The identification of the critical oscillatory modes in multimachine power system

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This paper proposes an iterative technique for the approximate computation of complex eigenvalues corresponding to electromechanical oscillations in a power system. By this technique, the critical oscillatory modes in multimachine power system can be identified. It enables to identify the natural frequencies and the least oscillatory stability margin of the synchronous generators. The effectiveness of the method is described through the application to a multi-machine power system dynamic mode.

هذا البحث يقدم طريقة رياضية تجريبية للحسابات التقريبية للجذور المركبة المتعلقة بالاهتزازات الكهروميكانيكية في الأنظمة الكهربائية. باستخدام هذه الطريقة يمكن تحديد الحالة الاهتزازية الحرجة للأنظمة الكهربائية متعددة الآلات وكذلك تحديد الترددات الطبيعية وحدود الاستقرارية الاهتزازية للمولدات التزامنية. تقدم الورقة أيضا وصفا لفعالية هذه الطريقة بتطبيقها على حالة ديناميكية لنظام كهربائي متعدد الآلات.

Keywords: Oscillatory stability, Multimachine power system, Dominant eigenvalues

1. Introduction

Power systems experience low frequency oscillations when subjected to disturbances. These oscillations may sustain and grow to cause system separation if no adequate damping is available. To enhance system damping, the generators are equipped with Power System Stabilizers (PSSs) that provide supplementary feedback stabilizing signals in the excitation systems. PSSs extend the power system stability limit by enhancing the system damping of low frequency oscillations associated with the electromechanical modes [1–3]. Some of the earliest power system stability problems included spontaneous power system oscillations at low frequencies. These LFOs are related to the small-signal stability of a power system and are detrimental to the goals of maximum power transfer and power system security [4].

2. Statement of the problem

Power system stability may be broadly defined as that property of a power system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance [5]. Eigenvalue determination has been an integral component of the analysis of power systems for many years [6,7]. The efficient computation of eigenvalues for large-scale problems requires that:

1. Only certain modes are significant from the perspective of stability (i.e., solving the complete eigen problem is not necessary), and
2. Sparsity is preserved throughout the computation.

Proper preservation of sparsity requires that the entire problem be formulated as an augmented problem. One of the first efforts in this regard was the AESOPS algorithm [7]. This algorithm was limited to electrom-echanical modes of a system and it finds the modes associated with each and every machine in turn by exciting the machine with a torque after shifting the system state matrix. This algorithm works well because the electrom-echanical modes are generally the most important modes as far as electromechanical stability is concerned.

The focus of this paper is the development of a method for determining the dominant eigenvalues of a linearized mathematical model of a multimachine power system. The method enables to identify the critical generators. The method can be used to develop an algorithm for determining the tuning parameters of excitation controllers for the synchronous generators in complex power systems. The significance of this method comes from the necessity for a rapid and precise calculation of the limitations on the maximum power transfer, this is in turn is connected with the development of new and fast-acting methods and programs for the calculation of the steady-state modes, dynamic stability and oscillatory stability evaluation.

3. The dominant eigenvalues

Once the state space system for the power system is written, the stability of the system can be calculated and analyzed. The performed analysis follows the traditional root-locus (or root-loci) methods such as those discussed in [8,9,10]. First, the eigenvalues p_i, $i = 1,2,\ldots,n$, are calculated for the A-matrix, which are the non-trivial solutions of the equation:

$$A\Phi = p_i \Phi . \qquad (1)$$

where Φ is an $nx1$ vector. Rearranging (1) to solve for p yields:

$$\det (A - p\, I) = 0. \qquad (2)$$

A is the nxn state matrix, the n solutions of (2) are the eigenvalues (p 1, p 2,..., p n) of A. These eigenvalues may be real or complex, and are of the form $\sigma \pm j\omega$. If A is real, the complex eigenvalues always occur in conjugate pairs. Eigenvalues associated with an unstable or poorly damped oscillatory mode are called dominant modes since their contribution dominates the time response of the system. The damped frequency of oscillation in Hertz is given by:

$$f = \omega / 2\pi . \qquad (3)$$

The damping ratio is given by:

$$\xi = - \sigma / \sqrt{(\sigma^2 + \omega^2)} . \qquad (4)$$

The dominant eigenvalues are the eigenvalue pair closest to imaginary axis if they are already on the left hand side of the complex plane [11,12]. Experience shows that with a change in system mode of operation or controller tuning, oscillatory stability is determined by the complex roots, whose imaginary part is found to be in the range from 0 to 10 rad/s or from 20 to 40 rad/s.

The connection of these roots with the transient process equations and the state variables can be formed with the use of the participation matrix:

$$M = |M_{ki}| ,$$

each element of which is determined by the formula $M_{ki} = W_{ki} . V_{ki}$ where W_{ki}, V_{ki} are *k-th* elements for the *i-th* right and left eigenvector matrix A of the system which are determined by the relations:

$$A.V_i = V_i . p_i , V_i \neq 0, W_i^T . A = p_i W_i^T , W_i \neq 0$$

where p_i is the *i-th* eigenvalue.

Left and right eigenvectors are overnormalized in such a manner that:

$$\sum_k W_{ki} V_{ki} = 1 .$$

4. Test system and analysis

To evaluate the effectiveness of the proposed method, tests were carried out for the three-machines power system shown in fig.1, with the following parameters (in pu): Z_{13}= 0.022+ j0.240; Z_{12} = 0.14 + j0.12; Z_{14} = 0.010+j0.02; Z_{34} = 0.0091 + j0.08; Z_{23} = 0.0053 + j0.07; Z_{24} = 0.047 + j0.13. The three pairs of dominant eigenvalue are: $P_{1,2}$= -0.03 ± j4.02; $P_{3,4}$ = -0.17 ± j6.5; $P_{5,6}$= -0.17 ± j5.3; a pair of high frequency roots are also present; $P_{7,8}$= -7.92 ± j29.97.

From table1 it is clear that the torque angle deviation $\Delta\delta$ and the bus power deviation ΔS introduce the greatest contribution in low-frequency roots, while internal generated voltage deviation ΔE_r and the variables participating in the descriptions of the controller common channel introduce the greatest contribution into the high-frequency roots.

Let the power system at small disturbances be described by the linearized differential and algebraic equation:

$$A(p)X = 0\,, \qquad (5)$$

where $X=(\Delta X_1, \Delta X_2,\ldots, \Delta X_m)$- vector of variables.

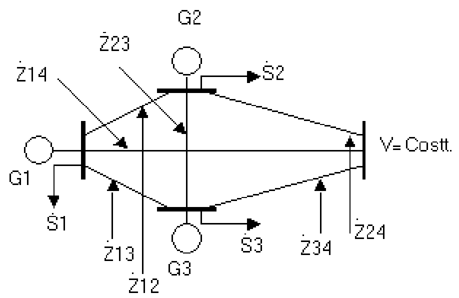

Fig. 1. The Scheme for a system of three interconnected machines.

Table 1
The fragment of the participation matrix for the diagram of fig. 1

The variables	Root value			
	$P_{1,2}$	$P_{3,4}$	$P_{5,6}$	$P_{7,8}$
ΔE_{g1}	0.1012	0.0036	0.0726	0.0545
ΔE_{g2}	0.0050	0.0442	0.0011	0.0385
ΔE_{g3}	0.0402	0.0132	0.0976	0.2262
ΔS_1	0.4308	0.0167	0.1831	0.0033
ΔS_2	0.0529	0.4365	0.0267	0.0048
ΔS_3	0.1425	0.0519	0.4554	0.0274
$\Delta\delta_1$	0.3563	0.0154	0.1233	0.0008
$\Delta\delta_2$	0.0596	0.3938	0.0265	0.0003
$\Delta\delta_3$	0.1308	0.0432	0.3559	0.0015
ΔE_{r1}	0.0042	0.0004	0.0028	0.4383
ΔE_{r2}	0.0002	0.0034	0.0001	0.0178
ΔE_{r3}	0.0016	0.0009	0.0054	0.0142

The elements of matrix $A(p)$ are polynomials, they are determined from the transfer function of the individual controller channels, the rotor speed equations and the damping winding calculation. Excluding from (5) all variables, except ΔX_k that has the greatest participation in the formation of the desired root, at $p = P_k^{(0)}$, we get:

$$p^2 \Delta X_k = B \Delta X_k\,, \qquad (6)$$

where B- complex number.

$P_k^{(1)} = \sqrt{B}$, is the next approximation for the desired eigenvalue related to the k-th unit of the power system; in which the parameter ΔX_k takes the greatest participation. Substituting the obtained value of $P_k^{(1)}$ in (5) and again excluding all variables, except ΔX_k. As a result, a precise value of $P_k^{(2)}$ will be obtained. This iterative process should be finished with the satisfaction of the condition $|Re(P_k^{(i)} - P_k^{(i+1)})| < \varepsilon$; where ε is the assigned accuracy of calcu-lation and $Re(p)$ is the real part of the root. To obtain the dominant eigenvalues for the rema-ining power system aggregates, analogous procedure is implemented.

To check the convergence of the proposed method, a system of two generators connected to an infinite power bus with the absence of automatic voltage regulation is considered. Disregarding the damping circuit, the system stability may be investigated using the following model of standard well known notations:

$$\begin{aligned} &(T_{J1} p^2 + S_{11})\Delta\delta_1 + S_{12}\Delta\delta_2 = 0; \\ &S_{21}\Delta\delta_1 + (T_{J2} p^2 + S_{22})\Delta\delta_2 = 0, \end{aligned} \qquad (7)$$

where; T_{j1}, T_{j2} the inertia constants, $\Delta\delta_1$, $\Delta\delta_2$ the torque angle deviations for generators 1 and 2, respectively; p is the differential operator, S_{11}, S_{12}, S_{21}, S_{22} the synchronizing torque coefficients. Transforming the system (7) into the form of (6) at $p^{(0)}$, we get:

$$(p^{(1)})^2 \Delta\delta_1 = \frac{S_{12}^2 T_{J1} T_{J2} - \mu_1\mu_2 - \mu_1 (p^{(0)})^2}{(p^{(0)})^2 + \mu_2} \Delta\delta_1, \quad (8)$$

where $\mu_1 = \dfrac{S_{11}}{T_{J1}}$; $\mu_2 = \dfrac{S_{22}}{T_{J2}}$.

Let the initial approximation $P^{(0)}$ differs from the actual eigenvalue P_a by ΔP, neglecting the small second-order values, the first approximation on ΔP^*, after a simple mathe-matical transformations, may be written in the following form:

$$\Delta P^* = \frac{\mu_1 + P_a^2}{\mu_2 + P_a^2} \Delta p, \quad (9)$$

or taking into account the actual eigenvalue:

$$P_a^2 = \frac{1}{2}\left[-(\mu_1 + \mu_2) \pm \sqrt{(\mu_1 \pm \mu_2)^2 + D}\right],$$

where $D = \dfrac{4S_{12}^2}{T_{J1}T_{J2}}$, then

$$\Delta P^* = \frac{\mu_2 - \mu_2 \pm 0.5\sqrt{D}}{\mu_1 - \mu_2 \pm 0.5\sqrt{D}} \Delta p.$$

To improve the convergence of the iterative process (6), the condition $|\Delta P^*| < |\Delta P|$ or $\left|\dfrac{\Delta p^*}{\Delta p}\right| = \left|\dfrac{\mu_2 - \mu_1 \pm 0.5\sqrt{D}}{\mu_1 - \mu_2 \pm 0.5\sqrt{D}}\right| < 1$ should be fulfilled.

Examining the case for $\mu_2 > \mu_1$, a stable process for the root is obtained

$$P_a = 0.5\left[-(\mu_1 + \mu_2) - \sqrt{(\mu_1 - \mu_2)^2 + D}\right].$$

For $\mu_1 > \mu_2$ a successful iteration will be noticed for the second pair of complex conjugate roots.

In case of pure imaginary roots, The solution of system (7) takes the following form:

$$\Delta\delta_1 = c_1 \cos(\omega_1 t + \varphi_1) + c_2 \cos(\omega_2 t + \varphi_2);$$
$$\Delta\delta_2 = c_1 m_{21} \cos(\omega_1 t + \varphi_1) + c_2 m_{22} \cos(\omega_2 t + \varphi_2),$$

Where c_1, c_2, φ_1, φ_2 - constants of integration; m_{21}, m_{22} – the distribution coefficients for the amplitude of oscillation, characterizing the relationship between the amplitude of oscillation for the angles $\Delta\delta_2$ and $\Delta\delta_1$ at frequencies of ω_1 and ω_2, respectively. The amplitude distribution coefficients are determined by the relations:

$$m_{21} = \frac{\omega_1^2 + \mu_1}{-S_{12}} T_{J1}; m_{22} = \frac{\omega_2^2 + \mu_1}{-S_{12}} T_{J2}$$

For $\mu_2 > \mu_1$, the convergence of iter-ative process is satisfying the conditon $|m_{21}| > 1$; $|m_{22}| < 1$.

Thus, the algorithm of iteration gives the solution for that root, in which the participation of state variable preserved in the rotational motion component of expression (8) is maximum.

For $\mu_1 = \mu_2$, in order to achieve the algorithm convergence to the root, characterizing the main motion of the system, and possesses smaller damping, the synch-ronizing power for only one of n generators relative to the outgoing bus is to be considered in (8).

The results, in table 2, illustrate the working of the algorithm at different assigned initial values $p^{(0)}$ for the example of fig. 1.

The calculations were carried out at an assinged accuracy of $\varepsilon = 0.0001$. It is obvious that the obtained result does not depend on the given initial values in the range from 0 to 30 rad/s. The greatest number of iterations is observed for those cases, when the assigned initial values are equal to the values of the roots. For practical calculation, the eigen-values obtained from the analysis of the preceded regimes may be taken as initial values. If they are absent, then from the frequency band characterized for the electromechanical oscillations, which does not usually exceed 8 rad/s.

Table 2
The results of the eigenvalues' calculation at various initial values

Generator	The results of the eigenvalues' calculation at various initial values				Number of iterations
	Initial values		Obtained values		
	Re	Im	Re	Im	
1	0	6.29	-0.0329	4.0220	7
	-0.1717	6.49	-0.0324	4.0216	12
	-0.1724	5.13	-0.0323	4.0243	17
	-7.9200	29.97	-0.0329	4.0220	16
2	0	6.28	-0.1715	6.4990	3
	-0.1724	5.13	-0.1715	6.4990	8
	-7.9200	29.97	-0.1715	6.4990	7
3	0	6.28	-0.1727	5.1300	7
	-0.1717	6.40	-0.1730	5.1300	8
	-7.9200	29.97	-0.1727	5.1300	12

5. Conclusions

1. The proposed approach gives the possibility of presenting the mathematical model of the power system in the form of transfer functions without the transformation into Cauchy's form.
2. The analysis of the eigenvalues obtained by the proposed algorithm enables to reveal the generator of a complex root with the smallest attenuation factor, and to determine the generators' natural frequencies.
3. The developed approache allows effectively using the state matrix *A(p)*, and consequently, carrying out investigation on the schemes of significant complexity in sufficiently short time.
4. The iterative procedure of the propsed method possesses a satisfactory convergence.

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