## New simplified equation for predicting ultimate strength of damaged tubular members

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New simplified equation is proposed to calculate the ultimate strength of damaged tubular structures subjected to combined axial compression and bending. The new equation allows the designers to take into account any type of damage such as perforation, dent or corrosion. Morever, the interaction between different damages may be considered. The ultimate strength of several tubular members, with different damages and slenderness ratios, subjected to combined axial compression and bending moment are determined based on the new equation and compared with those calculated using ABAQUS program. Results of analysis demonstrate considerable accuracy of the simplified equation and good agreements with those calculated using FEM.

تم اقتراح معادلة جديدة مبسطة لحساب المقاومة القصوي للمنشآت الأنبوبية ذات الأعطاب تحت تأثير حمل ضغط وثني. تسمح المعادلة الجديدة للمصممون أن يأخذوا في اعتبار هم أي نوع من الأعطاب مثل الثقوب، النقر أو التآكل. فوق ذلك ويمكن اخذ التداخل بين الأعطاب المختلفة في الاعتبار من خلال هذه المعادلة. المقاومة القصوي للأعضاء الأنبوبية وذات اعطاب ونحافات مختلفة معرضة لحمل ضغط وعزم ثني وتم حسابها باستخدام هذه المعادلة ومقارنتها بالنتائج المحسوبة باستخدام برنامج اباكوس. تنتبعة التحليلات توضح دقة معتبرة لنتائج المعادلة المبسطة وملائمة جيدة مع النتائج المحسوبة بواسطة طريقة العناصر المحددة.

**Keywords:** Ultimate strength, Damaged tubular members, Combined axial compression and bending, FEM.

#### 1. Introduction

Steel structures employed in offshore activity are usually constructed from tubular members. These structures are subjected to various types of loads. Besides the normal functional loads and environmental loads, loads due to accidents may occasionally act. These loads will cause several damages such as corrosion, cracks and/or dent. These damages range from total collapse of the structure to small damages which may not have serious effect at time of accident. Such small damages may later affect the ability of the structures to withstand extreme loads, thus having an influence on the safety of the structure in its functional time.

The assessment of structural integrity is carried out using two approaches. Firstly, maintenance of structural members through carrying out periodical in-service inspection and making the necessary maintenance for the intact and damaged structures. Secondly, the evaluation of the deteroration of the ultimate strength of damaged members which will be investigated in this research work.

Predicting ultimate strength of tubular members is not so easy because buckling and plasticity should be considered. Studies on the influence of damages on the ultimate strength have been performed in many literatures. In this regard, Rashed et al. [1-2] and Yao et al. [3] suggested theoretical equations of dented tubular members subjected to axial compression and bending. Ueda and Rashed [4] derived an ultimate strength interaction relationship of a dented cross section subjected to compression and bending in two perpendicular directions. Okada et al. [5] proposed a simplified equation for predicting deterioration of the ultimate strength of perforated tubular structures. In this approach the ultimate strength of perforated tubes is based on Carlsen' method. Daun and Chen [6] used moment-thrust curvature relationships for predicting ultimate strength and considering effect of local buckling on the strength of dented members. Since structural components are prone to corrosion damage due to environmental conditions, many approaches dealt with the assessment of the ultimate strength taking into account the

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degradation of structural members due to general corrosion [7-8]. However, these studies dealt with the damaged member as an individual damage deteriorating the ultimate strength. It is found that the interaction between different damages is necessary to predict more accurately the ultrimate strength since this effect will increase deterioration of the capacity of damaged members [9-10].

In this work, a new equation is proposed to calculate the ultimate strength of damaged members subjected to combined loads. It dealts with the formulation of the plastic condition of the damaged part and remainder of the tubular cross section seperately when the ultimate strength is calculated. The new equation allows the designers to take into account any type of damage such as perforation, dent or corrosion and the interaction among different damages. The ultimate strength of several tubular members, with different damages and slendreness ratios, subjected to combined axial compression and bending are evaluated using ABAQUS program and the new proposed equation. The performance of the new formulation is checked with those obtained using FEM and shows good agreement.

# 2. Ultimate strength interaction relationship of damaged tubular members

The total acting load, P on the tubular section is composed of two components.  $P_1$  is the acting load after yielding of the damaged part and  $P_2$  is the load after yielding of the rest of the cross section. These loads are calculated as follows,

1-Yielding of damaged part: When a tubular member subjected to compression and bending, the fibers in the damaged zone may be subjected to compressive stresses. In the elastic range the deformation of damaged zone is small and no interaction between the rest of the cross section of the tube. Then, the damaged zone may be separated from the undamaged part. Yielding starts at the



Fig. 1. Acting loads on damaged member.

damaged area where maximum stresses are developed due to compression and bending as shown in fig. 1. Yielding continues until plastic hinges are formed. At this stage the acting compressive load on the tube is,

$$P_1 = \sigma_i A_i \quad . \tag{1}$$

Where  $\sigma_i$  is the initial stress at the damaged zone.

2- Ultimate strength stage: After plastifiaction has occurred at the damged region, eccecntricity due to an additional acting load  $P_2$  starts developing overall bending moment acting on the tube. The lateral deflection is substaionally increased and yielding at the adjacent region to the damaged part starts and tube stifness decreases rapidly. When the acting load  $P_2$  reaches its maximum value and the bending moment is increased, the ultimate strength is attained. The developed moment Mdue to additional load  $P_2$  is expressed as follows [11].

$$M = P_2 w = P_2 \sum_{i=1}^{2} w_{oi} \sin(i\pi c/L) / \{1 - (1/i^2)(P_u/P_e)\}.$$
 (2)

Where

- $P_2$  is the  $P_u P_1$ ,  $P_u$  is the ultimate compressive load,
- $T_u$  is the utilitate compressive load,
- $P_e$  is the Euler buckling load= $E\pi^2 I/L^2$ ,
- $w_{oi}$  is the  $(e_p+w_o)A_i$ ,
- $e_p$  is the  $D \sin \alpha/(\pi \alpha \zeta_2)$ ,

 $A_i$  is the coefficient given in ref. [11], and  $\alpha$  an  $\zeta_I$  are the defined in table 1.

The maximum moment ocurrs when coefficient  $w_{oi}$  of eq. (2) is maximum [9]. Hence, M may be expressed as follows.

$$M = P_2 \,\delta_2 + P_1 \,\delta_1 \qquad (3)$$

Where

$$\delta_2 = (e_p + w_o) \phi, \quad \delta_1 = w_o \phi,$$

 $\phi = 1/(1-p_u/p_e).$ 

Based on the above assumption and when the fully plastic stress distribution on the cross section of the tubular member is as shown in fig. 2, the ultimate strength interaction equation may be expressed as follows;

$$\{M_u + P_u (w_{o+} e_p) + P_1 w_{o}\}\phi / M_p' + \zeta_1 p_u / p_p = 1, (4)$$

where,

$$M_{p}' = M_{p} \{ \zeta_{3} \zeta_{2} (2 \ \sigma_{i} / \sigma_{o}) \sin \alpha + \sin (\frac{1}{2} \ \pi [P_{T} - P_{u}] / P_{p} + \sin \alpha [\frac{1}{2} \ \zeta_{2} \ \zeta_{4} \ -1] \}.$$
(5)

From eqs. (4) and (5), the ultimate moment  $M_u$  is expressed as follows

$$M_u = \{1 - \zeta_1 p_u / p_o\} M_p' \phi - P_2 (w_{0+} e_p) - P_1 w_o.$$
 (6)

Where,

$$P_T = \sigma_0 Dt (\pi - \alpha \zeta_2) + \sigma_1 Dt \alpha \zeta_2.$$
$$M_p = \sigma_0 D^2 t, P_p = \sigma_0 \pi D t.$$

Finally, for damaged tubes subjected to only compression, the ultimate compressive load could be obtained by solving eq. (6). In order to apply eq. (6), the following parameters in table 1 are considered.



Fig. 2. Stress distribution over the cross section of damaged tubular member.

Parameter	Perforation	Dent	Corrosion	Dent with corrosion	
$\sigma_{\overline{i}}$	$\sigma_{o}$	$\sigma_0 \sqrt{4(\mu/t)^2 + 1} - 2\mu/t$	$\sigma_o t_c/t$	$\sigma_o \sqrt{4(\mu/t_c)^2 + 1} - 2\mu/t_c$	
$P_1$	$\sigma_{o} \ lpha Dt$	$\sigma_{i}$ ( $\pi$ - $\alpha$ ) $Dt$	$\sigma_0 Dt \left(\pi - \alpha t_c / t\right)$	σ <sub>i</sub> (π-α) Dt	
α	$\sin^{-1}(a/D)$	$\cos^{-1}(1-d/D)$	α	$\cos^{-1}(1-d/D)$	
$\zeta_1$	1.0	0.0	1.0	0.0	
ζ2	1.0	1.0	1.0	$t_c/t$	
ζ3	0.0	1.0	0.0	1.0	
ζ4	0.0	1.0	1.0	1.0	

Table 1 Parameters of the interaction equation

where,

 $\mu = D \left( \sin \alpha / \alpha - \cos \alpha \right) / 2$ 

d = depth of the dent

a = diameter of perforation

t =shell thickness

 $t_c$  = shell thickness after corrosion

 $\alpha$  = damaged angle on the tubular cross section.

#### Table 2

Properties of different tubular member models

	D/t	$\sigma_{o}$	Е	λ	L
		(N/mm <sup>2</sup> )	(N/mm <sup>2</sup> )		Mm
a- Perforated tube					
				0.52	8000
$K_{\rm s} = 0.10$	50	390	207000	1.0	16000
Ks = 0.20				1.6	24000
				2.0	30000
b- Dented tube					
<i>d/D</i> =0.05				0.75	2500
d/D = 0.1	50	360	207000	1.0	3400
				1.5	5000
				2.0	6600
c- Corroded tube					
$\gamma = 0.1$				1.0	3400
$\gamma = 0.5$	50	360	207000	1.5	5000
				2.0	6600
d- Dented-corroded tube					
$d/D = 0.05$ and $\gamma = 0.1$	50	360	207000	1.0	3400
$d/D = 0.1$ and $\gamma = 0.2$				1.5	5000
				2.0	6600

Where,

 $K_s = a/\pi D$ 

 $\gamma = \alpha / \pi$ 

a and D are the hole and tube diameters, respectively.

#### **3. Applications**

#### 3.1. Numerical models

Several damaged tubes with different geometrical properties are analyzed using the non-linear finite element program ABAQUS. Elastic-perfect plastic material properties is employed in the analysis. In addition, S4R5 element is used to model the different cases and the modified risk technique is employed in the solution. Amplitude of initial deflection of value equal L/1000 is assumed. The tubes are assumed to be simply supported at both ends and subjected to combined compression and bending. The geometrical and material properties of the damaged tubes are given in table 2. Results of FEM analyses are compared with results based on using the new eq. (6).

#### 3.2. Discussion of results of analyses

Series of analyses are performed to examine the effects of different parameters on the interaction curve of the ultimate strength of damaged tubular members.

#### 3.2.1.Ultimate strength of perforated tube

Figs. 3-a and 3-b illustrate ultimate strength interaction curves for perforated tubes subjected to combined compression and bending loads. It is observed from these figures that ultimate strength remarkably decreases as the perforation size and the slenderness ratio increases, as expected. Also, it is shown that the pesent equation simulates the behavior of the perforated tubes similar to that based on FEM.

#### *3.2.2. Ultimate strength of dented tubes*

Figs. 4-a and 4-b give the ultimate strength interaction curves for dented tubes, two different depths of d/D, subjected to combined compression and bending loads. The most important effect on the deterioration of the ultimate strength is the size of the dent and the tube slenderness ratio. Both compressive and bending ultimate strengths remarkably decrease as the size of dent increases and the deterioration in the ultimate strengths is increased as the slenderness ratio increases. In addition, it is seen that the results based on the present equation have a good agreement for most cases with those obtained using FEM.

#### 3.2.3. Ultimate strength of corroded tubes

Figs. 5-a and 5-b illustrate the ultimate strength of tubular members having corroded part and subjected to combined compression and bending. The corrosion depth is assumed to be half the thickness of the pipe and extends a distance equal the pipe diameter in the lengthwise direction. The deterioration of the ultimate strength due to corrosion is investigated through changing the circumferential length of corroded part an angle 2  $\alpha$  in the circumferential direction as shown in fig. 2. Two different magnitudes for both slenderness ratio  $\lambda$  and  $\gamma$ , such that  $\lambda = 1.0$  and 2.0 and  $\gamma = \alpha/\pi = 0.1$  and 0.5, are adopted to

investigate the variation in the ultimate strength. Both compressive and bending ultimate strengths remarkably decrease as ratio  $\gamma$  and slenderness ratio increase as shown in fig. 5. Fig. 6 shows relationships between ultimate compressive strength of tubes with the slenderness ratios for three different values of  $\gamma$  angle of the circumferential extension of the corroded part. It is seen from this figure the tendency of the deterioration of the ultimate compressive strength due to corrosion increases as  $\gamma$  increases and slenderness ratio  $\lambda$  decreases. As the slenderness ratio increases above 1.5, the deterioration of the ultimate compressive strength due to corrosion becomes insignificant compared with buckling of pipe effect. Moreover, one can notice from figs. 5 and 6 that the present equation simulates behavior of corroded tubes similar to those obtained using FEM.

### 3.2.4. Ultimate strength of dented and corroded tubes

This section investigates ultimate strength interaction curves of tubular members with combined denting and corrosion. The models of combined denting and corrosion have dent values of d/D=0.05 and 0.1 with the same corroded part condition described previously, such that  $\gamma = \alpha/\pi = 0.1$  and 0.2, and slenderness ratios  $\lambda$  equal 1.0 and 2.0.

Figs. 7-a and 7-b demonstrate effect of combined denting and corrosion on the deterioration of the ultimate strength. It is shown from fig. 7 that both the compressive and the bending ultimate strengths decrease when d/D and  $\gamma$  increase. It is found that the interaction between different damages is necessary to predict the deterioration of the ultimate strength. Also, the interaction of damages is calrified in fig. 8 when investigating variations of ultimate compressive strength deterioration with tube slenderness ratios. In addition, fig. 9 shows a comparison between the ultimate strength interaction curves for dented and dented corroded tubes based on the new equation and those obtained by FEM cases. Good agreement between the predicted results from the proposed equation can be noticed from these figures.



Fig. 3. Ultimate strength interaction curves for perforated tubes.



Fig. 4. Ultimate strength interaction curves for dented tubes.



Fig. 5. Ultimate strength interaction curves for corroded tubes.



Fig. 6. Relationship between ultimate compressive strength of tubes with slenderness ratios for different values of  $\gamma$ .



Fig. 7. Ultimate strength interaction curves for dented-corroded tubes.



Fig. 8. Relationship between ultimate compressive strength of tubes with slenderness ratios for different d/D and  $\gamma$ .

#### 4. Conclusions

From the present study, the following conclusions can be drawn:

1. A new equation is proposed to calculate the ultimate strength of tubes with different damages such as perforation, dent and/or corrosion.

2. Good agreement between the results calculated by the proposed equation and those obtained using FEM is confirmed for the considered cases.

3. The proposed equation offers the designers a good means to evaluate the effect of the interaction of different damages on the ultimate strength of tubular members subjected to combined axial compression and bending moment with acceptable accuracy.

4. The ultimate compressive strength of damaged tubes may be evaluated with acceptable accuracy. However, small differences are observed in the results of cases of damaged tubes having lower slenderness ratio.

#### References

 J. Taby, T. Moan and S.M.H. Rashed, "Theoretical and Experimental Study of the Behavior of Damaged Tubular Members in Offshore Structures",



Fig. 9. Comparison between ultimate strength interaction curves of dented and dented-corroded tubes for  $\lambda = 1$ 

Norwegian Maritime Research, Vol. 9 (2), (1981).

- [2] S.M.H. Rashed, "Behavior to Ultimate Strength Damaged of Tubular Offshore Structures by the Idealized Structural Unit Method", Report SK/R51, Division of Marine Structures, Norwegian Institute of Technology (1980).
- [3] T. Yao, J. Taby and T. Moan, "Ultimate Strength and Post Ultimate Strength Behavior of Damaged Tubular Members in Offshore Structures", OMAE, pp. 301-308 (1986).
- [4] Y. Ueda and S.M.H. Rashed, "Behavior of Damaged Tubular Members", OMAE, pp 528-536 (1985).
- [5] H. Okada and et al., "A Simplified Method for Estimating Deterioration of Collapse Strength of Damaged Jacket Structures Loads", Proc., 20<sup>th</sup> Int. Conference on OMAE (OMAE01), Paper No. OMAE01/S&R-2156, (CD-ROM) (2001).
- [6] L. Duan, W.F. Chen and J.T. Loh, "Analysis of Dented Tubular Members Using Moment Curvature", Thin-Walled Structures, Vol. 15, pp. 15-41 (1993).
- [7] C.G. Soares and Y. Gabatov, "Reliability of Maintained Ship Hulls Subjected to Corrosion", Journal of Ship Research, Vol. 40, (3), pp. 235-243 (1996).

- [8] B. Talei-Faz, F.P. Brennan and W.D. Dover, "Residual Static Strength of High Strength Steel Cracked tubular Joints.", Marine Structures, Vol. 17, Issue 3, pp. 291-309 (2004).
- [9] Y.A. Abdel-Nasser, K. Masaoka and H. Okada, "Ultimate Strength of Perforated Tubular Members Subjected to Compression and Bending" Alexandria Engineering Journal, Vol. 45, (1), pp. 27-36 (2006).
- [10] Y.A. Abdel-Nasser, K. Masaoka and H. Okada, "Ultimate Strength of Dented

Tubular Members Subjected to Compression and Bending", Alexandria Engineering Journal, Vol. 45 (1), pp. 37-46 (2006).

[11] K. Masaoka, A Non-Linear Finite Element System program, ULSTRUCT-Code, Osaka Perfecture University, Marine System Engineering Department, (2004).

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