Ultimate strength of perforated tubular structures subjected to compression and bending

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Tubular structures are used extensively in offshore structures, which are installed to facilitate offshore oil and gas production. Some members subjected to damage in the form of perforation due to environmental loading conditions. Perforated damage leads to deterioration of ultimate strength and shortening of life-time of the tubular members. The aim of this paper is to demonstrate and develop the ultimate strength analysis of tubular structures with perforated damages and subjected to combined compression and bending. The ultimate strength is calculated using a non-linear finite element system and a new simplified equation. In order to investigate a general applicability of the present equation, cylindrical tubes with different perforated damages and subjected to combined loads are analyzed. Results of analysis demonstrate considerable accuracy of the new equation and good agreements with those calculated by non-linear finite element method.

تستحدم المنشآت الأنبوبية بكثرة فى منشآت ما وراء الشواطئ للتنقيب واستخراج الغاز والبترول. بعض هذه المنشآت البترولية معرضة للتآكل والثقوب نتيجة تعرضها للأحمال البيئية. وجود مثل هذه الثقوب سوف يؤدى إلى تقليل المقاومة القصوى وكذلك طول عمر هذه المنشآت الأنبوبية. فى هذا البحث تم عرض وتحليل المقاومة القصوى لهذه المنشآت الأنبوبية المثقوبة عند تعرضها لحمل ضغط وانحناء معا. تم اقتراح معادلة بسيطة لحساب هذه المقاومة لعدة منشآت أنبوبية ذات خواص مختلفة وقورنت النتائج بمثيلاتها بطريقة الوحدة المنتهية وكانت نتيجة المقارنة جيدة.

Keywords: Ultimate strength, Cylindrical tubes, Combined compression and bending, FEM, Perforated damage

1. Introduction

Tubular members of jacket structures such as braces and columns are subjected to severe environmental and loading conditions. These tubular structures may contain large cracks and corrosion due to extreme loading conditions. Very little information is available about the residual strength of these damaged tubular structures [1, 2]. It becomes important to estimate the residual strength of these members with a higher degree of safety because of the deterioration of their ultimate strength and shortening of their lifetime. However, predicting ultimate strength of cylindrical tubes is not so easy because the effects of buckling and plasticity should be considered. This paper deals with the ultimate strength of cylindrical tubes with perforated damages throughout the tube length and subjected to combined compression and bending. Using the non-linear Finite Element Method (FEM) was time-consuming in the past. However, due to improvement of computer speed and capacity, it is easy to calculate the ultimate strength in a few minutes by using FEM [3]. In this work, the ultimate strength is calculated using simplified equations and the non-linear finite element method. Okada. et al. [1] developed a simplified equation by applying the Carlsen's method. The simplified equation had shown good agreements with FEM analysis for perforated tube subjected to axial compression [4]. However, it is seen that the simplified equation predicted higher ultimate strength than those analyzed by FEM for perforated tubes subjected to combined compression and bending loads. The differences between results become large for cases of cylindrical tubes with higher ratio of D/t and large perforation sizes. Inaccuracy of the results has been investigated and a new equation is suggested to take into account this effect in the ultimate strength calculation. In order to investigate the applicability of the present equation, cylindrical tubes with different geometrical properties and different degree of perforated damages are analyzed. Results of analysis are presented and compared with those calculated by non-linear finite element method. The comparison shows good agreements.

2. Prediction of ultimate strength using non-linear finite element system

Different numerical methods are available to analyze the behavior of structures until their ultimate strength is reached. The nonlinear Finite Element Method (FEM) is the most powerful method for analyzing complicated behavior until ultimate strength and it has a wide application in various structural fields. In this study, a non-linear finite element system is applied for calculating the ultimate strength of a cylindrical tube. Four-node isoparametric shell elements are used [5]. The developed finite element system is constructed by using the principle of virtual work for each incremental step and taking into account both geometrical and material nonlinearities of the shell elements. It is capable to analyze elastic- plastic behavior until and beyond the ultimate strength state. The Newton-Raphson method and the arc-length method are used to get converged solutions for each incremental step. The element can be used to accurately analyze a plate with an initial deflection and residual stresses. Different boundary and loading conditions can also be considered. Based on this system a source code (ULTSTRUCT) is used in the following analyses [6].

2.1. Boundary conditions and FEM modeling

Fig. 1 shows the assumed boundary and loading conditions of a cylindrical tube subjected to combined uniaxial compression and bending loads. Simply supported hinges are assumed at both ends of the tube and a perforated damage is located at the mid-length of the tube. An initial deflection of a simple pattern is assumed along the entire length of the tube.

Fig. 2 shows the FEM model of the cylindrical tube. Fine mesh divisions are applied near the perforated area at the middle of the tube, where higher stresses are developed while coarse mesh divisions are applied far away from the center of the tube. Rigid elements are assumed at both ends of the tube to prevent local failures near tube ends where external loads are acting.

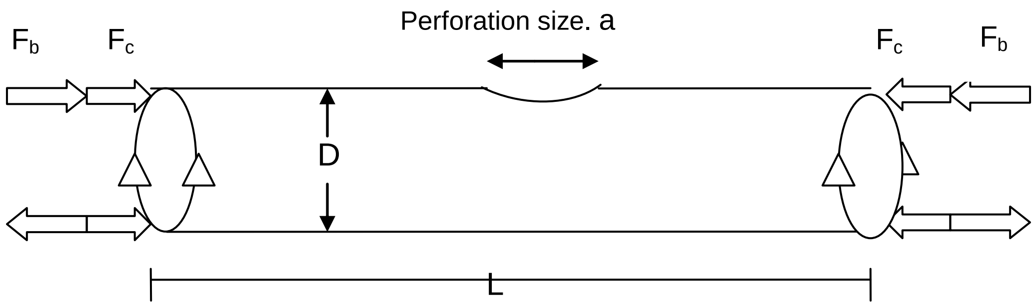

Fig. 1. Boundary and loading conditions of the perforated tube.

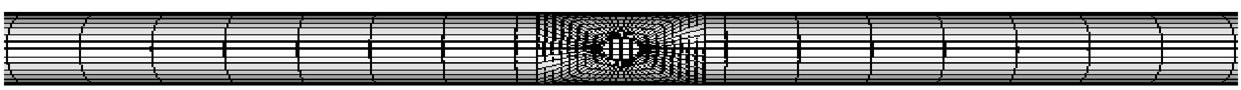

Fig. 2-a. FEM model of the cylindrical tube.

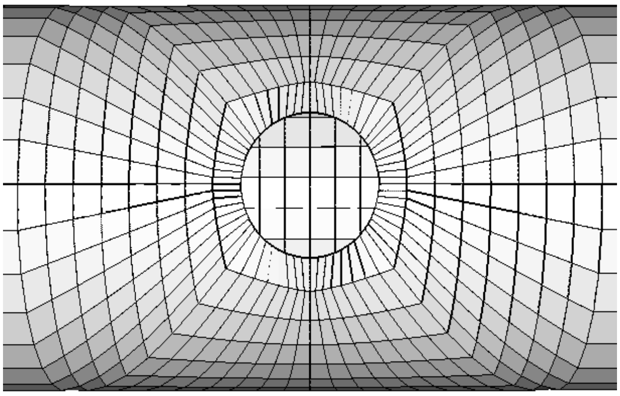

Fig. 2-b. Mesh division around the perforation.

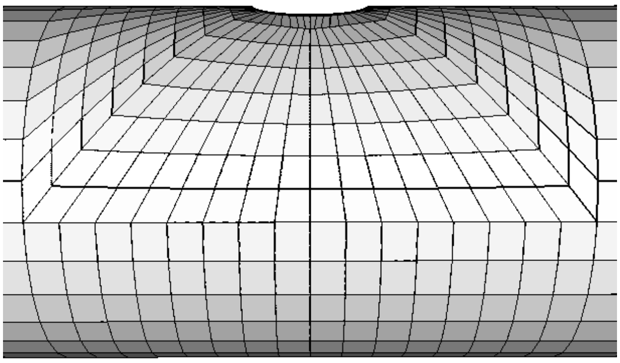

Fig. 2-c. Depth of the perforated damage.

2.2. Examples of numerical analyses

2.2.1. Geometrical properties and loading conditions

Cylindrical tubes with different geometrical properties modeled by the finite element method are analyzed. Several perforated damages such as K_s =0.0, 0.05, 0.1 and 0.2 in the middle of the tube length are considered. Initial deflection of a value equal $L/1000$ is assumed. As mentioned before the tube is assumed to be simply supported at both its ends and subjected to combined uniaxial compression and bending. The geometrical properties and loading combination factors are shown in tables 1 and 2.

where,
λ is the $(L/\pi r)(\sigma_o/E)^{.5}$,
r is the $(I/A)^{.5}$,
σ_o is the yield strength, N/mm^2,
L is the tube length,
t is the tub thickness, mm,
w_o is the initial deflection = $L/1000$, mm,
K_s is the $a/\pi D$, the perforated damage ratio,
a is the diameter of the hole, mm,
D is the diameter of the tube, mm,
F is the maximum acting load, N,
P_o is the fully plastic compressive load, and
M_o is the fully plastic bending moment.

Table. 1
Geometrical properties of the models

Damage-1 (K_s=0.0)	λ =0.5	λ =1.0	λ =1.5	λ =2.0
Geometrical properties	L =8000 D/t =50 σ_o =350	L =15000 D/t =50 σ_o =350	L =24000 D/t =50 σ_o =350	L =32000 D/t =50 σ_o =350
Damage -2 (K_s=0.05)	λ =0.47	λ =0.96	λ =1.41	λ =1.9
Geometrical properties	L =2500 D/t =15.4 σ_o =350	L =5000 D/t =15.4 σ_o =350	L =7500 D/t =15.4 σ_o =350	L =10000 D/t =15.4 σ_o =350
Damage -3 (K_s=0.05)	λ =0.52	λ =1.0	λ =1.6	λ =2.0
Geometrical properties	L =8000 D/t =50 σ_o =390	L =16000 D/t =50 σ_o =390	L =24000 D/t =50 σ_o =390	L =30000 D/t =50 σ_o =390
Damage -4 (K_s=0.10)	λ =0.52	λ =1.0	λ =1.6	λ =2.0
Geometrical properties	L =8000 D/t =50 σ_o =390	L =16000 D/t =50 σ_o =390	L =24000 D/t =50 σ_o =390	L =30000 D/t =50 σ_o =390
Damage -5 (K_s=0.20)	λ =0.52	λ =1.0	λ =1.6	λ =2.0
Geometrical properties	L =8000 D/t =50 σ_o =390	L =16000 D/t =50 σ_o =390	L =24000 D/t =50 σ_o =390	L =30000 D/t =50 σ_o =390

Table-2
Combination load factors

Combination load factors	*C*1	*C*2	*C*3	*C*4	*C*5
Compression , F_c	0.03F	0.33F	0.50F	0.66F	1.0F
Bending , F_b	0.97F	0.66F	0.50F	0.33F	0

2.2.2. Impact of Perforation size

The most important effect on the deterioration of the ultimate strength is the size of the perforation. Figs. 3-4 show relationships between the normalized load (P/P_o) and normalized displacement (δ/δ_o) for cylindrical tubes with different perforation sizes. Slenderness ratio such as λ = 0.52 and λ = 1.6 and the combination load factor of case 4 (see table 2) are adopted. The corresponding relationships between the ultimate bending moments and displacements are also presented. It is observed from these figures that ultimate strength remarkably decreases as the perforation size increases even at cases of sturdy columns. The deterioration in the ultimate strength is drastically increased as the slenderness ratio increases.

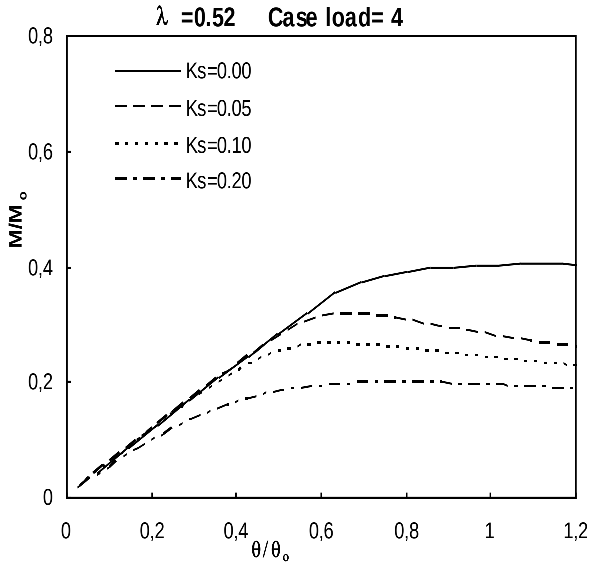

Fig. 3-b. M/M_o - θ/θ_o relationship.

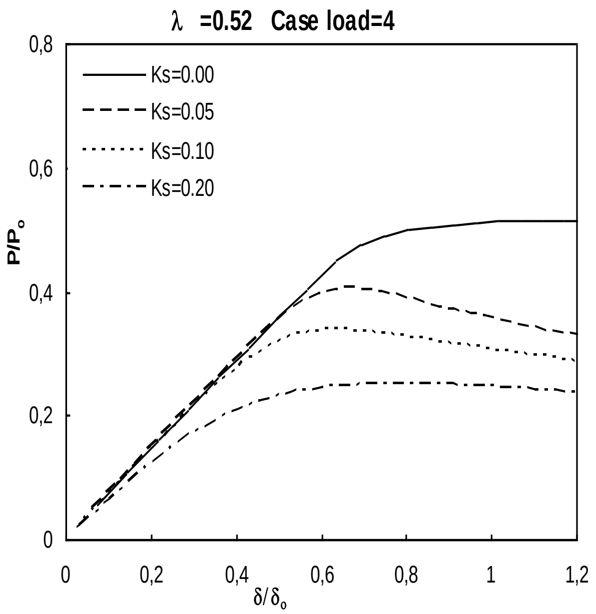

Fig. 3-a. P/P_o - σ/σ_o relationship.

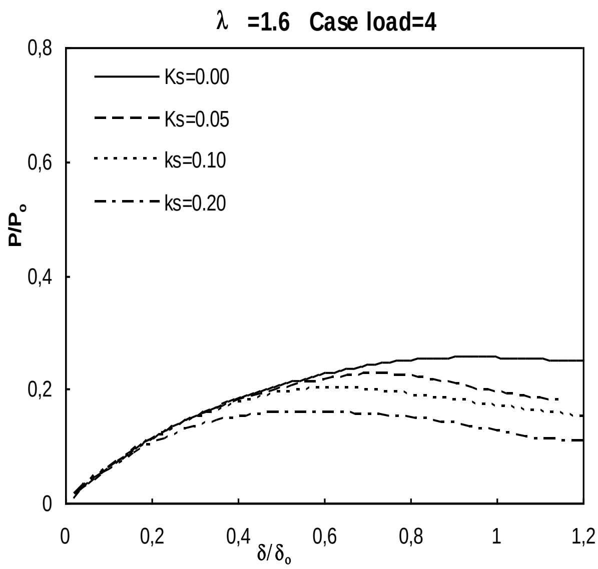

Fig. 4-a. P/P_o - σ/σ_o relationship.

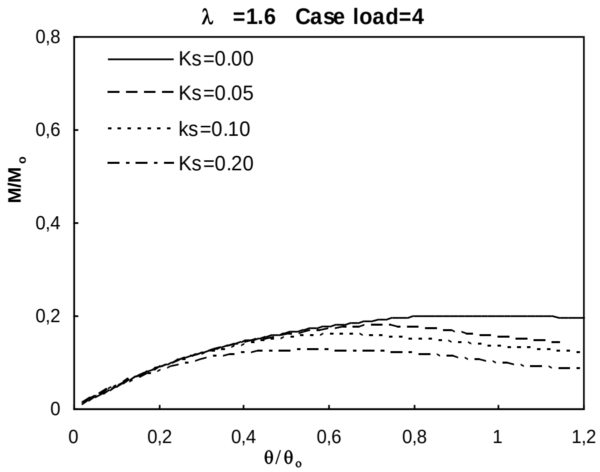

Fig. 4-b. M/M_o - θ/θ_o relationship.

2.2.3. Effect of combination load factors

Figs. 5 and 6 examine the influence of different combination load factors on the deterioration of ultimate strength. Cylindrical tubes with perforated damages such as K_s=0.1 and 0.2 and slenderness ratio of λ =1.6 are adopted to demonstrate this effect. The figures clarify the tendency of degradation in compressive ultimate strength as the bending moment and size of perforated damage increase as shown in figs. 5-a and 6-a. The influence of the load combination factor and perforation size are the two most important parameters.

3. Prediction of ultimate strength using simplified methods

Different approaches for predicting the ultimate strength of columns subjected to combined loads are presented in many literatures as in refs. [7 and 8]. Among those approaches, it is possible to estimate the ultimate strength of tubular structures with initial imperfection and subjected to combined compression and bending. However, very little literatures are available about the residual strength of damaged tubular structures under combined loads. Okada and et.al developed an approximate simplified equation taking into account the effect of the perforated damage on the ultimate strength calculation. The simplified equation predicted higher ultimate strength than those analyzed by FEM for

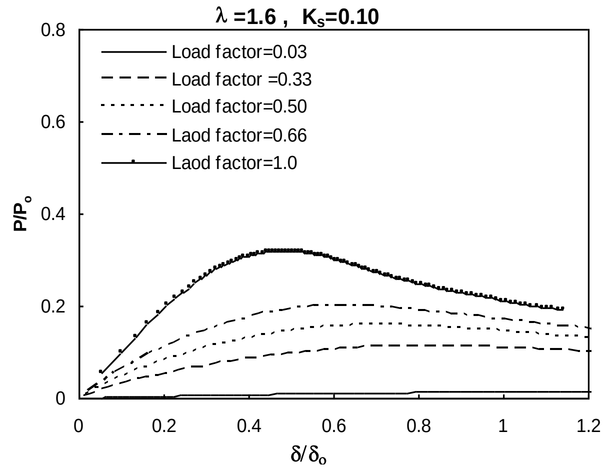

Fig. 5-a. P/P_o - σ/σ_o relationship.

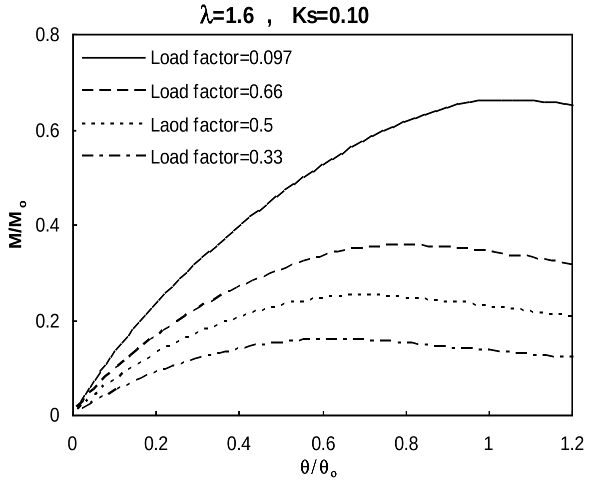

Fig. 5-b. M/M_o - θ/θ_o relationship.

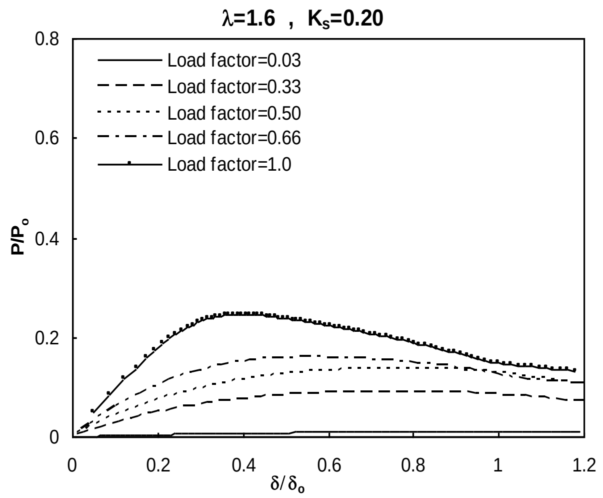

Fig. 6-a. P/P_o -σ/σ_o relationship.

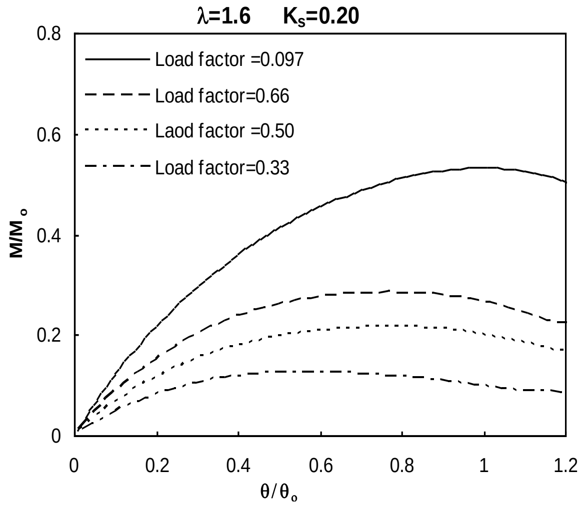

Fig. 6-b. M/M_o - θ/θ_o relationship.

perforated tubes with higher ratio of D/t and large perforation sizes. Differences of the results have been investigated and a new equation is suggested taking into account this effect in the ultimate strength calculation. The procedure of the present equation is presented as follows.

- Yielding of the perforation region: at the beginning, the axial compressive load transformed from intact tube section to the perforation region will cause compressive and bending stresses at the lower perforated region. Therefore, yielding starts at the perforation region until plastic hinges are developed. At this stage, the acting compressive load on the tube is assumed as follows,

$$P_{dp} = \sigma_o A', \tag{1}$$

where,

$A' = Dt\,\alpha$

- The ultimate strength stage: after plastifiaction has occurred at the perforation region, eccentricity due to an additional acting load P_2 increases which develops overall moment acting on the tube. The lateral deflection is substantionally increased and yielding at the adjacent region to the perforated area starts and tube stiffness decreases rapidly. When the acting load P_2 reaches its maximum value,the ultimate strength is attained. The ultimate strength stage may be discussed as follows,
- Assuming the tube has an initial deflection expressed by Fourier series as follows,

$$w_o(x) = \sum_{i=1}^{2} w_{oi}\sin(i\pi x/L), \tag{2}$$

where,

w_{oi} is the $(e_p+w_o)A_i$,

e_p is the shift in the natural axis = $D\sin\alpha/(\pi-\alpha)$, see fig. 7, and

w_o is the amplitude of the initial deflection.

The coefficient, A_i is given in ref. [1]

Substituting w_o in the following governing equation, and solving the equation,

$$EI\delta^4(w-w_o)/\delta x^4 + P_2\,\delta^2 w/\delta x^2 = 0. \tag{3}$$

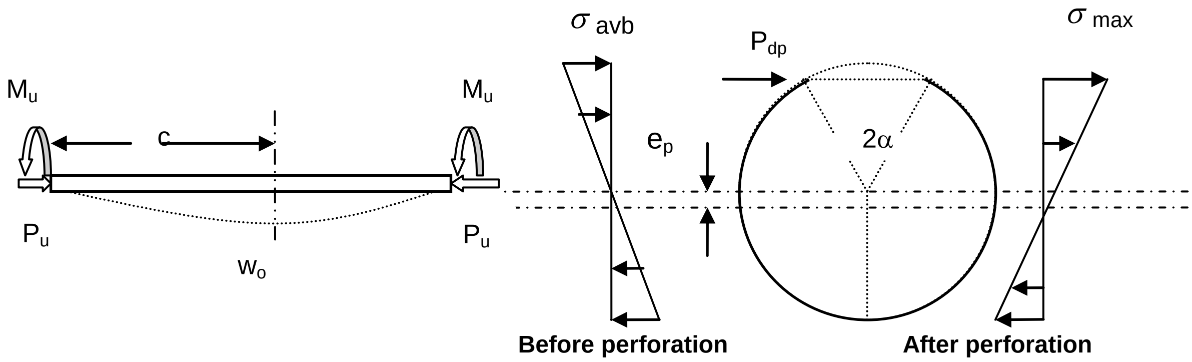

Fig. 7. Perforated tube with initial deflection under combined loads.

The induced moment, M_i due to the load P_2 is expressed by eq. (4) as follows,

$$M_i = P_2 w = P_2 \sum_{i=1}^{2} w_{oi} \sin(i\pi c/L)/\{1-(1/i^2)(P_u/P_e)\} \;, \qquad (4)$$

where, c is location of maximum perforation damage. The maximum moment ocurrs when coefficent w_{oi} of eq. (4) is maximum. It is assumed that maximum deflection at $x=L/2$, ($A_1=1$, $A_2=0$). Hence, M_i may be expressed as follows.

$$M_i = P_2\, \delta_2 + P_{dp}\, \delta_1, \qquad (5)$$

where,
$\delta_2 = (e_p + w_o)\, \phi$, $\quad \delta_1 = w_o \phi$
$\phi = 1/(1-p_u/p_e)$
P_e = Euler buckling load = $E\pi^2 I/L^2$
P_u = ultimate compressive load.

3.1. Ultimate strength interaction equation

When the perforated tube is subjected to combined compression load P_u and bending moment M_u, the maximum moment may be expressed as follows;

$$M_{max} = M_u/\cos\{(\pi/2)\sqrt{(P_u/P_e)}\} + P_2\,(e_p + w_o)\,\phi + P_{dp}\, w_o \phi \;. \qquad (6)$$

The above equation is approximated as follows,

$$M_{max} = \{ M_u + P_2\,(w_{o+}\, e_p) + P_{dp}\, w_o \}\phi \;. \qquad (7)$$

• Assuming the stress distribution acting on the tube at the perforation region is as shown in fig. 8, the interaction equation of the ultimate strength of the perforated tube subjected to combined compression load P_u and bending moment M_u is expressed as:

$$\{ M_u + P_u\,(w_{o+}\, e_p) + P_{dp}\, w_o)\}\, \phi / M_p^{\backslash} + p_u/p_o = 1, \quad (8)$$

where,

$$M_p^{\backslash} = M_p\{\sin(½\,\pi\,[P_1 - P_u]/\,P_p) - \sin\,\alpha\}\,. \qquad (9)$$

From eqs. (8) and (9), M_u is expressed as follows

$$M_u = (1 - p_u/p_o)\, M_p^{\backslash}/\phi - P_2\,(w_{o+}\, e_p) - P_{dp}\, w_o \;, \quad (10)$$

where,

$$P_1 = \delta_o Dt\,(\pi - \alpha) + \delta_o Dt\alpha$$

$$M_p = \delta_o\, D^2 t \qquad , P_o = \delta_o\, \pi\, Dt$$

Finally, from eq. (4), the ultimate compressive strength P_u (σ_u A) is given as follows [1],

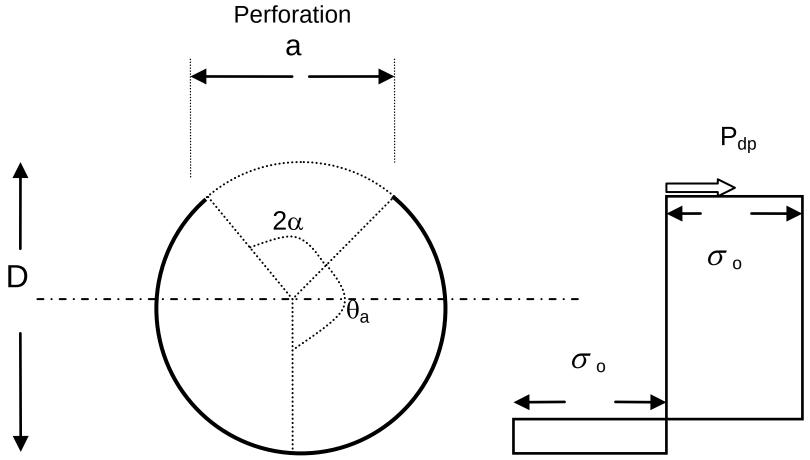

Fig. 8. Stress distribution at the ultimate strength stage through cross section.

$$\sigma_u / \sigma_o = [\{(\pi/\theta_a)(1+\lambda^2) + A(e_p+w_o)/Z\} - (\{(\pi/\theta_a)(1+\lambda^2) + A(e_p+w_o)/Z\}^2 - 4\lambda^2)^{1/2}] / (2(\pi/\theta_a)\lambda^2). \quad (10)$$

4. Applications

In order to investigate the applicability of the present equation, several cylindrical tubes are analyzed and results are compared with FEM results. Figs. 9 to 12 illustrate ultimate strength calculations for cylindrical tubes subjected to combined compression and bending loads. Perforation sizes such as K_s = 0.05, 0.1 and 0.2 are adopted with a tube slenderness ratio equals 1.0. It is seen that the results based on the simplified equation of ref. [1] have a considerable agreement with those by FEM for cases of sturdy columns (D/t=15.4). However, a difference in the ultimate strength is observed for cases of cylindrical tubes with higher D/t ratio as shown in figs. 11 and 12. It is noticed that the present equation shows adequate agreement with results based on FEM. Figs. 13 to 16 investigate variations of slenderness ratios on the ultimate strength of perforated tubes. Also, the results based on the present equation are compared with FEM results. It is shown that results based on the present equation show good satisfaction with those based on FEM analysis. However, some results show lower estimation of ultimate strength than those analyzed by FEM when the acting combined load is a dominant bending as shown in fig. 16. The difference is due to simulation difficulties of a pure bending moment behavior using FEM.

5. Conclusions

This paper has demonstrated the degradation of the ultimate strength of cylindrical tubes with perforated damages and subjected to combined compression and bending moment. The residual ultimate strength is estimated based on a simplified equation and a non-linear finite element method. Several examples are performed on perforated tubes with different geometrical properties. From the results of analyses, the following conclusions may be deduced:

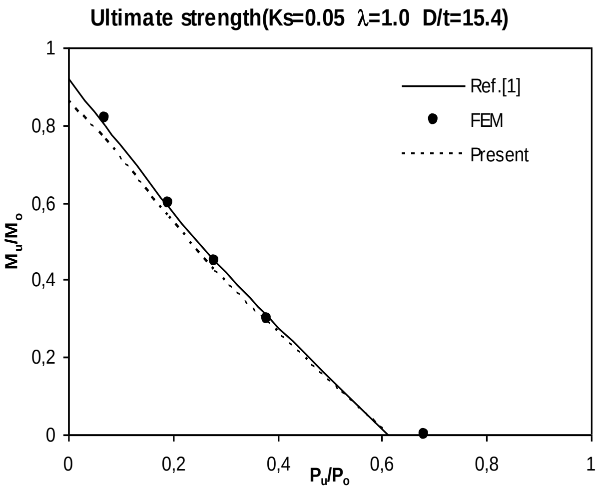

Fig. 9. Ultimate strength under combined loads.

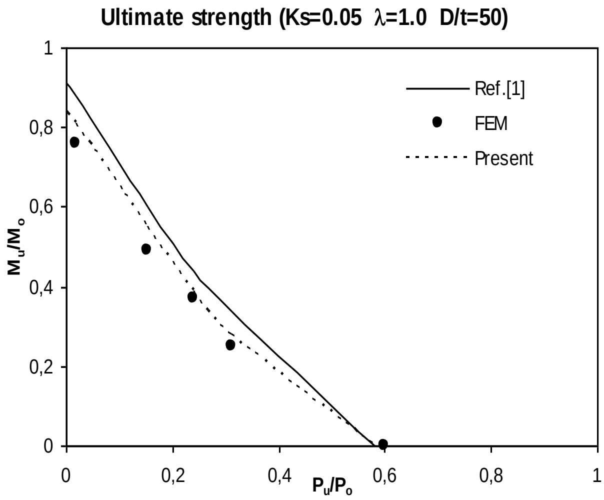

Fig. 10. Ultimate strength under combined loads.

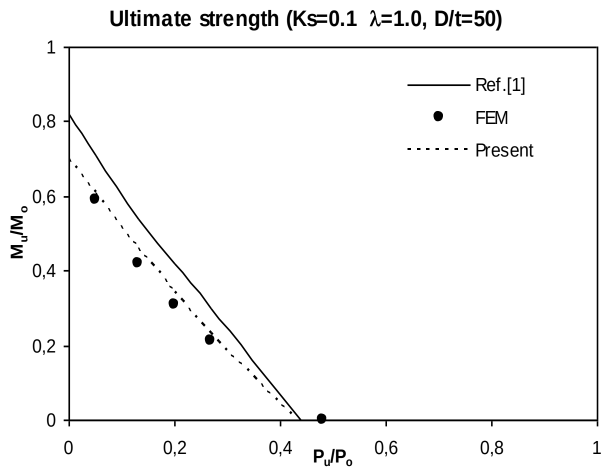

Fig. 11. ltimate strength under combined loads.

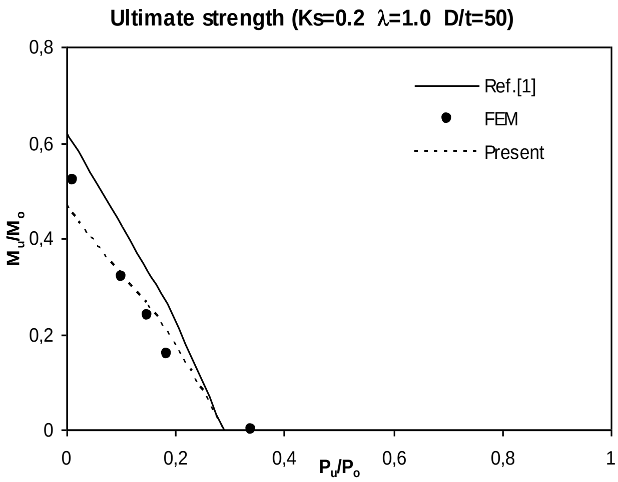

Fig. 12. Ultimate strength under combined loads.

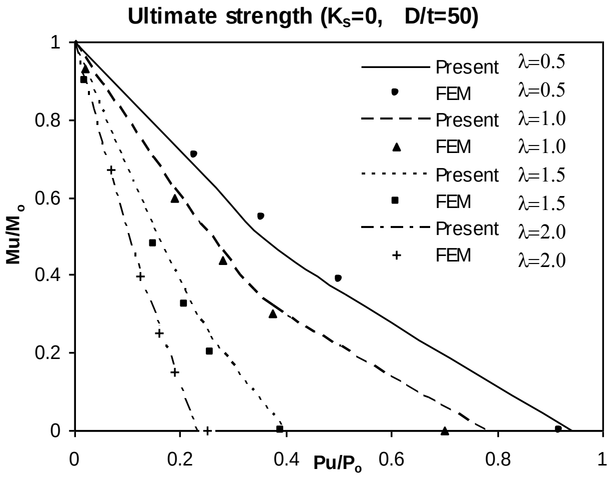

Fig. 13. Ultimate strength under combined loads.

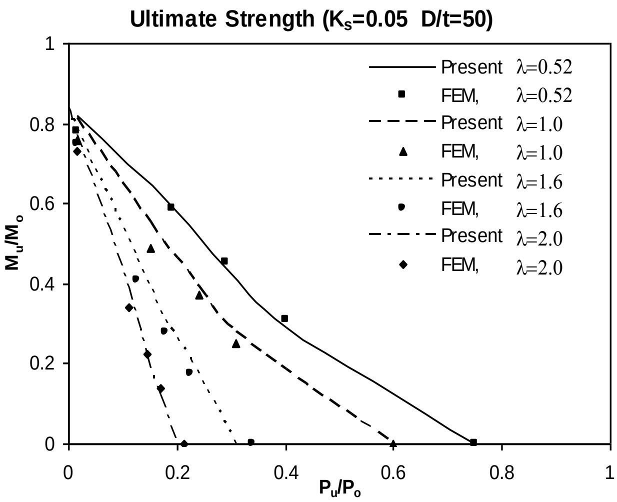

Fig. 14. Ultimate strength under combined loads.

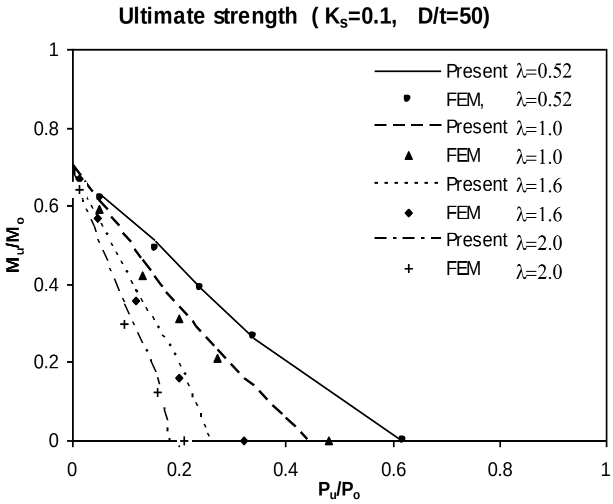

Fig. 15. Ultimate strength under combined loads.

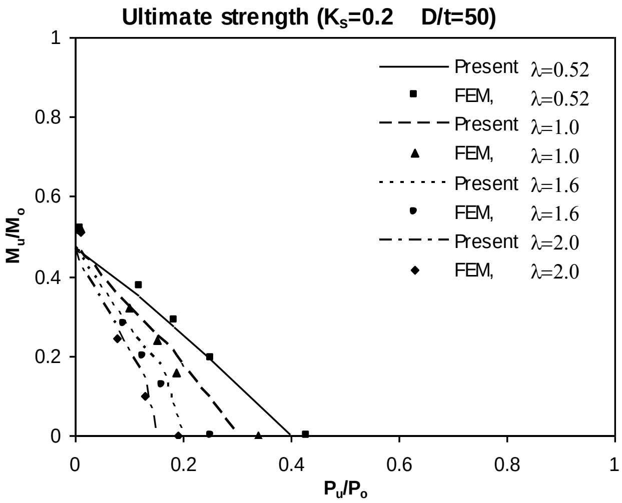

Fig. 16. Ultimate strength under combined loads.

1. The influences of *D/t* and perforation size on the degradation of the ultimate strength are the two most important parameters.
2. The results based on the present method show considerable satisfaction with those based on FEM.
3. When the combined load is a dominant bending moment, the present equation predicts lower estimation of ultimate strength than that based on the FEM.

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