Motion planning for autonomous mobile robots

Mahmoud Wahdan
Ministry of Communications & Information Technology, Cairo, Egypt
mwahdan@mcit.gov.eg

This paper describes a motion planning algorithm for an autonomous robot. The motion planning problem of a rigid body vehicle is divided into three subproblems in this algorithm: (i) decomposing a given free space into a finite number of simple shaped regions in such a way that the following two subproblems become natural and easy (we name this region as "W-region"), (ii) given a start and a goal configuration, planning an optimal global path using the region decomposition, and (iii) planning a detailed motion from the start position/ orientation to the goal using the aforementioned global path. In this algorithm, the most important part is the first one, that is, how to decompose a world into simpler regions. The method proposes several new concepts including borders and directed-borders. The planned path in part (ii) is a sequence of W-regions, which in turn specifies a sequence of directed borders belonging to W-regions. In part (iii), one of the significant problems is how to design a safe and smooth path inside a W-region. This is uniformly solved by the steering function method.

تشرح هذه الورقة خوارزم تخطيط مسار الحركة لروبوت (إنسان آلي) ذاتي الحركة. وتنقسم مشكلة تخطيط مسار الحركة لمركبة ذات جسم صلب إلى ثلاث مشكلات فرعية في هذا الخوارزم: (١) تجزئة مساحة خالية معينة إلى عدد محدد من المناطق ذات أشكال هندسية بسيطة بحيث تصبح المشكلتان التاليتان طبيعيتان وسهلتان (نطلق على هذه المنطقة اسم "منطقة و"). (٢) تخطيط المسار الكلى الأمثل باستخدام تجزئة المنطقة، مع وجود إحداثيات نقطتي البداية والهدف. (٣) تخطيط مسار حركة مفصل من وضع البداية/التوجيه إلى الهدف باستخدام المسار الكلى المذكور أعلاه. وفي هذا الخوارزم، يعد أهم جزء هو الجزء الأول، أي كيف يتم تجزئة عالم إلى مناطق أكثر بساطة. وتقدم الطريقة عدة مفاهيم جديدة منها الحدود والحدود الموجهة. والمسار المخطط في الجزء (٢) هو عبارة عن متتابعة مناطق من النوع "و"، التي بدورها تحدد متتابعة من الحدود الموجهة التي تنتمي إلى مناطق "و". وفي الجزء (٣) واحدة من أهم المشكلات هي كيفية تصميم مسار آمن وممهد داخل منطقة "و". ويتم حل هذه المشكلة بطريقة دالة التسيير.

Keywords: Global path planning, Local motion planning, Borders, Convex decomposition, Steering function method

1. Introduction

The problem of motion planning for rigid body robots has been considered one of the most difficult theoretical problems in robotics and, obviously, must be solved for a robot to perform real world tasks such as mine searching and processing.

For an autonomous vehicle, planning motions which avoid known and unknown objects in its environment is the most fundamental functionality. Given an arbitrary mission, for instance, mine searching and clearance, motion planning is an inevitable subproblem that needs to be solved. We use motion planning rather than path planning, because vehicles we are dealing with are not points, but rigid bodies.

Several concepts and theories have been developed which may lead to solving the motion planning problem. The configuration space approach is one global motion planning method using the concept of the vehicle configuration (x, y, θ) [1]. The artificial potential field method is a path planning method for a point robot. [2]. The Voronoi diagrams method is one of a robot path planning [3].

The other global motion planning ideas can be found in other research reports. Some of these focuses on the motion planning for manipulators [4] and others provide general ideas [5-10].

However, the author considers that the motion planning problem for a rigid body robot must be divided into at least two subproblems: The global path planning problem and detailed motion planning problem. The first is the problem of finding the best path class in terms of homotopy [11]. In that sense, this level is an abstract portion of the whole problem. The second is the problem of finding the best rigid body motion when a path class is defined by the first subproblem. We call this method a layered motion planning.

In order to make this approach possible, we also claim that an appropriate decomposition of the free space in the world is needed. Several decomposition methods have been proposed [12-15]. Instead of using one of these methods, we are proposing a new one which we name as the W-decomposition method. We divide the free space into a finite number of W-regions, each of which is a convex polygonal patch (see section 2).

Based on the *W*-decomposition method, the two layers are clearly defined and, as a result, each problem becomes much simpler than the original motion planning problem. The global path planning subproblem becomes a problem of finding the best sequence of W-regions, or equivalently, the best sequence of directed borders. The detailed motion planning subproblem becomes a problem of motion planning in a *W*-region given two directed borders, or one directed-border and one configuration. In this part, the steering function method [16] is used to make planned motion smooth.

2. W-decomposition

2.1. W-regions and W-decompositions

In this paper, a world W is;

$$W = \{B_0, B_1, \cdots, B_n\}, \tag{1}$$

consists of n simple polygons (holes) and another simple polygon B_0 which defines the outmost boundary. The free space of W is the inside of B_0 minus the union of the other n polygons inside it and is denoted by free (W) fig. 1. A polygon

$$R = R(v_1, \cdots, v_n), \qquad n \geq 3,), \tag{2}$$

defined by its vertices v_is, is called a *W*-region in W if (i) the interior of R lies completely in the free space free (W), (ii) all the vertices are on the boundary of the world W, and (iii) R is convex. Each portion of the boundary of a *W*-region R lies on the boundary of the world or in its free space. A physical boundary is the intersection of the boundary of R and the boundary of the world. A portion of the boundary of R is called a border if it lies in the free space of the world. Thus the union of all the physical boundaries and borders is equal to the boundary of R. Because of the condition (ii) above and the fact that the *W*-regions are polygons, a border of R is always a line segment.

A set $\{R_1, R_2, \ldots, R_q\}$ of *W*-regions is called a *W-decomposition* of W if the union of the q *W*-regions covers the free space free (W) and the intersection of an any pair (R_i, R_j) of the *W*-regions is the empty set except their borders. Fig. 1 shows an example of world. The white area indicates free space and the shaded areas indicate obstacles. Fig. 2 illustrates one of its *W*-decompositions. The broken lines in the figure are borders (A *border* in a world W is a straight line segment L, where (i) its both endpoints are on the boundary of the world, and (ii) the open segment L is a subset of the free side free(W) of the world). It is obvious that there is at least one *W*-decomposition for an arbitrary closed world. Furthermore, the way of decomposing a world into *W*-regions is not unique.

2.2. Representation of path classes

A *path* in W is a continuous function $f : [0, 1] \rightarrow \text{free}(W)$. The two points $f(0)$ and $f(1)$ are called its endpoints. If they are distinct, we usually denote $f(0)$ as a *start* s and $f(1)$ goal g. we assume that f is rectifiable (its length is finite). Two path f and f' with the same endpoints are said to be homotopic if f can be continuously transformed into f' without moving both endpoints and without running over any polygon. If f and f' are homotopic, we write $f \cong f'$ [6]. In fig. 3, $f_1 \cong f_2$ and $f_3 \cong f_4$. The relation $\cong$ is an equivalence relation and defines equivalence classes of

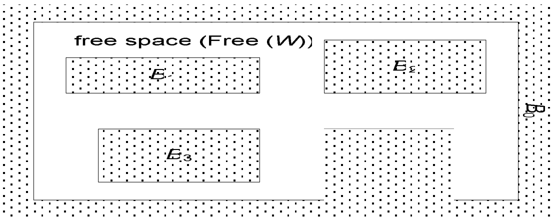

Fig. 1. A rectilinear polygonal world.

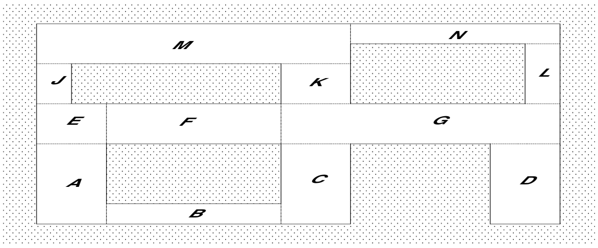

Fig. 2. *W*-Decomposition.

paths that share the same endpoints. These equivalence classes are called path classes in this paper.

$$P = (R_{i_1}, \cdots, R_{i_n}), \qquad n \geq 1. \tag{3}$$

Now, we consider the problem of finding an "optimal" path from a start configuration q_s to a goal configuration q_g in a *W*-decomposed world. A path class connecting q_s and q_g is a sequence of *W*-regions, where (i) R_{ij} and R_{ij+1} are adjacent for all $j \in [1, n-1]$, (ii) a specific *W*-region occurs in the sequence at most one time, and (iii) q_s and q_g belong to R_{i1} and R_{in} respectively. In a special case, where q_s and q_g belong to the same *W*-region, $n = 1$ and $R_{i1} = R_{in}$. An example of a path class for a given *W*-decomposition (shown in fig. 4) is

$P = (A, E, F, G, K, M, N)$.

A path class corresponds to a distinct homotopy class [17].

2.3. Graph representation

Given a *W*-decomposition of a world, its basic connectivity graph $G = (V, E)$ is defined as follow: V is a set of *nodes* or *vertices* and E a set of *edges*. Each *W*-region is considered as a node of G, and each border is considered as an edge in G. To each border β, we associate

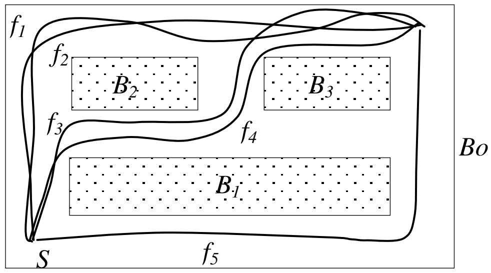

Fig. 3. Path classes.

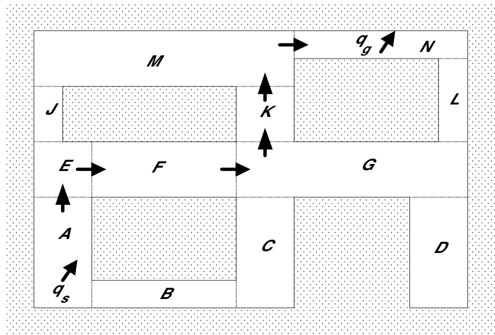

Fig. 4. Example of a path class.

two directed borders, β^+ and β^-. Assume that a border β belongs to two *W*-regions R_{i1} and R_{i2}. If β^+ is an edge from R_{i1} to R_{i2}, then β^- is an edge from R_{i2} to R_{i1}; and vice versa. Thus, the edges β^+ and β^- are said to be complementary. Fig. 5 shows the corresponding connectivity graph of the *W*-decomposition given in fig. 2.

3. Finding optimal path class

In order to find an optimal path class given two configurations, (q_s, q_g), a modified Dijkstra's algorithm is used. The algorithm uses a minimum heap to achieve an $O((V + E) \log E)$ complexity. We take both position and orientation of the initial and final configurations into consideration when performing the search for the minimum cost path class. The cost for edge is computed, for instance, as the energy (or time) needed to move the vehicle from one region to another with appropriate vehicle directions. In this cost evaluation, the cost for both transnational and rotational motions should be included. Relaxation of edges involving the start and the goal regions uses a cost determined by a bi-directional path generation algorithm. This algorithm results in achieving a better approximation of the total cost of the path class by considering

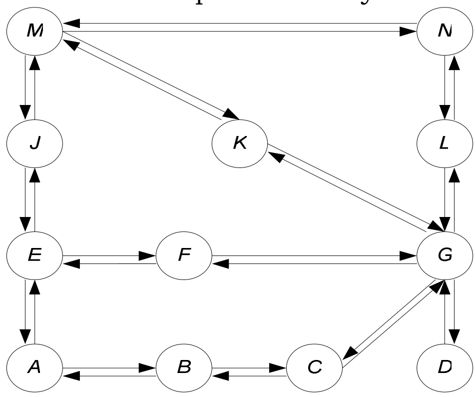

Fig. 5. Connectivity graph of *W*-decomposition.

the maneuvers conducted in the initial and final motions due not only to position but also to the orientation of (q_s, q_g).

4. Detailed motion planning

4.1. From global path plan to local motion plans

When a global path plan $P = (R_{i1},\ldots, R_{in})$ is obtained, we proceed to detailed motion planning. Specifying a path class does not specify any detailed motion plan, yet, we still need to specify exact motion plans in each W-region, R_{ij}. In this Section, we assume that $n \geq 2$, i.e., the start and goal belong to distinct W-regions. We call the motion planning task in R_{i1} *initial portion motion planning*, the ones in R_{ij}s for $2 \leq j \leq n - 1$ mid-portion motion planning, and the one in R_{in} final portion motion planning. We would like to make each planning task inside a region as independent as possible from other tasks. Unfortunately, these individual tasks cannot be independent in some cases.

4.2. Mid-portion motion planning

The fact that each *W*-region is a convex polygon makes the mid-portion motion planning simpler and straightforward. However even this problem has complex aspects. The input to this problem is one *W*-region and two borders belonging to it, one for entrance (border) and one for exit (border) of the *W*-region. Naturally, the purpose is to plan a safe and "reasonable" motion connecting the two borders. In this Section, we assume the world and its *W*-decomposition are both rectilinear, i.e., all edges in the world and all borders of the *W*-decomposition are both orthogonal.

Basically, there are three major types on the positioning of the entrance and exit borders to this *W*-region: (i) the entrance and exit borders are parallel fig. 6. (ii) The entrance and exit borders are on edges which share the same vertex of the *W*-region (As a special case, the entrance and exit share the same vertex of the *W*-region.) fig. 7. (iii) The entrance and exit borders are on the same edge of the *W*-region fig. 8.

Fig. 6. A *W*-region with parallel borders.

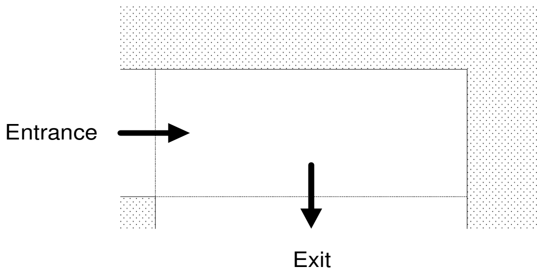

Fig. 7. A *W*-region with borders s sharing a vertex.

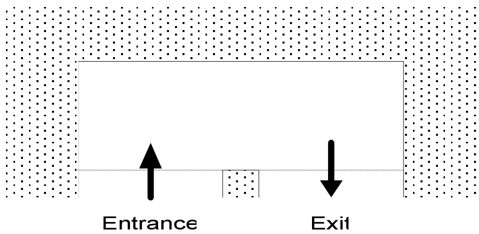

Fig. 8. A *W*-region with borders on the same edge.

At this moment we simplify the motion planning in this *W*-region independently of the situation in the neighboring *W*-region. In each positioning categorized as (i) to (iii), the fundamental planning algorithms are distinct.

1. If the entrance and exit borders are parallel, the vehicle motion should be basically a straight-path-following behavior, where the path is perpendicular to the borders and crosses the exit border. If there is at least one physical boundary parallel to the path, the wall following behavior can be used with sonar.

1. If the entrance are exit borders are parallel, the velicle motion should be basically a straight-path-following behavior, where the path is perpendicular to the borders and crosses the exit border. If there is at least one physical boundary parallel to the path, the wall following behavior can be used with sonar.

2. If the entrance and exit borders are on edges of the *W*-region which share the same vertex of the *W*-region, the motion should include a right angle turn to the left or right. If the borders share the same vertex, the motion is a *vertex-following* type one. If not, the motion may be a combination of path-following and vertex-following.

3. If the entrance and exit are on the same edge of the *W*-region, the behavior will be a simple u-turn type or a combination of vertex-following, path-following and another vertex-following.

We achieve this planning by using the steering function. In fig. 9, a vehicle $q = (p, \theta, k)$ is supposed to track a directed line L where k is the path curvature. It is obvious that we need a feedback about the vehicle heading θ and a damping factor, therefore the steering function for this problem must include both terms. In addition, we need a new feedback term for the "positional error". We propose a steering function,

$$\frac{dk}{ds} = -(ak + b(\theta - \theta_1) + c\Delta d), \quad (4)$$

where, a, b, and c are positive constants, Δd is the distance between p and L with a sign (+ or -), and $d\kappa/ds$ is the derivative of the curvature. If the vehicle is on the left side of the directed line, $\Delta d > 0$ and if it is on the right side of the line, $\Delta d < 0$. Eq. (4) works as negative feedback for the vehicle's trajectory with the positive constants a, b, and c. For more details see [16].

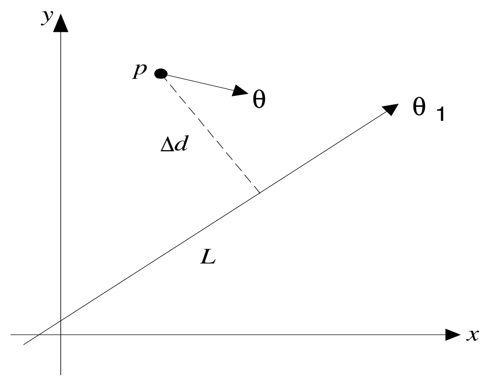

Fig. 9. Principle of path tracking.

4.3. Initial/final portion motion planning

The planning techniques needed for mid-portion motion planning are effective for the end portion planning problem because this is related to one border. However, we need theories which were not needed before:

- For the final portion, the vehicle must be stopped at the exact configuration. This task is far more difficult than the one to follow a path or follow a vertex (in these behaviors, where it should be precisely on the path is usually not specified).
- A bi-directional path generation algorithm provides a method to conduct the final parking maneuver.
- Backing up may be needed if the initial or final orientation is not favorable with respect to the next or previous border to be crossed.
- Collision checking is critical in determining whether a backing up motion is needed.

5. Simulation result

The motion planning algorithm described in this paper (section 4) is being tested and evaluated by simulation. Consider the problem of finding a path from a start configuration, q_s, to a goal configuration, q_g in a polygonal world W fig. 4. It is desired to connect the start configuration, q_s, to a goal configuration, q_g, using a continuous, smooth path. One of the path classes is shown in fig. 4. Fig. 10 shows the result of motion planning and execution from q_s to q_g where the initial configuration of the vehicle is q_s = (20, 50), $\pi/2$, 0). Fig. 11 shows another result of motion planning and execution from q_s to q_g.

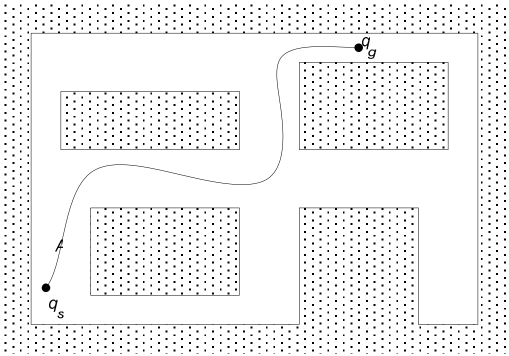

Fig. 10. Simulation result (I).

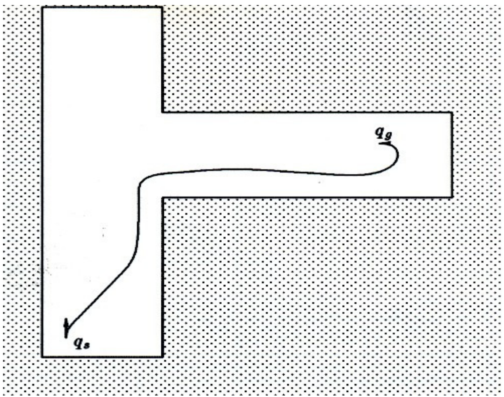

Fig. 11. Simulation result (II).

6. Conclusions

This paper gives answers to the following questions in robotics.

Q1: How to symbolically represent path classes in a polygonal world?

Q2: How to find the "optimal" path class for a given pair of robot configurations.

Q3: How to plan a detailed motion from the start position/orientation to the goal using the aforementioned global path in *Q2*.

The results are:

1. The set of "border sequences" in a "homotopically decomposed world" is an answer to *Q1*.
2. The "connectivity graph" of a homotopically decomposed world is an answer to *Q2*. This method takes a $O((V + E) \log E)$ complexity.
3. Convex decompositions (special kind of homotopic decompositions) are generally more useful in these tasks.
4. Planning the optimal motion of the vehicle at each point directly from the information of the world by computing the steering function $d\kappa/ds$ without calculating the track to follow is an answer to *Q3*.

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