Various wavelet transform techniques for improving the performance of LPA-DOA estimation beamformer

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Wavelet denoising is a nonlinear signal enhancement technique that involves the application of a wavelet transform to a noisy signal to eliminate much of the noise without causing a significant distortion of the signal. Since, the performance of Local Polynomial Approximation (LPA) degrades under low Signal-to-Noise Ratio (SNR) conditions and due to errors produced by Taylor series truncation. This paper employs Wavelet Transform (WT) techniques to enhance the progressive degradation in the performance of the Direction Of Arrival (DOA) estimation of the nonstationary targets location using the LPA beamformer. Three types of WT techniques namely; Discrete Wavelet Transform (DWT), Stationary Wavelet Transform (SWT) and Wavelet Packet Transform (WPT) are mentioned here. These denoising schemes will be applied to the output of each sensor of a multi-sensor array to enhance the SNR at the array output. The effectiveness of these three types of wavelet transform with the noisy signal before applying LPA will be compared. Also, the performance of the LPA beamformer with and without WT will be compared. It is shown that, denoising using SWT leads to a significant reduction in the Mean Square Error (MSE) of the DOA estimation and can be used with input SNR much less than that which can be used with both DWT or WPT. Also, using WT-LPA is superior to LPA without prior denoising; especially with reduced SNR.

تعتبر الطرق المتعددة للموجات المتناهية الصغر من الوسائل المستخدمة لإزالة الشوشرة من الإشارات. و تشمل العديد من الطرق منها الموجات المتناهية الصغر الثابتة، حزمة الموجات المتناهية الصغر و الموجات المتناهية الصغر المنفصلة. و نظرا لان مشكلة تشكيل الشعاع من المواضيع الهامة و الحديثة فان هذه المقالة البحثية تقدم دراسة للتقريب المحلى لمتسلسلة القوى عند استخدامه مع الطرق المتعددة للموجات المتناهية الصغر لتحسين أداؤه.مع تقليل نسبة الإشارة - إلى- الشوشرة المعرضة لها المصادر المتحركة إلى مقدار تشويش عالي لم يكن يستطيع التقريب المحلى لمتسلسلة القوى تحديد موقع و تعقب الأهداف المتحركة بكفاءة معها. إن هذا البحث يدرس مقدار أداء وسيلة مشكل الشعاع في التقريب المحلى لمتسلسلة القوى عند تحسين أداءه باستخدام الثلاثة طرق المتعددة للموجات المتناهية الصغر، كمايقارن كفاءة عملهم. وتم كذلك دراسة مقارنة بين هذه الوسيلة المطورة وبين مشكل الشعاع بدون ازالة الشوشرة . وتم استنتاج أن طريقة الموجات المتناهية الصغر الثابتة هي أفضلهم. وبالتالي تم تطوير وسيلة مشكل الشعاع للتقريب المحلى المعدل لمتسلسلة القوى و ذلك لإتاحة التتبع اللابارمترى فائق التميز للأهداف المتحركة متغيرة السرعة. وأثبت البحث أن هذه الوسيلة تقلل من متوسط مربع الخطأ.

Keywords: Wavelet transform, Direction-of-arrival estimation, Moving source localization, Local polynomial approximation

1. Introduction

Since, as a new way to represent a signal, wavelets have been widely used in time-series signal processing. In array processing, both temporal and spatial processes affect the received data. As there is an essential similarity between time-domain signal processing and spatial-domain signal processing, the array signal processing can benefit from the application of wavelets as well.

From the point of view of Direction Of Arrival (DOA) estimation, Local Polynomial Approximation (LPA) becomes a high resolution new technique for estimating the DOA, angular velocity and/or acceleration of multiple closely spaced nonstationary targets in a noisy environment [1-3]. It can be used in a wide-spread applications as in radar, sonar and mobile communication. Inevitably, the performance of the LPA estimator suffers a progressive degradation and can't locate the targets location correctly as the SNR is reduced and becomes lower than -4dB [4]. The Local Polynomial Approximation (LPA) performance may be improved by employing a preprocessor that enhances the Signal-to-Noise Ratio (SNR), before performing the LPA. In this paper, a denoising technique based on the WT is proposed for enhancing the output SNR of a Uniform Linear Array (ULA) of sensors receiving narrowband signals in the form of plane waves from different directions also it decreases the MSE. In this work the WT is applied to the received signal vector from the antenna array then the LPA algorithm is applied to the resultant vector to estimate the angle of arrivals and angular velocities of the incident signals. A comparison of the estimation results obtained by using raw and transformed data, gives the performance improvement of the approach.

The advantages of the varying-resolution scheme in signal DOA estimation using LPA is realized by applying the Interference Confidence Interval (ICI) rule [5] , where, its basic idea is to use large scale (window size); corresponding to a large sensor spacing, if the target oscillates slowly across the array. In contrast, for a target that oscillates quickly across the array, we must observe it in a small scale (i.e., a small sensor spacing). Therefore, the scale in a spatial signal is well defined based on the spatial sampling resolution. A new idea is added in this paper where using WT not only vary the scale as in the ICI-LPA beamformer but also changes the location of the window used. Also, the difference between using the WT and ICI rule is that in the ICI we choose the window size used in the LPA step by step i.e., we use ICI then LPA then ICI then LPA to select only the optimum scale (window size) at each time t , while here, we completely make denoising by WT then we apply the LPA beamformer.

In this work we explore the possibility of using a wavelet denoising technique to improve the performance of LPA in a very low SNR environment. The wavelet denoising algorithm is used to enhance the SNR at the output of each sensor. LPA is employed on the denoised data vector for DOA estimation. The effect of denoising on the performance of LPA is analyzed by evaluating and comparing: (1) both undenoised and denoised data for different wavelet transform techniques, and (2) the response of the LPA using different WT techniques at different low input SNR values. It is shown that denoising using the three different WT leads to a significant improvement in the performance of the LPA estimator especially using SWT.

2. Wavelets application in the array problem

The block diagram for our algorithm is shown in fig. 1. It can be described as:

2.1. Signal model

Assume ULA with m point sensors with spacing d between any two adjacent sensors. A plane wave from q different targets sources arrives at the array from directions $\{\theta_i(t)\}_{i=1}^{q}$. The output of the m^{th} , over time, sensor can be written as:

$$r_m(t) = \sum_{i=1}^{q} b_i(t) e^{j\omega t + j(m-1)kd\sin(\theta_i)} + e_m(t)\,, \qquad (1)$$

where $b_i(t)$ is the random amplitude of the signal from the i^{th} target, ω is the center frequency of the signal, k is the wavenumber and $e_m(t)$ is zero-mean additive white Gaussian noise of variance σ^2. The signal model in eq. (1) can be represented in a vector form as:

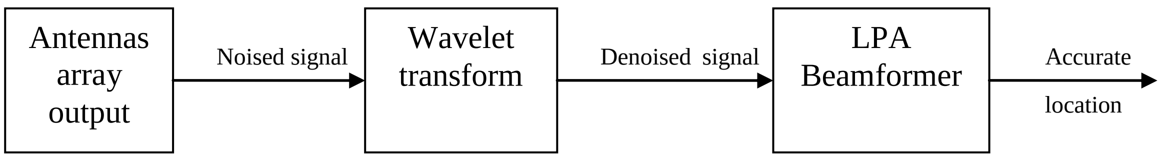

Fig.1. Block diagram for the LPA-WT algorithm.

$$\boldsymbol{r}(t) = \boldsymbol{A}(t)\boldsymbol{s}(t) + \boldsymbol{e}(t), \quad (2)$$

Where, $\boldsymbol{e}(t)$ is the $m\times1$ vector of sensor noise and $\boldsymbol{s}$ (t) is the vector of target signals at time t and is given by:

$$\boldsymbol{s}(t) = [b_1(t)e^{j\omega t}\ b_2(t)e^{j\omega t}\ \ldots b_q(t)e^{j\omega t}], \quad (3)$$

The $m\times q$ steering matrix $\boldsymbol{A}(t)$ is a time-varying direction matrix with the $m\times1$ steering vector,

$$\boldsymbol{a}(\theta) = \left[1, e^{-j\frac{2\pi}{\lambda}d\sin\theta}, \ldots\ldots\ldots, e^{-j(m-1)\frac{2\pi}{\lambda}d\sin\theta}\right]^T. \quad (4)$$

Therefore,

$$\boldsymbol{A}(t) = \lfloor \boldsymbol{a}(\theta_1(t)), \boldsymbol{a}(\theta_2(t)), \ldots, \boldsymbol{a}(\theta_q(t)) \rfloor. \quad (5)$$

The source motion within the observation interval, T, using Taylor series is,

$$\theta(t+kT) = \theta(t) + \theta^{(1)}(t)(kT) + \frac{\theta^{(2)}(t)}{2}(kT)^2 + \frac{\theta^{(3)}(t)}{6}(kT)^3 + \ldots$$
$$= c_0 + c_1 kT + c_2(kT)^2 + c_3(kT)^3 + \ldots \quad (6)$$

Assuming that the observation window is sufficiently short and, therefore, the third and later terms in eq. (6) are negligible, so we have

$$\theta(t+kT) = c_0 + c_1 kT\ , \quad (7)$$

with

$$c_0 = \theta(t),\ c_1 = \theta^{(1)}(t), \quad (8)$$

being the instantaneous source DOA and angular velocity, respectively. So, the problem is to estimate the vector $\boldsymbol{c} = (c_0, c_1)^T$ from the nonstationary array observation vector $\boldsymbol{r}$.

According to the previous block diagram in fig. 1, a denoising time-frequency analysis technique will be done first using WT as follow.

2.2. Wavelet analysis

WT is a powerful denoising technique for DOA estimation. It was used for enhancing the performance of MUSIC algorithm for DOA as in [6]. For denoising, WT decomposes a signal into a set of frequency bands (referred to as scales) by projecting the signal onto an element of a set of basis functions. Projection of the signal onto different scales is equivalent to bandpass filtering. The basis functions are called wavelets.

For wavelet analysis, assume N samples, so eq. (1) can be rewritten as:

$$r(i) = f(i) + e(i) \qquad i = 1,2,\ldots,N, \quad (9)$$

where, r, f and e represent $N \times 1$ column vectors containing the samples of each.

Let W represents $N \times N$ discrete wavelet transform matrix, so eq. (9) becomes

$$\boldsymbol{r}_W = \boldsymbol{f}_W + \boldsymbol{e}_W, \quad (10)$$

where $\boldsymbol{r}_W = W\boldsymbol{r}$, $f_W = Wf$ and $\boldsymbol{e}_W = W\boldsymbol{e}$.

A key property of DWT is that it approximates the KLT [Karhunen – Loeve Transform] transform for a large class of signals [7], and consequently, it tends to concentrate the signal energy into a relatively small number of large coefficients. The advantage of wavelet denoising over Wiener filtering is that, it is totally independent of the signal statistics and hence can he applied to signals of any kind.

The DWT is an orthnormal transform that compacts the signal into a few large coefficients in $\boldsymbol{f}_W$, while e is mapped on to $\boldsymbol{e}_W$. The process of wavelet denoising is to threshold the coefficients $\boldsymbol{r}_W$, to discard small values most likely due to the additive noise [7].

Similar to classical denoising methods (e.g., lowpass filtering), there is a tradeoff between noise reduction and oversmoothing of signal details.

Wavelet transform [8, 9] has several techniques which can be summarized as follow:

2.2.1. DWT

In the DWT the data under analysis are fed through a pair of low-pass and high-pass filters, and then down sampled, yielding the approximations and the details. The approximations are again fed through the same pair of low-pass and high-pass filters on the next level, producing another set of approximations and details. This process is repeated until the intended decomposition level is completed. With down-sampling, total data length of the approximations and details on each level remains the same so that the redundancy is removed and the fast algorithm can be achieved.

The DWT-based denoising method applies the DWT to the signal under analysis yielding a series of coefficients. Noise rejection can be achieved by keeping the coefficients associated with signal and discarding those caused by noise through thresholding.

2.2.2. WPT

It is a variation of the normally referred DWT, applies the decomposition not only to the approximations but also to the details on each level. This provides richer analysis but results in greater computation load. Where, in the case of the DWT, the information lost between two successive approximations is captured in the details. However, details produced on each of the levels are not analyzed any more. In contrast, the WPT applies the decomposition not only to the approximations but also to the details on each level. It divides the entire spatial frequency range into frequency bands of uniform width.

2.2.3. SWT

It is identical to the DWT in terms of the decomposition structure except that no down-sampling is involved. Down-sampling is crucial in the DWT and WPT as it removes the redundancy in the computation, making the fast algorithms to be realizable. It has the major disadvantage of very large data.

These three WT techniques will be used before LPA and their results will be compared.

2.3. LPA analysis

The DOA estimation using the powerful LPA beamformer will be applied after denoising using wavelet transform. For a single source assumption, the LPA beamformer function is found to be [1-4]:

$$P(t,c) = \frac{1}{m\sum_k \omega_h(kT)} \sum_k \omega_h(kT) \left| a^H(c,kT) r(t+kT) \right|^2 , \quad (11)$$

where $|.|$ stands for the absolute value and $k = 0,1,\ldots,N_l - 1,$ where N_l is the number of snapshots. This is a linear function with respect to the second order moments of the signal $\boldsymbol{r}(t)$. The summation interval in (11) is determined by the window function $\omega_h(kT)$. The dependence of $\boldsymbol{a}(\theta)$ is expressed via the vector c and the time kT. The window function is given by,

$$\omega_h(kT) = \left(\frac{T}{h}\right)\omega\left(\frac{kT}{h}\right). \quad (12)$$

Our algorithm for DOA estimation can be formulated in the following steps as:

1- Obtain the received time-varying sources vectors incident on the array [eq. (2)],

2- Approximate the time-varying function using part of the truncated Taylor series to acquire the source motion model [eq. (7)],

3- Apply the denoising technique using WT,

4- Use the weighted least squares approach to formulate the LPA beamformer [eq. (11)].

5- The LPA beamformer is then used to estimate the DOA (angle and angular velocity) of the sources.

The LPA beamformer is then used to estimate the DOA (angle and angular velocity) of the sources. This algorithm is used in the following section.

3. Simulation results

The first step of the simulation procedure is to get the received signal vector which is basically composed of signals incident on the ULA sensors caused by different sources. This simulation is performed by creating a response vector from the sensor array and then adding different levels of noise to the vector. The number of sensors is chosen as 10 sensors. Then WT is applied to the simulated received vector. Different wavelet techniques are tested with different basis functions to choose the best suitable WT technique that can be used. It is found that using the 'Daubechies' wavelet functions give the best performance. Then, we choose the best 'db' level with the best of decomposition after testing the response of the WT techniques at different levels as shown in fig. 2 and 3.

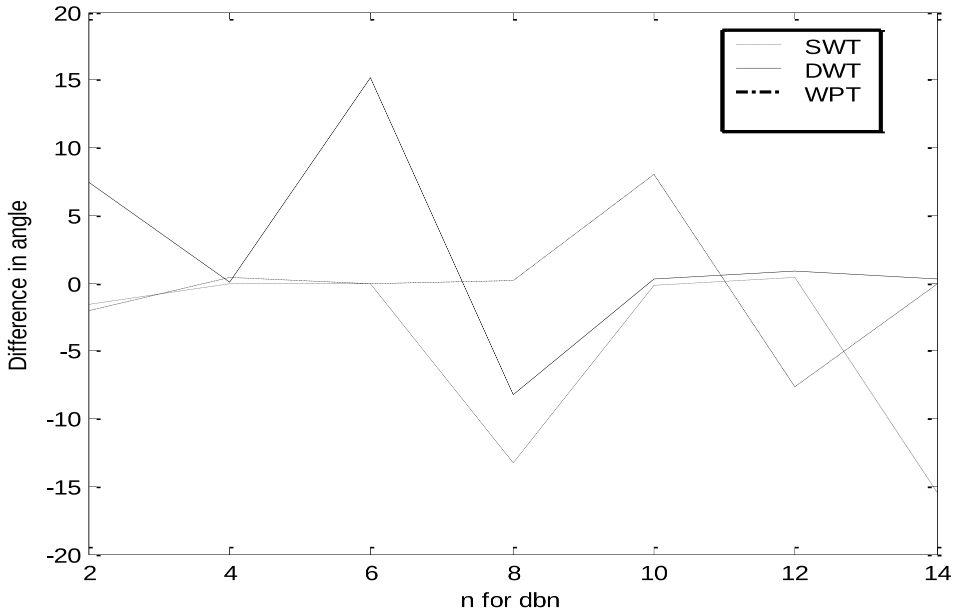

Fig. 2. The response of the WT techniques at different "Daubechies". The vertical axis shows the error in the angle estimation.

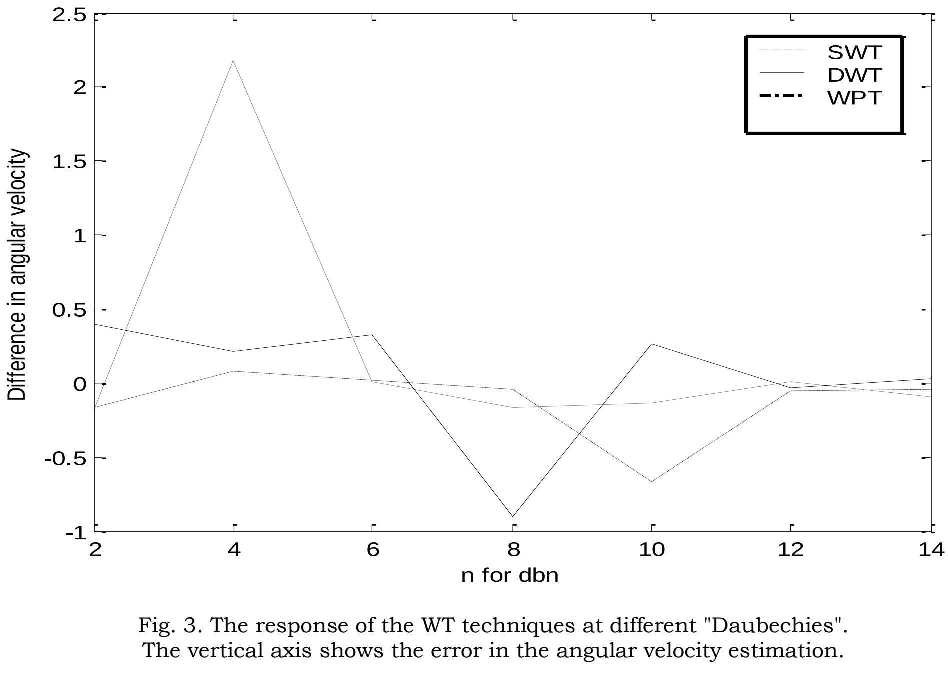

Fig. 3. The response of the WT techniques at different "Daubechies". The vertical axis shows the error in the angular velocity estimation.

Fig. 4 depicts the difference between the original signal vector s and each of $\boldsymbol{r}$, $\boldsymbol{r}_{SWT}$, $\boldsymbol{r}_{DWT}$ and $\boldsymbol{r}_{WPT}$ before using LPA (i.e., for the raw data before applying Taylor series) at different input SNR. It is clear that, denoising using WPT and DWT is superior to SWT, especially at low input SNR at the sensors output. Also, all the WT techniques give lower difference than that obtained using the noised form $\boldsymbol{r}$ itself. It is concluded that denoising using WT is effective for input SNR less than 4 dB.

The DOA's of the received signal vector are estimated by using the LPA beamformer algorithm and after creating the source motion model. It is clear from steps 2 and 3 in the previous section that, the truncated Taylor series is done before the denoising step to remove errors due to truncation when using WT. This will affect also the performance of the WT techniques and will improve the performance of the LPA beamformer where denoising will reduce the Mean Square Error (MSE) and makes the LPA beamformer acts with more decreased input SNR, as displayed in figs. 5, to 8. These figures compare the response of the LPA function with the undenoised and the denoised data using the three WT techniques (SWT, DWT and WPT) to improve its performance with lower input SNR. The figures legends refer to each curve in each figure. The vertical axis in figs. 5 and 7 show the error in the angle estimation i.e., $|\theta(t)-\hat{\theta}(t)|$, against the input SNR displayed at the horizontal axis for each plot. While, figs. 6 and 8 shows the error in the angular velocity; $|\theta^{(1)}(t)-\hat{\theta}^{(1)}(t)|$, against the input SNR displayed at the horizontal axis for each plot. Figs. 5 and 6 show the performance if a single source impinging from the location $(4^\circ + 1^\circ k)$ while figs. 7 and 8 for source incident from

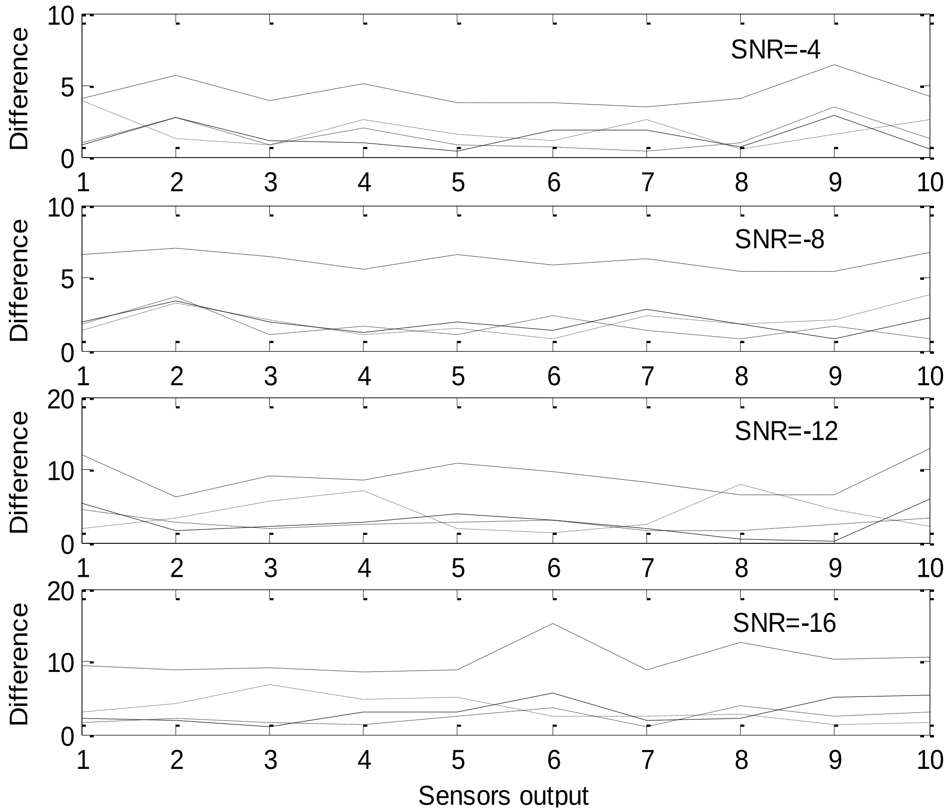

Fig. 4. The difference between the original signal vector $\boldsymbol{s}$ and each of r, r_{SWT}, r_{DWT} and r_{WPT} before using LPA. They are shown as dashed, dotted, solid and dash-dot; respectively.

$(0^\circ - 2^\circ k)$ location. It is shown from all these figures that, wavelet denoised data with LPA identify the two DOAs and angular velocities correctly under low input SNR. While, using the undenoised LPA gives degraded performance for input SNR less than -4dB. So, using WT prior to LPA beamformer enhances the performance of the LPA itself. The reason for the improved performance of the denoised data is that denoising reduces the MSE of the estimated target location.

It is clear from figs. 5-8 that using SWT is superior to the other WT techniques, where it gives accurate target location estimation till input SNR equals -12 dB, while using both DWT and WPT give accurate target location estimation till input SNR equals -10 dB. It can be observed; for example, from figs. 5 and 6 at -12dB input SNR:

1- denoising using WPT cannot estimate the angle of arrival correctly where it gives -10.613° error in the angle estimation and $4^\circ k$ error in the angular velocity estimation,

2- using DWT gives -3.505° error in the angle estimation and $4^\circ k$ error in the angular velocity estimation,

3- SWT gives us the chance to estimate correctly the DOA of this target with solely -0.3891° of the angle estimation error and $-0.1998^\circ k$ error in the angular velocity estimation.

The same conclusion is obtained from figs.7 and 8 at the other source location. So, when applying the WT techniques with the LPA, the SWT becomes the best method that can be used with the LPA beamformer till-12dB input SNR. This means that the error due to Taylor series truncation can be removed better using SWT technique, especially for input SNR =-10 to -12 dB. This is proved from tables 1 and 2 by using 30 trails to obtain this data for a source located at $(4^\circ + 1^\circ k)$ for input SNR equals -10 and -12 dB, where the MSE of the error in both the angle and angular velocity using the SWT is the least. And the DWT performs better than the WPT.

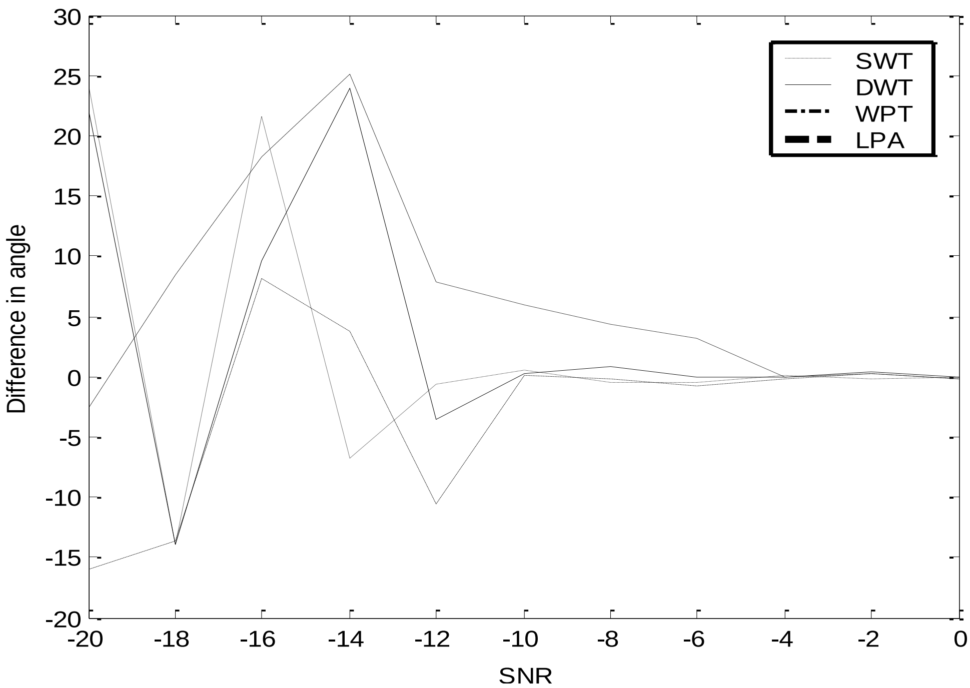

Fig. 5. The error in the angle estimation for the source located at $(4^\circ + 1^\circ k)$ for the undenoised (refer as LPA) and the LPA with denoised signals (refer by the name of the WT technique used). SNR is in dB.

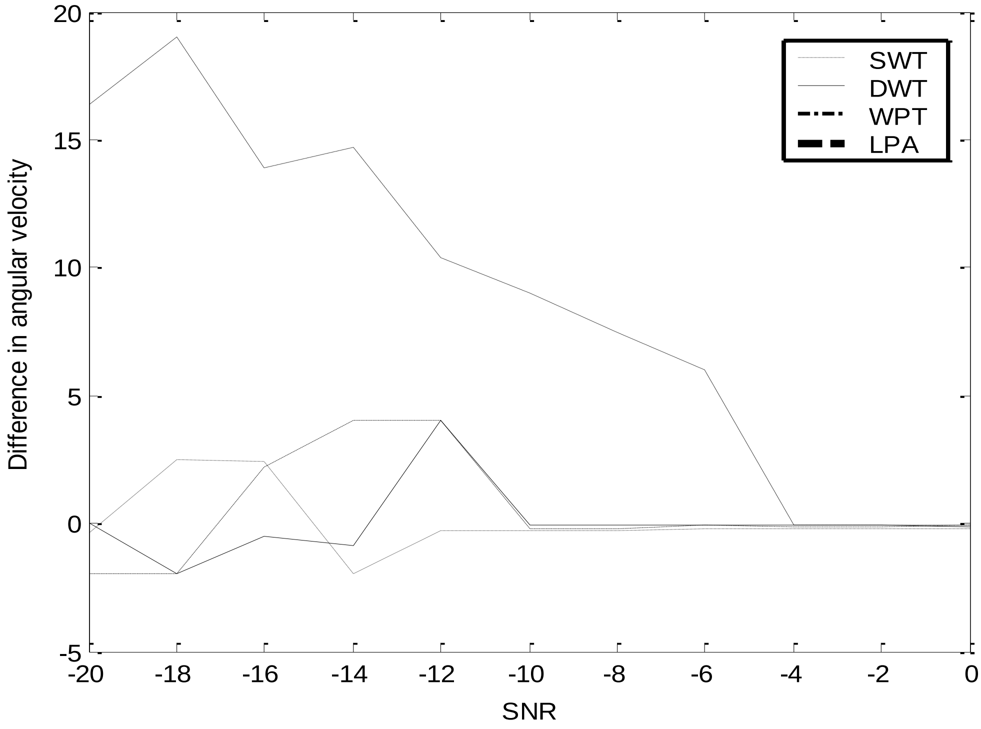

Fig. 6. The error in the angular velocity estimation for the source located at $(4^\circ + 1^\circ k)$.
SNR is in dB.

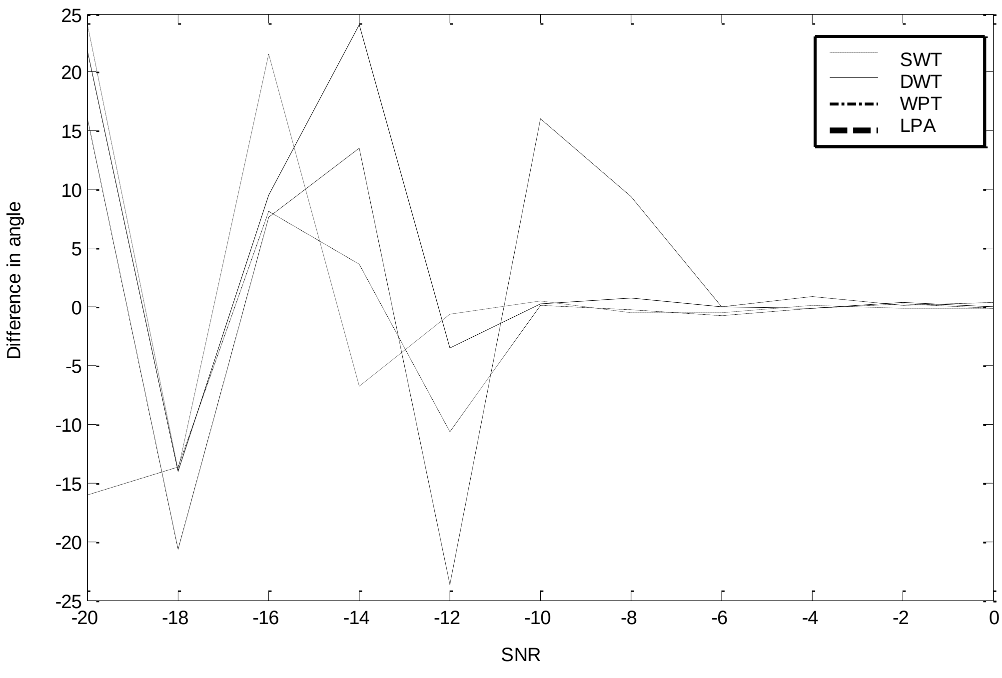

Fig. 7. The error in the angle estimation for the source located at $(0^\circ - 2^\circ k)$.
SNR is in dB.

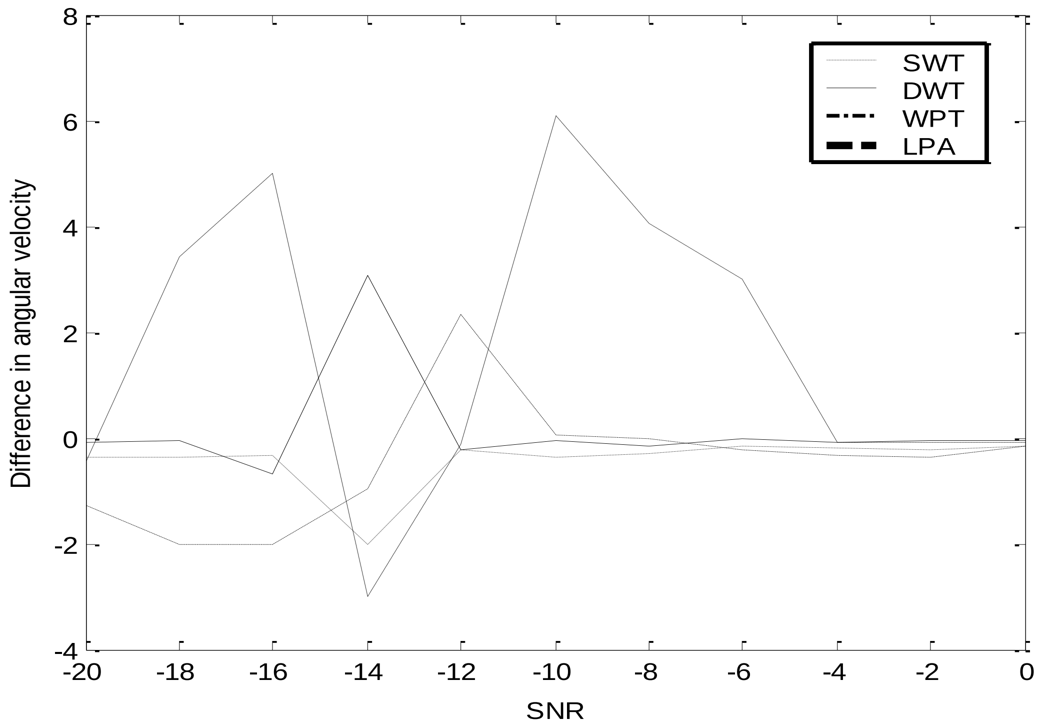

Fig. 8. The error in the angular velocity estimation for the source located at $(0^\circ - 2^\circ k)$.
SNR is in dB.

Table 1
The performance of each WT technique used with the LPA, for the error in angle estimation, for a source located at $(4^\circ + 1^\circ k)$.

	Input SNR=-10dB				Input SNR=-12dB			
	MSE	variance	Mean	Std	MSE	variance	Mean	Std
Denoised (LPA-SWT)	3.5651	0.5652	2.0186	4.0745	2.8050	0.1114	1.7865	3.1916
Denoised (LPA-DWT)	64.1648	-2.6906	8.0026	64.0415	114.0249	5.4280	9.8307	96.6419
Denoised (LPA-WPT)	120.3519	-1.3219	11.5512	133.4301	101.0112	-2.1670	10.4916	110.0746

Table 2
The performance of each WT technique used with the LPA, for the error in angular velocity estimation, for a source located at $(4^\circ + 1^\circ k)$.

	Input SNR=-10dB				Input SNR=-12dB			
	MSE	variance	Mean	Std	MSE	variance	Mean	Std
Denoised (LPA-SWT)	0.0791	-0.2803	0.0689	0.0047	0.0612	-0.1660	0.1960	0.0384
Denoised (LPA-DWT)	1.7868	0.3814	1.3588	1.8464	0.7883	0.0113	0.9491	0.9008
Denoised (LPA-WPT)	4.5567	1.0391	1.9778	3.9116	3.8701	0.2564	2.0852	4.3479

This work can be generalized to the different: LPA polynomial degrees (acceleration, ...) and for different array geometries, that has been studied in [1-4].

4. Conclusions

In this work, the use of wavelet denoising for plane wave DOA estimation has been investigated. Based on the idea that wavelet denoising improves the SNR of a noisy signal, we proceeded to perform wavelet denoising of the signal from each sensor of the array independently, prior to estimating the DOA. Wavelet denoising helps to reduce the estimation error of the sources location.

Since, denoising effect gets better when more decomposition levels are adopted but with increased computation load, in this work, we choose db11 with second level decomposition as the wavelet basis function to compare the response of the three techniques using it.

Also, it is evident in this work that with only white noise in presence, both the DWT and WPT techniques achieve the best denoising effect than the SWT, but some error exists when Taylor series truncation is performed to obtain the source motion models. Therefore, when the LPA is applied it adds error due to this truncation. At this instant, using the SWT gives superior performance compared to the other techniques with less estimation error. It gives the least MSE and makes our beamformer identify the sources location even if the input SNR becomes -12dB. In terms of computing time, WPT demand computing load, which makes it inapplicable in practice, especially in the case of high sampling rate and long sampling period. Among the three denoising techniques, the SWT method clearly gives the best trade-off between the denoising effect and the computing time.

Generally, the undenoised LPA beamformer performance is degraded for input SNR less than -4dB and gives maximum error level in the DOA estimation, while, the denoised LPA beamformer leads to a significant improvement in the LPA performance and increases the LPA ability for DOA estimation with very low input SNR.

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Received May 4, 2008
Accepted May 17, 2008