Design of a C-core single-phase transformer

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This paper addresses design, modeling, and simulation for a C-core single-phase transformer using analytical (MATLAB package). A power rating of 20kVA is assumed while the primary and secondary are separated by a big air-gap of 3mm are used as a case study. Optimum dimensions are selected for such case study to obtain higher performance. Numerical results using finite element analysis are compared with analytical ones to verify the optimum dimensions. These types of transformers are used in applications where power is required to be transferred via big air-gap.

تقدم هذه المقالة تصميما و نمذجة و محاكاة للمحول أحادي الطور من النوع الذي يشبة حرف C وذلك باستخدام الطرق تحليلية وهي حزمة برامج الماتلاب. بعطي المحول تحت الدراسة يعطى قدرة في حدود ٢٠ كيلو فولت أمبير. الجسم الابتدائي و الجسم الثانوي مفصولون عن طريق ثغرة هوائية تقدر بحوالي ٣ مليمتر وهذه الأرقام اختيرت لتكون حالة تحت الدراسة. اختيرت الأبعاد المثلي للحالة التي تحت الدراسة لتحقق أفضل أداء .وتم تحقيق هذه النتائج باستخدام الطرق العددية باستخدام حزمة برامج العناصر المحددة لتتوافق مع الطرق التحليلية. يمكن استخدام هذه النوعية من المحولات في التطبيقات التي تتطلب نقل قدرة كهربية خلال ثغرة هوائية كبيرة.

Keywords: Single-phase transformer design, C-core, Numerical, And finite element analysis

1. Introduction

Basically, the transformer has two circuits magnetically coupled, electrically isolated. Elementary iron core is wound by a primary coil of turns, “N_1”, and secondary coil of turns, “N_2”. Both circuits are wound on a common magnetic circuit. By assumption of idealized case, when a time varying voltage “v_1" is applied to the primary winding a core flux is established and a counter voltage, “e_1”, is produced. Core flux (useful flux) links the secondary turns and produces an induced voltage, “e_2”. If a passive load is connected to the secondary terminals, current will path through that load [1-3]. The proposed system, shown in fig. 1-a and fig. 1-b, operates on the same theory. The main difference between the ordinary transformer and the proposed system that iron core for both primary and secondary are C-cores separated by g-mm air gap. They are constructed from iron material that has high permeability. Primary and secondary windings are fitted as illustrated in fig. 1-b. The design should lie in resolving the conflict for space between iron, copper, insulation and thermal aspects [1], [4-7].

For the proposed system, design, modeling, and simulation for the magnetic circuit, (iron core), and electric circuit, (copper winding) is presented in this paper. The design of insulation, heat flow and ventilation aspects are out of scope here. The proposed system is usually used in applications, which require big air-gap between primary and secondary such as a charging unit for electric vehicles. The primary is fixed and the secondary is fitted in the body of the vehicles. The power transferred is rectified and used for charging batteries [8]. Also, this type of transformers could be used in robots to transfer high power from fixed parts to movable ones [9].

Analytical method for design process using MATLAB package is employed. An m-file calculates the electric parameters and dimensions of the proposed system. Numerical solution using finite element software is utilized to verify MATLAB results and to confirm the values of specific electric and magnetic loadings already chosen.

The finite element analysis has the advantage that geometric variations, irregularities, saturation and eddy current effects can be modeled with a high degree of accuracy [10]. Also, it allows the designer to obtain a clearer

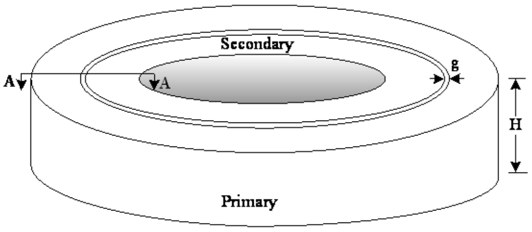

Fig. 1-a. Isometric view of proposed system.

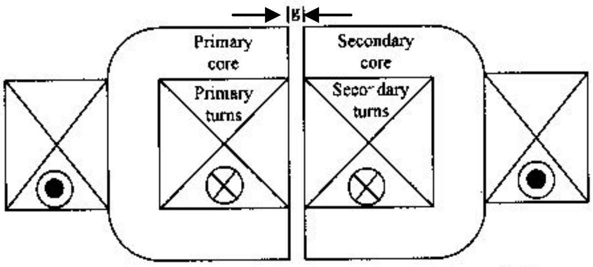

Fig. 1-b. Elevation view of proposed system section A-A.

understanding of the flux distribution in the electromagnetic device.

2. Design process

Electromagnetic devices require essentially two linked circuits, where the magnetic circuit is, as far as possible, constructed of ferromagnetic material. The electric circuit is used to produce main or useful flux [1]. The design process deals with those two main parts, (iron core and copper winding). It starts by assigning suitable values for specific magnetic loading "*B*" Wb/m^2 and specific electric loading "*J*" A/mm^2. By applying specified voltage at certain frequency for the transformer under design, dimensions, parameters and performance can be evaluated [6]. The effect of the air-gap on the magnetic circuit should be considered. Fig. 2 shows the equivalent magnetic circuit for the proposed system by neglecting the reluctance of the iron paths.

Where, N_1I_1 and N_2I_2 represent the primary and secondary magneto-motive force respectively R_g represents the air gap reluctance and R_L represents the leakage reluctance. Hence, transformer parameters can be evaluated using the equivalent magnetic circuit. There are some assumptions used to ease the analysis and solution:

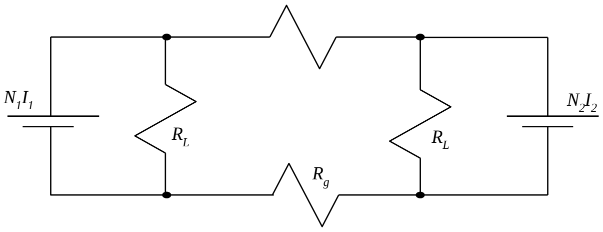

Fig. 2. Equivalent magnetic circuit.

- The transformer has the same number of turns "*N*" on both primary and secondary.
- To minimize the number of variables the circumference of the primary is chosen to be 1m.
- The load is resistive.
- The performance does not consider the transient state.
- The power rating (20kVA) and air gap (3 mm) are chosen to give definite solution for the design routines while these routines can be adapted to any other power ratings and reasonable air gap required.

Hence, the leakage inductance of both primary and secondary is calculated as:

$$L_{l1} = L_{l2} = \frac{N^2}{R_L}, \tag{1}$$

and, the magnetizing inductance is

$$L_m = \frac{N^2}{2R_g} . \tag{2}$$

The resistances of both primary and secondary is calculated from:

$$r_1 = r_2 = N\frac{\rho\, l}{A_i}, \tag{3}$$

Where, ρ is the resistivity in Ω.m, l represents the length of the winding turns in m, and A_i is the cross section area in m^2. The maximum allowable current in the primary side must be lower than or equal to the maximum current [1].

$$I_{\max} = \frac{J\, A_w}{N}, \tag{4}$$

where, A_w is the area of the window.

The number of turns is calculated from induced voltage equation at no-load case where

$$N = \frac{V_1}{4.44\,\phi f}, \qquad (5)$$

and the flux "ϕ" is calculated as

$$\phi = B A_i$$

Eqs. (1) to (6) lead to the transformer equivalent circuit. It is shown in fig. 3.

3. Analytical technique

The m-files under MATLAB software [8] are used in analytical solution for the case under study. The programs are carried out for the C-core shown in fig. 4.

The symbol "k" represents the dimension factor and varied from 1.5 up to 3.5 in steps of 0.5. The variable "x" represents the height of C-core arm in mm. To determine dimensions and electrical performance, the m-files run in two modes. First mode is the no-load case, which gives indication about number of turns required, conductor area, leakage and magnetizing inductances, and hence, magnetizing current (no-load current). Second mode is the loading case, which gives indication about transformer performance for different dimension factors.

These results lead to search for the resistive load, which takes 20kW, hence the dimensions according to that load with data already calculated from no-load case are estimated. This will be achieved by comparing the primary current to the allowable current as in (4) when the load resistance varied from no-load point to short circuit point. By assigning the supply voltage to 415V and 50Hz, the specific magnetic loading to 1.4 Wb/m^2 and specific electric loading to 4 A/mm^2, results can be obtained.

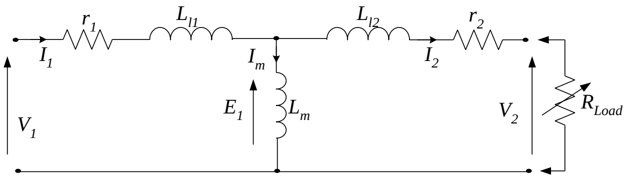

Fig. 3. Transformer equivalent circuit referred to the primary side.

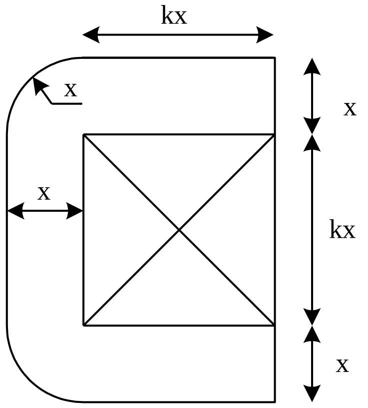

Fig. 4. Side view of magnetic structure.

Fig. 5 shows the number of turns and height of C-core arm versus dimension factor. Fig. 6 shows the primary, secondary and magnetizing current against dimension factor. Fig. 7 illustrates the efficiency and power factor for different dimension factors. It is clear that when dimension factor "k" increases from 1.5 up to 3.5 the number of turns required increases while the height of the C-core arm decreases from 23 mm to 13 mm. Also, as the dimension factor increases, the magnetizing current decreases from 60A to 25A while the load current (secondary current) increases from 54A to 73A. The primary current is the vectorial summation of magnetizing current and secondary current. This shows that primary current has a minimum value around "k = 2.25". Fig. 7 indicates that when dimension factor increases the efficiency decreases. The

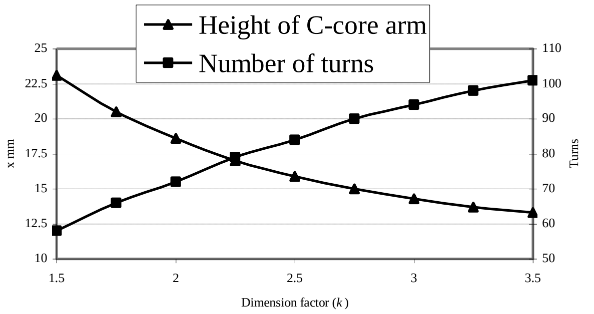

Fig. 5.Height of C-core arm and number of turns versus dimension factor.

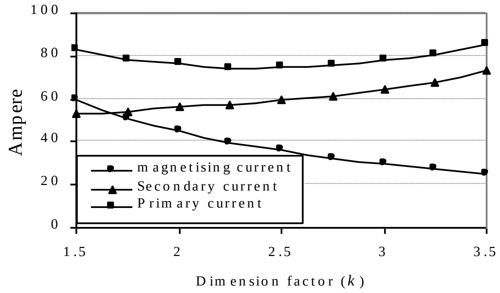

Fig. 6. Magnetizing current, secondary current and primary current versus dimension factor.

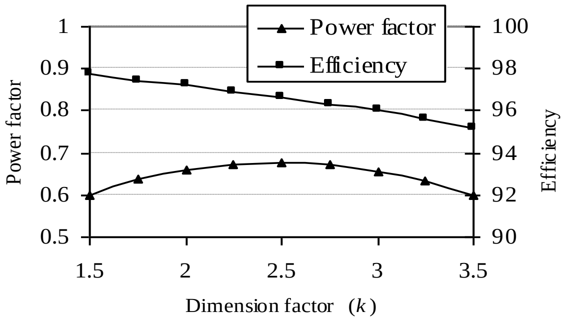

Fig. 7. Power factor and efficiency versus dimension factor.

input power factor has a maximum value around "*k* = 2.5". It could be concluded that optimum design for that transformer lies between "*k*=2.25" and "*k*=2.5". The results for both dimension factors are tabulated in Table 1 in the appendix. A comparison between both dimension factors, shows that at "*k*=2.25" there are lower conductor area, lower primary and secondary currents, lower ratio of leakage inductance to magnetizing one and higher efficiency. At "*k*=2.5", lower iron volume, higher conductor area, lower magnetizing current and higher power factor.

3. Numerical technique

Generally, numerical techniques are computationally intensive requiring the use of computers capable of carrying out the large number of operations in a reasonable time. Finite element analysis is one possible numerical technique, which has became popular for magnetic design. It can cope with magnetic materials of a linear and non-linear nature as well as modeling permanent magnets. It basically requires the problem area to be subdivided into a large number of well-defined regions, normally known as elements, an example being a triangle. In order to obtain the field distribution relating the differential equations defining the problem, energy functional is derived from the energy of the problem concerned. This functional is then minimized with respect to the nodes defining the elements and then integrated over these elements [10], [11]. Thus, the magnetic potential vector varies linearly across each element and the flux density is constant within elements. To optimize the distribution of magnetic material in the C-core under study for both primary and secondary, two dimensional finite element analysis is used. Choosing "*k* = 2.25", region plot and flux plot are shown in figs. 8 and fig. 9 respectively.

There are some flux lines, (fringing flux), spreads into surrounding air and there are other flux lines which take short cut across the space between the poles, instead of crossing the air gap (leakage flux). To display flux density against distance a contour line must be defined. In this model a contour line is taken through the middle of the air-gap.

Figs. 10 to fig. 12 illustrates the normal component, tangential component and magnitude of flux density against distance taken by the contour respectively.

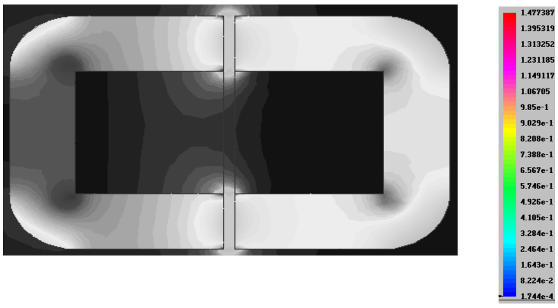

Fig. 8. Region plot.

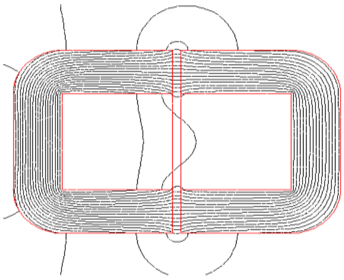

Fig. 9. Flux plot..

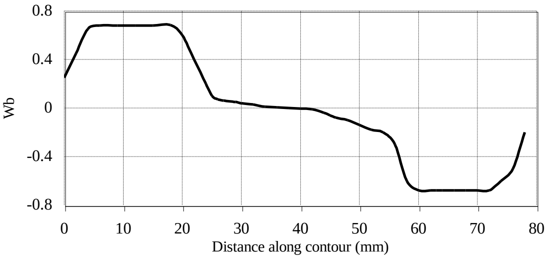

Fig. 10. Normal component of flux density.

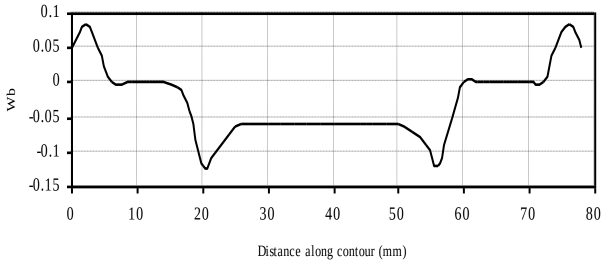

Fig. 11. Tangential component of flux density.

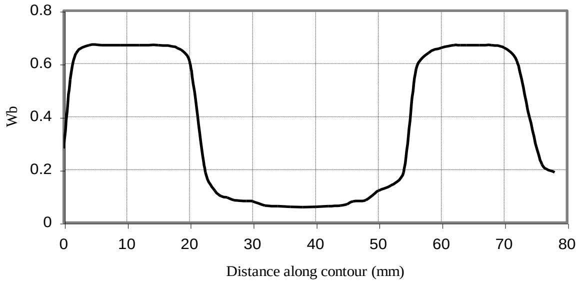

Fig. 12. Magnitude of flux density.

These figures indicate that maximum flux density in the air-gap is in range of 0.7 Wb/m^2 while at the C-core arm the maximum flux density is in range of 1.38 Wb/ m^2 which verifies the designed value in the analytical solution. At the edge corners, the flux density is slightly higher than 1.4 Wb/ m^2.

The self-inductance of the primary side at no-load is given by the finite element software and equals 36.433mH while it is calculated using analytical technique, (MATLAB), and equals 32.4mH (see table 1). This means, finite-element results show accepted matching with analytical results.

5. Conclusions

Design, modeling and simulation of C-core single-phase transformer with big air-gap, (3mm), and transferring electric power in the range of 20kVA is illustrated in this paper. Simulation results show that as the dimension factor “*k*” increases, the number of turns required increases, the height of C-core arm decreases, the magnetizing current decreases, load current increases, and efficiency decreases. Optimum design lies between “*k*=2.25” and “*k*=2.5”.

Dimension factor equal 2.25 provides minimum copper losses and better voltage regulation but “*k*=2.5” provides minimum no-load losses and smaller dimensions. Numerical technique using finite-element soft-ware is used to confirm the simulation results which shows accepted matching.

Appendix

Dimension factor “*k*”	2.25	2.5
The height of C-core arm by mm	17	15.9
Side area *1000 by mm^2	3.99	3.98
Height by mm	72.25	71.55
Width by mm	55.25	55.65
Iron volume *10^6 by mm^3	2.405	2.29
Number of turns	79	84
Conductor area by mm^2	18.52	18.81
Ratio between leakage inductance to magnetizing inductance by %	10.84	11.32
Magnetizing current by ampere	39.2	35.9
Primary current at full load by ampere	74.1	75.2
Secondary current at full load by ampere	57	59.7
Minimum load resistance by Ω	6.2	5.74
Input power factor	0.672 lag	0.678 lag
Efficiency by %	96.9	96.65
Primary and secondary resistance by Ω	0.073	0.077
Primary and secondary leakage inductance by mH	3.2	3.55
Magnetizing inductance by mH	29.2	31.3

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