Nonparametric approach for direction-of-arrival estimation of accelerated moving sources using cubic array

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A modified-windowed Local Polynomial Approximation LPA beamformer is developed for nonparametric high-resolution tracking of rapidly accelerated moving targets. Exploiting the acceleration of the source is used to distinguish between many nonstationary sources that have the same position and velocity. In addition, it decreases the Mean Square Error (MSE). Applying a cubic array geometry with this quadratic LPA beamformer enhances its performance and allows it to scan the main beam toward any point in space. In this article, quadratic LPA-beamformer performance expressions and the problem of the optimal window choice are studied. A comparison to the conventional beamformer response is also introduced.

تم تطوير وسيلة مشكل الشعاع للتقريب المحلى المعدل لمتسلسلة القوى و ذلك لإتاحة التتبع اللابارمترى فائق التميز للأهداف المتحركة معجلة السرعة. وذلك من خلال استغلال عجلة سرعة المصدر للتمييز بين العديد من المصادر المتحركة المتشابهة في الموقع و السرعة. بالإضافة إلى ذلك، أثبت البحث أن هذه الوسيلة تقلل من متوسط مربع الخطأ. وتم تطبيق نموذج المصفوفة المكعبة في حالة مشكل الشعاع التربيعى للتقريب المحلى المعدل لمتسلسلة القوى فحسن أدائه، كما عزز قدرة مسح الشعاع الرئيسي تجاه أي نقطة في الفراغ. وتضمن هذا البحث دراسة مقدار أداء مشكل الشعاع التربيعى بالإضافة إلى الاختيار الأمثل لطول النافذة. وتم كذلك دراسة مقارنة بين هذه الوسيلة المطورة وبين مشكل الشعاع التقليدي.

Keywords: Array signal processing, Direction-of-arrival estimation, Source localization, Local Polynomial Approximation, LPA estimator

1. Introduction

Smart antennas are urgently needed in the expanding field of communications. Thus, the problem of beamforming is primarily considered. Recently, there has been considerable interest in developing efficient algorithms for tracking the Direction-Of-Arrival (DOA) of multiple moving targets. As well, the DOAs at any time can be estimated by the method of nonlinear least squares [1, 2]. Aiming to improve the tracking performance, Rao et al. [2] introduce the dynamic model governing the motion of different objects. This enables to predict the state (position, velocity, and acceleration) of each object in any time interval using the estimated state in the previous interval. With the assumption that the DOA change is negligible within each interval.

Since the source motion may severely vary with time within the observation interval as in many source tracking applications, this assumption appears to be nonoptimal because it may lead to a very poor tracking performance. To relax this situation, Elkamchouchi et al. [3] and Ashour et al. [4] track the motion of the accelerated targets within a sliding window using quadratic Local Polynomial Approximation (LPA) beamformer. This beamformer is a modification of the linear LPA beamformer presented by Katkovnik and Gershman [5,6]. The quadratic LPA beamformer exploits the third term in source motion expansion using Taylor series. This term expresses the sources acceleration which decreases the MSE of the angle estimation.

Since the quadratic LPA beamformer is quite efficient for beamforming and DOA estimation of accelerated sources, applying it to cubic array geometry becomes vital requirement as a generalization. The cubic array is used to scan the main beam toward any point in space. Indeed, it produces high LPA function value which improves the beamformer performance.

A model for the output signal of the receiving sensor array will be derived. Additionally, asymptotic expressions for the bias and variance of the LPA beamformer are also achieved.

2. Problem formulation

Consider a cubic array of $n = MM \times NN \times MN$ sensors with equidistant interelement spacing $d_x = d_y = d_z = d$. This cubic array represent an example of 3D array applied to track an incoming signal which is characterized by the vector of DOA parameters $\psi = [\varphi, \theta]^T$, where φ and θ are azimuth and elevation, respectively. The p^th^ sensor is located at (x_p, y_p, z_p) for $p = 0,1,2,\ldots,n-1$. The time delay can be defined as:

$$\tau_p = \frac{(x_p \cos\varphi \sin\theta + y_p \sin\varphi \sin\theta + z_p \cos\theta)/v_0}{v_0} \quad (1)$$

Without loss of generality, let the center of the coordinate system be located at the phase center, i.e. at the axes origin (0,0,0).

Assume that q narrowband signal sources, $s_1(t),\ldots, s_q(t)$, located in the far-field of the array, impinge on the cubic array from distinct unknown directions, $(\varphi_i(t), \theta_i(t))$, $i = 1,2,\ldots,q$ where $-\pi \le \varphi_i \le \pi$, $0 \le \theta_i \le \pi/2$. The array observation vector at the output of the array will have the following form:

$$r(t) = A(\varphi(t), \theta(t))s(t) + e(t)\,, \quad (2)$$

where,

$$A(t) = [a(\varphi_1(t), \theta_1(t)), a(\varphi_2(t), \theta_2(t)), \ldots, a(\varphi_q(t), \theta_q(t))], \quad (3)$$

is the $n \times q$ time-varying direction matrix, $s(t)$ is the $q \times 1$ vector of the source waveforms and $e(t)$ is the $n \times 1$ vector of sensor noise. Using the assumption that the noise is white zero-mean Gaussian random process with a variance of σ^2, it could be shown that the $n \times 1$ steering vector may be written as:

$$a(\varphi_i, \theta_i) = \begin{bmatrix} 1 \\ \exp\{-j\frac{2\pi}{\lambda} d(x_1 \cos\varphi_i \sin\theta_i + y_1 \sin\varphi_i \sin\theta_i + z_1 \cos\theta_i)\} \\ \ldots\ldots \\ \ldots\ldots \\ \exp\{-j\frac{2\pi}{\lambda} d(x_{n-1} \cos\varphi_i \sin\theta_i + y_{n-1} \sin\varphi_i \sin\theta_i + z_{n-1} \cos\theta_i)\} \end{bmatrix} \quad (4)$$

The main goal is to apply the LPA beamformer for estimating the DOAs of the sources and to locate them correctly.

To exploit the sources acceleration, expand the source motion within the observation interval using Taylor series as follows:

$$\varphi(t+kT) = \varphi(t) + \varphi^{(1)}(t)(kT) + \frac{\varphi^{(2)}(t)}{2}(kT)^2 + \frac{\varphi^{(3)}(t)}{6}(kT)^3 \ldots = z_0 + z_1 kT + z_2 (kT)^2 + z_3 (kT)^3 \ldots \quad (5)$$

$$\theta(t+kT) = \theta(t) + \theta^{(1)}(t)(kT) + \frac{\theta^{(2)}(t)}{2}(kT)^2 + \frac{\theta^{(3)}(t)}{6}(kT)^3 + \ldots = l_0 + l_1 kT + l_2 (kT)^2 + l_3 (kT)^3 + \ldots \quad (6)$$

Here, T is the sampling interval and $k = 0,1,\ldots,N-1$, where N is the number of snapshots. Assuming that the observation window is sufficiently short, therefore, the fourth and the later terms in the previous two equations are negligible. So, the source motion can be expressed in the form [3, 4]:

$$\varphi(t+kT) = z_0 + z_1 kT + z_2 (kT)^2$$
$$\theta(t+kT) = l_0 + l_1 kT + l_2 (kT)^2\,, \quad (7)$$

$$z_0 = \varphi(t)\,,\ z_1 = \varphi^{(1)}(t)\,, z_2 = \varphi^{(2)}(t)/2$$
$$l_0 = \theta(t)\,,\ l_1 = \theta^{(1)}(t), l_2 = \theta^{(2)}(t)/2\,, \quad (8)$$

where (z_0, l_0), (z_1, l_1) and (z_2, l_2) are the instantaneous source DOA, angular velocity and acceleration, respectively. Therefore, the problem is to estimate $c = (z + 1)$ from the nonstationary array observation vector $\mathrm{r}(t)$. Where,

$$z = (z_0, z_1, z_2)^T,\ 1 = (l_0, l_1, l_2)^T, \mathrm{c} = (c_0, c_1, c_2)^T. \tag{9}$$

According to eqs. (5) and (6), the source trajectories are assumed to be arbitrary functions of time which belong to the nonparametric class of piecewise continuous α-differentiable functions:

$$F_\alpha = \{\left|\varphi^{(\alpha)}(t)\right| \le L_\alpha,\ \varphi^{(\alpha)}(t) = \frac{d^\alpha \varphi(t)}{dt^\alpha},$$
$$FF_\alpha = \{\left|\theta^{(\alpha)}(t)\right| \le LL_\alpha,\ \theta^{(\alpha)}(t) = \frac{d^\alpha \theta(t)}{dt^\alpha}. \tag{10}$$

The 'piecewise' assumes that a small number of discontinuities in the functions $(\varphi^{(\alpha)}(t), \theta^{(\alpha)}(t))$ or their derivatives can exist on observation interval. In fact, the local expansion is applied in order to calculate the estimate for a single time-instant t only. For the next time-instant the calculations should be repeated.

3. LPA beamformer

Use the weighted least squares approach to formulate the LPA beamformer for a single source which can be extended to the multiple sources case. Minimize the following LPA function [5,6]:

$$G(t, \mathrm{c}) = \frac{1}{\sum_k \omega_h(kT)} \sum_k \omega_h(kT)$$
$$\left\|\mathrm{r}(t+kT) - \mathrm{a}(\mathrm{c}, kT)s(t+kT)\right\|^2, \tag{11}$$

where $\|.\|$ stands for the norm. The summation interval in (11) is determined by the window function $\omega_h(kT)$ and the dependence of $\mathrm{a}(\varphi, \theta)$ is expressed via the vector c and the time kT. The window function is given [7].

$$\omega_h(kT) = (\frac{T}{h})\omega(\frac{kT}{h}), \tag{12}$$

where $\omega(v)$ is a real symmetric function $[\ \omega(v) = \omega(-v)\ ]$ satisfying the conventional properties;

$$\omega(v) \ge 0, \quad \omega(0) = \max_v \omega(v), \quad \int_{-\infty}^{\infty} \omega(v)dv = 1, \tag{13}$$

and the scaling parameter h determines the window length.

Minimize (11) with respect to the unknown deterministic waveform $s(t+kT)$. Thus,

$$\frac{\partial G}{\partial s^*(t+kT)} = \frac{-\omega_h(kT)}{\sum_k \omega_h(kT)} \mathrm{a}^H(\mathrm{c}, kT),$$
$$\{\mathrm{r}(t+kT) - \mathrm{a}(\mathrm{c}, kT)s(t+kT)\} = 0. \tag{14}$$

The estimate of the waveform $s(t+kT)$ is obtained as:

$$\hat{s}(t+kT) = \frac{\mathrm{a}^H(\mathrm{c}, kT)\mathrm{r}(t+kT)}{n}, \tag{15}$$

where the property $\mathrm{a}^H(\mathrm{c}, kT)\mathrm{a}(\mathrm{c}, kT) = n$ is exploited. Inserting (15) into (11), the following function is obtained.

$$G(t, \mathrm{c}) = \frac{1}{\sum_k \omega_h(kT)} \sum_k \omega_h(kT)$$
$$\left\{\mathrm{r}^H(t+kT)\mathrm{r}(t+kT) - \frac{\left|\mathrm{a}^H(\mathrm{c}, kT)\mathrm{r}(t+kT)\right|^2}{n}\right\}, \tag{16}$$

Eq. (16) should be minimized over the vector parameter c. This is equivalent to the maximization of,

$$P(t,\mathrm{c}) = \frac{1}{n\sum_{k}\omega_h(kT)}\sum_{k}\omega_h(kT)\left|\mathrm{a}^H(\mathrm{c},kT)\mathrm{r}(t+kT)\right|^2, \tag{17}$$

where $|.|$ stands for the absolute value. Let us refer to the function (17) as the quadratic LPA beamformer. Here, the maximization of the LPA function requires 6D search instead of 3D search as in [3] or 2D search as in [5] ,or 1D search as in conventional beamformer using beamforming function in [6]. As in conventional beamformer, the response to multiple sources represent a direct superposition of particular responses to each source. Therefore, the LPA beamformer (17) can be applied to the scenarios with multiple well-separated sources as well.

4. Asymptotic LPA beamforming

The asymptotic analysis of the LPA beamformer for single accelerated moving source is also valid for multiple sources that can have two identical values of parameters (i.e., angle, angular velocity, or acceleration). The following formulas are utilized in order to demonstrate that the window width selection is important for accurate estimation.

Let the estimation error vector be,

$$\begin{aligned}\Delta\mathrm{z} &= (\Delta z_0, \Delta z_1, \Delta z_2)^T \\ &= \hat{\mathrm{z}} - \mathrm{z}\,,\ \Delta\mathrm{l} = (\Delta l_0, \Delta l_1, \Delta l_2)^T = \hat{\mathrm{l}} - \mathrm{l},\end{aligned} \tag{18}$$

where $\hat{\mathrm{z}}$ and $\hat{\mathrm{l}}$ represent the estimate of eq. (9) obtained via the maximization of the LPA-beamforming function. Using the source waveform remain constant within the observation interval, short-time asymptotic, and the superposition principle to solve φ and θ separately. Thereby,

$$\begin{aligned}E\{\Delta\mathrm{c}\} &= E\{\Delta\mathrm{z}\} + E\{\Delta\mathrm{l}\},\ var\{\Delta\mathrm{c}\} \\ &= var\{\Delta\mathrm{z}\} + var\{\Delta\mathrm{l}\}.\end{aligned} \tag{19}$$

Consequently, the biases and variances of the source motion parameters estimation are:

$$E\{\Delta c_0\} = [F_{zz}\varphi^{(4)}(t) + H_l\theta^{(4)}(t)].\ \frac{[(\int\omega(\upsilon)\upsilon^4 d\upsilon)^2 - \int\omega(\upsilon)\upsilon^2 d\upsilon\int\omega(\upsilon)\upsilon^6 d\upsilon]}{[\int\omega(\upsilon)\upsilon^4 d\upsilon - (\int\omega(\upsilon)\upsilon^2 d\upsilon)^2\,]}, \tag{20}$$

$$E\{\Delta c_1\} = [F_{zz}\varphi^{(3)}(t) + H_l\theta^{(3)}(t)].\ \frac{h^2}{6}\frac{\int\omega(\upsilon)\upsilon^4 d\upsilon}{\int\omega(\upsilon)\upsilon^2 d\upsilon}, \tag{21}$$

$$E\{\Delta c_2\} = [F_{zz}\varphi^{(4)}(t) + H_l\theta^{(4)}(t)].\ \frac{[\int\omega(\upsilon)\upsilon^6 d\upsilon - \int\omega(\upsilon)\upsilon^2 d\upsilon\int\omega(\upsilon)\upsilon^4 d\upsilon]}{[\int\omega(\upsilon)\upsilon^4 d\upsilon - (\int\omega(\upsilon)\upsilon^2 d\upsilon)^2\,]}, \tag{22}$$

$$var\{\Delta c_0\} = 0\,, \tag{23}$$

$$var\{\Delta c_1\} = \frac{TJ_1}{h^3}\frac{\int\omega^2(\upsilon)\upsilon^2 d\upsilon}{(\int\omega(\upsilon)\upsilon^2 d\upsilon)^2}, \tag{24}$$

$$var\{\Delta c_2\} = \frac{TJ_1}{h^5}\frac{[\int\omega^2(\upsilon)\upsilon^4 d\upsilon - 2\int\omega^2(\upsilon)d\upsilon\int\omega(\upsilon)\upsilon^4 d\upsilon + \int\omega^2(\upsilon)d\upsilon(\int\omega(\upsilon)\upsilon^2 d\upsilon)^2\,]}{[(\int\omega(\upsilon)\upsilon^4 d\upsilon)^2 - 2\int\omega(\upsilon)\upsilon^4 d\upsilon(\int\omega(\upsilon)\upsilon^2 d\upsilon)^2 + (\int\omega(\upsilon)\upsilon^2 d\upsilon)^4\,]} \tag{25}$$

where,

$$J_1 = \frac{k_z}{\sin^2\theta(t)} + k_{ll},\ \text{and} \tag{26}$$

$$k_z = \frac{\lambda^2(1+\frac{1}{nSNR})}{4(2\pi d)^2 SNR\left[\mathrm{b}_{1z}^H\mathrm{b}_{1z} - \frac{1}{n}\mathrm{b}_{1z}^H\mathrm{a}\mathrm{a}^H\mathrm{b}_{1z}\right]},$$

$$k_{ll} = \frac{\lambda^2\gamma_{ll}}{4(2\pi d)^2|s(t)|^2[\mathrm{b}_{1ll}^H\mathrm{b}_{1ll}\mathrm{a}^H\mathrm{a} - \mathrm{b}_{1ll}^H\mathrm{a}\mathrm{a}^H\mathrm{b}_{1ll}\,]^2}, \tag{27}$$

and the signal-to-noise ratio (SNR) is defined as:

$$SNR = \frac{|s|^2}{\sigma^2} . \quad (28)$$

This proposition role out the following facts: firstly, they clarify an explicit dependence of the achieved accuracy on the array geometry and source parameters. Secondly, the biases of the DOA and acceleration estimate are proportional to the fourth derivatives $\left(\varphi^{(4)}(t), \theta^{(4)}(t)\right)$ in both the azimuth and elevation directions, and therefore, it depends on the source motion parameters. Thirdly, the variances of the angular velocity and acceleration estimates in both directions do not depend on the derivatives of $\varphi(t)$ or $\theta(t)$ at all, but on their angles only. Fourthly, the variances of the LPA beamformer are not affected by a possible source nonstationarity, while its bias is affected.

4. Optimal window size

Considering the conventional case with the rectangular window, therefore

$$\omega(v) = \begin{cases} 1 & , \quad -\frac{1}{2} \le v \le \frac{1}{2} \\ 0 & , \quad otherwise, \end{cases} \quad (29)$$

and $N = \frac{h}{T}$ is the number of snapshots.

To determine the optimal parameter h, the MSE of the estimates should be minimized. Therefore, the optimal parameter h of the angle estimate is found to be $h_{0,opt} = 0$ which has the advantage over $h_{0,opt}$ obtained in [5]. When h tends to zero, the MSE decreases but requires to increase the number of samples.

The optimal window length of the angular velocity estimate which minimizes the MSE is expressed as:

$$h_{1,opt} = [\frac{27TJ_1 \int \omega^2(v) v^2 dv}{\left(\left\{F_{zz}\varphi^{(3)}(t) + H_l \theta^{(3)}(t)\right\} \int \omega(v) v^4 dv\right)^2}]^{1/7} . \quad (30)$$

Indeed, the results presented in eq. (30) clearly deduce that the optimal window size depends on the third derivative of both azimuth and elevation directions. The bias-to-variance tradeoff is:

$$\beta_1 = \frac{|E\{\Delta c_1\}|_{opt}}{\sqrt{var_{opt}\{\Delta c_1\}}} = \sqrt{\frac{3}{4}} . \quad (31)$$

It follows from eq. (31) that the optimal bias-to-variance tradeoff for the angular velocity estimate is independent of the source parameters, irrespective of the array geometry or the source motion model. Also, it corresponds to the situation where the bias squared and the variance have the same order. Additionally, it has the same value obtained when linear LPA beamformers used as has been introduced by Elkamchouchi et al. [8] and quadratic LPA by Elkamchouchi et al. [3] and Ashour et al. [4]. Furthermore, the parameter β_1 is independent of the azimuth-elevation directions or their derivatives.

The optimal window length of the acceleration estimate is expressed as:

$$h_{2,opt} = [\frac{720JT_1(\int \omega(v)v^4 dv - [\int \omega(v)v^2 dv]^2)^2 (\int \omega^2(v)v^4 dv - 2 \int \omega(v)v^4 dv \int \omega^2(v) dv + \{\int \omega(v)v^2 dv\}^2 \int \omega^2(v) dv)}{\left\{F_{zz}\varphi^{(4)}(t) + H_l \theta^{(4)}(t)\right\} \{\{\int \omega(v)v^6 dv - \int \omega(v)v^2 dv \int \omega(v)v^4 dv\}\}^2}]^{1/9} . \quad (32)$$

It is clear that the optimal window size $h_{2,opt}$ depends on the fourth derivative of both the azimuth and elevation directions. The bias-to-variance tradeoff for the acceleration estimate is presented as:

$$\beta_2 = \frac{|E\{\Delta c_2\}|_{opt}}{\sqrt{var_{opt}\{\Delta c_2\}}} = \sqrt{\frac{5}{4}} [\int \omega(v) v^4 dv - (\int \omega(v) v^2 dv)^2] . \quad (33)$$

It follows that the optimal bias-to-variance tradeoffs for both the angular velocity and

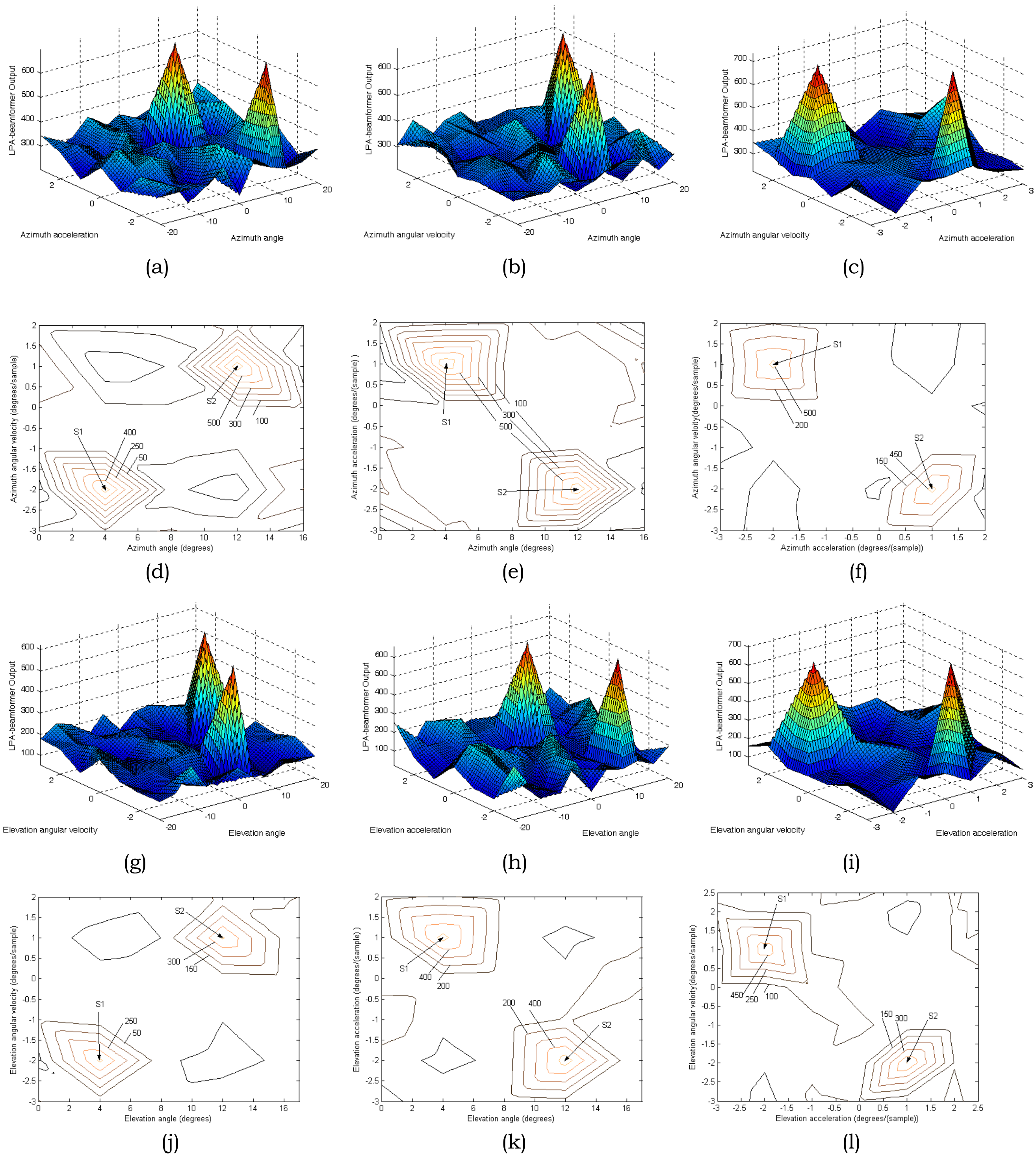
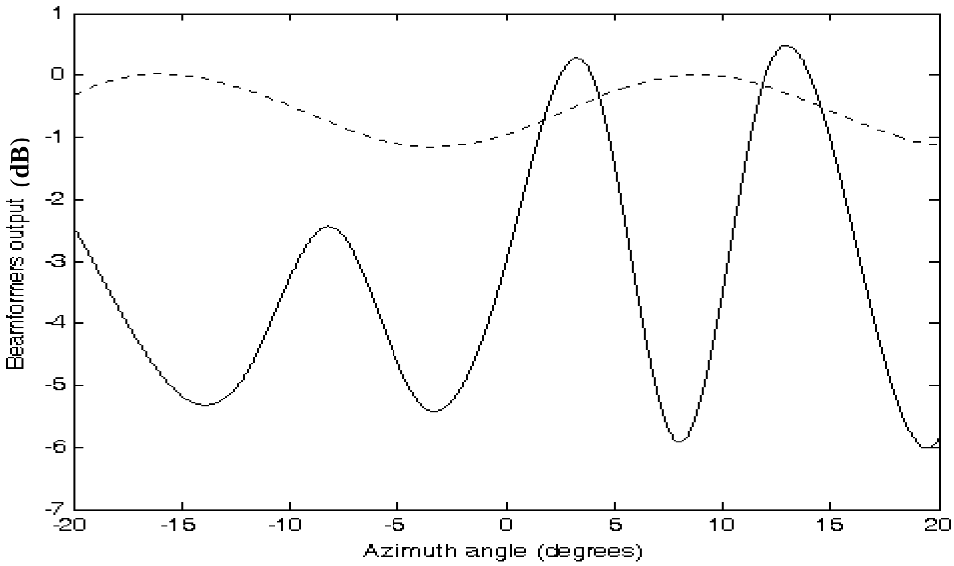
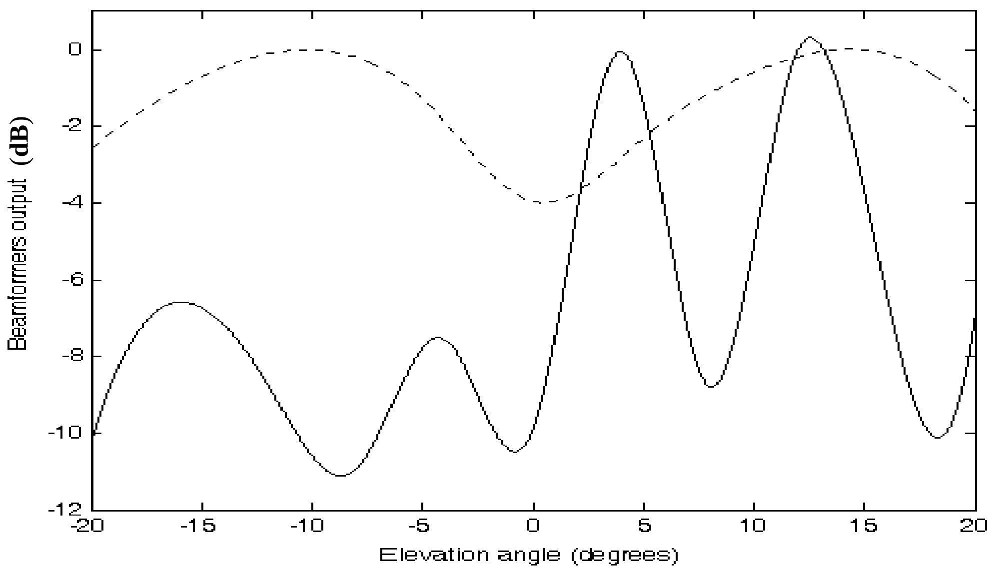


Fig. 1. The output of the quadratic LPA beamformer for the well separated sources (case 1) in pairs as surface and contour plots, (a-f), for azimuth direction, (g-l), for elevation direction.



(a)



(b)

Fig. 2. Comparison between the 1D output of the LPA beamformer (solid curve) and the conventional beamformer (dotted curve) for the well separated sources (case 1), (a), for azimuth direction, (b) for elevation direction.

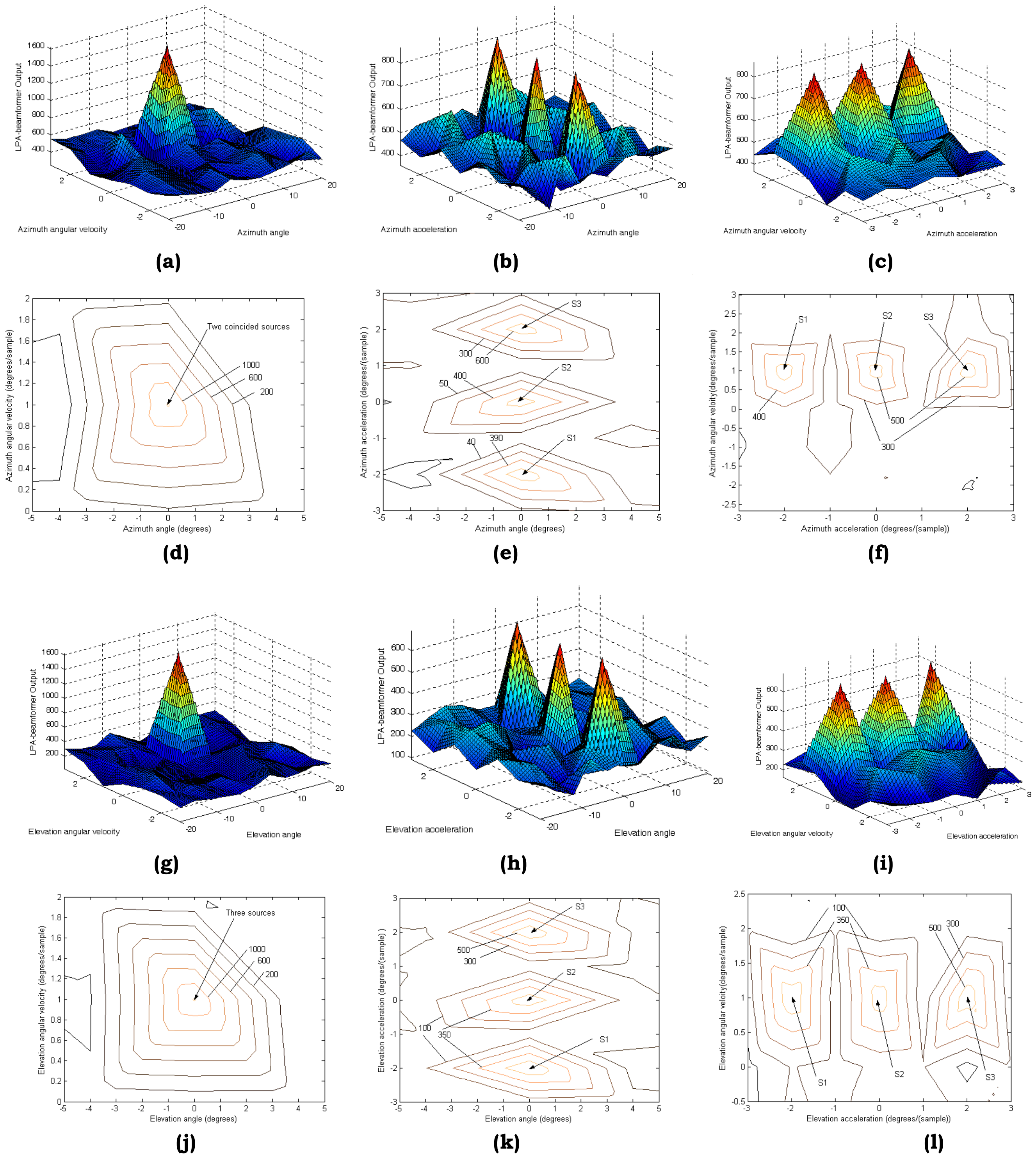


Fig. 3. The output of the quadratic LPA beamformer for three sources have the same angle and velocity (case 2) in pairs as surface and contour plots, (a-f), for azimuth direction, (g-l), for elevation direction.

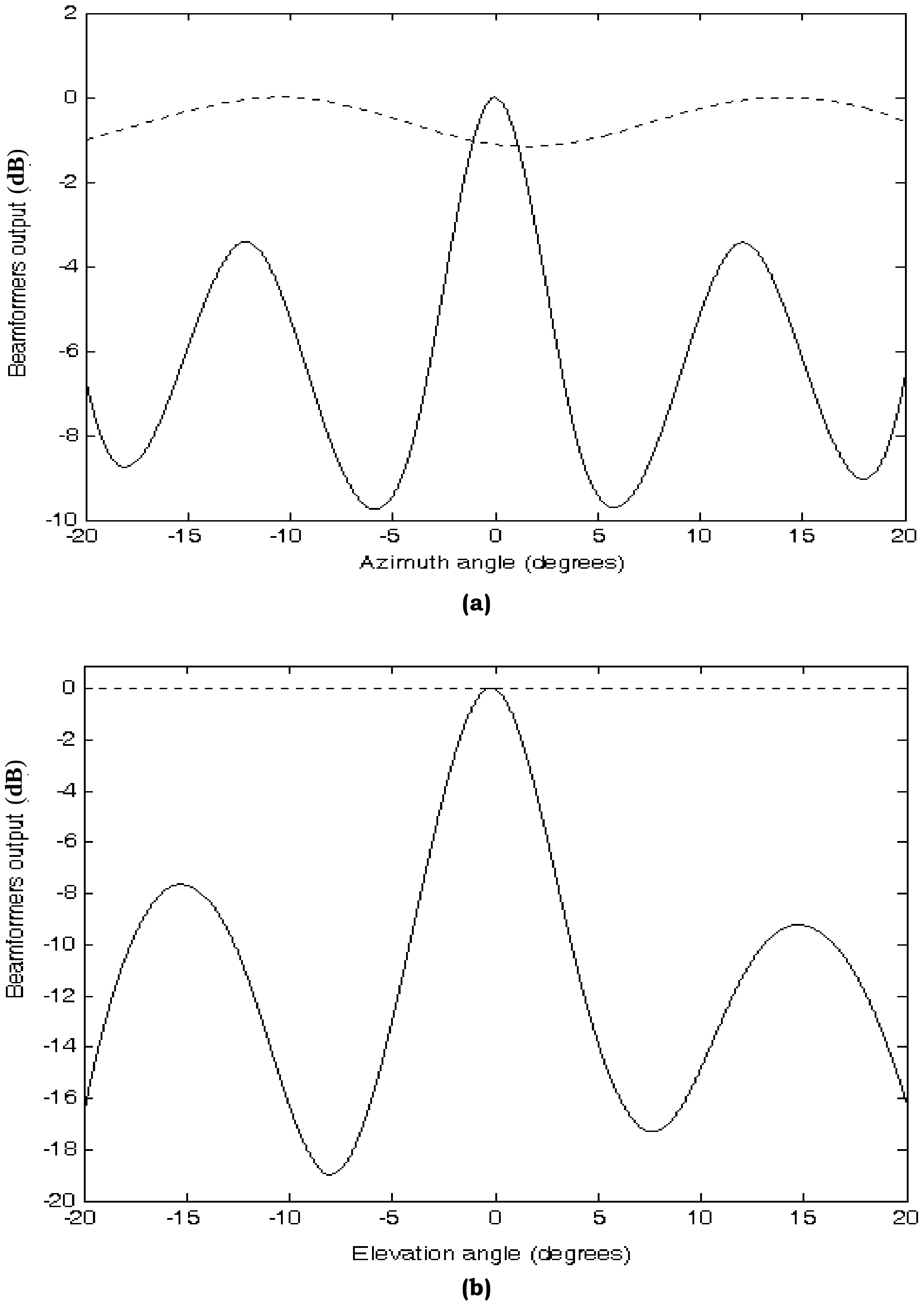


Fig. 4. Comparison between the 1D output of the LPA beamformer (solid curve) and the conventional beamformer (dotted curve) for three sources have the same angle and velocity (case 2), (a), for azimuth direction, (b), for elevation direction.

acceleration estimates are independent of the source parameters. They correspond to the situation where the bias squared and the variance have the same order, but in the acceleration case the bias-to-variance tradeoff depends on the window parameters.

5. Simulation results

Assume a cubic array of n = 3× 3 × 3 = 27 sensors spaced half-wavelength apart, and uncorrelated moving sources with $SNR = 10dB$ in a single sensor. The rectangular window with $N = 20$ snapshots is considered in the following cases.

Case 1:

Consider two sources located at the following directions:

$\varphi_1(k) = \theta_1(k) = 4 - 2^\circ k + 1^\circ k^2$, and

$\varphi_2(k) = \theta_2(k) = 16^\circ + 1^\circ k - 2^\circ k^2$.

The two sources are well separated where the separation between their angles are sufficient. Accordingly, the LPA beamformer can distinguish and localize the correct sources location as illustrated in fig. 1. Whereas the conventional beamformer cannot distinguish or localize them, indicating that the LPA superimpose the conventional beamformer as shown in fig. 2.

Case 2:

Consider three sources that have the same angle and angular velocity as (0^o, 1^o /sample) in both directions and their accelerations, respectively, are: [(-2^o, 0^o, 2^o)/ (sample)^2]. Note that, the abbreviation (sample)^2 is used in the figures to refer to (sample)^2. Fig. 3 shows that the LPA beamformer can distinguish between through their different accelerations; a phenomenon that is completely abolished in both the linear LPA and the conventional beamformers where they cannot resolve this case. Consequently, the source acceleration isexploited to improve the source localization in nonstationary situations. And to resolve sources that have the same angle and angular velocity, where the uniform velocity LPA cannot resolve it. At the same time, the conventional beamformer is neither able to exploit the source motion nor to resolve closely spaced sources. In addition, the 1D representation of the quadratic LPA versus the angles does not resolve the sources where they have the same angles but indicates the correct angles location as clear in fig. 4.

6. Conclusions

This article propose a modified nonparametric approach based on quadratic LPA beamformer of time-varying DOA for angle, angular velocity, and acceleration estimations. By exploiting the acceleration of the moving source, the MSE of the angle estimation is decreased with small window size which improve the LPA beamformer performance. Also, the values of the LPA function is very high using cubic arrays compared to the corresponding values when using planar array [4] or ULA as in [3]. The variance and the bias of the beamformer estimates are derived for short-time (h= cost, $T \to 0$) asymptotic behavior. The related velocity part of these expressions has a relationship with that of the uniform velocity LPA. The simulations show that the nonparametric technique can resolve closely spaced sources. Taking into considerations that their accelerations are sufficiently different even if they have the same velocity and angle of arrival.

References

[1] C.R. Sastry, M. Simaan, and E.W. Kamen, "An Efficient Algorithm for Tracking the Angles of Arrival of Moving Targets," IEEE Trans. Acoust., Speech, Signal Processing, Vol. 39, pp. 242-246 (1991).

[2] C.R. Rao, C.R. Sastry, and B. Zhou, "Tracking the Direction of Arrival of Multiple Moving Targets," IEEE Trans. on Signal Processing, Vol. 42 (5), pp. 1133-1144 (1994).

[3] H. Elkamchouchi, M.E. Nasr and A.S. Ashour, "Modified LPA beamformer for localizing and tracking rapidly accelerated moving sources," 21th National Radio Science conference (NRSC), B4 (2004).

[4] A.S. Ashour, H. Elkamchouchi and M.E. Nasr "Planar Array for Accelerated Sources Tracking Using Local Polynomial

Approximation Beamformer," 2004 IEEE AP-S/URSI Symposium, Vol. 1, pp. 431-434 (2004).

[5] V. Katkovnik and A.B. Gershman, "Performance Study of the Local Polynomial Approximation Based Beamforming in the Presence of Moving Sources," IEEE Trans. on Antennas and Propagation, Vol. 50, pp. 1151-1157 (2002).

[6] V. Katkovnik and A.B. Gershman, "A Local Polynomial Approximation Based Beamforming for Source Localization and Tracking in Nonstationary Environments," IEEE Signal Processing Lett., Vol. 7, pp. 3-5 (2000).

[7] V. Katkovnik, "Discrete-Time Local Polynomial Approximation of Instantaneous Frequency," IEEE Trans. on Signal Processing, Vol. 46, pp. 2626-2637 (1998).

[8] H. Elkamchouchi, M.E. Nasr and A.S. Ashour, "Planar array for signal tracking using sliding window approach," 21th National Radio Science conference (NRSC), B10 (2004).

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