Delay assignment and optimal rate allocation for OFDMA based cellular networks

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In this paper, we derive an analytical model for the delay bound of the down link (base station to mobiles), in a single cell Orthogonal Frequency Division Multiple Access (OFDMA) cellular system. The delay bound is exploited in formulating a dynamic subcarrier assignment strategy that equalizes both the queuing, and the channel throughput performances in response to the time-varying nature of frequency selective fading channels. Optimal subcarrier's transmission power allocation is found by solving a constrained optimum rate allocation problem. Results have indicated that the proposed strategies exploit the time varying channel conditions in an efficient way, and have the prospects to improve the queuing delay while maximizing the data transmission rates in real life networks.
يقدم هذا البحث نظام ديناميكى لتخصيص عدد الموجات المستخدمة فى نظام الموجات الحاملة المتعامدة. كذلك قدم البحث مود يل لتعظيم كفاءة تخصيص معدلات الارسال و القدرة المستخدمة لتحقيق جودة خدمة معينة. اظهرت النتائج فاعلية النظم المقترحة فى التعامل مع التغيرات التى تطرأ على قنوات التراسل اللاسلكية مما ادى الى تحسين الأداء فى هذا النظام.

Keywords: Delay Assignment (DA), Delay Bound for Reverse Link (DBRL), OFDMA, Cellular Networks (CN), Optimal Rate Adaptation (ORA), Power Allocation (PA)

1. Introduction

Joint subcarrier and power allocations in Orthogonal Frequency Division Multiple Access (OFDMA) networks is a complex problem [1]. Usually, the problem is simplified by separating subcarrier allocation and power allocation [2]. Subcarrier allocation provides more capacity gain than does power allocation [3, 4]. However, the overall capacity achieved by these subcarrier allocation algorithms that are solely based on channel conditions does not necessarily translates into throughput gain especially when the input traffic is bursty in nature. For example, [5] shows that some packet scheduling schemes can lead to unstable queues even for low packet arrival rates, since it does not utilize the buffer state in its allocation systems. Buffering of bursty traffic can take advantage of multi-user diversity across time, and improves throughput [6]. Queuing delay analysis for the IEEE 802. 16 networks was conducted in [3, 4]. A vacation queuing model was adopted in [7] to analyze the queuing performance of OFDMA-TDMA systems. Although previous work in [3, 4] examined different scenarios by adopting the average queuing delay as the performance measure, there has been little work on other relevant performance measures such as the delay bound and the delay violation probability, which are indicative measures of the worst-case delay behavior. In particular, in an OFDMA network, due to the time-varying channel conditions, only a fraction of transmitted packets is accepted at the receiver as received correctly (i.e., without or with an acceptable number of bits in errors). Packets that are received with an unacceptable number of errors are retransmitted. Due to retransmissions, and even, multiple of retransmissions in many cases, temporary congestion, and consequently packet delays and buffer overflows tend to occur.

In this paper, we derive an analytical model for the delay bound for the reverse (down) link from base station to mobiles in a single cell of an OFDMA cellular network (section 2). The derived delay-bound is used in formulating a dynamic subcarrier assignment strategy (section 4). In order to support traffic loads with service rates that are very close to the maximum channel capacity. We solved a constrained rate optimization problem (section

5). This way, the proposed dynamic subcarrier assignment; optimal rate and power strategies allocate users for each subcarrier based on a combination of the current estimate of the delay bound and the time varying state of the channel conditions. Numerical results are presented in section 6.

2. Delay bound model

Assume that a traffic stream arrives at the First-In-First-Out (FIFO) buffer shown in fig. 1 below.
If the time-varying server process, $U(t)$, is kept to meet an average queuing delay of $\overline{D}$. Then, the maximum delay of any data bit passing through the buffer is upper bounded by [10].

$$\overline{D} < \frac{L_k}{U(t)}, \quad (1)$$

where L_k is the length in bits of packet k.

For purpose of introducing to the delay bound modeling and analysis, let us consider the following "introductory" example.

Assume an OFDMA environment. Here, the server process in fig. 1, is a discrete-time random process $U(n)$, where n is the OFDM symbol index. Let $U(n)$ refers to the number of bits/symbol. Therefore, if a user k is assigned $J(k)$ subcarriers, then $\sum_{n \in J(k)} q_{k,n}$ bits can be served, where $q_{k,n}$ denotes the number of bits/symbol allocated to subcarrier n. For a Rayleigh fading channel, the maximum number of bits in a symbol to be transmitted for the k^{th} user's n^{th} subcarrier is [9].

$$q_{k,n} = \log_2(1+\frac{\gamma_{k,n}}{\Gamma}) , \quad (2)$$

where,

$\gamma_{k,n}$ is the Signal-to-Noise Ratio (SNR) for k^{th} user's n^{th} subcarrier signal,

$$\gamma_{k,n} = \frac{p_{k,n} \ |h_{k,n}|^2}{\sigma^2} = p_{k,n} \ H_{k,n}. \quad (3)$$

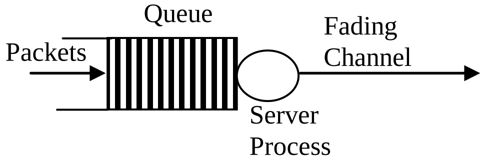

Fig. 1. server model.

$\Gamma = -\ln(5BER)/1.5$,

$p_{k,n}$ is the transmission power allocated to k^{th} user's n^{th} subcarrier,

$h_{k,n}$ is the random variable representing the channel fading factor for n^{th}, subcarrier between base station and k^{th} user's receiver,

$\sigma^2 = N_0 \frac{B}{J(k)}$ is the variance of Additive White Gaussian Noise (AWGN),

N_0 is the noise power spectral density,

B is the available bandwidth,

$J(k)$ is the number of subcarriers allocated to user k, and

$H_{k,n}$ is the Channel-to-Noise Ratio (CNR) for the k^{th} user's n^{th} subcarrier.

This ends the "introductory" example.

We are now, ready to derive the delay bound defined by eq. (1). In an OFDMA network with K users, and N total number of subcarriers. Randomly arriving packets are buffered in the FIFO buffer of user k. The buffer size is determined by the absolute delay bound for the corresponding user. Since channel conditions are time varying, we assume that the base station transmitter has knowledge of the channels of all users. This information is assumed to be updated periodically with the help of a feedback channel. From the "introductory" example, the maximum achievable rate for user k is.

$$U(t) = \sum_{n \in J(k)} q_{k,n} \text{ bits/symbol}. \quad (4)$$

Substituting eq. (4) into eq. (1) gives

$$\overline{D} < \frac{L_k}{\sum_{n \in J(k)} q_{k,n}} . \quad (5)$$

Of importance here to note that with L_k as the length in bits of packet k, $U(t)$ in bits per symbol. Then $\overline{D}$ here refers to the buffer (occupancy) length i.e., number of OFDM symbols inqueued in the buffer. Furthermore, in order to account for some burstiness degree, $\delta = 0, 1, 2, \ldots$, in packet lengths. We assume that the number of bits in the packets, L_k, is Poisson distributed with mean m bits/packet, therefore we can replace L_k by $\tilde{L}_k$ where.

$$\tilde{L}_k = m + \sqrt{\delta . m} \; . \tag{6}$$

In next section, we find the buffer occupancy bound in terms of the probability of bits in error.

3. Server rate

Consider the queuing model depicted in fig. 1. The throughput of this queue can be characterized by the probability of bits in error. Due to retransmissions, the actual throughput changes with time depending on the (fading, shadowing) channel conditions. assume that M-QAM modulation method is being utilized. The probability of bit in error can be expressed as [10].

$$P_b = C_1 \exp\left(\frac{C_2 \, p_{k,n} H_{k,n}}{2^{q_{k,n}} - 1}\right), \tag{7}$$

where $C_1 = 0.2$, $C_2 = 1.5$.

Consequently, the maximum achievable rate for the traffic backlogged at user's k-queue is.

$$U(t) = \Big(\sum_{n \in J(k)} q_{k,n}\Big)(1 - P_b) \;, \tag{8}$$

combining eqs. (5-6), and eq. (8) gives the upper delay bound.

$$\overline{D} < \frac{\tilde{L}_k}{\Big(\sum_{n \in J(k)} q_{k,n}\Big)(1 - P_b)} . \tag{9}$$

In next section, we propose a dynamic subcarrier assignment strategy based on the delay bound in eq. (9).

4. Dynamic subcarrier assignment

As mentioned early, subcarrier allocation strategies that are solely based on channel conditions does not necessarily translates into throughput gain, especially when the input traffic is bursty in nature. From eqs. (9), (7), it can be seen that the channel conditions, $H_{k,n}$, is embedded in the delay bound model. This, therefore, prompts us to formulate the following dynamic subcarrier allocation strategy aiming to equalize the user's delay, and throughput performances in response to varying traffic loads as well as varying channel conditions.

Assume that the number of active mobile users (i.e., having data to receive from base station) is J_a. The dynamic subcarrier allocation strategy compares the obtained delay bound estimates for the J_a users, and calculates the amount of subcarriers $J(k)$ to be assigned to user k as follows.

$$J(k) = 1 + Int\left[N \frac{\tilde{L}_k}{\sum_{\forall k \in J_a} \tilde{L}_k} + 0.5 \right], \tag{10}$$

where Int. stands for integer. After subcarriers are being allocated, still we need to assign their powers but in an optimal way. This is carried out in next section.

5. Optimum power allocation

The optimal power allocation per subcarrier is achieved by optimizing the bit allocation in eq. (2) under constrained total power per user k:

$$\text{Maximize} \sum_{n \in J(k)} log_2\Big(1 + \frac{\gamma_{k,n}}{\Gamma}\Big) \,,$$

$$\text{subject to} \sum_{n \in J(k)} p_{k,n} \le P_k \; .$$

This problem can be solved by writing the following Lagrangian cost function,

$$\Phi = \Big(\sum_{n \in J(k)} log(1 + \frac{\gamma_{k,n}}{\Gamma}) \Big) + \lambda \Big(\sum_{n \in J(k)} p_{k,n} - P_k \Big), \tag{11}$$

where λ denotes the Lagrange multiplier. Optimal solution to eq. (11) can be obtained from the following set of necessary conditions,

$$\frac{\partial \Phi}{\gamma_{k,n}} = 0 \qquad \forall\, n \in J(k).$$

Which yields the following iterative equations,

$$p_{k,n} = p_{k,m} + \Gamma \frac{H_{k,n} - H_{k,m}}{H_{k,n} H_{k,m}} \quad \forall \{n, m\} \in J(k). \tag{12}$$

To find the initial power allocation, let m=1, and substitute for all n into the total power constraints, we obtain.

$$p_{k,1} = \frac{1}{j(k)} \Big(P_k - \Gamma \sum_{n=2}^{J(k)} \frac{H_{k,n} - H_{k,1}}{H_{k,n} H_{k,1}} \Big). \tag{13}$$

Having obtained $p_{k,1}$, the power for other subcarriers can be calculated using eq. (12). Next section presents numerical evaluation results.

6. Numerical evaluations

To investigate the performance of the strategies presented in this paper. A down link of an OFDMA network is modeled with the following characteristics: each subcarrier observes a frequency selective multipath fading with zero mean Gaussian variable in the logarithmic scale and maximum delay spread of 5 microseconds. Shadowing effect is simulated with standard deviation of 8 *dB*. The path loss $h_{k,n}$ for the n^{th} subcarrier of user k is the product of the $\alpha_{k,n}$ factor, and d_k^{-4}, where d_k is the distance between the transmitter, and receiver of link *k*, and α_k is the Rayleigh fading factor with a Chi-square probability density function. The power spectral density of the AWGN is –80 dB [12].

Fig. 2, compares the analytical delay (i.e., buffer occupancy)bound (solid line) derived in section 3, with a sample path of the channel gain (short dashes). As can be seen, buffer occupancy increases at subcarriers (e.g., #40) experiencing deep fade which imply large number of backlogged (symbol) traffic. However, at other locations, users may experience better channel conditions (e.g., #140) and hence, their estimated buffer occupancy is much less. Another aspect shown in fig. 2 is the bit allocation (long dashes). As can be seen, bit allocations adapt smoothly with respect to the channel gains observed by each subcarrier.

Figs. 3-4 show the case of constant power per subcarrier assignment at BER=1.E-4. As can be seen in fig. 3, increasing the transmission power reduces the buffer occupancy as a result of improving the SNR, which, in turn, leads to improving the bit allocations as seen in fig. 4. The conclusion up to this point is that per subcarrier power.
control is an efficient technique to combat the time varying wireless channel impairments. The major problem with this approach is that it reduces the frequency reuse property of the OFDMA system. It is exactly here, where adaptive (subcarriers, power, bits) techniques come into play.

Figs. 5-7 present numerical results for the dynamic subcarrier, delay assignment, with and without optimal power allocation.

On one hand, we see that optimal power assignment optimizes the SNR, hence improves the overall performance of the OFDMA system in terms of the buffer occupancy in fig. 5, bits/symbol allocation in fig. 6, as well as the achieved BER in fig. 7. On the other hand, the dynamic delay, and subcarrier assignments exploit the states of the channel gains to.

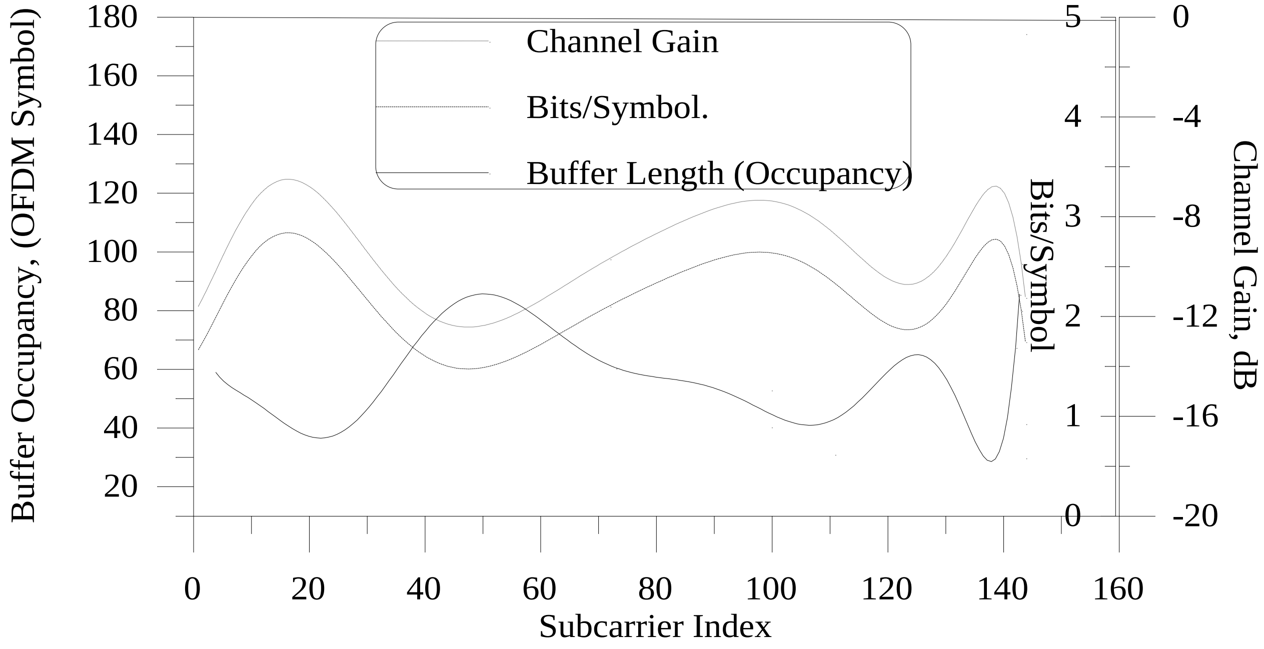

Fig. 2. Performance of analytical delay occupancy as a result of improving the.

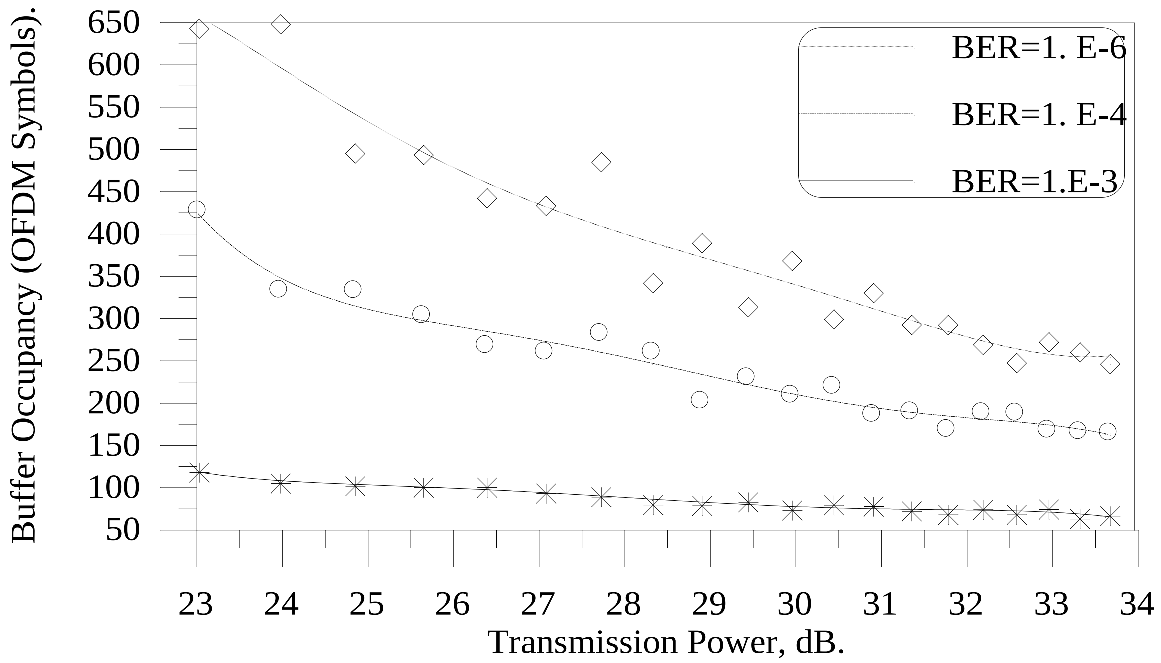

Fig. 3. Effect of transmission power, and BER on the analytical buffer occupancy performance.

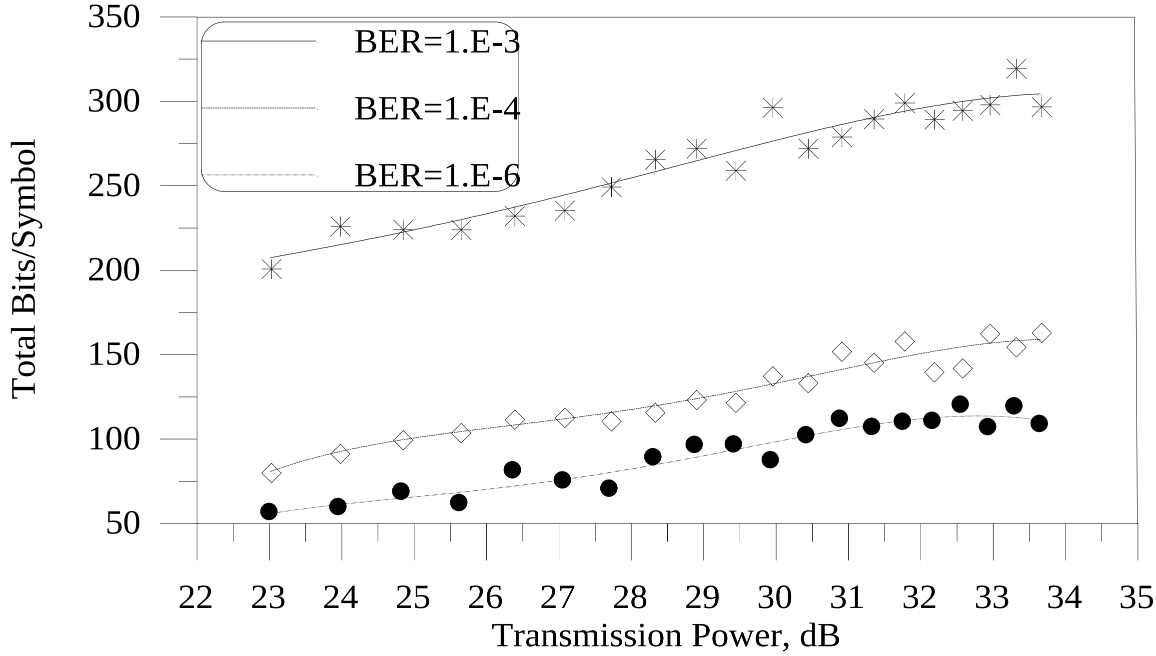

Fig. 4. Effect of transmission power, and BER on bit allocation.

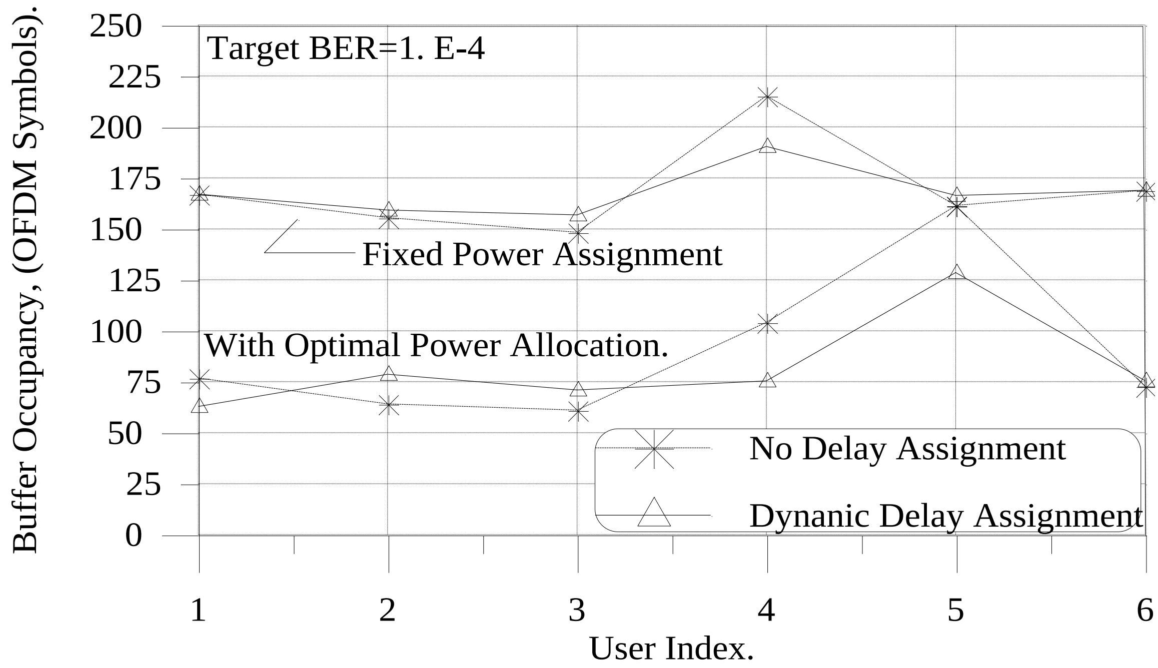

Fig. 5. Delay assignment and optimal power control.

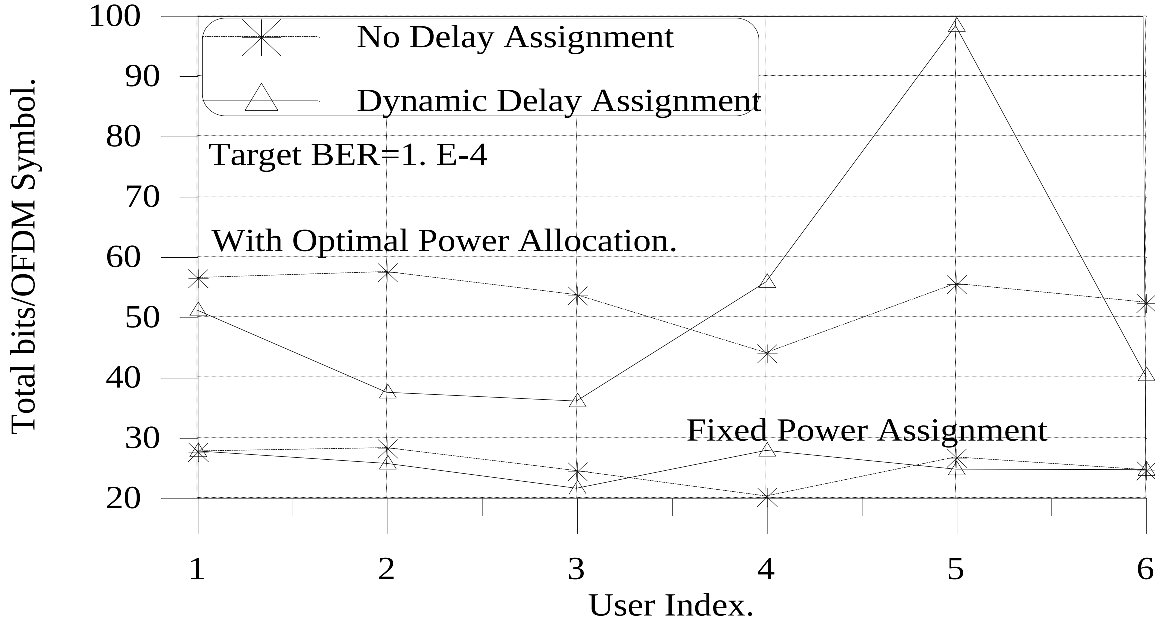

Fig. 6. Delay assignment, and optimal power control.

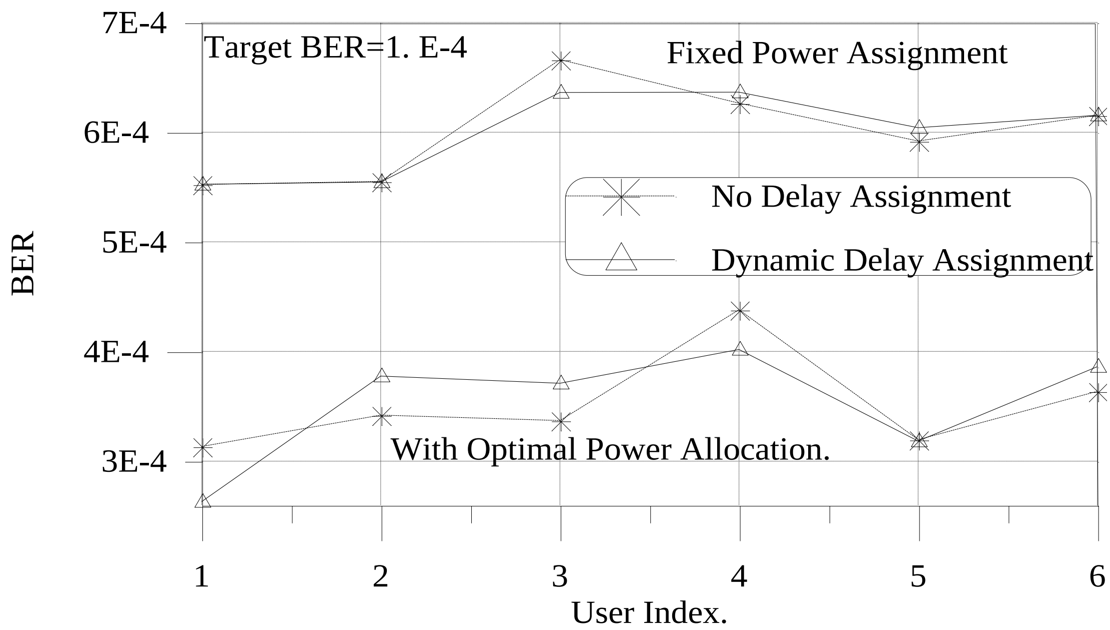

Fig. 7. Delay assignment, and optimal power control.

Table 1

User No.	1	2	3	4	5	6
Fixed subc. assignment	24	24	24	24	24	24
Dynamic assignment	20	17	16	28	43	20

From table 1, we can see that user #5, for example, is observing a deep fading channel. Under fixed subcarriers assignment. He is assigned 24 subcarriers. This have led to a high delay(i.e.,buffer occupancy) fig. 5, less bits/symbol fig. 6. However, under the dynamic subcarriers assignment, he is assigned 43 subcarriers table 1 which have led to reduced buffer occupancy fig. 5, much better bits/ symbol allocation fig. 6. By comparing this case (i.e., user #5), with the case of, for example, user #2 who happened to be experiencing better channel gains. We can easily realize that the strategies proposed in this paper allocate the number of subcarriers, transmission power, and bits/symbol to the user(s) who needs it most at the expense of slight reduction in the performance of other users who observe better channel conditions.

7. Summary and conclusions

In this paper, we derived an analytical model for the delay bound of the down link in a single cell of an OFDMA cellular network. A dynamic delay assignment is formulated, and used to assign the number of subcarriers per user in response to the time varying channel gains. Optimal power assignment per subcarrier is found by solving a constrained optimal rate allocation problem. Numerical results have shown that our delay bound model encompasses the time varying nature of the fading, shadowing channel conditions. This means that, our dynamic subcarrier assignment strategy can efficiently allocate the subcarriers, transmission powers as well as the bits/symbol to the user(s) who needs it most at the expense of slightly degrading the performances of other users who are seen to be experiencing better channel conditions.

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