9507	−0.7647
0.5	1.0819	0.6167	−0.275	0.9629	−0.8344
0.6	1.0548	0.6918	−0.28	0.9656	−0.8479
0.7	1.0119	0.7637	−0.29	0.9714	−0.8738
0.720	1.001	0.7779			
0.722	1.0001	0.7794	−0.292	0.9725	−0.8794
0.723	0.9996	0.7801			
0.73	0.99566	0.7851	−0.293	0.9732	−0.8815
0.74	0.9898	0.7922			
0.75	0.9838	0.7993			
0.8	0.9507	0.8354	−0.2933	0.9733	−0.8827
0.9	0.9116	0.9725			
0.95	0.8653	0.9114			
1.00	0.7434	1.00			
1.05	0.6566	1.0563			
1.10	0.5253	1.1380			
1.12	0.4222	1.2029			

Table . 8 : R-L_r-C Circuit. No signal applied, L_r is inside the differentiation sign. (K positive)

K	R/w_oL	h	g	w/w_c	wt_s
0.1	0.00	0.1801	0.0000	0.9782	0.0000
	0.01	0.1888	-0.0580	0.9783	0.0990
	0.02	0.1838	-0.1161	0.9786	0.2073
	0.03	0.1706	-0.1727	0.9801	0.3395
	0.035	0.1567	-0.1989	0.9819	0.4241
	0.0375	0.1463	-0.2101	0.9834	0.4779
0.2	0.00	0.3305	0.0000	0.9639	0.0000
	0.01	0.3311	-0.0576	0.9636	0.0361
	0.02	0.3327	-0.1156	0.9625	0.0737
	0.03	0.3354	-0.1745	0.9608	0.1148
	0.04	0.3389	-0.2347	0.9585	0.1620
	0.05	0.3425	-0.2976	0.9557	0.2209
	0.055	0.3437	-0.3311	0.9542	0.2587
	0.06	0.3435	-0.3671	0.9527	0.3073
	0.065	0.3380	-0.4082	0.9522	0.3813
0.3	0.00	0.4415	0.0000	0.9567	0.0000
	0.01	0.442	-0.0498	0.9563	0.0141
	0.02	0.4433	-0.0993	0.9552	0.0286
	0.04	0.4487	-0.1999	0.9509	0.0607
	0.06	0.4583	-0.3025	0.9435	0.1011
	0.08	0.4733	-0.4115	0.9325	0.1605
	0.09	0.4838	-0.4739	0.9251	0.2083
0.4	0.00	0.5347	0.0000	0.9559	0.0000
	0.01	0.5349	-0.0408	0.9556	0.00477
	0.02	0.5353	-0.0817	0.9547	0.00968
	0.04	0.5374	-0.1636	0.9511	0.0206
	0.06	0.5412	-0.2462	0.9451	0.0340
	0.08	0.5470	-0.3300	0.9365	0.0519
	0.1	0.5568	-0.4152	0.9254	0.0771
	0.12	0.5717	-0.5085	0.9105	0.1168
	0.13	0.5826	-0.5600	0.9011	0.1479

Table 8 :(Continue)

k	R/w_oL	h	g	w/w_o	wt_s
0.5	0.00	0.6167	0.0000	0.9614	0.0000
	0.02	0.6166	-0.0661	0.9604	0.00127
	0.04	0.6167	-0.1328	0.9575	0.003
	0.06	0.6169	-0.1997	0.9527	0.00577
	0.08	0.6177	-0.2678	0.9459	0.0102
	0.1	0.6195	-0.3372	0.9370	0.0170
	0.12	0.6230	-0.4082	0.9261	0.0275
	0.14	0.6292	-0.4816	0.9128	0.0437
	0.16	0.6397	-0.5584	0.8968	0.0689
	0.18	0.6583	-0.6453	0.8760	0.1137
0.6	0.0000	0.6918	0.0000	0.9739	0.0000
	0.0200	0.6915	-0.0536	0.9729	-0.0025
	0.04	0.6906	-0.1078	0.9705	-0.00486
	0.06	0.6891	-0.1623	0.9665	-0.00675
	0.08	0.6873	-0.2181	0.9608	-0.00805
	0.10	0.6852	-0.2749	0.9535	-0.083
	0.12	0.6833	-0.3333	0.9444	-0.0072
	0.14	0.6820	-0.3938	0.9335	-0.0042
	0.16	0.6820	-0.4558	0.9208	0.0018
	0.18	0.6841	-0.5202	0.9062	0.0118
	0.20	0.6896	-0.5872	0.8893	0.0281
	0.22	0.7008	-0.6589	0.8694	0.0549
	0.24	0.7244	-0.7438	0.8428	0.1079
0.7	0.00	0.7637	0.0000	0.9941	0.0000
	0.04	0.7619	-0.0883	0.9913	-0.0083
	0.08	0.7567	-0.1779	0.9829	-0.0158
	0.12	0.7486	-0.2728	0.9687	-0.0219
	0.16	0.7388	-0.3748	0.9480	-0.0250
	0.20	0.7304	-0.4847	0.9209	-0.0216
	0.24	0.7291	-0.6042	0.8867	-0.0054

Table 8 : Continued

K	R/w_cL	h	g	w/w_o	wt_s
0.8	0.00	0.8354	0.000	1.0256	0.000
	0.04	0.8335	-0.0716	1.023	-0.00933
	0.08	0.8275	-0.1461	1.0153	-0.01878
	0.12	0.8179	-0.2230	1.0023	-0.0274
	0.16	0.8047	-0.3063	0.9834	-0.0352
	0.20	0.7887	-0.3986	0.9582	-0.041
	0.24	0.7730	-0.5000	0.9267	-0.0418
	0.28	0.7625	-0.6131	0.8885	-0.0328
	0.32	0.7679	-0.7354	0.84385	-0.0024
0.9	0.00	0.9114	0.0000	1.0748	0.0000
	0.04	0.9094	-0.0587	1.0723	-0.0095
	0.08	0.9034	-0.1180	1.0647	-0.0188
	0.12	0.8933	-0.1807	1.0517	-0.0282
	0.16	0.8787	-0.2488	1.0330	-0.0378
	0.20	0.8618	-0.3196	1.0094	-0.04596
	0.24	0.8449	-0.39	0.986	-0.054
		0.8605	-0.3223	1.0087	-0.0464
		0.8596	-0.3249	1.0079	-0.04708
		0.8606	-0.3219	1.0089	-0.0463
		0.8594	-0.3253	1.0078	-0.0472
		0.8608	-0.3213	1.009	-0.04616
	0.22	0.8533			
0.95	0.00	0.9532	0.000	1.11063	0.0000
	0.02	0.9527	-0.0267	1.1100	-0.0048
	0.04	0.9512	-0.0525	1.1080	-0.0093
	0.06	0.9487	-0.0786	1.1047	-0.0137
	0.08	0.9451	-0.1061	1.1000	-0.0185
	0.10	0.9405	-0.1335	1.0940	-0.0231
	0.12	0.9350	-0.1615	1.0866	-0.0276
	0.13	0.9318	-0.1761	1.0824	-0.0299
	0.14	0.9287	-0.1904	1.0777	-0.0321
0.99	0.00	0.9901	0.0000	1.1484	0.0000
	0.02	0.9896	-0.0235	1.1478	-0.0043
	0.04	0.9881	-0.0471	1.1457	-0.0087
	0.06	0.9856	-0.0717	1.1421	-0.0131
	0.08	0.9820	-0.0956	1.1372	-0.0176
	0.10	0.9772<	-0.1214	1.1305	-0.0223
	0.12	0.9716	-0.1469	1.1229	-0.0268
	0.14	0.9650	-0.1732	1.1135	-0.0312

Table 9 : R-L_t-C Circuit; L_t is inside the differentiation sign.

(k is negative)

K	R/w_oL	h	g	w/w_o	wt_s
-0.05	0.00	-0.1233	0.0000	1.0128	0.0000
	0.01	-0.1131	-0.0356	1.0120	-0.1864
	0.02	-0.0729	-0.0775	1.0084	-0.4308
-0.1	0.00	-0.2699	0.0000	1.0250	0.0000
	0.01	-0.2656	-0.0251	1.0248	-0.0770
	0.02	-0.2518	-0.0519	1.0242	-0.1587
	0.03	-0.2249	-0.0826	1.0228	-0.2525
	0.04	-0.1730	-0.1236	1.0194	-0.3790
-0.15	0.00	-0.4357	0.0000	1.0330	0.0000
	0.02	-0.4274	-0.0342	1.0331	-0.0851
	0.04	-0.3985	-0.0731	1.0331	-0.1790
	0.06	-0.3295	-0.1283	1.0320	-0.3040
	0.07	-0.2345	-0.1854	1.0282	-0.4250
-0.2	0.00	-0.6071	0.0000	1.03357	0.0000
	0.01	-0.6060	-0.0109	1.0336	-0.0261
	0.02	-0.6029	-0.0223	1.0338	-0.0527
	0.03	-0.5976	-0.0338	1.0341	-0.0797
	0.04	-0.5899	-0.0460	1.0346	-0.1075
	0.06	-0.5654	-0.0731	1.0360	-0.1671
	0.08	-0.5221	-0.1078	1.0380	-0.2378
	0.10	-0.4222	-0.1697	1.0401	-0.3477
-0.25	0.00	-0.7647	0.0000	1.0256	0.0000.
	0.01	-0.7640	-0.0074	1.0257	-0.0181
	0.04	-0.7563	-0.0300	1.0267	-0.0731
	0.06	-0.7451	-0.0464	1.0281	-0.1113
	0.08	-0.7282	-0.0645	1.0301	-0.1515
	0.12	-0.6641	-0.1134	1.0369	-0.2473
	0.14	-0.5920	-0.1571	1.0427	-0.3182
	0.15	-0.4694	-0.2226	1.0484	-0.4062
-0.275	0.00	-0.8344	0.0000	1.0191	0.0000
	0.01	-0.8346	-0.006	1.0192	-0.0154
	0.02	-0.8334	-0.1220	1.0194	-0.0311
	0.03	-0.8314	-0.0185	1.0197	-0.0468
	0.04	-0.8287	-0.0248	1.0201	-0.0626
-0.29	0.00	-0.8738	0.0000	1.0146	0.0000
	0.05	-0.8659	-0.0280	1.0161	-0.0722
	0.10	-0.8394	-0.0599	1.0206	-0.1466
	0.15	-0.7810	-0.1046	1.0298	-0.2389
	0.175	-0.7217	-0.1416	1.0378	-0.2996
-02933	0.00	-0.8827	0.0000	1.0136	0.0000
	0.025	-0.8803	-0.0135	1.0139	-0.0352
	0.05	-0.8747	-0.0272	1.0150	-0.0707
	0.075	-0.8648	-0.0418	1.0168	-0.1072
	0.10	-0.8492	-0.0583	1.0195	-0.1457
	0.125	-0.8270	-0.0773	1.0232	-0.1869
	0.15	-0.7943	-0.1010	1.0284	-0.2331
	0.175	-0.7407	-0.1349	1.0360	-0.2901

in section 3.1. Tables 8 and 9 give h,g, w/w_o and "wt_s" for different values of K and R/w_oL. The closed form parameteric solution of (24) is $x(t) = A \cos \theta$, $\theta = wt-wt_s -(h/2) \sin 2\theta+(g/2) \cos 2\theta$; h,g,w/w and "wt " are given in tables 8 and 9 and Fig.3.

If "wt" is replaced by $(wt+\pi/2)$, (24) reduced to $d/dt[(1-k \cos 2wt). x(t)]+(R/L)x(t)+w_o^2\int x(t)dt=0$ (24-b). Therefore, for solution of (24-b), same parameters $h,g,w/w_o$ given in tables 8 and 9 are used, only "wt_s" is to be replaced by $(wt_s+ \pi/2)$.

Tables 8 and 9 are very useful in the design of parameteric power generators and other similar applications.

5.Conclusion

The known methods used in the analysis of periodically varying systems, with no dissipation, are limited to special $f_1(t)$, $f_2(t)$ and $f_3(t)$ functions and used the techniques of series expansion and continued tractions.The method presented in the paper is general and not limited to special periodic functions $f_1(t)$, $f_2(t)$ and $f_3(t)$. Closed form parametric solutions with simple closed form parametric characteristic relations are attained for three cases, namely: $L-C_t$ circuit, L_t-C (L_t outside the differentiation sign circuit) and L_t-C(L_t inside the differentiation sign circuit). In the three cases, the solution is $x(t) = A \cos \theta = wt - wt_s-(h/2) \sin 2\theta$.

There are two solutions for each case, one solution corresponds to "wt_s" = 0 and the other solution corresponds to "wt_s" = $\pi/2$. For a given k, h and w_o^2/w^2 ($=x_1$) are obtained from the following characteristic relations for each case, "wt_s" = 0; the corresponding differential equations are also stated. For "wt_s" = $\pi/2$, same relations are used, only the sign of "k" is reversed.

1. **L-C_t circuit** $dx/dt + w_o^2 (1+k \cos 2wt) \int x \, dt = 0$

$$1/x_1 = (1-k \sin h)(1+2h/3)$$

$h = -1 + \sqrt{1+3[1-1/x_1(1-k)]}$, Table 3 and Curve"1", Fig.1

2. **L_t-C circuit** : $(1+k \cos 2wt)dx/dt + w_o^2 \int x dt = 0$

$$x_1 = (1-k \sin h)/(1+2h/3)$$

$h = -1 + \sqrt{1+3[1-(1-k)/x_1]}$, Table 6 curve "2", Fig. 1.

3. **L_t-C circuit**: $d/dt\,[(1+k \cos 2wt).x] + w_o^2 \int x \, dt = 0$

$$x_1 = (1-k \sinh+2k \cos h)/(1+2h/3)$$

$h = -1 + \sqrt{1+3\ [1-(1-k)/x_1]}$, table 7 and curve "3", Fig.1.

Fig.1 shows a plot of the characteristic relations for three cases, for wt_s=0 and for "wt_s "π = /2. It is clear from this figure that the instability range bounded by the two stable curves (wt_s =0 and wt_s =$\pi/2$) is largest for the L-$C_t$circuit (curves "1" and "1'") and smallest for the L_t -C circuit with L_t inside the differentiation sign (curves "3" and "3'"). This means that the system can generate periodic signals with the largest resistive loads if , the capacitance is periodically varying (Tables 4,5,8 and 9) and hence, the R-L-C_t circuit is more suitable for parametric power generators, Fig. 1 and 3.

Dividing the differential equations (33) and (34) by (1+k cos 2wt) gives, respectively:

$$dx/dt+w_o^2[a_o+\sum_{n=1,2..} a_n \cos 2nwt]\int x dt=0 \qquad (33\text{-}a)$$

$$dx/dt-2kw\ \{a_o \sin 2wt+(a_1/2)\sin 4wt+\sum_{n=2,3}(a_n/2)$$
$$[\sin 2(n+1)wt-\sin 2(n-1)wt]\}\ x+w_o^2[a_o+\sum_{n=1,2..} a_n$$
$$\cos 2nwt]\int x dt = 0 \qquad (34\text{-}a)$$

where $a_o = 1/\sqrt{1-k^2}$, $a_1=(2/k)(1-a_o)=-k-(3/4)k^3-..,$

$a_2= -2a_1/K-2\ a_o$, and for $n > 2$, the recurrence relation is, $a_n+(k/2)(a_{n+1}+a_{n-1})=0$

It is clear that (33-a) is similar to (32) except that $w_o^2/\sqrt{1-k^2}$ replaces w_o^2 and the sign of k (to the first approximation) is reversed and thus the characteristic curves are interchanged and the instability range is reduced. However, equation (34-a) includes, over and above, a periodically varying resistance term, playing a big role in signal generation and changing drastically the corresponding characteristic curves (3 and 3', Fig. 1).

The problem of power exchange between the input electrical source and the mechanical (or otherwise) sources producing the periodic variations of the system is also studied. Table 1 indicates how the angle "α" between the input electrical source and the capacitance variation controls this exchange. The response amplitude "A" varies between maximum at "α "= 50° and minimum at "α" = 142.5°. For a given "α", the energy of the mechanical source may compensate completely the energy dissipated in the resistance at a certain "k" and the amplitude "A" approaches infinity, table 2 k=1.99 at $\alpha = 0$.

More accurate results, regarding the whole analysis given in the paper,can be attained if more harmonic terms in the dt/dθ series are considered.

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