Optimization of preventive maintenance schedules via cost analysis

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Optimization of Preventive Maintenance (PM) schedules for equally important components is analyzed and evaluated based on cost of failure and maintenance. An algorithm is designed to calculate the minimum time to repair for optimum PM schedules. Recent Weibull distributions that take into account different phases and failure modes, are used as probability density function for component failure. These distributions are appropriate and recommended to be used in most practical applications.

يناقش البحث أمثلية جدولة الصيانة الوقائية لمكونات نظام هندسى من خلال تقدير امثل لتكلفة الصيانة. قدم البحث نمودج حسابى لاستخدامه فى حساب التكلفة المثلى للصيانة وقد تم تطبيقه على بعض التوزيعات الحديثة لدالة ويبل حيث وجد انها مناسبة و قابلة للتطبيق فى نواحى شتى.

Keywords: Preventive maintenance, Cost, Optimization, Maintenance schedules

1. Introduction

The problem of Preventive Maintenance (PM) schedules is extremely important in the area of safety analysis and cost management. In most practical situations, the one and two parameter(s) Weibull distributions for probability density function (pdf) are found appropriate to fully describe the situation. Depending on failure mode(s) (single/ double/triple), and time region of application (phase) (single/ double/ triple), six forms of Weibull distribution could be assigned.

The important point in the present work is the introduction of the advanced Weibull distributions to an adapted cost model to determine the optimum time between two preventive maintenance actions of the component at which the total cost records a minimum value.

A computer code written in MATLAP has been designed to carry out all computations involved in the cost model.

2. The cost model with weibull pdf

2.1. Analytical model

As clearly shown in eq. (1), the total cost of maintenance per unit time can be calculated in terms of the failure cost and the preventive maintenance cost.

$$C_T = \frac{Cp\int_T^{\infty} f(t) + C_f\int_0^T f(t)dt}{Cp\int_T^{\infty} f(t)dt + \int_0^T f.f(t)dt}, \qquad (1)$$

where
- C_T is the total cost of maintenance,
- C_p is the preventive maintenance cost,
- C_f is the failure cost,
- T is the time between preventive maintenance actions, and
- $F(T)$ is the failure probability density function.

In many practical situations, $f(t)$ can be adapted from the different advanced forms of Weibull distribution.

For more simplicity, eq. (1) will be reformed as follows:

$$C_T = \frac{Cp.I_1 + C_f.I_2}{T.I_1 + I_3}. \qquad (2)$$

It should be noted that $C_p.\ I_1$ represents the PM cost, C_f. I_2, represents the failure cost, and the denominator is the normalized time factor.

2. Single Phase Weibull Model (SPSM)

The probability density function $f(t)$ of the single phase Weibull distribution can be defined by the following equation,

$$f(t) = \beta \frac{(t-\delta)^{\beta-1}}{\theta^{B}} . \exp\left[-\left(\frac{t-\delta}{\theta} \right)^{\beta} \right], t \geq 0 , \quad (3)$$

where
β is the he shape parameter,
θ is the scale parameter, and
δ is the he location parameter.
β, θ and δ are continuous and the acceptable ranges for these variables are $0 < \beta < \infty$, $0 < \theta < \infty$ and $-\infty < \delta < \infty$.

When applying eq. (3) to the cost model given by eq. (2), the integral terms (I_1, I_2, and I_3) can be calculated as follows:

$$I_1 = \int_T^{\infty} \frac{\beta(t-\delta)^{\beta-1}}{\theta^{\beta}} . \exp\left[-\left(\frac{(t-\delta)^{\beta}}{\theta} \right) \right] .dt$$
$$= \exp\left[-\left(\frac{(T-\delta)^{\beta}}{\theta} \right) \right] . \quad (4)$$

$$I_2 = \int_o^{T} \frac{\beta(t-\delta)^{\beta-1}}{\theta^{\beta}} . \exp\left[-\left(\frac{(t-\delta)^{\beta}}{\theta} \right) \right] .dt$$
$$= 1 - \exp\left[-\left(\frac{(T-\delta)^{\beta}}{\theta} \right) \right] . \quad (5)$$

$$I_3 = \int_o^{T} t . \frac{\beta(t-\delta)^{\beta-1}}{\theta^{\beta}} . \exp\left[-\left(\frac{(t-\delta)^{\beta}}{\theta} \right) \right] .dt$$
$$= -\delta e^{-\left(\frac{(T-\delta)^{\beta}}{\theta} \right)} - e^{-\left(\frac{(-\delta)^{\mu}}{\theta} \right)}$$
$$+ \theta \left[\Gamma\left(1+\frac{1}{\beta}\right) - \right] \Gamma\left(1+\frac{1}{\beta}, \left(\frac{\delta}{\theta}\right)^{\beta} \right) - \Gamma(1+\frac{1}{\beta}, \left(\frac{T-\delta}{\theta} \right)^{\beta}) \Big] . \quad (6)$$

Similarly,(double/ triple) mode(s) and (single/double/triple) phase(s) Weibull distributions could be applied to the model depending on the problem being discussed. Table 1 gives the probability density function $f(t)$ of the failure rate for each form of the advanced Weibull distributions. The derived forms of the integral terms (I_1, I_2 , I_3) are given in the same table.

2.2. Computer code

A computer code written in Matlab programming language has been recommended to incorporate all cases addressed in this paper in the cost model proposed in eq. (2).

3. Case study and sample results

The cost model with all suggested probability density function have been applied to some components of special interest in most practical applications. Table 2 provides (type/ failure mode/input value) for some selected components.

The computer code explained in section (2.2) has been applied to get the optimum mean time to repair MTTR for such components, the results are provided by figs. 1-3, and 4.

It should be noted that all computations are based on C_f = 1000, C_P = 25 (arbitrary units). For any change because of currency index, the values of C_f and C_p should be updated accordingly.

4. Conclusions

Based on the problems investigated and the results obtained, the following conclusions could be withdrawn:
1. Weibull distribution have been successfully applied to the adapted cost model for optimizing maintenance schedules.
2. The distribution takes into consideration, the component failure mode and phasing changes.
3. Optimum MTTR is significantly affected by the component failure rate. Final decision should take the importance of the component into consideration.

Table 1
The advanced Weibull distributions considering cost model

Dist. type	$f(t)$	I_1	I_2	I_3
Single Mode Single Phase (SMSP)	$\frac{\beta t^{\beta-1}}{\theta^{\beta}} e - \left(\frac{t}{\theta}\right)^{\beta}$	$\exp\left[-\left(\frac{T}{\theta}\right)^{\beta}\right]$	$1-\exp\left[-\left(\frac{T}{\theta}\right)^{\beta}\right]$	$-T.\ \exp$ $\left[-\left(\frac{T}{\theta}\right)^{\beta}\right]+\frac{\theta}{\beta}\left[\Gamma(\frac{1}{\beta})-\Gamma\left(\frac{1}{\beta},\left(\frac{T}{\theta}\right)^{\beta}\right)\right]$
Double Mode Single Phase (DM SP)	$[\frac{\beta_1 t^{\beta_1-1}}{\theta_1^{\beta_1}}+\frac{\beta_2 t^{\beta_2-1}}{\theta_2^{\beta_2}}]$ $\exp\left[-\left\{\left(\frac{t}{\theta_1}\right)^{\beta_1}+\left(\frac{t}{\theta_2}\right)^{\beta_2}\right\}\right]$	$\exp\left[-\left\{\left(\frac{T}{\theta_1}\right)^{\beta_1}+\left(\frac{T}{\theta_2}\right)^{\beta_2}\right\}\right]$	$1-\exp\left[-\left\{\left(\frac{T}{\theta_1}\right)^{\beta_1}+\left(\frac{T}{\theta_2}\right)^{\beta_2}\right\}\right]$	$\int_0^T t.\left[\frac{\beta_1 t^{\beta_1-1}}{\theta_1^{\beta_1}}+\frac{\beta_2 t^{\beta_2-1}}{\theta_2^{\beta_2}}\right].$ $\exp\left[-\left\{\left(\frac{T}{\theta_1}\right)^{\beta_1}+\left(\frac{T}{\theta_2}\right)^{\beta_2}\right\}\right]dt$
Triple Mode Single Phase (TMSP)	$[\frac{\beta_1 t^{\beta_1-1}}{\theta_1^{\beta_1}}+\frac{\beta_2 t^{\beta_2-1}}{\theta_2^{\beta_2}}+\frac{\beta_3 t^{\beta_3-1}}{\theta_3^{\beta_3}}].$ $\exp\left[-\left\{\left(\frac{t}{\theta_1}\right)^{\beta_1}+\left(\frac{t}{\theta_2}\right)^{\beta_2}+\left(\frac{t}{\theta_3}\right)^{\beta_3}\right\}\right]$	$\exp\left[-\left\{\left(\frac{T}{\theta_1}\right)^{\beta_1}+\left(\frac{T}{\theta_2}\right)^{\beta_2}+\left(\frac{T}{\theta_3}\right)^{\beta_3}\right\}\right]$	$1-\exp\left[-\left\{\left(\frac{T}{\theta_1}\right)^{\beta_1}+\left(\frac{T}{\theta_2}\right)^{\beta_2}+\left(\frac{T}{\theta_3}\right)^{\beta_3}\right\}\right]$	$\int_0^T t$ $\left[\frac{\beta_1\ t^{\beta_1-1}}{\theta_1^{\beta_1}}+\frac{\beta_2 t^{\beta_2-1}}{\theta_2^{\beta_2}}+\frac{\beta_3 t^{\beta_3-1}}{\theta_3^{\beta_3}}\right].$ $\exp\left[\left\{\left(\frac{T}{\theta_1}\right)^{\beta_1}+\left(\frac{T}{\theta_2}\right)^{\beta_2}+\left(\frac{T}{\theta_3}\right)^{\beta_3}\right\}\right]dt$
Double Phase Single Mode (DPSM)	$f_1(t)=\frac{\beta_1(t-\delta)^{\beta_1-1}}{\theta_1^{\beta_1}}\cdot$ $\exp\left[-\left(\frac{t-\delta_1}{\theta_1}\right)^{\beta_1}\right]$ $f_2(t)=\frac{\beta_2(t-\delta_2)^{\beta_2-1}}{\theta_2^{\beta_2}}$ $\cdot\exp\left[-\left(\frac{t-\delta_2}{\theta_2}\right)^{\beta_2}\right]$	$\exp\left[-\left(\frac{T-\delta_1}{\theta_1}\right)^{\beta_1}\right]$ $\exp\left[-\left(\frac{T-\delta_2}{\theta_2}\right)^{\beta_2}\right]$	$1-\exp\left[-\left(\frac{T-\delta_1}{\theta_1}\right)^{\beta_1}\right]$ $1-\exp\left[-\left(\frac{T-\delta_2}{\theta_2}\right)^{\beta_2}\right]$	$I_3=-\delta_1\left[e^{-\left(\frac{T-\delta_1}{\theta_1}\right)^{\beta_1}}-e^{-\left(\frac{-\delta_1}{\theta_1}\right)\beta_1}\right]$ $+\theta_1\left[\Gamma\left(\frac{1}{\beta_1}+1\right)-\Gamma\left(\frac{1}{\beta_1}+1,\left(-\frac{\delta_1}{\theta_1}\right)^{\beta_1}\right)-\Gamma\left(\frac{1}{\beta_1}+1,\left(\frac{T-\delta_1}{\theta_1}\right)^{\beta_1}\right)\right]$ $I_3=-\delta_2\left[e^{-\left(\frac{T-\delta_2}{\theta_2}\right)^{\beta_2}}-e^{-\left(\frac{-\delta_2}{\theta_2}\right)^{\beta_2}}\right]+$ $\theta_2\left[\Gamma\left(\frac{1}{\beta_2}+1\right)-\Gamma\left(\frac{1}{\beta_2}+1,\left(-\frac{\delta_2}{\theta_2}\right)^{\beta_2}\right)-\Gamma\left(\frac{1}{\beta_2}+1,\left(\frac{T-\delta_2}{\theta_2}\right)^{\beta_2}\right)\right]$

Table 1 Cont.

Triple phase single mode (TP SM)	$f_1(t) = \frac{\beta_1 (t-\delta)^{\beta_1 - 1}}{\theta_1^{\beta_1}} \exp\left[-\left(\frac{t-\delta_1}{\theta_1}\right)^{\beta_1}\right]$ $f_2(t) = \frac{\beta_2 (t-\delta_2)^{\beta_2 - 1}}{\theta_2^{\beta_2}} \exp\left[-\left(\frac{t-\delta_2}{\theta_2}\right)^{\beta_2}\right]$ $f_3(t) = \frac{\beta_3 (t-\delta_3)^{\beta_3 - 1}}{\theta_3^{\beta_3}} \exp\left[-\left(\frac{t-\delta_3}{\theta_3}\right)^{\beta_3}\right]$	$\exp\left[-\left(\frac{T-\delta_1}{\theta_1}\right)^{\beta_1}\right]$ $\exp\left[-\left(\frac{T-\delta_2}{\theta_2}\right)^{\beta_2}\right]$ $\exp\left[-\left(\frac{T-\delta_3}{\theta_3}\right)^{\beta_3}\right]$	$1-\exp\left[-\left(\frac{T-\delta_1}{\theta_1}\right)^{\beta_1}\right]$ $1-\exp\left[-\left(\frac{T-\delta_2}{\theta_2}\right)^{\beta_2}\right]$ $1-\exp\left[-\left(\frac{T-\delta_3}{\theta_3}\right)^{\beta_3}\right]$	$I_3 = -\delta_1\left[e^{-\left(\frac{T-\delta_1}{\theta_1}\right)^{\beta_1}} - e^{-\left(\frac{-\delta_1}{\theta_1}\right)\beta_1}\right] + \theta_1\left[\Gamma\left(\frac{1}{\beta_1}+1\right) - \Gamma\left(\frac{1}{\beta_1}+1, \left(-\frac{\delta_1}{\theta_1}\right)^{\beta_1}\right) - \Gamma\left(\frac{1}{\beta_1}+1, \left(\frac{T-\delta_1}{\theta_1}\right)^{\beta_1}\right)\right]$ $I_3 = -\delta_2\left[e^{-\left(\frac{T-\delta_2}{\theta_2}\right)^{\beta_2}} - e^{-\left(\frac{-\delta_2}{\theta_2}\right)^{\beta_2}}\right] + \theta_2\left[\Gamma\left(\frac{1}{\beta_2}+1\right) - \Gamma\left(\frac{1}{\beta_2}+1, \left(-\frac{\delta_2}{\theta_2}\right)^{\beta_2}\right) - \Gamma\left(\frac{1}{\beta_2}+1, \left(\frac{T-\delta_2}{\theta_2}\right)^{\beta_2}\right)\right]$ $I_3 = -\delta_3\left[e^{-\left(\frac{T-\delta_3}{\theta_3}\right)^{\beta 3}} - e^{-\left(\frac{-\delta_3}{\theta_3}\right)^{\beta 3}}\right] + \theta_3\left[\Gamma\left(\frac{1}{\beta_3}+1\right) - \Gamma\left(\frac{1}{\beta_3}+1, \left(\frac{-\delta_3}{\theta_3}\right)^{\beta_3}\right) - \Gamma\left(\frac{1}{\beta_3}+1, \left(\frac{T-\delta_3}{\theta_3}\right)^{\beta_3}\right)\right]$

Table 2
Results and comments of some selected cases

Case #	Component type	Failure mode	$F(t)$	Input value	Fig.	Comment
1	Motor Operated Valve(MOV)	Failure to operate	SM	β = 2.5 , θ = 181, δ = 0.	1	The shape parameter (β) is a sensitive factor affecting the optimum time between preventive maintenance. Referring to the case represented by Figure 1, T_{opt} days when β = 2.5. As β increases to 6, the hazard decreases and hence T_{opt} increases. A value of T_{opt} = 74.66 days was obtained for β = 6 as shown in fig. 2.
2	Limit switches	Failure to operate	SM	β = 6,θ =181, δ = 0.	2	
3	Pump	Failure to start	SP	β = 2.5,θ =181, δ =1.3	3	The time between PM in this case is 40.91days.The case fits any component that has a failure rate given by $\frac{1}{t_f - t_i}\int_{t_i}^{t_f} h(t).dt$ derived and have increasing hazard (wear out period). *hint:* $[t_i, t_f]$ is arbitrary time interval (operation time).

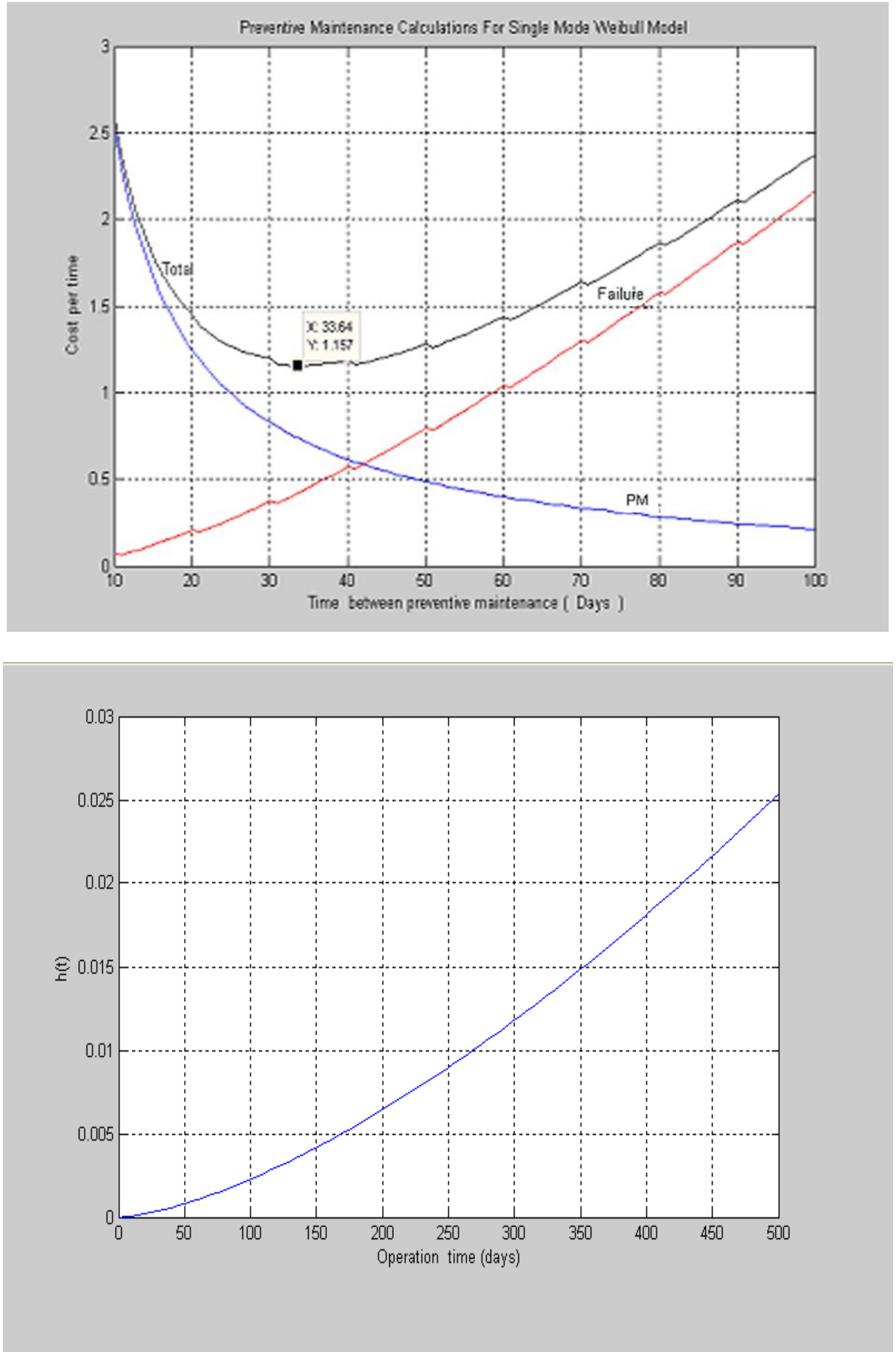


Fig.1. Cost-time between PM for a component having Weibull hazard in SM version (β = 2.5, θ = 181, δ = 0, C_P = 25, C_f = 1000).

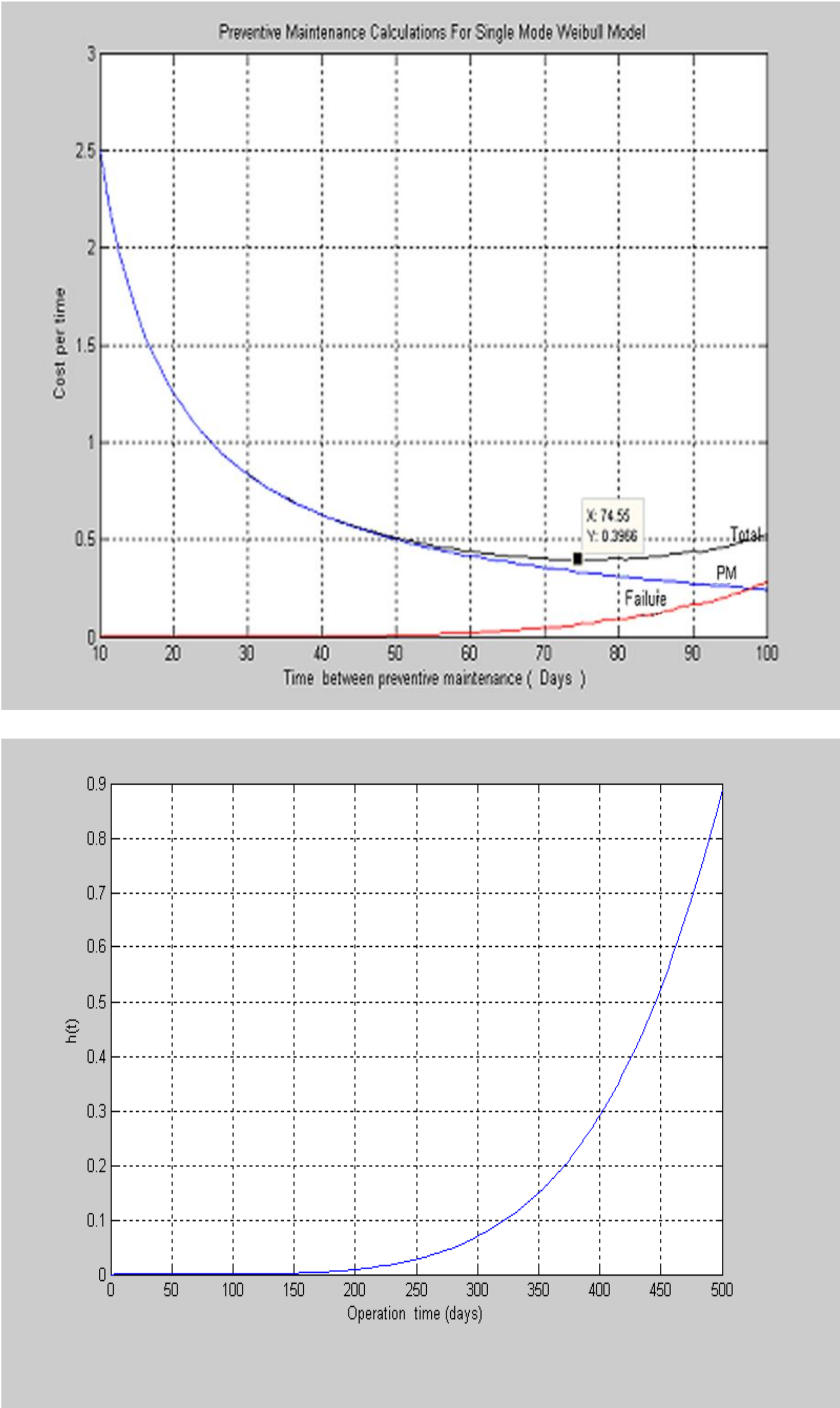
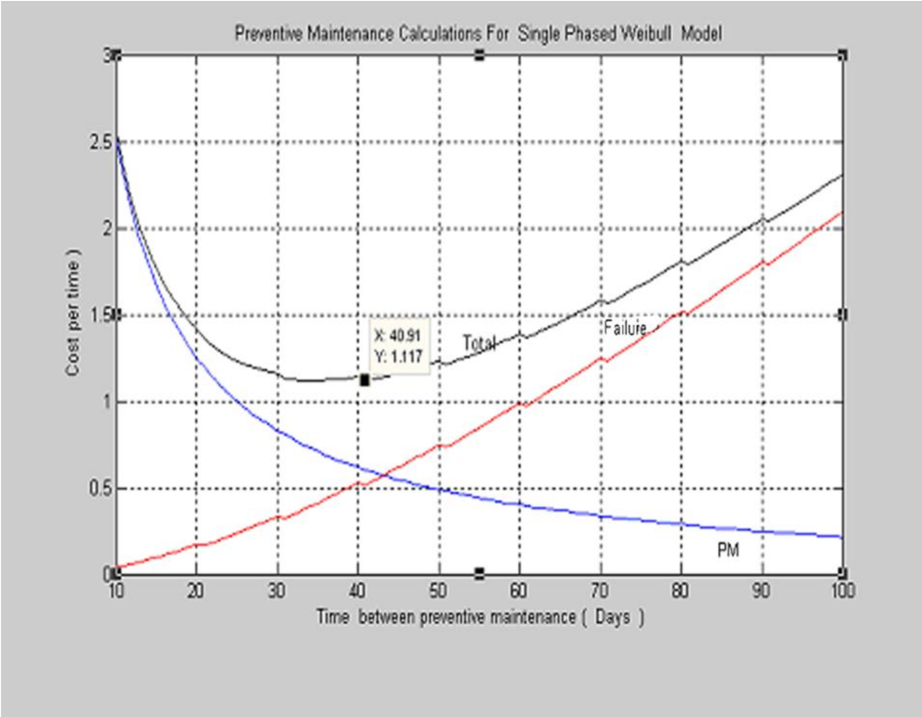


Fig. 2. Cost-time between PM for a component having Weibull hazard SM version (β = 6, θ = 181, δ = 0, C_P = 25, C_f = 1000).



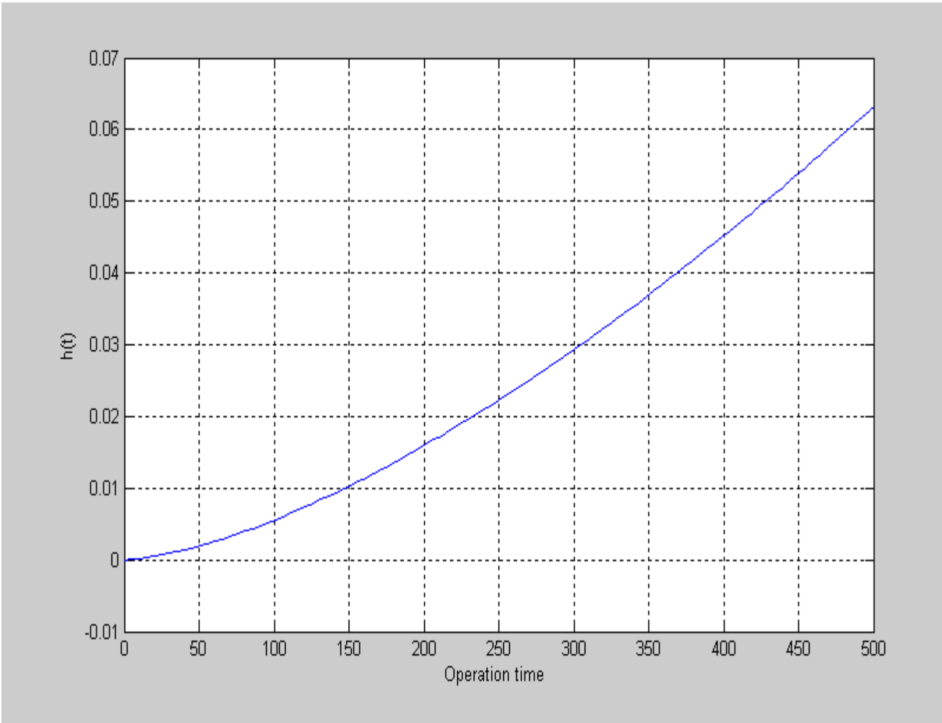


Fig. 3. Cost-time between PM for a component having Weibull hazard in SP version (β = 2.5, θ = 181, δ = 1.3, C_P = 25, C_f = 1000).

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