A simulation-based meta-heuristic search for optimal facility layout considering batch-size and stochastic product flows

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This paper tackles the eminent case of a facility layout which is usually modeled in literature as a Quadratic Assignment Problem (QAP). In the first part of the paper, two models were investigated, the first considers only the material handling cost, and the second elaborates the effect of dividing the product flows between cells into batches. Operational and design attributes were engaged to the two models by means of a spine driveway layout. Effects of using different types of material handling transporters were also explored. A meta-heuristic procedure combined with simulation search for the optimal layout assignment is considered. The two models were analyzed and solved using a set of other well known techniques, namely; a greedy, a random, and an explicit enumeration algorithm. A comparison was held to illustrate the superiority and the speed of the suggested meta-heuristic procedure. In the second part of the paper, stochastic products flows (demand) between cells were assumed to imitate more pragmatic environment. Five different scenarios for the demand were analyzed. The effects of dependencies between product flows, in terms of different correlation coefficients, were studied. Finally, optimal assignments under those scenarios and the corresponding minimum total cost for the two models were calculated. The results evidently demonstrate the soundness of the proposed approach.

تتناول هذه الورقة المسألة الكلاسيكية لتخطيط المنشأت الصناعية والتى عادة ما يتم إستخدام نماذج (QAP) التحليلية لمعالجتها. ويقدم الجزء الأول من الورقة نموذجين، حيث يعرض النموذج الأول تكلفة مناولة المواد فقط داخل منظومة التصنيع كدالة للهدف بينما يقدم النموذج الثانى تأثير تقسيم الطلب بين خلايا التصنيع إلى دفعات مختلفة. فى كلا النموذجين يجرى إدماج الظروف العملية للتصنيع وإعتبارات تصميم المنشأة من خلال إفتراض مخطط طريق الحبل الشوكى (Spine Driveway). ضمن هذا الإطار فقد تم دراسة تأثير استخدام وسائل متعددة لمناولة المواد على دالة الهدف، ومن ثم إختبار النموذجين سالفى الذكر، وعقد مقارنة بين مجموعة من أساليب حل المسائل المتشابهة مع الطريقة المقترحة للوصول للحل الأمثل فى ضوء أقل التكاليف وتحت مجموعة القيود الجديدة. حيث أشارت النتائج إلى جدوى وفعالية أسلوب ما وراء الحلول التقريبية المبنية على المحاكاة مقارنة بالطرق التقليدية لهذه النوعية من المشكلات صعبة الحل. فى الجزء الثانى تم معالجة الطلب (التدفقات) ما بين خلايا التصنيع كمتغيرات إحتمالية، وذلك من خلال سيناريوهات مختلفة تتراوح فيها نسب هذه المتغيرات بين (20%) و (100%) من إجمالى التدفقات. ونظراً لأهمية إدراج تأثير إعتماد موارد التصنيع على بعضها فقد تم أيضاً دراسة هذا الموضوع وأثاره على النماذج الأصلية وإمكانيات الوصول للحل وبالتالى المخطط الأمثل لهذه النوعية من نظم الانتاج فى ضوء الظروف الجديدة. لقد أثبتت نتائج هذا البحث جدوى وكفاءة الأساليب المقترحة.

Keywords: Quadratic assignment problem, Facility design, Meta-heuristics, Stochastic demand

1. Introduction

In today's highly competitive industrial environment, a process facility layout is characterized by its flexibility and suitability to handle a wide range of volume-variety applications. However, it suffers from: planning issues complexity, intra-traffic congestions, and costs related problems. Many researchers with overwhelming number of articles were devoted, for decades, to solve the aforementioned problems and keep, at the same time, the advantages which could not be overlooked by other strategic advances in this connection. Solutions related to flexible manufacturing systems, cellular manufacturing and group technology adoption are quite some few examples in the field. Nevertheless, classical process layout proved its superiority and still being adopted in many production systems allover the world.

1.1. Motivation for the problem of facility layout

The problem of process facility layout is concerned with finding the relative locations of physical manufacturing facilities (departments, cells, machines, etc.) to each other. The goal is to generate a block plan showing the relative positioning of the cells. The problem is often tackled by considering materials flows between cells and distances between locations as input data.

The dilemma is mostly treated by trying to reduce the materials handling and other interrelated costs that link respective manufacturing cells to locations. With that framework, the problem is classically modeled as a Quadratic Assignment Problem (QAP), Burkard et al. [1].

In its general form the case could be illustrated by assuming (n) to be the number of cells, with known flows between them, to be assigned to (n) locations (with known distances between them). The problem is reduced to find the minimum cost assignment of the (n) cells to the (n) locations, as described by the following minimum cost objective function, Jensen and Bard [2]:

$$\text{minimize} \quad Z = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} C_{ijkl}x_{ij}x_{kl} \,. \tag{1}$$

Subject to

All locations must be used, $\forall\, k$

$$\sum_{i=1}^{n} x_{ik} = 1 \,. \tag{2}$$

All cells must have locations, $\forall\, i$

$$\sum_{k=1}^{n} x_{ik} = 1 \,. \tag{3}$$

Here, (**C_{ijkl}**) is the cost to assign cell (**i**) to location (**k**) and cell (**j**) to location (**l**). Also, (**C_{ijkl}**) represents material handling and implicitly other cost interactions between cells (**i**) and (**j**). The decision variables are (**x_{ik}**), where (**x_{ik}**) equals 1 if cell (i) is assigned to location (k) and 0 otherwise. Also (**x_{ik}**) are between 0 <(**x_{ik}**) < 1 and integers for i=1,2,..., n , and k = 1,2,...,n.

The preceding treatment is a usual simplifying assumption since it disregards the stochastic nature of departmental flows and assuming deterministic, known for certain, flows (demand). Further, the model is focused on finding the cells assignment to locations that would minimize only the materials handling costs between cells. Justifications for such a treatment is attributed to the high share (20-50%) of the material handling costs, as non-value added costs, in the total production cost of a manufacturing facility , Tompkins et al. [3]. As stated in a survey by Meller and Gau [4], such an approach was the subject matter of numerous research trends when attempting to workout the facility layout problem. However, even with the deterministic versions of the problem, it remains hard to solve. This is due to its combinatorial nature, yet for fewer numbers of cells and locations; the number of alternative configurations is enormous. Classified as an NP-hard problem, Garey and Johnson [5], the QAP is considered as one of the difficult to solve problems in the area of combinatorial optimization. It carries (n^2) decision variables, with ($n!$) different assignment permutations for the objective function.

Another feasible approach is to consider the flows between cells to be dynamic in nature under the same material handling constraints and objective, Conway and Venkataramanan [6]. This is a step towards a more pragmatic formulation of the problem, since demand is subject to change with time. With some degree of flexibility in the layout design configuration, a dynamic facility layout model could further enhance the suitability of the model to application, Balakrishnan and Cheng [7]. Other research work attempted to add some operational, technical aspects and physical improvement to the situation, such as; different cell design configuration, Filho and Tiberti [8], layout with aisles, Gómez, et al. [9], flexible layout under uncertainty, Yang and. Brett [10], effect of various material flow patterns in a manufacturing environments, Elbaz [11], and the use of product and process parameters to increase routing flexibility, Castillo and Peters [12].

On the other hand, most of the research work in the area focused on using different search techniques to solve the problem. Since, its appearance as a sound challenge in the literature for industrial facility design, operations research, and operations management, several methods were implemented.

1.2. Optimization search procedures

As far as the facility layout design is concerned, the QAP problem has been tackled by numerous algorithms to find near optimal, local and/or global optimal solutions. A partial list of applicable algorithms is displayed here after. In an early attempt, Hosni [13] suggested the use of craft exchange algorithm that applied the steepest-descent pair-wise procedures to find a near optimal assignment. In a most recent work by Ioannou [14] an optimal solution is provided with the adoption of explicit enumeration technique. Foulds and Wilson [15] used a branch and bound algorithm for an assignment problem subject to a special set of side constraints. The resulting model represents a special case of a restricted facilities layout problem in which it is forbidden to locate any facility in certain zones. Ashayeri et al. [16] presented a modified simple yet effective generic search approach to the classical facility layout problem as applied to design warehouses and production systems.

As more practical and operational-based issues incorporated, the need for additional powerful solution procedures escalates, to handle such additional complexities. Researches faced the dilemma of whether to deal with relatively simplified problems to get optimal results, or rather to reach sub-optimal, second-best, solutions for multifaceted situations. Under that framework, heuristics are being used comprehensively to provide approximate solutions to complex facility layout problems, Chan, et al. [17].

Alternatively, the area of meta-heuristics arose, as valid search methodologies, with the goal of providing better environment, based on integrating high-level intelligent procedures and fast computing capabilities, Solimanpure et al. [18].

With such innovations, the same problem was extensively tackled. Few examples are illustrated here as the use of; genetic algorithm, Lee et al. [19], tabu search, Chiang and Chiang [20], simulated annealing, Wang et al. [21], ant colony, McKendall and Shang [22], and many others.

Generally, such broad concepts are based on not to separate the problem modeling realm from the optimization procedure domain. However, this made the solutions very limited to specific tailored cases. A proper treatment of that situation would consider building the model first and use a proper environment to link it with an optimizer Glover et al. [23]. For instance, a Monte Carlo simulation platform, built with MS Excel, would be used to take inputs from the model, change it and then feed it to the optimizer. The optimization procedure uses the outputs from the simulation model, evaluates the outcomes of the inputs, using a search procedure, and then feed it back into the model. The process continues until some termination criterion is satisfied. This repeatedly, and successively generated output would breed a highly efficient path to the optimal solutions.

1.3. Operational characteristics

Despite the aforementioned contributions in the problem solution domain, its implementation in real and world class industry is not widely adopted. Reasons for such denial attitude could be attributed to the impracticality and inapplicability of the suggested methods. The lack of the proposed approaches, for example, to reflect the challenge, in today's manufacturing environment, for product demand variability, and small batch size considerations are other prime reasons. It is well known that, most of the current obstacles that would face industrialists are those raised due to the continual changing operational conditions and not to design principles.

The main objective of this paper is to consider more practical and operational issues than the case when solving the facility layout as a problem of system design only. Important aspects like the inclusion of batch size considerations when material flows between cells, which generates setups and work-in-process inventory are studied. Constraints are placed on initial conditions to further allow for flexibility and modularity. A useful configuration, in this regard, is the spine driveway approach to facility arrangement. In addition to a deterministic or even a dynamic facility problem, a stochastic material flow pattern is then considered. On the other hand, a meta-heuristic solution procedure rather than traditional optimization search procedures is adopted.

2. Model formulation

2.1. The spine driveway approach

Instead of assuming a completely random initial layout design, it is more practical, as in real life situations; to start the analysis by having a general flow pattern. Such a bound on the solution space is usually dictated by physical and technical constraints. The spine approach refers to a central driveway to conduct traffic-material, utilities, information, and people, Askin and Sandrdidge [24]. Fig. 1 illustrates the spine concept where cells expand out from the central driveway core, and aisles can be used to conduct flows into cells. Utilities can be carried overhead to simplify the network of pipes and cables. Material is stored along the spine driveway, and with each department has an input and output storage area along the spine. This departmental storage concept reduces material flow as compared to a system's single centralized warehouse visited by all parts and materials after each operation.

Distance between the centroids of any two cells along the spine configuration is usually measured rectilinearly, as in fig. 1, by the equation:

$$d(p_k,p_h) = [\Delta y_{kh1} + \Delta y_{kh2} + \Delta x_{kh}] = [\,|y_k - y_h| + |x_k - x_h|\,]. \quad (4)$$

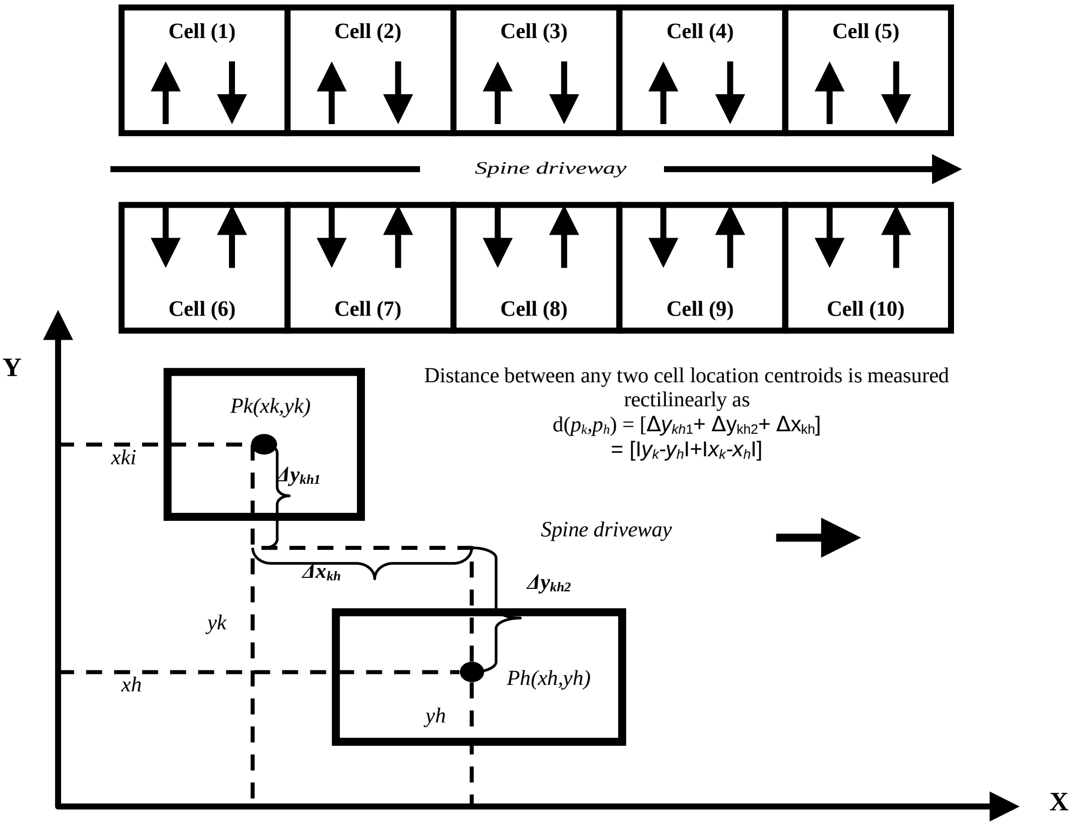

Fig. 1. The spine driveway configuration for cellular manufacturing.

2.2. Batch size considerations

During a certain planning horizon, product flows between cells could be divided into batches with known sizes, in order to utilize the main advantage of the spine approach of distributing the system main storage area among different cell locations. On the other hand, and as inspired by a preliminary modified EPQ inventory model due to Irani and Liu [25], such an assumption would amend the generalized QAP model to include other cost terms. Accordingly, in this research, a certain batch size (Q_{ij}) is only allowed at a time to transfer between cells (i), (j), when cells (i,j) are assigned to locations (k,h), respectively. The difference in product quantities between the flows (Dij) connecting cells (i), and (j) and the transferred batch size (Q_{ij}) will be held in inter-departmental (Buffer) storage for that batch to be processed in the corresponding cell (i) with a certain holding cost (H_{ij}). Further, a setup cost (S_{ij}) will be allocated to cells for products in a rate that matches the number of travels between cells.

The material handling cost in the objective function is the sum of material flows, or demands, (D_{ij}) between cells multiplied by the distances (d_{kl}) between all locations. If the material flows are assumed to occur on batch size basis, the cost function can be expanded to include other cost terms. For instance, demand (D_{ij}) is divided to flow in batches (Q_{ij}) between cells, the number of material handling moves will be (D_{ij}/Q_{ij}). Thus instead of having one material flow between each two cells, the number of moves will increase, causing the material handling cost to increase. Further, such material handling movements depend on the type of material transporter used. However, such increase in material flows is accompanied by the generation of instantaneous work-in-process (or a buffer) inventory to compensate for the difference between the demands (D_{ij}) and the actual flow quantities (Q_{ij}). As already has been stated, the spine drive way layout configuration could easily allow for that assumption. On the other hand, another cost term would be added to portray the batch consideration setup cost. Hence, the previous QAP objective function in eq. (1) above could be expanded to be:

$$\text{minimize} \quad Z = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n}[C_{ijkl}(D_{ij}d_{kl})\frac{D_{ij}}{Q_{ij}} + \frac{H_{ij}}{2}Q_{ij}(1-\frac{D_{ij}}{Q_{ij}}) + S_{ij}\frac{D_{ij}}{Q_{ij}}]\; x_{ij}x_{kl}\,. \qquad (5)$$

Where; for a certain part, and under a planning horizon (T):

C_{ijkl} = Material flow cost per travel between cells (i) and (j) when they are assigned to locations (k) and (l) respectively ($/travel)

It should be noted that (C_{ijkl}) is a function of the product unit load (u_{ij}) (parts/travel), (d_{kl}) distance traveled between locations (k) and (l) in meters , (V_{ij}) velocity of travel between cells (i), (j) (meter/min) , and the material handling transporter operational cost (O_t), (Cost of moving one part a unit distance at a velocity of 1 meter/min). Thus a compelling expression for (C_{ijkl}) would be;

$$C_{ijkl} = \varphi\ (u_{ij}, d_{kl}, V_{ij}, O_t) = O_t\,\frac{d_{kl}}{u_{ij}V_{ij}}\,. \qquad (6)$$

To show the effect of using different material handling transporter, eq. (6) is depicted in fig. 2 for six different types by changing the velocity as a parameter in that equation. In addition to the model constraints shown in eqs. (3 and 4), the batching process dictates that (D_{ij}/Q_{ij}) is greater or equals to 1, otherwise 0.

2.3. Application of meta-heuristics search

Classical optimization search techniques, like linear programming or integer linear programming solvers are efficient for solving less intricate problems. However, they generally lacked the power to provide high quality solutions to complex problems. When dealing with the optimization of complex systems, as in the case of the QAP facility layout, specialized heuristic procedures are used which, in general, rely on approximate course of action to reach a better solution.

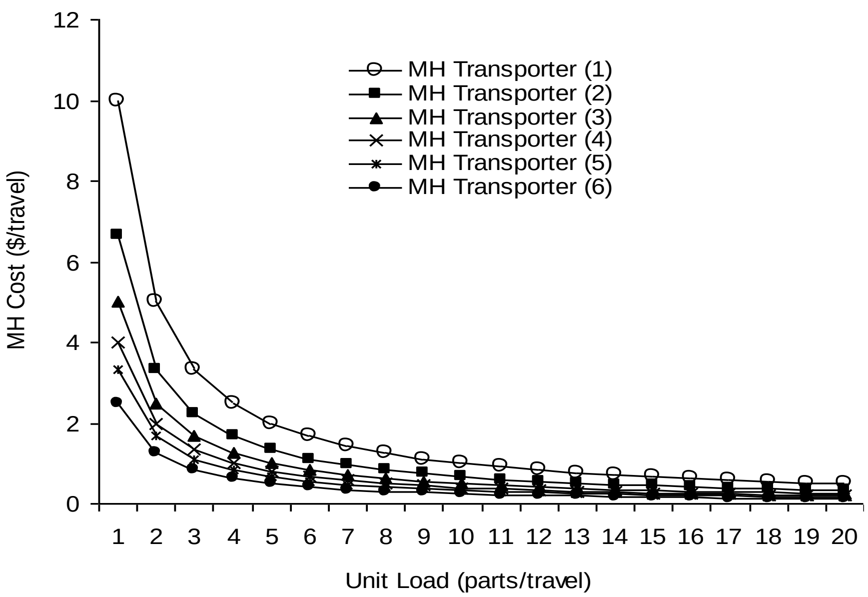

Fig. 2. Effect of using different material handling transporters on (C_{ijkl}) coefficient.

For an optimal solution, meta-heuristics provided a way of considerably improving the performance of simple heuristic procedures. Like their heuristic counterpart, meta-heuristics are also approximate techniques. The general form of an iterative meta-heuristic based on local search is given in Appendix 1 (Randall M.C. [26]).

The search strategies implemented with meta-heuristic result in iterative-procedures with the ability to escape local optimal points. Consequently, the optimization problem is defined outside the system, which is represented in this case by a simulation model. Therefore, the simulation model can change and evolve to incorporate additional elements, while the optimization routines remain the same. Hence, there is a complete separation between the model that represents the system and the procedure that is used to solve optimization problems defined within this model.

The optimization procedure uses the outputs from the simulation model which evaluate the outcomes of the inputs that were fed into the model. On the basis of this evaluation, and on the basis of the past evaluations which are integrated and analyzed with the present simulation outputs, the optimization procedure decides upon a new set of input values (see fig. 3). The process continues until some termination criterion is satisfied (usually by a time limit or a number of simulation runs set by the user).

An implementation of the meta-heuristic system described above is released under *Opt Quest* for *Crystal Ball*, Glover et al. [27]. In its current version, *Opt Quest* has been specifically customized to help users find optimal input parameter settings to simulation models built with *Crystal Ball*, (A registered trademark of *Decisioneering*, *Inc.*) In order to use *Opt Quest* the user first creates a *Crystal Ball*, Excel-based spreadsheet model. Once the simulation model has been created, an option can be selected within the system to access the optimization procedure.

A brief description of the search procedure and the major components of the *Opt Quest* system algorithm are outlined henceforth.

Two types of meta-heuristic procedures are used; Scatter Search (SS) and Tabu Search (TS). Scatter search is designed to operate on a set of points, called reference points that constitute good solutions obtained from the suggested initial solution. The approach systematically generates linear combinations of the reference points to create new points, each of which is mapped into an associated feasible point. TS is then superimposed to control the composition of reference points at each stage. TS is often done by defining suitable attributes of moves or solutions, and imposing restrictions on a set of the attributes, depending on the search history. The search process is intelligently guided to forbid certain duplicate or past solutions. More detailed information related to SS and TS could be found in Laguna [28], and Glover [29] respectively. Also check appendix 1 for pseudo code of both the TS and the SS algorithms.

A neural network filter (accelerator) is implemented to increase the power of the system's search engine. The concept behind embedding a neural network is to filter values of decision variables that are likely to result in a very poor value of the objective function. It is a prediction model that helps the system accelerate the search by avoiding simulation runs whose results can be considered as inferior. Information is usually collected about the objective function values obtained by different optimization variable settings. This information is then used to train the neural network during the search. The system automatically determines how much data is needed and how much training should be done, based once again on both the time to perform a simulation or the optimization time limit provided by the user, Glover et al. [30].

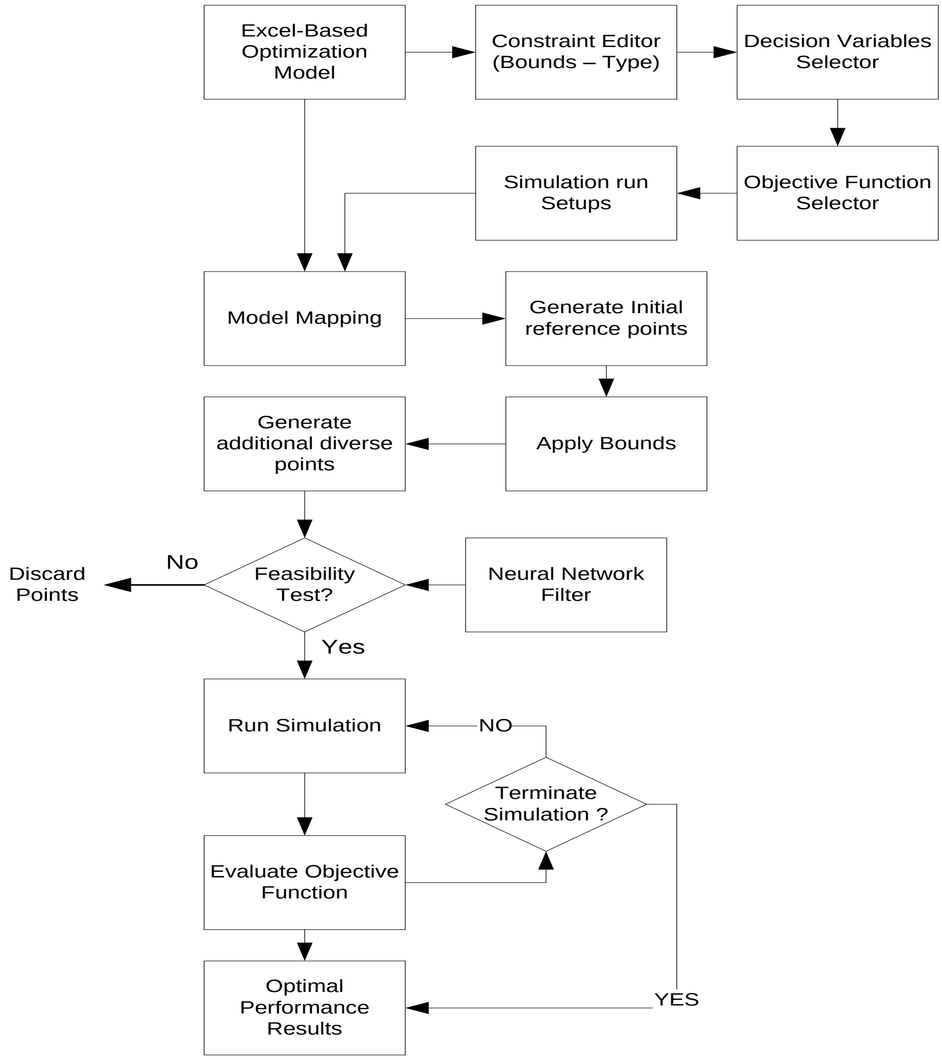

Fig. 3. Flow of the meta-heuristic search logic procedure.

3. Product deterministic flows

3.1. Problem initial investigation

To investigate the soundness of the facility layout problem model, as described in the previous sections, a medium sized case is assumed which comprises eq. (10) cells that must be assigned to eq. (10) different locations. Materials or product flows (demand) between cells are assumed to be symmetric. Distances are measured rectilinearly between cell centroids, in a spine driveway configuration and an initial assignment as depicted in fig. 4, to denote respective locations. Instead of generating a random layout, an initial layout is assumed to be (12345678910). Cells are arranged in a spine driveway (i.e. starting from upper left corner with cell 1 in location 1, and up to cell 5 in location 5, and then cell 6 from lower left corner in location 6 up to cell 10 in location 10. Tables 1 and 2 display the material flows (D_{ij}) and distances (d_{kl}) between cells (n =10). As already what has been stated, the problem as described has (3,628,800) objective functions to evaluate to reach an optimal solution.

With the initial layout assignment, a presentation of the total cost, as given by eq. (5), at different batch sizes and using different types of material handling transporters is given in fig. 4. Results show that there is an optimum batch size, for each type of transporter, at which total cost retains a minimum value as shown in table 3.

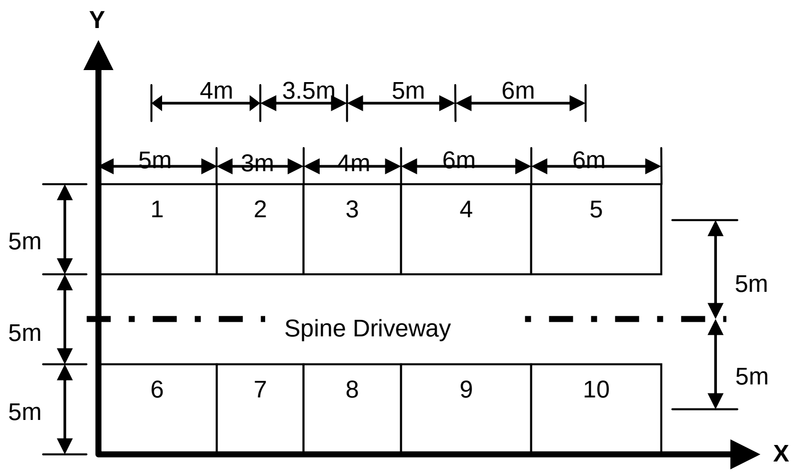

Fig. 4. Initial assignment and layout configuration.

Table 1
Flows (demand) between cells

D_{ij}	j =1	2	3	4	5	6	7	8	9	10
i =1	0	78	96	0	81	52	0	0	65	61
2	78	0	0	87	88	93	0	54	0	76
3	96	0	0	98	63	95	56	76	0	70
4	0	87	98	0	88	0	0	85	0	0
5	81	88	63	88	0	93	0	52	0	74
6	52	93	95	0	93	0	0	52	52	0
7	0	0	56	0	0	0	0	84	0	64
8	0	54	76	85	52	52	84	0	84	58
9	65	0	0	0	0	52	0	84	0	78
10	61	76	70	0	74	0	64	58	78	0

Table 2
Distances between corresponding locations

d_{kl}	k =1	2	3	4	5	6	7	8	9	10
l =1	0	14	17.5	22.5	28.5	10	14	17.5	22.5	28.5
2	14	0	13.5	18.5	24.5	14	10	13.5	18.5	24.5
3	17.5	13.5	0	15	21	17.5	13.5	10	15	21
4	22.5	18.5	15	0	16	22.5	18.5	15	10	16
5	28.5	24.5	21	16	0	28.5	24.5	21	16	10
6	10	14	17.5	22.5	28.5	0	14	17.5	22.5	28.5
7	14	10	13.5	18.5	24.5	14	0	13.5	18.5	24.5
8	17.5	13.5	10	15	21	17.5	13.5	0	15	21
9	22.5	18.5	15	10	16	22.5	18.5	15	0	16
10	28.5	24.5	21	16	10	28.5	24.5	21	16	0

Table 3
Minimum cost and corresponding batch size with different MH transporters under deterministic flow pattern between cells (assignment 12345678910) with total cost = $ 6099144 @ batch size = D

MH Transporter	1	2	3	4	5	6
Min cost ($) (*)	139152	282111	392598	485683	567879	642224
Batch size	34	50	62	70	78	86

(*) Conversion factor 1 $ ≈ 5.72 LE (December 2005 Prices)

3.2. Solving the problem

The problem, as initially analyzed, is solved for an optimum assignment using four different search strategies, namely; a greedy algorithm, a random search, an explicit enumeration, and a meta-heuristic approach.

It should be noted that a problem like the one under investigation is hard to solve. The greedy method could provide an acceptable solution for this problem, and so does, with lesser degree, the random generation algorithm. Improvements can be applied to the randomly generated solutions, and given sufficient time, better solutions to the problem are possible, Jensen and Bard [2]. Despite the overwhelming iterations, explicit enumeration can reach an optimal solution. However, it is certainly irrational for problems with more than 10 cells unless the allowed assignments are highly restricted.

A deterministic material flows between cells are considered with two basic models; (a) with no batching consideration according to eq. (2 and b) with batching process as described by means of eq. (5). Algorithms and search techniques were conducted on a personal computer with an AMD processor of 2.66 GHz CPU speed and a 512 MB RAM, using MS Excel and a set of QAP add-ins that were originally developed by Jensen and Bard [2], and extensively modified by the author to suit the application, as a test bed. *Opt Quest* was the optimization platform environment for the meta-heuristic method. A comparative analysis between the results obtained using the different search techniques is exhibited in table 4. Generally, when flows are allowed to move in batches, a noticeable decrease in the system total cost is achieved. Using different material handling transporters in transferring batches between cells, also affect total minimum costs and its corresponding batch size. A greedy algorithm provides a fast, yet a non-guaranteed optimal solution to the problem, as compared to the random search which suggests a best found alternative within the search space. Explicit enumeration affords an optimal solution; however it suffers from extensively large number of search runs (over 3.6 million runs) and considerable long search time (about 3.7 hours) for model (b) problem type. On the other hand, Meta-heuristics provides the same confident optimal solution and lower minimum cost levels with significantly lesser search time (about 5 minutes) and total solution seek runs (1350 simulation runs). Best (optimal) solutions were reached after (530 and 612) runs for model (a and b) respectively. Figs. 5 shows the relationship between the minimum total cost values versus the number of simulation runs with the meta-heuristic approach generated for the two models (a and b).

Table 4
Results summary for the different search methods

Search method	Search time (Sec)	No. of runs	Min cost ($) @ Batch size =D	Optimal	Min. cost ($) with MHT1	Min. cost ($) with MHT2	Min. cost ($) with MHT3	Min. cost ($) with MHT4	Min. cost ($) with MHT5	Min. cost ($) with MHT6	Optimal layout
Initial [(a)]	--	--	6099144	--	--	--	--	--	--	--	12345 678910
Initial [(b)]	--	--	6093544	--	139152 (Q=34)	282111 (Q=50)	392598 (Q=62)	485683 (Q=70)	567879 (Q=78)	642224 (Q=86)	12345 678910
Greedy [(a)]	28.2	387	5492774	Greedy	--	--	--	--	--	--	98267 310145
Greedy [(b)]	324	387	5399077	Greedy	118724 (Q=34)	253426 (Q=46)	357217 (Q=58)	444860 (Q=66)	522202 (Q=74)	592258 (Q=82)	105389 47261
Random [(a)]	42.3	1575	5612954	Best found	--	--	--	--	--	--	27148 351096
Random [(b)]	554	1575	5589401	Best found	124261 (Q=34)	261611 (Q=46)	368955 (Q=58)	456270 (Q=66)	534922 (Q=74)	606033 (Q=82)	24187 693105
Explicit [(a)]	7390	3628800	5171754	Optimal	--	--	--	--	--	--	31862 710954
Explicit [(b)]	13550	3628800	5166304	Optimal	111876 (Q=34)	243303 (Q=46)	345174 (Q=58)	430749 (Q=66)	506470 (Q=74)	574735 (Q=78)	86317 210954
Metah [(a)]	300	530	5171754	Optimal	--	--	--	--	--	--	31862 710954
Metah [(b)]	300	612	5166304	Optimal	111876 (Q=34)	243303 (Q=46)	345174 (Q=58)	430749 (Q=66)	506470 (Q=74)	574735 (Q=78)	86317 210954

(a) No batch size consideration
(b) With Batch size consideration and different material handling transporters (MHT_m)

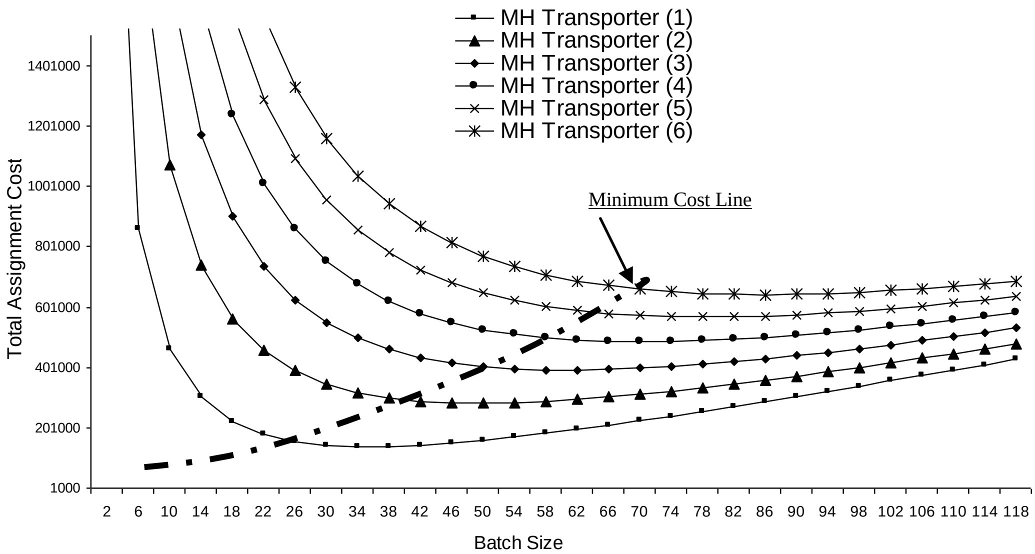

Fig. 5. Total cost as functions of traveled batch size for different types of Material Handling (MH) transporters.

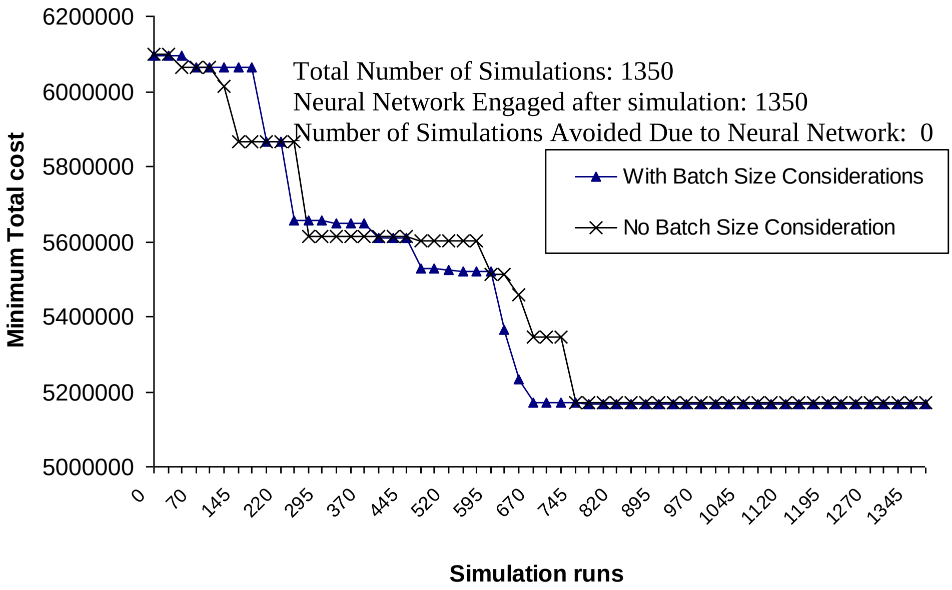

Fig. 6. Minimum total cost with meta-heuristic search output using simulation for models (a and b).

4. Product flows under uncertainty

Previous work in the area, mostly consider a deterministic product flow quantities between cells to find an optimal layout assignment. Deterministic product flows assume that all flows and consequently related costs are constant and known for certain over a given planning horizon. A more realistic formulation of the problem would consider such flows to be stochastic in nature. Demand conditions, in this context, are presented by means of expected values and standard deviations, assuming normal distribution conditions. This problem assumption allows more actual situation effectiveness. However, it would rather complicate the search for an optimal assignment.

4.1. Sources of uncertainties

In real situations, the allotted product flows between cells are subject to be under or overestimated due to system external variability. It is almost impractical to assume that those flows (demands) will meet their predicted values when seeking an optimal assignment. Lots of factors will cause that variability on a production system. For instant, effects of the general economic conditions, seasonal and random variations, customer behavior, competitive environment, supply chain responsiveness, etc. A major reason of uncertainties is not to consider the dependencies between each pair of flows in the system. Such circumstances would generate due to, for instance, sharing the resources between cells. For example, same operators, fixtures, material handling and tooling may be used for more than a single cell. In spite of that, it is rather difficult to capture every correlation coefficient between all products flows. An acceptable solution for that endemic situation is to assume a general correlation coefficient to study the effect of such a factor.

4.2. Problem initial investigation

The investigation starts by assuming product flows between cells to be stochastic under a set of five different scenarios. With a preset level of 99% degree of confidence, the five scenarios are classified into (20%, 40%), (60%, 80% and 100%) of the total flows are stochastic. Such operational scenarios reproduce different conditions of product flow uncertainties which the production system would face during parts processing. For instance, a 20% scenario for the stochastic flows of the total demand would imitate a case of relatively high certainty environment. While, a 100% stochastic product flows represent the case of low certainty (high uncertainty) of demand circumstances. The latter case would be considered, for example, in conditions of highly competitive situations.

The problem is preliminary investigated for the two models (a and b) under the initial assignment, (i.e. 12345678910). Input product flows are assumed to be normally distributed with mean = its predicted value, as shown in table 1, and with a standard deviation = 10% of its mean. For zero product flows, standard deviation is considered to equal 1. Simulation runs are conducted to analyze the effect of the aforementioned assumptions on the system total cost values in both cases.

Results for each run are expressed in terms of a frequency distribution, and the cumulative probability distribution of the total cost function. Another useful output is given by the percentiles (Px's) of the forecast probability distribution for the total cost function. Where, A Px is the x^{th} percentile of a probability distribution. This indicates that there is an x% probability that the total cost is less than its value at Px.

4.3. Analysis of results

Considering the initial layout, figs. 7 and 8 are system total cost example outputs of the simulation runs under the aforementioned (20%) scenario of product flows. Additional outputs are summarized in fig. 9 which groups the relationships between the total costs and the percentile of the total cost probability distributions under the five different stochastic product flow scenarios for model (a). Fig. 10 is the same output as generated for model (b). Results clearly indicate that the relationship is relatively steeper in cases for low level of uncertainties in product flows, thus those levels approaching a deterministic flow cases. This reflects that as the level of uncertainty in demand increases (e.g. the 100% scenario) the total costs variability increases – the range of probable total cost values becomes wider. The pivot point at which the relationships are intersecting is at the ($620,000) level. The probability not to exceed that level is as low as (35%) for all scenarios. While there is a (90%) probability that the total cost will not exceed ($6356831), and ($6521367) for the (20%) and the (100%) product flows scenarios respectively.

Similar results are drawn for model (b), in fig. 10, with improved total cost levels. The ($6200000) cost level is achievable at a probability of (80%) for the (20%) product flow scenario and at (58%) for the (100%) scenario. Further, a probability of (90%) is observed to achieve a cost level of ($6301481) for the worst scenario of (100%) stochastic flow as compared to the ($6521367) obtained for model (a). This is an improvement of about (3.4%).

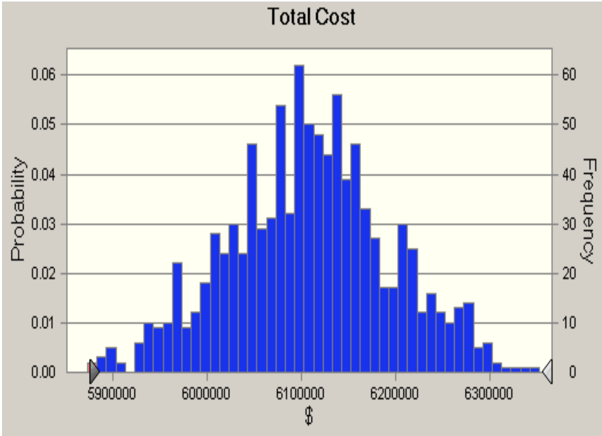

Fig. 7. Total cost frequency distribution at 20% scenario of product flows – simulated output.

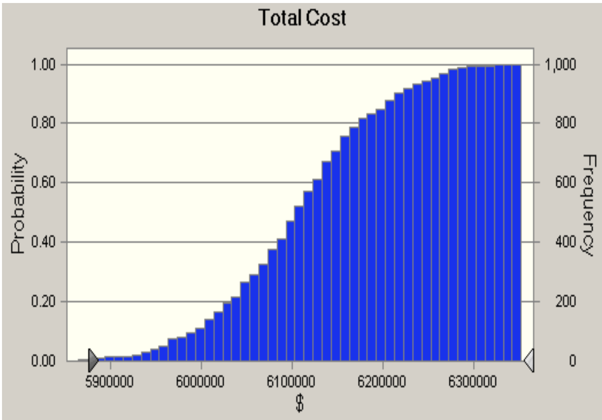

Fig. 8. Total cost cumulative probability distribution at 20% scenario of product flows – simulated output.

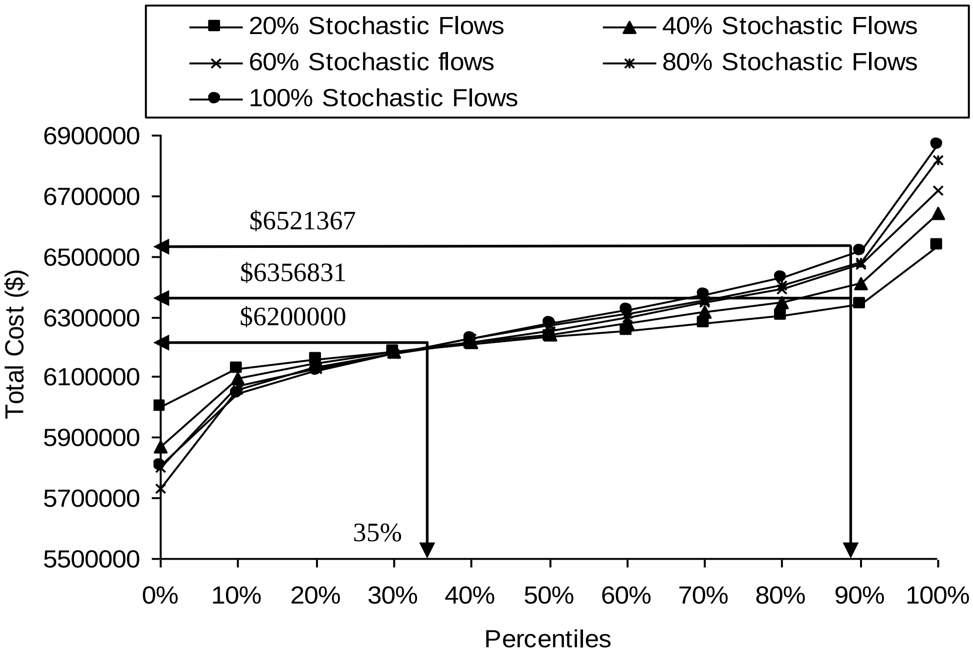

Fig. 9. Percentile of the total cost probability distribution at different stochastic demand scenarios – no batch size consideration – model (a).

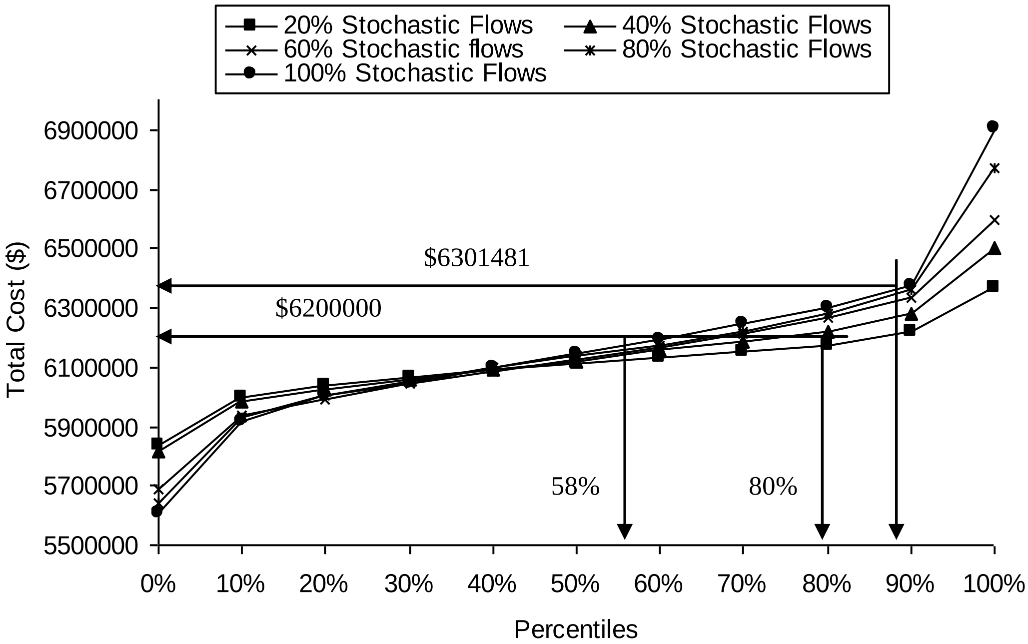

Fig. 10. Percentile of the total cost probability distribution at different stochastic demand scenarios – with batch size consideration – model (b).

4.4. Effect of dependencies between product flows

Dependencies between product flows are experimented by assuming three levels of total correlation coefficients. Although it is not totally adequate, assuming an inclusive correlation coefficient is satisfactory here for the sake of study. Such correlation coefficients are: No correlation condition resembles the case of no dependencies between product flows, a (0.3), and a (0.7) correlation coefficients represent the cases of relatively moderate and strong dependencies respectively. Results for those cases are summarized by means of figs. 11 and 12 which display the outcome of the different correlation coefficients on the mean value of the system total cost for the two models (a and b). Figs. 13 and 14 show the same effects on the standard deviation of the total cost, again for models (a and b). Experiments are conducted under the previously mentioned five levels of stochastic product flows; (20%, 40%, 60%, 80%, and 100%) scenarios. The results indicate that the higher the correlation coefficient the higher is the system total cost mean value. Such conclusion holds for both models (a and b). Also the effects of the correlations are amplified as the product flows become more stochastic in nature (i.e. Scenarios of higher stochastic level). Higher values of standard deviations, which resemble higher system variability, are noticeable for model (b) as compared to model (a). This lead to the inference that the system total cost become more uncertain and amplified due to the combined effects of high correlation, high stochastic flows, and batch size considerations.

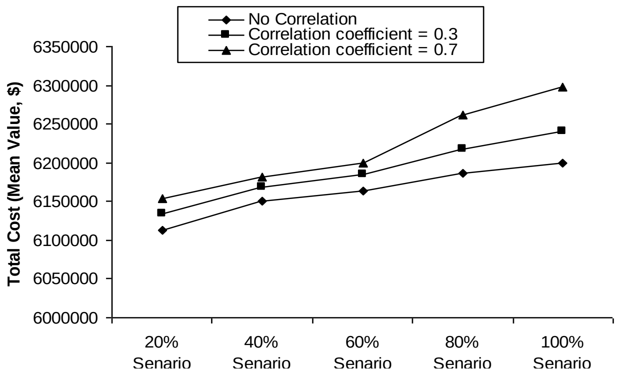

Fig. 11. Effect of correlation on total cost mean under different stochastic demand scenarios – no batch size consideration – model (a).

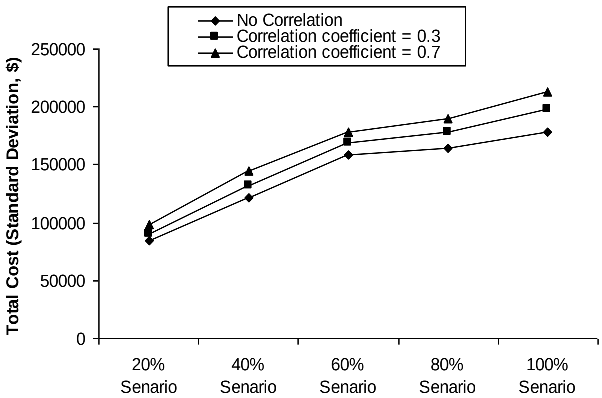

Fig. 13. Effect of correlation on total cost standard deviation under different stochastic demand scenarios – no batch size consideration - model (a).

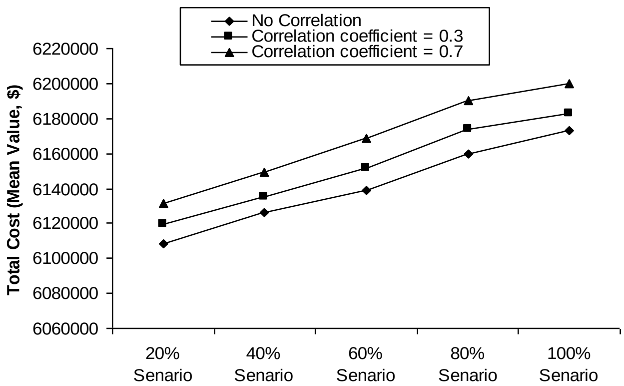

Fig. 12. Effect of correlation on total cost mean under different stochastic demand scenarios – with batch size consideration - model (b).

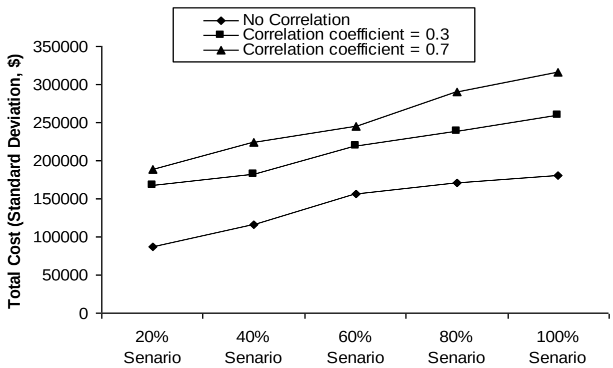

Fig. 14. Effect of correlation on total cost standard deviation different stochastic demand scenarios – with batch size consideration - model (b).

4.5. Finding the optimal assignment

The meta-heuristic search procedure is then implemented under simulation, as in the case of deterministic models, with a termination criterion of 1000 simulation trials to find optimal assignment solutions that guarantee minimum total costs.

Fig. 15 presents the optimization search outputs as the minimum total cost obtained with the simulation runs using the meta-heuristic search for model (a). The figure includes such an output for the five different proposed scenarios of product stochastic flows. It shows also the number of simulation runs at which the solution is reached for each scenario, i.e. the point at which no more improvement in the solution is attainable (best solution). Fig. 16 gives the same output but for model (b) of the problem.

Table 5 summarizes the results of the search experiments and the optimal layout under each product flow scenario for the two models (a and b) that represent, correspondingly, the cases of no batch size consideration and with batch size considerations.

Both table 5 and fig. 17 indicate that, as compared to the optimal solution for the deterministic models, a stochastic flow pattern leads to higher values of minimum cost and variant layout assignments. Such a conclusion is augmented as the level of uncertainty in product flows increases.

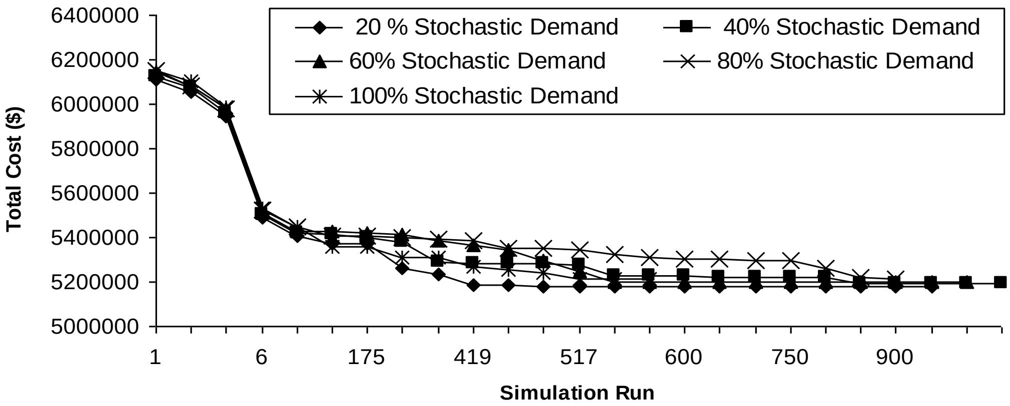

Fig. 15. Minimum total cost with the meta-heuristic search output using simulation under different demand scenarios for model (a).

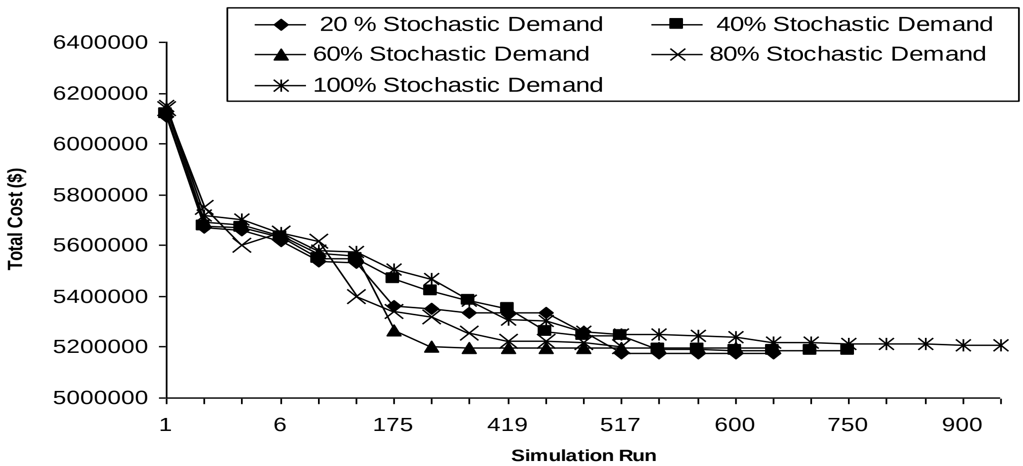

Fig. 16. Minimum total cost with the meta-heuristic search output using simulation under different demand scenarios for model (b).

Table 5.
Optimal assignment and the corresponding minimum total cost under different demand scenarios for both models (a and b)

	Under Deterministic Demand	20% Scenario	40% Scenario	60% Scenario	80% Scenario	100% Scenario
Model (a) - no batch size considerations						
Min total cost mean	5171754	5192400	5199328	5199300	5211000	5213900
Best found @ run #	530	465	891	546	754	564
Optimal assignment	31862 710954	263107 45189	641109 25387	641109 25387	254107 61389	24189 653107
Model (b) - with batch size considerations						
Min total cost mean	5166304	5174347	5183893	5195111	5202366	5207624
Best found @ run #	612	345	345	167	459	650
Optimal assignment	86317 210954	463109 25187	461107 25389	65110 94387	251107 46389	25189 643107

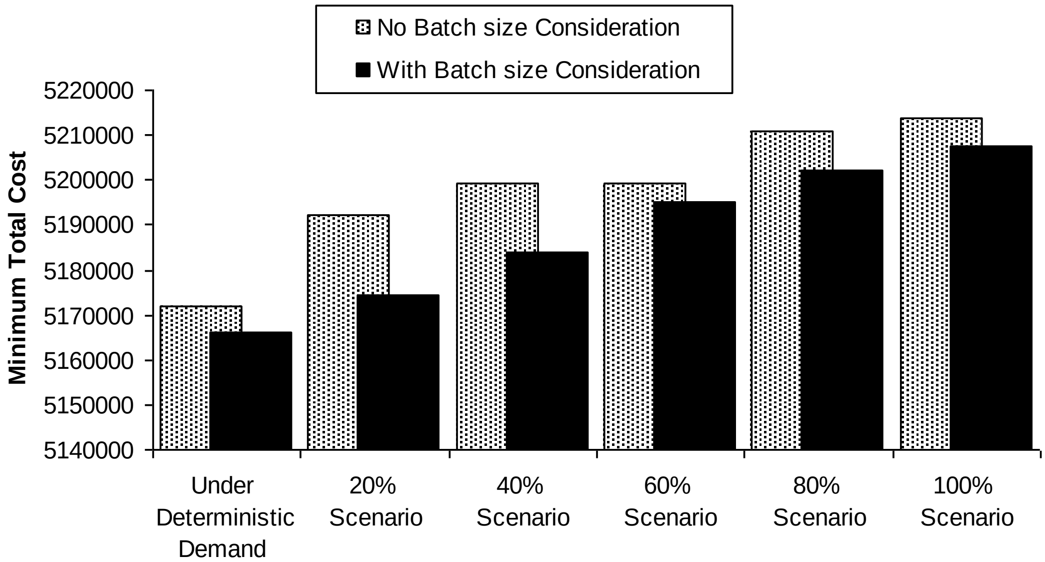

Fig. 17. Minimum total cost under different demand scenarios for both models (a and b).

5. Conclusions

Process oriented facility layout is widely used in many production environments due to its flexibility and suitability to the ever-changing nature of today's industry. However, assigning the manufacturing facilities to locations and minimizing its associated costs under the planned inter-facilities flow, is usually considered a hard to solve combinatorial optimization problem. With the case modeled as a quadratic assignment problem, many search techniques have been considered to tackle this problem, but they lacked the practicality and they suffer from the lengthy computational time and procedures. Meta-heuristics techniques combined with Monte Carlo simulation platform can significantly be used to overcome such drawbacks. Operational characteristics, as the spine driveway layout, could be easily incorporated into an Excel based model to replicate real life configurations. Dividing the product flows (demand) into batches is a second approach that would enhance the practicality of the models. Such an approach leads also to lower cost levels. As compared to the traditional optimization search strategies, such as;

Greedy, random and explicit enumeration techniques, the proposed meta-heuristic procedure (a combined Scatter and Tabu searches) proved its superiority and speed to reach an optimal solution. A further step towards more pragmatic treatments is to consider a stochastic demand (product flows) between facilities (manufacturing cells). To append such experiments to the proposed models, one may assume that product flows may occur in different scenarios (e.g. a 20% of the total flows are stochastic in nature with a normal distribution), 40%, 60%, 80% and 100%. The problem is treated using the same approach, as in the case of deterministic flows, and it shows that with the inflated degree of uncertainty, notably lower cost levels are achievable with batching considerations. Dependencies between product flows are common in any manufacturing facility due to shared or limited resources and other planning issues. Consequently, it is sensible to apply some sort of general correlation factors between product flows. Despite the fact of that such an assumption is difficult to calculate and implement in reality, however it is reasonable for the sake of experimentation. Different global correlation coefficients are assumed to study the aforementioned case; flows with No correlation, a correlation coefficient = 0.3, and 0.7). Experiments reveal that with increased altitudes of correlation the assignment cost increases. However, mean values of the costs are less for models that consider batch sizes as compared to those without batching. Further, the degree of uncertainty (i.e. risk) presented by the output cost standard deviations have escalated. Finally, the problem was solved under the stochastic flow considerations, presented with the different scenarios, using the meta-heuristic approach to get the optimal layout assignments. Once again, the proposed approach, models and procedures proved highly adequate to handle such type of difficult to resolve circumstances. For instance, one may consider the resulted layout assignment in high stochastic scenarios for heavy loaded, uncertain demand conditions where competitiveness is largely affected by those factors.

Appendix

Pseudo Code for the Greedy Search (GS) [31]

```
X = Generate Initial Feasible Solution
C(X) = Compute initial cost of X;
Continue = TRUE;
While (continue = TRUE)
    Transition = Select a transition from Neighborhood (X);
    X' = Apply Transition(X, Transition);
    ΔC = Compute Change in Cost (X, X', Transition);
    If (minimization problem and ΔC < 0)
        X = X';
        C(X) = C(X) + ΔC;
    Else
        Continue = FALSE;
    End If;
End while;
Output C(X);
End.
```

Pseudo Code for the Tabu Search (TS) [26]

```
X = Generate Initial Feasible Solution
C(X) = Compute initial cost of X;
Best_cost = C(X);
Initialize tabu list T = Ø;
While (stopping criterion not met)
      For (s Є N(X))
            X' = Apply Transition(X, s, Transition);
            ΔC = Compute Change in Cost (X, X', s);
      End For;
      While (suitable neighbor not found)
            s Є N(X);
            If (s does not belong to T)
                 X = Apply Transition(X, s, Transition);
                 T = T ∪ s;
                  C(X) = Compute Cost of X;
                  found suitable neighbor = TRUE;
            Else
                 If (aspiration(s) = TRUE)
                      X = Apply Transition(X, s, Transition);
                      C(X) = Compute Cost of X;
                      found suitable neighbor = TRUE;
                 End If;
             End If;
       End While;
       If (minimization problem and C(X) < Best_cost)
       Best_cost = C(X);
End While;
Output Best_cost;
End.
```

Pseudo code of an iterative meta-heuristic search algorithm [26]

```
X = Generate Initial Feasible Solution
C(X) = Compute initial cost of X;
While   stopping criterion not met)
      Transition = Select a Transition from Neighborhood (X);
     X' = Apply transition Operator(X, Transition);
     ΔC = Compute Change in Cost (X, X', Transition);
     If   (accept)
            X = X';
            C(X) = C(X) + ΔC;
     End If;
     If (minimization problem and C(X) < C_best)
          C_best = C (X);
End While;
Output C_best;
End.
```

A pseudo code of the Scatter Search [30]

```
Procedure Scatter Search;
        Begin
        Repeat
                Create Population (InitP op,InitP opSize);
                Repeat
                        Generate Reference Set (RefSet,RefSetSize);
                        Repeat
                                Select Subset (SubSet,SubSetSize);
                                Combine Solutions (SubSet, CurSol);
                                Improve Solution (CurSol, ImpSol);
                        Until (Stopping Criterion1);
                        Update Reference Set (RefSet);
                Until (Stopping Criterion2);
        Until (StoppingCriterion3);
End.
```

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