Direct adaptive fuzzy control for a class of MIMO nonlinear uncertain systems

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This paper proposes a new design methodology of direct adaptive fuzzy controllers for a class of MIMO nonlinear dynamical systems with unknown nonlinearities. The unknown nonlinearities are approximated by fuzzy system with a set of fuzzy IF-THEN rules whose parameters are adjusted on-line according to derived adaptive control laws. The fuzzy adaptive laws ensure stability, convergence of the controlled output, and 'boundedness' of the adaptation parameters. The goal is to control the output of a class of nonlinear systems (encountered mainly in robotics) in order to track some given trajectories. Theoretical results are illustrated through a simulation example. They show the effectiveness of the proposed control scheme.

نظم التحكم المهايئه لاتعتمد على معرفة مسبقه بنموذج رياضى مؤكد للأنظمه الديناميكيه. الفكره الأساسيه تعود الى تقدير الأشياء غير المؤكده فى الوحده المراد التحكم فيها بناءا على القياسات المتاحه. وبصوره عامه فأن الوحده المراد التحكم فيها (plant) تحتوى عادة على أحد نوعين من اللامحققية (uncertainty). النوع الأول هو اللامحققيه فى بنية النموذج الرياضى الديناميكى والنوع الثانى هو اللامحققيه فى عناصره. ويكون الهدف الأساسى للتحكم المهايىء فى كلا الحالتين هو المحافظه على أداء سليم للوحدة المراد التحكم فيها. وبما أن نظرية التحكم المهايىء التقليدية تتعامل فقط مع النوع الثانى من عدم المحققيه (اللامحققيه فى عناصر الوحده) وجب البحث عن طرق أخرى للتحكم المهايىء تستطيع التعامل مع اللامحققيه فى بنية النموذج الرياضى للوحده المراد التحكم فيها. فى الآونه الأخيره تم أجراء العديد من الدراسات التى تتناول نظريه الداله المشوشه التقريبيه وأستخداماتها بواسطة المحكمات المهايئه. على وجه التحديد يتم أستخدام الدوال المشوشه كدوال تقريبيه للخصائص اللاخطيه للوحده المراد التحكم بها. هذه الدوال المشوشه يمكن أعتبارها دوال خطيه يمكن ضبط عناصرها أثناء عمل الوحده المراد التحكم بها. الهدف من هذه الدراسة تقديم مقترح جديد بمنظومة تحكم مهايئة مشوشه ومباشره (أى لا تعتمد على نموذج رياضى مرجعى Reference model) لمجموعه من الأنظمه اللاخطيه متعددة المدخلات والمخرجات. طريقة التحكم المقترحه لا تعتمد على معلومات عن الوحده المراد التحكم بها بل على قوانين مهايئة تم أستنتاجها خلال هذه الدراسه. هذه القوانين تفرض الألتقاء(convergence) لعناصر المنظومه المشوشه. بالأضافه ألى ذلك تقوم القوانين المهايئه بحساب القيمه التقديريه للخطأ (estimation error) لبنية الوحده المراد التحكم بها بطريقه منفصله عن التشويش الخارجى. وحيث أن أحد مركبات التحكم تعتمد على هذه القيمة التقديريه للخطأ فأن التحكم يكون من النوع التعويضى. وقد خلصت الدراسه ألى الأستنتاجات الآتيه فيما يخص منظومة التحكم المقترحه فى هذه الدراسة: أنها قادره على دمج منظومه مشوشه لوصف الوحده المراد التحكم بها. تجدد بأستمرار عناصر التحكم وبالتالى تجعله قادرا على التغلب على الخصائص اللاخطيه والامحققيه للوحده المراد التحكم بها وكذلك التغلب على التشويش الخارجى. أنها تضمن الأستقرار الكلى (global stability) لدائرة التحكم المغلقه بمعنى أن كل الأشارات تكون داخل حدود منتظمه. المنظومه المشوشه التى أستخدمت كدوال تقريبيه للخصائص اللاخطيه تعتبر بسيطه بالنسبه ألى الدراسات السابقه. نتائج المحاكاة الرياضيه لروبوت ثنائى الأذرع أثبتت جدارة نظام التحكم المقترح.

Keywords: Fuzzy systems, Adaptive fuzzy control, Lyapunov synthesis, Robotic systems

1. Introduction

Adaptive control is a model-free approach for controlling uncertain dynamic systems. The basic idea is to estimate the uncertainties in the plant on-line based on the measured signals. In principle, the system under control can be uncertain in terms of its dynamic structure (nonparametric uncertainty), or its parameters (parametric uncertainty). Generally, the basic objective of adaptive control is to maintain consistent performance of the control system in the presence of these uncertainties. Conventional adaptive control theory, however, can only deal with the systems with known dynamic structure, but unknown parameters [1]. This drawback has been the main reason for seeking other adaptive control methodologies which can tackle the nonparametric uncertainty.

Recently, the analytical study of adaptive nonlinear control systems using universal function approximators has received much attention [2-14]. Typically, these methods use fuzzy logic systems as approximation models for unknown system nonlinearities. Using the approximation capability of fuzzy systems, which is the linear function of adjustable parameters, the design schemes of some stable adaptive fuzzy controllers were proposed in literature. In [3-4], the tracking error convergence depended upon the assumption that the approximation error should be square-integrable. On the bases of Sanner and Slotine [2] and Wang [4], Su and Stephanenko [5] proposed an adaptive fuzzy controller, which relaxes the condition that the approximation error should be square-integrable. The control scheme however is suitable only for nonlinear control systems with unity/constant control gain.

In this paper, a new direct adaptive fuzzy control method is proposed for a class of MIMO nonlinear plant encountered in robotics. The proposed method "relaxes" the knowledge of the plant upper bounds by introducing adaptive control laws. These laws ensure the convergence and boundedness of adaptation parameters of the fuzzy systems. Furthermore, it computes on-line the estimation error on the plant structure by means of an adaptive algorithm independent of the external disturbances. As the compensatory sliding term itself depends on this estimation error, it leads to an adaptive compensation.

The paper is organized as follows: Section 2 describes the features of the adjustable fuzzy systems used in the sequel. Section 3, presents the control problem statement. The error dynamics is elaborated in Section 4. In Section 5, the adaptive control laws are derived. Section 6 demonstrates how to design an adaptive fuzzy controller for two link planar robot. Simulation results are also demonstrated and discussed. Section 7 offers the concluding remarks.

2. Description of the implemented fuzzy system

Fuzzy logic systems performs a mapping from $U_1 \times U_2 \cdots U_n \subset R^n$ to R where each $U_i \subset R$, $i = 1,2,\cdots n$. Here, we use the implication and the reasoning method suggested by Takago and Sugeno (T-S), [15]. Consequently, the fuzzy IF-THEN rules of zero order type are expressed as:

$$R_k\text{:}\mathit{if}\ x_1\ \mathit{is}\ A_1^{l1}\ \text{and} \cdots x_n\ \mathit{is}\ A_n^{ln}\ \text{then}\ z_k = a^k , \quad (1)$$

where $x = (x_1, \cdots x_n)^T \in R^n$ and $z_k \in R$ are, respectively, the input of the fuzzy logic system and the consequent of the k[th] rule. Here, the label A_i^{li} associated to input x_i, $i = 1,2,\cdots n$, is a fuzzy set in U_i where the index li takes a value in $\{1,\cdots,m_i\}$ and m_i is the number of fuzzy sets characterizing the input x_i . The coefficient a^k (for $k = 1,2,\cdots,M$) is an adaptable coefficient of the consequent part for the k^{th} fuzzy rule. The number of rules M is defined by the Cartezian product as: $M = m_1 \otimes m_2 \cdots \otimes m_n$.

In this article, the product operation for fuzzy implication and T-norm are employed. The definition of the product operation is the same as in [16]. Besides, the singleton fuzzifier and weighted average defuzzification are used. The overall output value is

$$z(x) = \frac{\sum_{k=1}^{M} \alpha_k a^k}{\sum_{k=1}^{M} \alpha_k} , \quad (2)$$

where α_k denotes the firing strength of the R_k rule, which is evaluated by using the product inference and implication as:

$$\alpha_k = \prod_{i=1}^{n} \mu A_i^{li}(x_i) \quad \text{with} \quad l_i \in \{1,\cdots\cdots,m_i\} , \quad (3)$$

where $\mu A_i^{li}(x_i)$ is the membership grade of x_i associated to fuzzy set A_i^{li} .

If the antecedents A_i^{li} 's of the rule base are fixed and the a^k 's form the adjustable parameters; $z(x)$ in (2) can be rewritten as:

$$z(x) = \zeta^T(x)\theta \,, \tag{4}$$

where θ is the parameter vector given by:

$$\theta = \left[a^1, a^2, \cdots, a^M\right]^T \,, \tag{5}$$

and $\zeta^T(x) = [\zeta_1, \zeta_2, \cdots, \zeta_M]$ where

$$\zeta_i = \frac{a_i}{\sum_{k=1}^{M} a_k} \,, \; i = 1, \cdots M \,. \tag{6}$$

In the sequel, the fuzzy logic system for multi-input single output is represented by the mathematical expression (4).

3. Problem formulation

Our goal is to build a fuzzy adaptive control system for a certain class of MIMO nonlinear dynamic systems encountered mainly in robotics. This class is of the form:

$$\begin{aligned} u &= f(x)x^{(p)} + g(x) \\ y &= x \,, \end{aligned} \tag{7}$$

where $\mathrm{x} = [(x^{(p-1)})^T, \cdots, \dot{x}^T, x^T]^T$, $x = [x_1, \cdots x_n]^T$, $\mathbf{x} \in R^{n,p\times 1}$ is the state vector and assumed to be available from measurements, $f(\mathrm{x}) \in R^{n\times n}$ and $g(\mathrm{x}) \in R^{n\times 1}$ are unknown continuous vector functions, $u = [u_1, u_2, \cdots u_n]^T \in R$ and $y \in R$ are respectively, the input and output of the system. In order for eq. (7) to be controllable, we require that $g(\mathrm{x}) \neq 0$ for x in certain controllability region, $U_c \subset R^n$. This class of nonlinear systems is called square system since the number of inputs is the same as the number of outputs [1].

Now, the control objective is to force the output $y(t)$ to track a given bounded reference trajectory $y^d(t)$ under the constraint that all signals are bounded.

Assumption A1. We assume that the function $f(\mathrm{x}) \in R^{n\times n}$ is a positive-definite matrix fulfilling:

$$\left\|\dot{f}(\mathrm{x})\right\| < f_o \|\mathrm{x}\| \quad \forall\, \mathrm{x} \in \Omega_c \text{ with } f_o > 0 \,, \tag{8}$$

where $\Omega_c \subseteq R^{n.p}$ is a subspace through which the state trajectory may travel under closed-loop control and f_o is unknown.

The function $g(\mathrm{x}) \in R^{n\times 1}$ is a nonlinear function; it is composed of ill-known but bounded continuous functions.

Remark 1. Notice that in robotics, the function $f(\mathrm{x})$ is the inertia matrix, which is positive definite. The bounded function $g(\mathrm{x})$ represents globally the effects of Coriolis and centrifugal forces, the gravitational torques (or forces), viscous and/or dynamic friction, unstructured friction effects such as static friction terms, disturbances and unmodeled dynamics.

By exploiting the approximation property of fuzzy systems defined in (4), the unknown functions f and g are approximated by:

$$f(\mathrm{x}) = \phi_f(\mathrm{x}) \text{ and } g(\mathrm{x}) = \phi_g(\mathrm{x}) \,. \tag{9}$$

The unknown functions ϕ_f and ϕ_g, representing the system uncertainties are approximated by two zero-order T-S functions with rules of eq. (1) in the form of eq. (4) as follows:

$$\phi_f(\mathrm{x}) = \hat{\phi}_f(\mathrm{x}, \theta_f^*) + \delta_f(\mathrm{x}) = \psi^T(\mathrm{x})\theta_f^* + \delta_f(\mathrm{x}) \,. \tag{10}$$

$$\phi_g(\mathrm{x}) = \hat{\phi}_g(\mathrm{x}, \theta_g^*) + \delta_g(\mathrm{x}) = \psi^T(\mathrm{x})\theta_g^* + \delta_g(\mathrm{x}) \,, \tag{11}$$

where θ_f^* and θ_g^* are some unknown optimal parameter vectors, δ_f and δ_g represent the reconstruction (approximation) error for each fuzzy system. In general, increasing the number of rules by increasing the number of fuzzy sets for each variable reduces the reconstruction error.

The optimal parameter vectors θ_f^* and θ_g^* are artificial quantities required only for analytical purposes. Typically [9, 17], θ_f^* and θ_g^* are chosen as the values of θ_f and θ_g, respectively, minimize the reconstruction errors; i.e.,

$$\theta_f^* = \arg\min_{\theta_f}\left[\sup_{x\in U_c}\left|\hat{\phi}_f(x,\theta_f)-\phi_f(x)\right|\right]. \quad (12)$$

$$\theta_g^* = \arg\min_{\theta_g}\left[\sup_{x\in U_c}\left|\hat{\phi}_g(x,\theta_g)-\phi_g(x)\right|\right]. \quad (13)$$

The rest of terms in (10 and 11) are defined as follows:

$$\psi^T(x) = \begin{bmatrix} \zeta^T & 0 & 0 & 0 & \cdots 0 & 0 \\ 0 & \zeta^T & 0 & 0 & \cdots 0 & 0 \\ \cdots & \cdots & \cdots & & & \\ 0 & 0 & 0 & 0 & \cdots 0 & \zeta^T \end{bmatrix}, \quad (14)$$

where $\zeta^T(x) = [\zeta_1, \zeta_2, \cdots, \zeta_M]$,

$$\theta_f^* = \begin{bmatrix} \theta_f^*(1,1) & \theta_f^*(1,2) & \cdots & \theta_f^*(1,n) \\ \theta_f^*(2,1) & \theta_f^*(2,2) & \cdots & \theta_f^*(2,n) \\ \cdots & \cdots & \cdots & \\ \theta_f^*(n,1) & \theta_f^*(n,2) & \cdots & \theta_f^*(n,n) \end{bmatrix}, \quad (15)$$

where

$$\theta_f^*(i,j) = [a_{ij}^1 \; a_{ij}^2 \; \cdots \; a_{ij}^M]^T, \quad i,j = 1,2\cdots n, \quad (16)$$

and

$$\theta_g^* = [(\theta_{g1}^*)^T \; (\theta_{g2}^*)^T \cdots (\theta_{gn}^*)^T]^T, \quad (17)$$

where

$$\theta_{gi}^* = [a_1^{gi} \; a_2^{gi} \; \cdots \; a_M^{g}]^T, \quad i = 1,2\cdots n. \quad (18)$$

4. Error dynamics

Using the theory of sliding mode control, let us introduce the sliding surfaces (filtered tracking errors) as:

$$S = [s_1, s_2 \cdots s_n]^T, \quad (19)$$

with

$$s_i = \left(\frac{\partial}{\partial t} + \lambda_i\right)^{(p-1)} e_i \text{ for } \lambda_i > 0, \quad (20)$$

where λ_i is positive coefficient, $e_i = y_i^d - x_i$, with $i = 1,2\cdots,n$, and y_i^d stands for the desired i^{th} output. Now the control objective is to force the system trajectories to stick to the sliding surfaces (19), thus achieves tracking and nullifying errors. Reconsidering (20), we obtain

$$s_i = \lambda_i^{(p-1)} e_i + (p-1)\lambda_i^{(p-2)}\dot{e}_i + \cdots (p-1)\lambda_i e_i^{(p-2)} + e_i^{(p-1)}. \quad (21)$$

So that, asymptotic tracking can be achieved when roots of the following polynomial is Hurwitz.

$$h_i(s) = \lambda_i^{(p-1)} + (p-1)\lambda_i^{(p-2)} s + \cdots (p-1)\lambda_i s^{(p-2)} + s^{(p-1)}, \quad (22)$$

where s is the Laplace operator via the condition $\lambda_i > 0$ with $i = 1,2\cdots,n$.

To simplify the presentation, the relation (21) can be rewritten in the following compact form:

$$s_i = \Lambda_i^T E_i, \quad (23)$$

with

$$\Lambda_i = [\lambda_i^{(n-1)}, (n-1)\lambda_i^{(n-2)}, \cdots (n-1)\lambda_i, 1]^T, \quad (24)$$

and

$$E_i = [e_i, \dot{e}_i, \cdots e_i^{(p-1)}]^T . \quad (25)$$

Consequently, the vector S of (19) takes the following form:

$$S = \Lambda^T E , \quad (26)$$

where:

$$\Lambda^T = diag[\Lambda_1^T, \Lambda_2^T, \cdots, \Lambda_n^T]_{n\times p,n} , \quad (27)$$

and

$$E = [E_1^T, E_2^T, \cdots E_n^T]_{(p,n\times 1)}^T . \quad (28)$$

The first derivative of (23) is given by:

$$\dot{s}_i = \Lambda_{ri}^T E_i + e_i^{(p)} \text{ and } i = 1,2 \cdots n , \quad (29)$$

where

$$\Lambda_{ri}^T = [0, \lambda_i^{(p-1)}, (p-1)\lambda_i^{(p-2)}, \ldots\ 0.5(p-1)(p-2)\lambda_i^2, (p-1)\lambda_i] . \quad (30)$$

Therefore the dynamics of S can be written into the following compact form:

$$\dot{S} = \Lambda_r^T E + e^{(p)} , \quad (31)$$

where

$$\Lambda_r^T = [\Lambda_{r1}^T, \Lambda_{r2}^T \ldots \Lambda_{rn-1}^T, \Lambda_{rn}^T]_{n\times p.n} , \text{ and}$$
$$e = [e_1, e_2, \cdots e_n]^T . \quad (32)$$

From (7) we obtain:

$$y_d^{(p)} - x^{(p)} = y_d^{(p)} - f^{-1}(\mathrm{x})[u(t) - g(\mathrm{x})] , \quad (33)$$

or

$$e^{(p)} = y_d^{(p)} - f^{-1}(\mathrm{x})[u(t) - g(\mathrm{x})] . \quad (34)$$

Let us substitute $e^{(p)}$ given by (34) in the expression (31); it follows that:

$$\dot{S} = \Lambda_r^T E + y_d^{(p)} - f^{-1}(\mathrm{x})[u(t) - g(\mathrm{x})] . \quad (35)$$

Now, let us define the filtered reference

$$Y_{ref} = \Lambda_r^T E + y_d^{(p)} . \quad (36)$$

So that

$$\dot{S} = Y_{ref} - f^{-1}(\mathrm{x})[u(t) - g(\mathrm{x})], \quad (37)$$

which is equivalent to:

$$f(\mathrm{x})\dot{S} = f(\mathrm{x})Y_{ref} + g(\mathrm{x}) - u(t) . \quad (38)$$

Using (9, 10 and 11), the filtered tracking error dynamic (38) can be transformed into the final form:

$$f(\mathrm{x})\dot{S} = \psi^T(\mathrm{x})\theta_f^* Y_{ref} + \psi^T(\mathrm{x})\theta_g^* +$$
$$\delta_f(\mathrm{x})Y_{ref} + \delta_g(\mathrm{x}) - u(t) . \quad (39)$$

5. The control synthesis

In this Section, we develop an adaptive control law which is able to force the plant to follow the desired trajectory y^d. The procedure is based on Lyapunov direct method.

Proposition 1. If the nonlinear system (7) is conducted by the following adaptive control law:

$$u(t) = k_d S + \psi^T \hat{\theta}_f Y_{ref} + \psi^T \hat{\theta}_g + \frac{1}{2}\hat{f}_o \|\mathrm{x}\| . S + u_s , \quad (40)$$

where

$$u_s = \hat{\delta}_f \|Y_{ref}\| sign(S) + \hat{\delta}_g sign(S) , \quad (41)$$

the vector parameters $\hat{\theta}_g$ and $\hat{\theta}_f$ are updated by:

$$\dot{\hat{\theta}}_g = \gamma_1 \psi . S . \quad (42)$$

$$\dot{\hat{\theta}}_f = \gamma_2 \psi . S . (Y_{ref})^T , \quad (43)$$

and the parameter bounds $\dot{\hat{f}}_o$, $\hat{\delta}_f$ and $\hat{\delta}_g$ are updated such that:

$$\dot{\hat{f}}_o = \eta_1 \|x\|.\|S\| . \quad (44)$$

$$\dot{\hat{\delta}}_f = \eta_2 \|Y_{ref}\|.\|S\| . \quad (45)$$

$$\dot{\hat{\delta}}_g = \eta_2 \|S\| , \quad (46)$$

where $\gamma_1, \gamma_2, \eta_1, \eta_2 > 0$.

Therefore, under assumption (A1):

1. E, x and u are bounded.
2. $E \to 0$ as $t \to \infty$ and $\hat{\theta}_f \to \theta_f^*$; $\hat{\theta}_g \to \theta_g^*$ as $t \to \infty$.

Proof. The following Lyapunov function is considered:

$$V = \frac{1}{2} S^T f(x) S + \frac{1}{2\gamma_1} \tilde{\theta}_g^T \tilde{\theta}_g + \frac{1}{2\gamma_2} trace(\tilde{\theta}_f^T \tilde{\theta}_f) + \frac{1}{2\eta_1}(\tilde{f}_o)^2 + \frac{1}{2\eta_2}(\tilde{\delta}_f)^2 + \frac{1}{2\eta_2}(\tilde{\delta}_g)^2 , \quad (47)$$

with

$$\tilde{\theta}_f = \theta_f^* - \hat{\theta}_f \;, \tilde{\theta}_g = \theta_g^* - \hat{\theta}_g , \quad (48)$$

$$\tilde{f}_o = f_o - \hat{f} \;, \;\; \tilde{\delta}_f = \bar{\delta}_f - \hat{\delta}_f , \text{ and } \tilde{\delta}_g = \bar{\delta}_g - \hat{\delta}_g , \quad (49)$$

where $\bar{\delta}_f$ and $\bar{\delta}_g$ are nominal values for δ_f and δ_g. Differentiating the Lyapunov function with respect to time we obtain:

$$\dot{V} = \frac{1}{2} S^T \dot{f}(x) S + S^T f(x) \dot{S} - \frac{1}{\gamma_1} \tilde{\theta}_g^T \dot{\hat{\theta}}_g - \frac{1}{\gamma_2} trace(\tilde{\theta}_f^T \dot{\hat{\theta}}_f) - \frac{1}{\eta_1} \tilde{f}_o \dot{\hat{f}} - \frac{1}{\eta_2} \tilde{\delta}_f \dot{\hat{\delta}}_f - \frac{1}{\eta_2} \tilde{\delta}_g \dot{\hat{\delta}}_g \;. \quad (50)$$

Substituting in (50) $f(x)\dot{S}$ by its expression (39), $\dot{V}$ becomes

$$\dot{V} = \frac{1}{2} S^T \dot{f}(x) S + S^T [\psi^T \theta_f^* Y_{ref} + \psi^T \theta_g^* + \delta_f Y_{ref} + \delta_g - u(t)] - \frac{1}{\gamma_1} \tilde{\theta}_g^T \dot{\hat{\theta}}_g - \frac{1}{\gamma_2} trace(\tilde{\theta}_f^T \dot{\hat{\theta}}_f) - \frac{1}{\eta_1} \tilde{f}_o \dot{\hat{f}} - \frac{1}{\eta_2} \tilde{\delta}_f \dot{\hat{\delta}}_f - \frac{1}{\eta_2} \tilde{\delta}_g \dot{\hat{\delta}}_g . \quad (51)$$

Now, we introduce in (51) the adaptive control law $u(t)$ given by (40); we obtain:

$$\dot{V} = \frac{1}{2} S^T \dot{f}(x) S + S^T \psi^T \theta_f^* Y_{ref} + S^T \psi^T \theta_g^* + S^T \delta_f Y_{ref} + S^T \delta_g - S^T k_d S - \frac{1}{2} S^T \hat{f} \|x\|.S - S^T \psi^T \hat{\theta}_f Y_{ref} - S^T \psi^T \hat{\theta}_g - S^T \delta_f \|Y_{ref}\| sign\;(S) - S^T \hat{\delta}_g sign\;(S) - \frac{1}{\gamma_1} \tilde{\theta}_g^T \dot{\hat{\theta}}_g - \frac{1}{\gamma_2} trace(\tilde{\theta}_f^T \dot{\hat{\theta}}_f) - \frac{1}{\eta_1} \tilde{f}_o \dot{\hat{f}} - \frac{1}{\eta_2} \tilde{\delta}_f \dot{\hat{\delta}}_f - \frac{1}{\eta_2} \tilde{\delta}_g \dot{\hat{\delta}}_g . \quad (52)$$

Introducing the parameter adaptive laws (42) and (43) in expression (52) leads to:

$$\dot{V} = -k_d S^T S - \frac{1}{2} S^T \dot{f}(x) S - \frac{1}{2} S^T \hat{f}_o \|x\|.S - \frac{1}{\eta_1} \tilde{f}_o \dot{\hat{f}}_o + S^T(\delta_f Y_{ref} + \delta_g) - S^T(\hat{\delta}_f \|Y_{ref}\| + \hat{\delta}_g).sign\;(S) - \frac{1}{\eta_2} \tilde{\delta}_f \dot{\hat{\delta}}_f - \frac{1}{\eta_2} \tilde{\delta}_g \dot{\hat{\delta}}_g + S^T \psi^T \tilde{\theta}_f Y_{ref} - trace\;[\tilde{\theta}_f^T \psi_f S (Y_{ref})^T] + S^T \psi^T \tilde{\theta}_g - \tilde{\theta}_g^T \psi_g S . \quad (53)$$

As $S^T \psi^T \tilde{\theta}_f Y_{ref} = trace\;[\tilde{\theta}_f^T \psi_f S (Y_{ref})^T]$, $\dot{V}$ is reduced to:

$$\dot{V} = -k_d S^T S - \frac{1}{2} S^T \dot{f}(x) S - \frac{1}{2} S^T \hat{f}_o \|x\|.S - \frac{1}{\eta_1} \tilde{f}_o \dot{\hat{f}}_o + S^T(\delta_f Y_{ref} + \delta_g) - S^T(\hat{\delta}_f \|Y_{ref}\| + \hat{\delta}_g).sign\;(S) - \frac{1}{\eta_2} \tilde{\delta}_f \dot{\hat{\delta}}_f - \frac{1}{\eta_2} \tilde{\delta}_g \dot{\hat{\delta}}_g . \quad (54)$$

The following inequality is always fulfilled:

$$\dot{V} \le -k_d S^T S + \frac{1}{2} S^T f_o \|\mathrm{x}\|.S - \frac{1}{2} S^T \hat{f}_o \|\mathrm{x}\|.S - \frac{1}{\eta_1} \tilde{f}_o \dot{\hat{f}}_o + \bar{\delta}_f \|Y_{ref}\|.\|S\| - \hat{\delta}_f \|Y_{ref}\|.\|S\| - \frac{1}{\eta_2} \tilde{\delta}_f \dot{\hat{\delta}}_f + \bar{\delta}_g \|S\| - \hat{\delta}_g \|S\| - \frac{1}{\eta_2} \tilde{\delta}_g \dot{\hat{\delta}}_g \; . \qquad (55)$$

This is equivalent to:

$$\dot{V} \le -k_d S^T S + \frac{1}{2} S^T \tilde{f}_o \|\mathrm{x}\|.S - \frac{1}{\eta_1} \tilde{f}_o \dot{\hat{f}}_o + \tilde{\delta}_f \|Y_{ref}\|.\|S\| - \frac{1}{\eta_2} \tilde{\delta}_f \dot{\hat{\delta}}_f + \tilde{\delta}_g \|S\| - \frac{1}{\eta_2} \tilde{\delta}_g \dot{\hat{\delta}}_g . \qquad (56)$$

Using the adaptive laws (44 – 46) of the parameter bounds $\hat{f}_o$, $\hat{\delta}_f$ and $\hat{\delta}_g$, we obtain:

$$\dot{V} \le -k_d S^T S < 0 \qquad \forall S \ne 0 \; . \qquad (57)$$

Therefore, the function V in (47) is the Lyapunov function for the closed-loop system (40, 41 – 46). So ($S, \hat{\theta}_f, \hat{\theta}_g, \hat{f}_o, \hat{\delta}_f, \hat{\delta}_g$) are bounded and $S \to 0$ as $t \to 0$. As $S \to 0$ then from (25 and 28) $E = [e_1 \; \dot{e}_1 \dots e_1^{(n-1)}, \dots e_n \; \dot{e}_n \dots e_n^{(n-1)}]^T \to 0$ and as y^d and its derivative are bounded, we have $x, \mathrm{x} \in L_\infty$.

The boundedness of the control law $u(t)$ is directly deduced from the boundedness of $(x, \mathrm{x}, \hat{\theta}_f, \hat{\theta}_g, \hat{f}_o, \hat{\delta}_f, \hat{\delta}_g)$.

To show that $\tilde{\theta}_f, \tilde{\theta}_g, \tilde{f}_o, \tilde{\delta}_f, \tilde{\delta}_g \to 0$ as $t \to \infty$ let us define:

$$V_\theta = \frac{1}{2\gamma_1} \int_0^\infty \tilde{\theta}_g^T \tilde{\theta}_g d\tau + \frac{1}{2\gamma_2} \int_0^\infty trace\,(\tilde{\theta}_f^T \tilde{\theta}_f) d\tau + \frac{1}{2\eta_1} \int_0^\infty (\tilde{f}_o) d\tau + \frac{1}{2\eta_2} \int_0^\infty (\tilde{\delta}_f) d\tau + \frac{1}{2\eta_2} \int_0^\infty (\tilde{\delta}_g) d\tau. \qquad (58)$$

which can be written as:

$$V_\theta = \int_0^\infty \dot{V} d\tau - \frac{1}{2} \int_0^\infty S^T f(\mathrm{x}) S \, d\tau \; . \qquad (59)$$

As $f(\mathrm{x})$ is positive-definite, the following inequality is fulfilled:

$$S^T f(\mathrm{x}) S \ge \sigma . \|S\|^2 \text{ with } \sigma = \left\| f^{-1} \right\|_\infty . \qquad (60)$$

Therefore, we can write

$$V_\theta \le \int_0^\infty \dot{V} d\tau - \frac{\sigma}{2} \int_0^\infty \|S\|^2 d\tau \; . \qquad (61)$$

Note that the expressions (58), and (48)-(49) mean that V is bounded and none is increasing with time; hence it has a finite limit:

$$\lim_{t \to \infty} V(S, \tilde{\theta}_f, \tilde{\theta}_g, \tilde{\delta}_f, \tilde{\delta}_g) = V_\infty < \infty \; , \qquad (62)$$

and

$$\int_0^\infty \dot{V} d\tau \in L_\infty \; . \qquad (63)$$

Moreover, the inequality (57) leads to:

$$\int_0^\infty \|S\|^2 d\tau \le -\frac{1}{k_d} \int_0^\infty \dot{V} d\tau \; . \qquad (64)$$

$$\int_0^\infty \|S\|^2 d\tau \le \frac{1}{k_d} (V - V_\infty) \in L_\infty \; . \qquad (65)$$

The conditions (63 and 65) entail that $V_\theta \in L_\infty$ which means that $\tilde{\theta}_f, \tilde{\theta}_g, \tilde{f}_o, \tilde{\delta}_f, \tilde{\delta}_g \in L_2$. Because ϕ_f, ϕ_g, Y_{ref} and $S \in L_\infty$, it follows from (45-46) that $\dot{\tilde{\delta}}_f$ and $\dot{\tilde{\delta}}_g \in L_\infty$ which, together with $\tilde{\delta}_f$ and $\delta_g \in L_2$ implies, using Barbalat lemma [1], that $\tilde{\delta}_f$ and $\tilde{\delta}_g \to 0$ as $t \to \infty$.

Remark 2. In practice, because the control law contains *sign*(S) , a discontinuous term, applying (41) will cause a chattering problem. The coming results are obtained after replacing the *sign*(S) by a saturation function of the form:

$$sat(S/\kappa) = \begin{cases} sign(S/\kappa) & if\ S \le \kappa \\ S/\kappa & if\ S > \kappa \end{cases}, \quad (66)$$

where κ is a positive constant. So that the sliding control term in (41) becomes:

$$u_s = \hat{\delta}_f \left\| Y_{ref} \right\| sat\ (S/\kappa) + \hat{\delta}_g sat(S/\kappa). \quad (67)$$

6. Numerical tests

In this Section, we present a computer simulation to examine the validity of the proposed control algorithm using the two link planar manipulator shown in fig. 1, carrying a load.

6.1. The design of direct adaptive control system for the two link robot:

Consider the two-link robot moving in a vertical plane, whose inverse dynamics are given by:

$$\begin{aligned} u_1 = {} & m_2 l_2^2 (\ddot{x}_1 + \ddot{x}_2) + m_2 l_1 l_2 (2\ddot{x}_1 + \ddot{x}_2) \cos x_2 + \\ & (m_1 + m_2) l_1^2 \ddot{x}_1 - m_2 l_1 l_2 \dot{x}_2^2 \sin x_2 - \\ & 2 m_2 l_1 l_2 \dot{x}_1 \dot{x}_2 \sin x_2 + m_2 l_2 g \cos(x_1 + x_2) + \\ & (m_1 + m_2) l_1 g \cos x_1, \end{aligned} \quad (68)$$

$$\begin{aligned} u_2 = {} & m_2 l_1 l_2 \ddot{x}_1 \cos x_2 + m_2 l_1 l_2 \dot{x}_1^2 \sin x_2 + \\ & m_2 l_2 g \cos(x_1 + x_2) + m_2 l_2^2 (\ddot{x}_1 + \ddot{x}_2), \end{aligned} \quad (69)$$

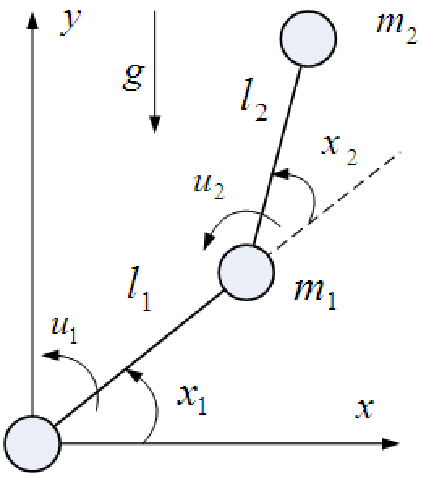

Fig. 1. Two link manipulator model.

which is equivalent to plant model (7) where the joint position vector x and the state vector x denote, respectively, $x = [x_1\ x_2]^T$ and $\mathrm{x} = [\dot{x}_1\ \dot{x}_2\ x_1\ x_2]^T$.

For simplicity, the control signals are viewed as two independent subsystems, one for each joint. Therefore, the dynamic plant can be rewritten as:

$$u_1 = f_1(\mathrm{x}_1)\ddot{x}_1 + g_1(\mathrm{x}_1) \text{ with } \mathrm{x}_1 = [x_1\ \dot{x}_1]^T, \quad (70)$$

$$u_2 = f_2(\mathrm{x}_2)\ddot{x}_2 + g_2(\mathrm{x}_2) \text{ with } \mathrm{x}_2 = [x_2\ \dot{x}_2]^T. \quad (71)$$

The functions $f_1(\mathrm{x}_1)$, $f_2(\mathrm{x}_2)$, $g_1(\mathrm{x}_1)$ and $g_2(\mathrm{x}_2)$ are modeled as zero-order T-S fuzzy system. Each input variable is described by two fuzzy sets.

- With respect to function f_1 and g_1, the rule base incorporates 4 rules of the form:

$$R_k^{f_1}: \text{IF } x_1 \text{ is } A_1^{l1} \text{ and } \dot{x}_1 \text{ is } A_2^{l2} \text{ then } F_k^1 = \alpha_k^{F_1}. \quad (72)$$

$$R_k^{g_1}: \text{IF } x_1 \text{ is } A_1^{l1} \text{ and } \dot{x}_1 \text{ is } A_2^{l2} \text{ then } G_k^1 = \alpha_k^{g_2}, \quad (73)$$

for l_1, $l_2 \in \{1,2\}$ and $k = \{1 \ldots 4\}$. The overall output is given by:

$$\hat{f}_1(x, \theta_{f_1}) = \frac{\sum_{k=1}^{4} \alpha_k^1 . f_k^1}{\sum_{k=1}^{4} \alpha_k^1} = \psi_1^T . \hat{\theta}_{f_1}, \quad (74)$$

$$\hat{g}_1(x, \theta_{g_1}) = \frac{\sum_{k=1}^{4} \alpha_k^1 . g_k^1}{\sum_{k=1}^{4} \alpha_k^1} = \psi_1^T . \hat{\theta}_{g_1}, \quad (75)$$

with:

$$\alpha_k^1 = \mu A_1^{l_1}(x_1) . \mu A_2^{l_2}(\dot{x}_1). \quad (76)$$

$$\hat{\theta}_{f_1} = [\alpha_1^{f_1} \ldots \alpha_4^{f_1}]^T. \quad (77)$$

$$\hat{\theta}_{g_1} = [a_1^{g_1} \ldots a_4^{g_1}]^T . \quad (78)$$

- Similarly, the rule base of f_2 and g_2 incorporates 4 rules of the form:

$$R_k^{f_2}: \text{IF } x_2 \text{ is } A_1^{l1} \text{ and } \dot{x}_2 \text{ is } A_2^{l2} \text{ then } F_k^2 = a_k^{F_2} . \quad (79)$$

$$R_k^{g_2}: \text{IF } x_2 \text{ is } A_1^{l1} \text{ and } \dot{x}_2 \text{ is } A_2^{l2} \text{ then } G_k^2 = a_k^{g_2} , \quad (80)$$

for l_1 , $l_2 \in \{1,2\}$ and $k = \{1 \ldots 4\}$. The overall output is given by:

$$\hat{f}_2(x, \theta_{f_2}) = \frac{\sum_{k=1}^{4} \alpha_k^2 . f_k^2}{\sum_{k=1}^{4} \alpha_k^2} = \psi_2^T . \hat{\theta}_{f_2} , \quad (81)$$

$$\hat{g}_2(x, \theta_{g_2}) = \frac{\sum_{k=1}^{4} \alpha_k^2 . g_k^2}{\sum_{k=1}^{4} \alpha_k^2} = \psi_2^T . \hat{\theta}_{g_2} , \quad (82)$$

with:

$$\alpha_k^2 = \mu A_1^{l_1}(x_2) . \mu A_2^{l_2}(\dot{x}_2) . \quad (83)$$

$$\hat{\theta}_{f_2} = [a_1^{f_2} \ldots a_4^{f_2}]^T . \quad (84)$$

$$\hat{\theta}_{g_2} = [a_1^{g_2} \ldots a_4^{g_2}]^T . \quad (85)$$

6.2. Results and discussions

The control law (40) is constituted by an adaptive fuzzy model term $\psi^T \hat{\theta}_f Y_{ref} + \psi^T \hat{\theta}_g$, the metric term $k_d S + \frac{1}{2} \hat{f}_o \|\mathrm{x}\| . S$ and the sliding compensatory term u_s . It is imperative to look at the control coefficients such that the adaptive fuzzy model term is preponderant. After few trails, satisfactory results have been obtained for the control coefficients and bounds initial values ($\hat{f}_o, \hat{\delta}_f, \hat{\delta}_g$), which are set up as indicated in table 1.

A time step of 0.0005 second has been incorporated in the simulation tests. The desired trajectories for x_1 and x_2 were set as: $y_1^d(t) = -b_1 \sin(\omega\, t)$, $y_2^d(t) = b_2 \sin(\omega_2 t)$, with $b_1 = 0.6 \ rad$, $b_2 = 0.8 \ rad$, $\omega_1 = 0.5\pi \ rad/s$ and $\omega_2 = \pi \ rad/s$. The input torque of joint one is saturated to $\pm 300 \ N.m$.

The simulations are conducted, first, when the robot starts from rest with initial position errors $\pm\pi/6$; (test 1: see figs. 2-6); second, where a mass of 10 kg is added to the tip of link 2 after one second, (test 2: see fig. 7); third, where masses of the links are randomly changed with time, (test 3: see figs. 8, 9); and fourth, when random noise with amplitude $5 \ N.m$ is added to the gravity torque, (test 4: see figs. 10, 11).

The obtained results show that the tracking regime is effectively established with acceptable tracking error and the control inputs (u_1, u_2) appear feasible. Fig. 6 shows that the sliding torque component evolves during motion within maximum value of $\pm 15 \ N.m$. Consequently, the adaptive fuzzy model term remains preponderant in the control law as it can be anticipated from a comparison with fig. 5. Moreover, the inputs remain continuous. However, in test 2, fig. 7, the control increases somewhat relative to test 1 and presents an acceptable discontinuity at $t = 1$ sec when the 10 kg mass is added to the tip of link 2. These results reveal that the proposed direct adaptive fuzzy control law is highly robust in the face of internal uncertainty and external disturbances.

Table 1
Control coefficients and initial bounds

Joint	K_d	λ	γ	η	$\hat{f}_o$	$\hat{\delta}_f$	$\hat{\delta}_g$
1	1500	5	1500	40	5	15	10
2	100	5	500	40	5	15	10

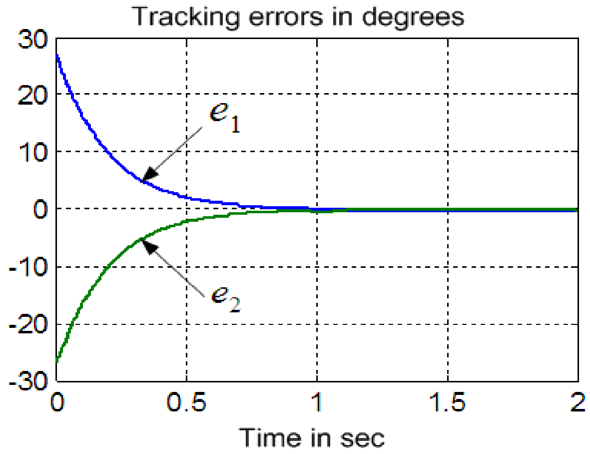

Fig. 2. The tracking errors (test 1).

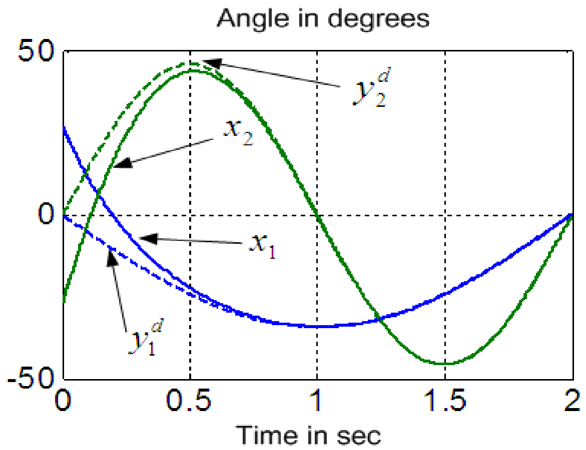

Fig. 3. Actual (solid) and desired (dashed) trajectories (test 1).

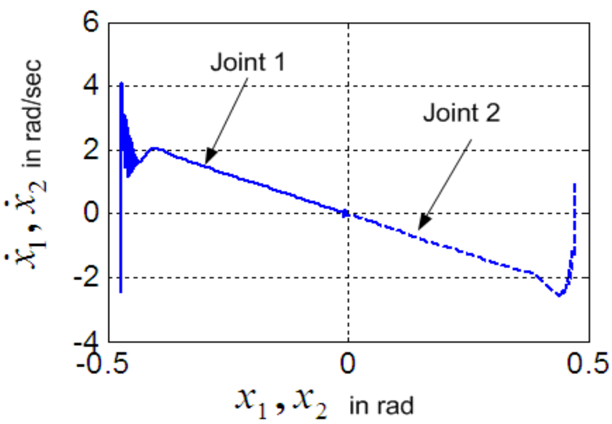

Fig. 4. The phase plots (test 1).

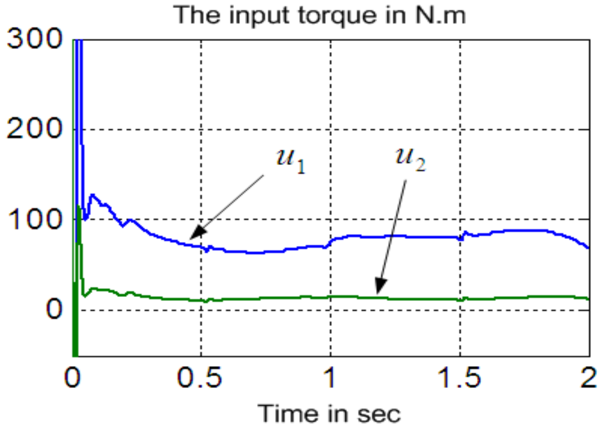

Fig. 5. The input torque (test 1).

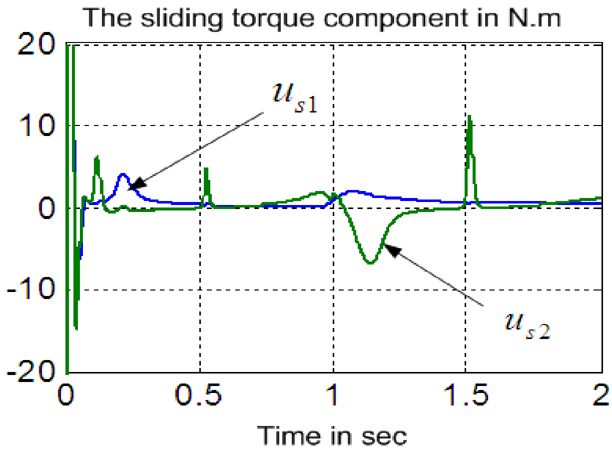

Fig. 6. The sliding torque component (u_s), (test 1).

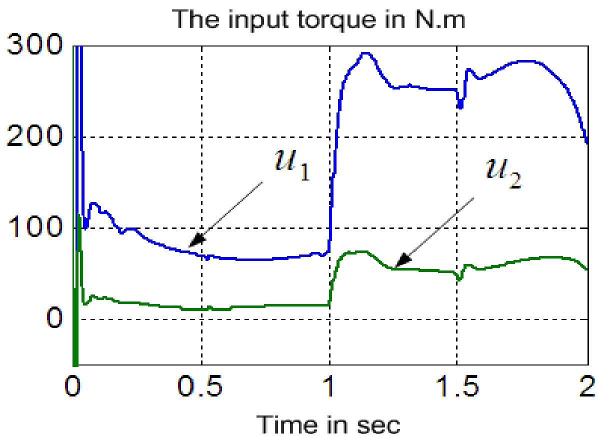

Fig. 7. The input torque when a pay load of 10 kg is added at the tip of link 2 at $t = 1$ sec (test 2).

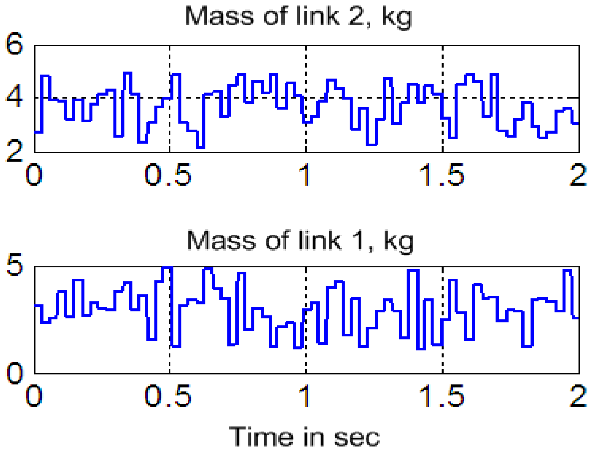

Fig. 8. Variations of masses of link one and two during motion (test 3).

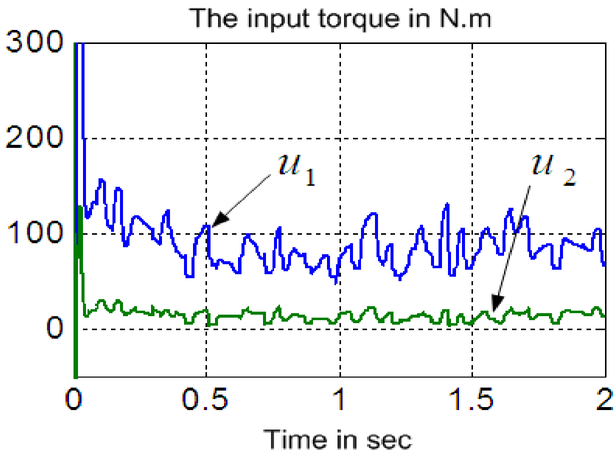

Fig. 9. The input torque under random masses of link one and two (test 3).

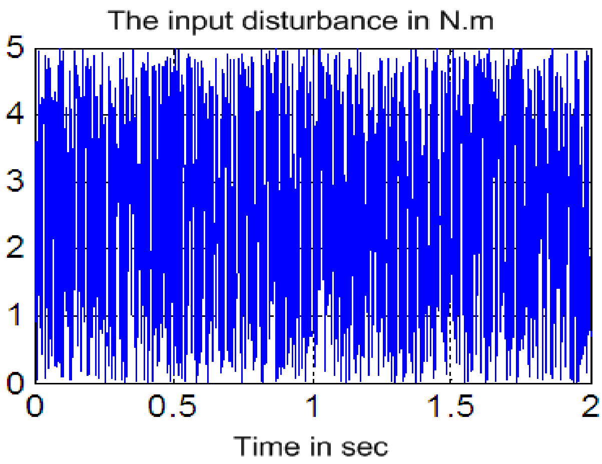

Fig. 10. The input disturbance (test 4).

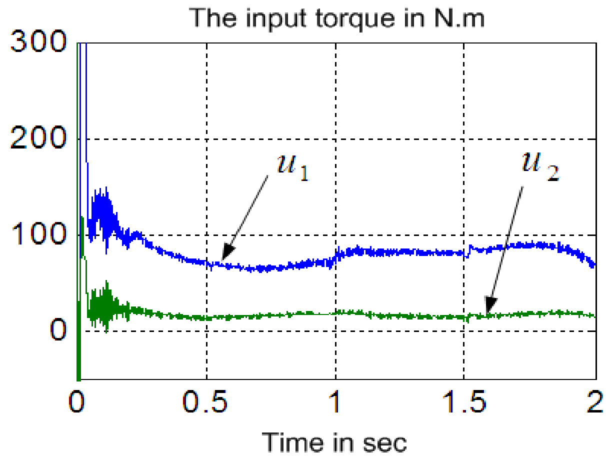

Fig. 11. The input torque under random disturbance added to the gravity terms (test 4).

7. Conclusions

In this paper, a new direct adaptive fuzzy controller is proposed which:
(a) is capable of incorporating fuzzy (IF-THEN) rules describing the system directly into the controller,
(b) updates on-line the control parameters so that unknown nonlinearity, uncertainties and external disturbances can be overcome.
(c) guarantees the global stability and robustness of the resulting closed-loop systems in the sense that all the signals are uniformly bounded.

The number of membership functions which approximates the nonlinearities can be extremely small and the results are less conservative than the methods presented in previous works.

The obtained simulation results clearly reveal that this direct adaptive fuzzy controller maintains the tracking errors in acceptable interval with feasible control inputs in the presence of hard parameter variations and external disturbances.

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