# A new flexibility distribution model for beam-column elements used in seismic analysis of steel frame buildings

Hamdy Abou-Elfath

Structural Eng. Dept., Faculty of Eng., Alexandria University, Alexandria, Egypt

A new flexibility distribution model is developed for simplified beam-column elements used in the seismic analysis of steel frame buildings. The developed model is used in representing the distributions of the cross section flexibility coefficients in the inelastic parts of the beamcolumn elements subjected to seismic loadings. The model facilitates selecting the proper flexibility distribution shape along the element length. It enables the user of the simplified beam-column elements to select any suitable flexibility distribution shape by determining the value of a flexibility factor ( $\eta$ ). The value of the flexibility factor ranges from zero to one. The value of one corresponds to uniform flexibility distribution shape, the value of zero corresponds to zero flexibility distribution shape, while the value of 0.5 corresponds to triangular (linear) flexibility distribution shape. The developed model eliminates the need to formulate the element flexibility matrix each time there is a change in the flexibility distribution along the beam-column length. It facilitates calibrating the simplified beamcolumn elements with the exact finite elements to find a proper selection of the flexibility factor ( $\eta$ ) that produces flexibility distribution shapes with good match with the complex actual flexibility distributions of the finite elements. The new flexibility distribution model is employed in a simplified beam-column element, which is implemented into the generalpurpose computer Program DRAIN-2DX. A numerical study has been carried out using the new flexibility distribution model. The results indicated the effectiveness of the new model. تم في هذا البحث تطوير نموذج جديد لتوزيع الانثنائية للعناصر الكمريق-العمودتي المبسطة المستخدمة في التُحليل الزلزالي م ي مد مجمع مسرير معود عميد معوريع المسابية معتاصر الممري العمودي المبسطة المستخدمة في النحليل الزلزالي للإطار ات الحديدية. ويستخدم النموذج المطور في تمثيل توزيع معاملات انتثاثية القطاعات في الأجزاء اللامرنة من العناصر الكمريق-العمودي المعرضة إلي أحمال الزلازل. ويسهل هذا النموذج اختيار شكل توزيع الانتثائية المناسب علي طول العنصر فهو يساعد مستخدمي العناصر الكمريق-العمودي المبسطة على اختيار الشكل المناسب لتوزيع الانتثائية عن طريق تحديد قيمة معامل للانتثائية (η). و قيمة معامل الانتثائية تتراوح من صفر الى واحد. وتناظر القيمة واحد لمعامل الانتثائية شكل التوزيع المنتظم و تتاظر القيمة صفر شكل توزيع صفري أما القيمة 0.5 فتناظر شكل توزيع مثلثي (خطى). ويتميز هذا النموذج المطور بأنه يقلل الحاجة إلى عمل اشتقاق جديد لمصفوفة الانتنائية في كل مرة يتم فيها اختيار شكل جديد لتوزيع الانتنائية. كما انه يسهل عملية معايرة العناصر الكمريخ العمودي المبسطة مع العناصر المحددة الدقيقة وذلك لاختيار قيمة مناسبة لمعامل الانثنائية (η) والتي تعطى أشكال لتوزيع ًالانثنائية للعناصر الكمّريتي–العمودتي المبسطة مشابهة لأشكال الانثنائية المحسوبة

بطريقة العُناصر المحددة. وقد تم توظيف هذا النموذج الجديد في عنصر كمرّي-عمودي مبسط يعمل في برنامج – DRAIN 2DX والمستخدم في التحليل اللاخطي للمنشئات تحت تأثير أحمال الزلازل. وقد تم عمل دراسة عددية باستخدام النموذج المطور حيث ثبت منها دقة و فاعلية النموذج المطور.

**Keywords:** Flexibility distribution model, Simplified beam-column element, Nonlinear analysis, Steel frames, Earthquakes, Buildings

## 1. Introduction

Intense research has been dedicated to develop beam-column elements for cyclic analysis of building structures in order to achieve high levels of accuracy and efficiency. Available beam-column elements in the literature range from the very simplified lumped plasticity elements to the computationally demanding finite element models. Lumped plasticity elements rely on the fact that inelastic behavior of frames subjected to lateral loading is usually concentrated at the ends of the beam-column members. Consequently, rotational or curvature springs are assumed at the member ends to model the inelastic flexural deformations. The models developed by (Giberson [1]; Otani, [2]) consist of two inelastic rotational springs at the ends of an elastic element. While, the models

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proposed by (Meyer et al. [3]; Park et al. [4]) depend on using two curvature springs at the member ends to represent the inelastic flexural deformations. The rotational and curvature spring modeling approaches are simple and require little amount of computations; however, several limitations and discrepancies are associated with them which significantly limit their accuracy (Abou-Elfath [5]).

In the finite element approach, the structural element is divided longitudinally into a number of segments and the cross sections are divided into a number of fibers. The element response is calculated by numerically integrating the fiber responses over the cross section and the segment responses over element length. Two types of finite element techniques are used in the literature to model beam-column element in framed structures. The first technique is displacement-based (stiffness-based) and requires predefined displacement shapefunctions to interpolate the displacements along the element length with respect to the nodal displacements (Keck [6]). The second technique is force-based (flexibility-based) and requires using interpolation functions to estimate the forces along the element length with respect to the nodal forces (Taucer et al. [7]). Contrarily to the rotational spring approach, the finite element approach is accurate, however, it requires substantial amount of computations for monitoring the

fiber responses of the various cross sections along the element length.

Despite the great developments in the field of digital computer, the finite element approach still is an unpopular tool to accomplish the seismic evaluation of building structures in reasonable time. This may be attributed to the huge computation demands in seismic evaluation process as it often requires repeated solutions of the responses of multi degrees of freedom systems. The simplicity of the modeling approach is a very important issue to achieve the seismic evaluation of building structures in reasonable time. For this reason, in the field of seismic analysis of building structures, the rotational spring approach is still a popular approach despite of the limitations associated with it.

A new modeling approach is proposed by Abou-Elfath [5] which is based on the assumption of linear (triangular) flexibility distribution shown in fig. 1. This presetflexibility model is proved to strike a good balance between the accuracy of the finite element approach and the simplicity of the lumped plasticity approach. In this model, only the end sections are divided into a number of fibers. The responses of only the fibers of the two end cross sections are monitored, which results in a significant reduction in computations in comparison with the finite element approach. The model is implemented into the general-purpose computer Program DRAIN-2DX [8].



Fig. 1. The preset-flexibility model proposed by Abou-Elfath [5].

The preset-flexibility element has the same advantages of the finite element models. For example, it considers the moment-axial force interaction effects and the inelastic axial deformations. Also, the fiber strains can be obtained as an output of the model. These strains can be used for performing seismic damage evaluation of the frame members. Moreover, the element accounts for the spread of plasticity and is capable of producing the gradual change of the member stiffness in the post yield range. The solution time of the preset-flexibility element is found less than one third of the solution time of the finite elements. The preset-flexibility element with triangular distribution of flexibility is found applicable only to I-shape cross sections. Applying the model to other cross section shapes requires modifying the flexibility distribution diagrams to new forms other than the triangular one shown in fig. 1. This main shortcoming requires inventing a new general flexibility distribution model that has the ability to produce various flexibility distribution forms in order to make the presetflexibility element applicable for the whole cross section shapes.

The objective of this paper is to develop a new flexibility distribution model for the preset-flexibility element. The developed model facilitates selecting the flexibility distribution element length shapes along the and eliminates any lengthy derivations of the tangent flexibility matrix each time there is a change in the distribution of the flexibility shapes. The developed model uses a flexibility factor  $(\eta)$  which controls the distribution of the flexibility shapes along the element length and facilitates calibrating the response of the preset-flexibility element with the performance of the finite element models. The new flexibility distribution model is employed in the preset-flexibility element developed for the seismic analysis of steel frame buildings by Abou-Elfath [5], which is implemented into the general-purpose computer program DRAIN-2DX. A numerical study is conducted to examine the advantages of using the new flexibility distribution model. The outcomes obtained from the numerical analysis indicated the efficiency of the new flexibility distribution model.

#### 2. The preset-flexibility modeling approach

The preset-flexibility modeling approach is flexibility-based following the approach presented by Taucer et al. [7]. It is assumed that the axial force distribution is uniform and the moment distribution is linear along the element length. Plane sections are assumed plane remain and normal to to the of the axis longitudinal element after deformation. The member deformations are assumed to be small and shear deformations are neglected. Each end section is divided into a number of fibers as shown in fig. 1. A material model that accounts for yielding and strain hardening of steel is assigned for each fiber. The behavior of each fiber is monitored at its center. The lengths of the plastic zones at the member ends,  $X_1$  and  $X_2$ , are calculated at every load increment using the approach proposed by Abou-Elfath [5].

The cross section response (tangent stiffness, force increments) is determined by integrating the fiber responses over the cross section. Similarly, the element response is obtained by integrating the cross section responses along the element length assuming a preset flexibility distribution shape over the plastic zone regions at the member ends.

### 3. Cross section tangent flexibility matrix

The tangent stiffness and flexibility matrices of the two end sections are denoted  $S_j$ ,  $D_j$ , respectively. The subscript j is equal to 1 for the left end section and 2 for the right end section. The two matrices are defined as:

$$S_{j} = \begin{bmatrix} s_{1,j} & s_{2,j} \\ s_{2,j} & s_{3,j} \end{bmatrix}, D_{j} = \begin{bmatrix} d_{1,j} & d_{2,j} \\ d_{2,j} & d_{3,j} \end{bmatrix}, (j = 1, 2).$$
(1)

The two matrices relates the  $j^{th}$  cross section incremental deformation vector  $\{d\varepsilon_j, d\phi_j\}$  with the  $j^{th}$  cross section incremental force vector  $\{dp_j, dm_j\}$ . Where,  $d\varepsilon_j$  is the axial strain increment at the center of the cross section,  $d\phi_j$  is the cross section curvature increment,  $dp_j$  is the axial force increment and  $dm_j$  is the moment increment. The stiffness coefficients  $s_{1,j}, s_{2,j}, s_{3,j}$  are calculated as:

$$s_{1,j} = \sum_{k=1}^{Nfib} E_{k,j} A_{k,j} \qquad s_{2,j} = \sum_{k=1}^{Nfib} E_{k,j} A_{k,j} Y_{k,j}$$
  
$$s_{3,j} = \sum_{k=1}^{Nfib} E_{k,j} A_{k,j} Y_{k,j}^{2}, \quad (j = 1, 2). \quad (2)$$

Where,  $A_{k,j}$ ,  $E_{k,j}$  and  $Y_{k,j}$  are the area, the tangent modulus of elasticity and the Ycoordinate of the  $k^{th}$  fiber at the  $j^{th}$  cross section. *Nfib* is the total number of fibers over the cross section. The cross section flexibility matrix  $D_j$  can be obtained by inverting the cross section stiffness matrix  $S_i$ . The distributions of the flexibility coefficients  $d_{1,j}$ ,  $d_{2,i}$ ,  $d_{3,i}$  along the element length are defined using six flexibility distribution functions  $B_{i,i}$ as shown in fig. 2. The subscript i is equal to 1 for the axial flexibility coefficient  $d_{1,j}$ , i=2 for the compound flexibility coefficient  $d_{2,i}$  and i=3for the flexural flexibility coefficient  $d_{3,j}$ . The distribution functions  $B_{i,j}$  relates the values of the flexibility coefficients (the function output) with the variable x or x'' (the function input). The lengths of the flexibility distribution shapes are defined by Xj (j = 1 for the left end and 2 for the right end) as shown in fig. 2. The heights of the *i*<sup>th</sup> flexibility distribution

shape at the  $j^{th}$  end are defined by  $(d_{i,j}-d_{i,0})$  as shown in fig. 2.  $d_{i,0}$  represents the  $i^{th}$  flexibility coefficient of the elastic part of the beamcolumn member  $(d_{1,0}=1/\text{EA}, d_{2,0}=0 \text{ and} d_{3,0}=1/\text{EI})$ , where, E is the member modulus of elasticity, A is the member cross section area and I is the moment of inertia of the member cross section.

Three shape constants  $(C_1, C_2 \text{ and } C_3)$  are calculated for each flexibility distribution function. The constants  $C_1$ ,  $C_2$  and  $C_3$  are related to the area of the flexibility distribution shape  $(A_{i,j})$ , the distance between the center of the flexibility shape and the nearest member end  $(R_{i,j})$  and the inertia of the flexibility shape about a central axis perpendicular on the beam-column element  $(I_{i,j})$ . The flexibility functions of the beam-column element are assumed to be selected in a manner that makes all of them have the same shape constants. In this situation, the three shape constants can be estimated as:

$$C_1 = \frac{A_{i,j}}{X_j (d_{i,j} - d_{i,0})}, C_2 = \frac{R_{i,j}}{X_j}, C_3 = \frac{I_{i,j}}{A_{i,j}X_j^2}.$$
 (3)



Fig. 2. Flexibility distribution functions along the beam-column element.

#### 4. Tangent flexibility matrix of the presetflexibility element

The frame element has three local deformation components  $\{U, \theta_1, \theta_2\}$  and three local force components  $\{P, M_1, M_2\}$ .  $\theta_1$  and  $\theta_2$  represent the nodal rotations, U represents the axial displacement,  $M_1$  and  $M_2$  represent the end moments and P represents the axial force. The incremental deformation vector  $\{dU, d\theta_1, d\theta_2\}$  and the incremental force vector  $\{dP, dM_1, dM_2\}$  of the element are related by a symmetric tangent flexibility matrix, according to:

$$\begin{cases} dU \\ d\theta_1 \\ d\theta_2 \end{cases} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{cases} dP \\ dM_1 \\ dM_2 \end{cases} .$$
 (4)

The data required for determining the element tangent flexibility matrix are, (a) the tangent flexibility matrices of the two end cross sections, (b) the lengths of the two plastic zones  $(X_1, X_2)$ , (c) the assumed flexibility distribution function and, (d) the properties of the elastic part of the member (EA, EI and  $X_3$ ). The flexibility coefficients  $f_{ij}$ are calculated using the elastic weight method. In the elastic weight method, the flexibility coefficients  $f_{ij}$  are the local reactions of the beam-column element when loaded with the elastic loads. The elastic loads are obtained by integrating the flexibility distribution diagrams shown in figs. 2 with the internal force diagrams shown in figs. 3. The flexibility coefficients  $f_{ij}$  are calculated as follows:

$$\begin{split} f_{11} &= A_{1,0} + A_{1,1} + A_{1,2} \\ f_{12} &= A_{2,1}(1-Z_1) + A_{2,2}Z_2 \\ f_{13} &= A_{2,1}Z_1 + A_{2,2}(1-Z_2) \\ f_{22} &= A_{3,0} / 3 + A_{3,1}(1-2Z_1) + A_{3,1}Z_1(Z_1+Q_1) + A_{3,2}Z_2(Z_2+Q_2) \\ f_{23} &= A_{3,0} / 6 + A_{3,1}Z_1(Z_1+Q_1-1) + A_{3,2}Z_2(Z_2+Q_2-1) \\ f_{33} &= A_{3,0} / 3 + A_{3,1}Z_1(Z_1+Q_1) + A_{3,2}(1-2Z_2) + A_{3,2}Z_2(Z_2+Q_2). \end{split}$$
(5)

The areas of the flexibility diagrams in eq. (5),  $A_{ij}$ , can be defined as follows:



Fig. 3. Distribution of axial and bending moment increments along the beam-column.

$$A_{1,0} = L/EA$$

$$A_{2,1} = C_1 X_1 d_{2,1}$$

$$A_{2,2} = C_1 X_2 d_{2,2}$$

$$A_{3,1} = C_1 X_1 (d_{3,1} - 1/EI)$$

$$A_{3,2} = C_1 X_2 (d_{3,2} - 1/EI)$$

$$A_{3,0} = L/EI .$$
(6)

The values of  $Z_1$ ,  $Z_2$ ,  $Q_1$  and  $Q_2$  are dependent on the shape constants, the inelastic lengths and the member length, they are described as follows:

$$Z_{1} = C_{2}X_{1} / L$$

$$Z_{2} = C_{2}X_{2} / L$$

$$Q_{1} = (C_{3} / C_{2})X_{1} / L$$

$$Q_{2} = (C_{3} / C_{2})X_{2} / L$$

$$Q_{3} = (C_{3} / C_{2})X_{2} / L .$$
(7)

Eqs. (5-7) indicates that the flexibility shapes are totally represented in the element flexibility coefficient formulas only by the length ( $X_1$  or  $X_2$ ), height ( $d_{i,j}$ - $d_{i,0}$ ) and the shape constants ( $C_1$ ,  $C_2$  and  $C_3$ ). This indicates that knowing the values of the three shape constants along with the heights and the lengths of the flexibility shapes of a beamcolumn element eliminates the need for further information about the flexibility shapes.

# 5. Selecting the flexibility distribution functions

The distributions of the flexibility coefficients  $d_{1,j}$ ,  $d_{2,j}$ ,  $d_{3,j}$  along the element length are defined using six flexibility distribution functions  $B_{i,j}$  as shown in fig. 2. Three shape constants  $(C_1, C_2 \text{ and } C_3)$  are calculated for each flexibility distribution function. The values of the three shape constants are assumed to be the same for all the six flexibility distribution functions. This assumption requires selecting one form for all the six flexibility distribution functions. However, each function may need different coefficients in order to satisfy its boundary conditions.

If the number of the boundary conditions at each flexibility distribution shape is considered equal to two (the coordinates of the two end points of the flexibility distribution shape), then only uniform and linear functions can be used to represent the flexibility distributions. However, more complicated functions can be utilized to represent the flexibility distributions, when considering extra boundary conditions other than the preliminary ones.

In the current study, the bilinear flexibility diagram shown in fig. 4 is adopted to represent the flexibility distribution shapes of beam-column elements. The bilinear shape is defined by the broken line m-q-n using a flexibility factor  $\eta$ . The flexibility factor  $\eta$  defines the position of the middle point q. The point q coincides with the point o when  $\eta$ =1, while it coincides with p when  $\eta$ =0, as shown in fig. 4.

The value of the flexibility factor  $\eta$  ranges from zero to one. The value of one corresponds to uniform flexibility distribution shape, the

value of zero corresponds to zero flexibility distribution shape, while the value of 0.5 corresponds to triangular (linear) flexibility distribution shape. The higher is the value of the flexibility factor  $\eta$ , the lower is the stiffness of the inelastic part of the beam-column element.

The shape constants ( $C_1$ ,  $C_2$  and  $C_3$ ) of all the bilinear flexibility distribution shapes can be calculated as follows:

$$C_1 = \eta, \quad C_2 = \frac{(1+2\eta)}{6}, \quad C_3 = \frac{(2-\eta+2\eta^2)}{36}$$
 (8)

Eq. (8) indicates that, in case of adopting the bilinear flexibility distribution shape, the values of the shape constants are only dependent on the value of the flexibility factor  $\eta$ . Assuming a constant flexibility factor  $\eta$  for all the flexibility distribution shapes, leads to identical shape constants ( $C_1$ ,  $C_2$  and  $C_3$ ) for all the flexibility shapes.

The proposed flexibility model presented in eqs. (5-8) eliminates the need to formulate the element flexibility matrix each time there is a change in the flexibility distribution along the beam-column length. The various levels of the flexibility factor  $\eta$  provide tremendous number of flexibility distribution selections that are expected to approximately fit any actual flexibility distribution shape. Values of the shape constants corresponding to some levels of the flexibility factor  $\eta$  are summarized in table 1.



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Fig. 4. Selected flexibility distribution diagram.

Shape constants corresponding to some levels of the flexibility factor  $\eta$ .

Table 1

η	$C_1$	$C_2$	$C_3$
0.00	0.00	0.000	0.0000
0.25	0.25	0.250	0.0521
0.50	0.50	0.333	0.0555
0.75	0.75	0.417	0.0660
1.00	1.00	0.500	0.0833

# 6. Numerical verification of the flexibility distribution model

The new flexibility distribution model is employed in the preset-flexibility beamcolumn element developed for the seismic analysis of steel frame buildings by Abou-Elfath [5], which is implemented into the general-purpose computer program DRAIN-2DX [8]. The proposed model is evaluated on the level of one element. The model prediction is compared with the fiber model of Taucer et al. [7]. The element considered in the current study is a 2.0 m cantilever beam (shown in fig. 5). The steel yield stress is 300 MPa and the modulus of elasticity is 200,000 MPa. The beam is modeled using 10 fibers over the cross section and 10 segments at each end of the element.

The cantilever beam is assumed to be subjected to a lateral static load (P) and an axial static load (F) at the free end. The accuracy of the proposed model is evaluated bv measuring one global performance parameter which is the level of lateral load at 2.0% lateral drift ratio of the free end and one local performance parameter which is the absolute value of the maximum strain at the fixed end. The prediction errors of the proposed model are measured as the absolute differences between the predictions of the



Fig. 5. Cantilever beam.

proposed model and those of the fiber model and are normalized with respect to the fiber model predictions. The effects of the cross section type, level of axial loading and the dynamic loading on the accuracy of the proposed model are investigated in details in the following subsections.

#### 7. Effect of cross section type

Two types of steel cross sections are considered in this study. The first is a rectangular shape cross section, while the second is a W-shape cross section. The strainhardening ratio is considered to be 1.0% and the axial load (F) is assumed to be equal to zero. A displacement controlled analysis is conducted until the lateral deflection of the free end reaches 2.0% of drift ratio.

For the case of using a rectangular cross section with 8 cm width and 40 cm height, fig. 6 shows the relationships between the lateral drift of the free end and the lateral load in case of using  $\eta$ =0.22,  $\eta$ =0.50 and the fiber model. The maximum strain outputs at the fixed end of the proposed model are  $2.12 \times 10^{-2}$  in case of using  $\eta$ =0.22 and  $1.21 \times 10^{-2}$  in case of using  $\eta$ =0.50. The corresponding value given by the fiber model is  $2.12 \times 10^{-2}$ . The results presented in fig. 6 as well as the strain outputs indicate the good performance of the proposed model in case of rectangular cross section when using  $\eta$ =0.22.

For the case of using a W21×122 cross section, fig. 7 shows the relationships between the lateral drift ratio of the free end and the lateral load in case of using  $\eta$ =0.22,  $\eta$ =0.50 and the fiber model. The results presented in fig. 7 indicate the good agreement between the proposed model and the fiber model in predicting the global behavior of the Wsections when using  $\eta$ =0.5. The maximum strain outputs at the fixed end of the cantilever reached 5.04×10<sup>-2</sup> in case of using  $\eta$ =0.22 and 3.02×10<sup>-2</sup> in case of using  $\eta$ =0.50, while the corresponding value given by the 3.0×10<sup>-2</sup>. fiber model is These strain measurements indicate the good performance

of the proposed model when using  $\eta$ =0.50 for the W-sections.



Fig. 6. Lateral load-lateral drift relationship in case of the rectangular section.



2 Lateral disp., Cm

3

4

Fig. 7. Lateral load-lateral drift relationship in case of the W-section.

1

 Table 2

 Global and local prediction errors of the proposed model

W-Sections, $\eta$ =0.5			Rectangular Sections, η=0.22			
Section	Height/width	Global error (	%) Local error (%)	Height/width	Global erro	or (%) Local error (%)
W44x285	3.73	0.16	0.98	0.1	0.01	0.17
W30x211	2.05	0.99	5.33	0.5	0.01	0.17
W21x122	1.77	0.10	0.67	1.0	0.01	0.17
W18x130	1.72	0.36	2.40	5.0	0.01	0.17
W14x159	0.96	0.37	3.43	10.0	0.01	0.17

0 **#** 0

both the global and the local responses of the steel frame members subjected to a monotonic static loading when selecting a proper level of the flexibility factors  $\eta$ . Local and global prediction errors are calculated for a variety of W and rectangular cross section shapes using  $\eta$ =0.5 for the W-section and  $\eta$ =0.22 for the rectangular section. The local and global prediction errors are summarized in table 2. The results presented in table 2 indicate that, for all the selected cross sections, the higher global prediction error is equal to 0.99 % and the higher local prediction error is equal to 5.33 %.

## 8. Effect of axial loading

The free end of the cantilever beam is subjected to lateral static load (P) and axial static load (F). The strain-hardening ratio is considered 1.0%. A displacement controlled analysis is conducted until the lateral deflection of the free end reaches a 2.0% of drift ratio.

Fig. 8 shows the relationships between the lateral load P and the lateral deflection of the free end when using the rectangular cross section and a level of axial load equal to 50% of the axial load capacity of the cross section. The results presented in fig. 8 indicate the good agreement between the proposed model and the fiber model in predicting the global behavior of structures having rectangular sections when using  $\eta$ =0.22. The obtained maximum strain levels at the fixed end of the cantilever are 2.3×10<sup>-2</sup> for the proposed model and 2.48×10<sup>-2</sup> for the fiber model.

Fig. 9 illustrates the relationship between the lateral load P and the lateral deflection of the free end when using the W21×122 section and a level of axial load equal to 50% from the axial load capacity of the cross section. The results presented in fig. 9 show the accuracy of the proposed model in predicting the global behavior when using  $\eta$ =0.50 for W-sections. The obtained maximum strain levels at the fixed end of the cantilever are  $3.36 \times 10^{-2}$  for the proposed model and  $3.59 \times 10^{-2}$  for the fiber model. Local and global prediction errors of the W21×122 section and the rectangular section are also calculated for a level of axial load equal to 75% of the axial load capacity of the cross. Table 3 summarizes the global and the local prediction errors corresponding to the 50% and the 75% levels of the axial load (F). The results presented in table 3 indicate that, for the two axial load levels considered, the higher global prediction error is equal to 2.32% and the higher local prediction error is equal to 8.93%.

### 9. Effect of dynamic loading

The proposed model response is calculated under the effect of an earthquake loading. A lumped mass is assumed at the free end of the cantilever beam. The mass value is determined in order to produce a 1.0 sec. period in the lateral direction of the cantilever beam. The earthquake excitation considered in this study is the SOOE component of El Centro record which has been recorded during the Imperial Valley, California earthquake of May 18, 1940.

The dynamic analysis of the cantilever is performed using a 3.0% viscous damping and a time step increment of 0.005 second. The ground motion record is scaled to different Peak Ground Acceleration (PGA) levels. The maximum selected PGA level is 0.18 g. The relationship between the peak lateral deflection of the cantilever and the PGA of the earthquake is presented in fig. 10 for the rectangular section and in fig. 11 for the W21×122 section.

The results presented in figs. 10 and 11 indicate the good agreement between the proposed model and the fiber model in predicting the global seismic behavior of the rectangular section with  $\eta$ =0.22 and the W-sections with  $\eta$ =0.5.

At maximum PGA level (0.18 g), the predicted peak lateral displacement at the free end and peak strain at the fixed end of the cantilever with W21×122 section are 5.67 cm and 0.039, respectively. The corresponding values obtained from the fiber model are 5.78 cm and 0.039, respectively. For the rectangular section, at maximum PGA level, the predicted peak lateral displacement at the free end and peak strain at the fixed end are 4.44 cm and 0.024, respectively. The corresponding values obtained from the fiber model are 4.46 and 0.024, respectively. These results indicate the effectiveness of the proposed model in predicting the global and the local responses of the rectangular section with  $\eta$ =0.22 and the W-sections with  $\eta$ =0.5.



Fig. 8. Behavior of rectangular section to lateral load (axial load ratio=50%).



Fig. 9. Behavior of W21×122 section to lateral load (axial load ratio=50%).

Table 3							
Global and local	prediction	errors	due	to	axial	loadi	ng

Axial load ratio	W21×122 s	ection, $\eta$ =0.50	Rectangular section (40×6 cm ), $\eta$ =0.22			
	Global error (%)	Local error (%)	Global error (%)	Local error (%)		



Fig. 10. Global seismic response of the cantilever with rectangular section.

# 10. Computer time demands and storage requirements of the proposed model

The solution of the cantilever beam is carried out on a personal computer. The solution time of the proposed model has been found approximately one third the corresponding time of the fiber model. The number of integer and real variables required for one element of the proposed modeling approach is approximately one fourth the corresponding number of the fiber modeling technique.

#### **11. Conclusions**

A new flexibility distribution model is developed for simplified beam-column elements used in the seismic analysis of steel frame buildings. The developed model is employed in the preset-flexibility element developed by Abou-Elfath [5]. The new model significantly improved the preset-flexibility element performance. It provided the presetflexibility element with the capability of modeling any cross section shape by the proper selection of a flexibility factor  $(\eta)$ .

The numerical study conducted in this paper on the element level indicated that adequate accuracy can be obtained when using the preset-flexibility element in both static and dynamic analysis with  $\eta$ =0.22 for



Fig. 11. Global seismic response of the cantilever with  $$W21{\times}122$$  section.

rectangular cross sections and  $\eta$ =0.5 for W-sections.

There is a need to test the preset-flexibility element on the structure level and also to provide adequate data regarding the proper values of  $\eta$  for cross section shapes other than the rectangular and the W shapes.

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