Modeling of rectangular submerged hydraulic jumps

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Submerged hydraulic jump is one of the most interested and encountered phenomenon that formed downstream of single vent regulators. The basic characteristics of these jumps were covered well in literature both experimentally and theoretically. This paper presents a simplified solution based on the application of the 1-D momentum equation. The results of the developed equation were compared to those available in the literature using the results of the experimental data obtained by the present study. Similar models are developed for the energy loss. Also, empirical models are developed for the basic characteristics of this type of jumps based on the present showy experimental data.

تتكون القفزة الهيدروليكية المغمورة خلف المنشآت الهيدروليكية وهى من الظواهر الهامة التى حظيت باهتمام الباحثين فى كل أنحاء العالم. فى هذه الورقة العلمية تم استخدام المعادلات الهيدروليكية الاساسية (معادلة الاستمرارية، معادلة كمية الحركة، معادلة الطاقة) لاشتقاق معادلات دقيقة ومبسطة لحساب العمق النسبى (أونسبة الغمر) والفاقد فى الطاقة بدلالة متغيرات السريان مثل رقم فرويد. وقد تم التأكد من صحة ودقة المعادلات المشتقه بمقارنتها بالقياسات المعملية الحالية. وتتميز المعادلات المشتقة (لاسيما معادلة العمق النسبى) بأنها أبسط من معادلة جوفيندا وراجراتنام وادق من معادلة العزيزى وذلك في مجال البيانات المعملية الحالية. هذا وقد استخدمت البيانات المعملية التى تم الحصول عليها من الدراسة الحالية فى التوصل الى نماذج احصائية لحساب خصائص القفزات الهيدروليكية المغمورة فوق القيعان الملساء.

Keywords: Submerged hydraulic jump, Stilling basins

1. Introduction

Submerged hydraulic jump has attracted many researchers one of the poineer studies on the submerged hydraulic jump is that of Govinda Roa and Rajaratnam [1]. They discussed the characteristics of the submerged hydraulic jump in rectangular channels. The momentum and continuity equations were applied to have the following equation.

$$\frac{y_3}{y_1}=\left[(1+S_r)^2\phi^2-2F_1^2+\frac{2F_1^2}{(1+S_r)\phi}\right]^{0.5}. \qquad (1)$$

In which, y_3, is the backup water depth, y_1, is the supercritical flow depth, S_r, is the Govinda Roa and Rajaratnam submergence ratio and equals $S_r=(y_4-y_2)/y_2$, y_4 is the tailwater depth, y_2, is the sequent depth for the free jump, and F_1, is the supercritical Froude number and $\phi=0.5\left(\sqrt{1+8F_1^2}-1\right)$.

In addition, they defined the relative energy loss as:

$$\frac{E_L}{E_1}=\left[\left(\frac{y_3}{y_1}-(1+S_r)\phi\right)+0.5F_1^2\left(1-\frac{1}{(1+S_r)^2\phi^2}\right)\right]\left[\frac{y_3}{y_1}+\frac{F_1^2}{2}\right]^{-1}. \qquad (2)$$

Rajaratnam [2] Studied the submerged jump as the case of a plane turbulent wall jet under an adverse pressure gradient over which a backward flow had been placed. Narayanan and Bhargara [3] studied the pressure fluctuation in submerged jump downstream of a sluice gate. They showed that the increasing of tail water depth up to certain submergence causes a rise in the magnitudes of pressure fluctuations, beyond that, it begins to decrease. El-Azizy [4] studied the effect of different intensities of bed roughness on the rectangular submerged hydraulic jump. A theoretical equation was developed for smooth case. The control volume was starting at the gate opening and ending at the end of the jump. It was noticed that, the theoretical curve for all verification cases is almost lower than the experimental one.

In addition to the above, McCorquodal and Khalifa [5], Abdel Gawad et al. [6,7], Smith [8], Ohtsu et al. [9] and Negm et al. [10] analyzed the submerged hydraulic jump formed in a radial stilling basin provided with sudden drop both theoretically and experimentally. Both the experimental results and the developed equations indicated that at a particular relative location of the drop, the relative water depth, relative energy loss and relative length of jump increase by increasing Froude number keeping the submergence unchanged. Negm et al. [11] studied the effect of end sill on the main characteristics of the submerged radial jump. It was found that the presence of sill at the end of the radial basin produces a small effect on the jump characteristics. Negm et al. [12] analyzed both theoretically and experimentally the submerged flow in radial stilling basin provided with negative step and an end sill. The effects of various parameters such as submergence, relative height of sill, relative height of step and inlet Froude number were presented and discussed.

The present paper aims at developing a more accurate equation than that developed by El-Azizy [4] and more simplified than that developed by Govinda Roa and Rajaratnam [1].

2. Experimental work

The experimental data were collected (El-Azizy [4]) in the hydraulic laboratory of Faculty of Engineering, Ain Shams University. Re-circulating rectangular flume was used. The flume is 13.92 m long, 0.53 m wide, and 0.70 m deep. A moving point gauge was used to measure the water depth. The rate of flow was measured by using a pre-calibrated orifice meter. A sluice gate was used to control the upstream flow while the tailgate was used to control the tailwater level. The experiments were conducted to cover a range of supercritical Froude number ranging from1.1 to 3.5. The following test procedure was followed to complete any particular run in a horizontal flume. (a) the gate opening was set, (b) the power is switched on and the control valve was adjusted to a specific discharge. (c) a submerged hydraulic jump was formed using the tailgate (c) once the stability conditions were attained, the flow rate, length of the jump, water depth just downstream of the gate and the tail water depth were recorded. The length of jump was taken to be the section at which the flow depth becomes almost horizontal (d) the position of the tailgate was changed to obtain another submergence ratio and then step (c) was repeated. (e) step (d) was repeated several times then the procedure was repeated for another discharge till the required discharges were covered (f) the procedure was repeated for another gate opening and so on till the range of the experimental data were covered.

3. Theoretical approach

In order to apply the momentum equation on control volume b-c, the following assumptions are involved. (a) the velocity distribution at the two sections b and c are unifrom. (b) the pressure distribution at both sections b and c is hydrostatic. (c) the flow is steady and is incompressible. (d) the effects of bed friction, side wall effect, air entraibment are all ignored.

Based on these asumptions and based on fig. 1 which shows the phenomenon under investigation, the related terms and pressure forces to be used in the momentum equation are being defined in table 1 based on two control volumes. The first one a-c was used by El-Azizy [4] and the second one b-c is used by the present study.

Applying the momentum equation and continuity equation on the control volume b-c, yields:

$$P_1 - P_2 = \frac{\gamma}{g} Q\left(V_{out} - V_{in}\right). \qquad (3)$$

In which, P_1 is the back up hydrostatic force, P_2 is the hydrostatic pressure force at the end of the jump, γ is the specific weight, g is the gravitational acceleration, Q is the flow rate, V_{out} is the velocity of flow at the end of control volume, and V_{in} is the velocity of flow at the beginning of the control volume.

Table 1
different parameters affecting on rectangular submerged jump

Control Volume	a-c	b-c
Y_o	$\frac{y_4}{G}$	$\frac{y_4}{y_1}$
V_{in}	$V_G = \frac{Q}{BG}$	$V_1 = \frac{Q}{By_1}$
V_{out}	$V_2 = \frac{Q}{By_4}$	$V_2 = \frac{Q}{By_4}$
P_1	$\frac{\gamma}{2} y_3^2 B$	$\frac{\gamma}{2} y_3^2 B$
P_2	$\frac{\gamma}{2} y_4^2 B$	$\frac{\gamma}{2} y_4^2 B$
E_1	$y_3 + \frac{V_G^2}{2g}$	$y_3 + \frac{V_1^2}{2g}$
E_2	$y_4 + \frac{V_2^2}{2g}$	$y_4 + \frac{V_2^2}{2g}$

By substituting all defined parameters from table 1 into eq. (3), gives:

$$\frac{\gamma}{2} y_3^2 B - \frac{\gamma}{2} y_4^2 B = \frac{\gamma}{g} Q(V_2 - V_1). \tag{4}$$

In which, B is the width of flume.

By substituting V_{out}, and V_{in} from table 1 (control volume b-c) into eq. (4), and dividing it by $\gamma B y_1^2$ the following equation is obtained:

$$\left(\frac{y_3}{y_1}\right)^2 - \left(\frac{y_4}{y_1}\right)^2 = 2F_1^2\left(\frac{y_1}{y_4} - 1\right). \tag{5}$$

This equation can be put in the following form:

$$S^2 - Y_o^2 = 2F_1^2\left(\frac{1}{y_o} - 1\right). \tag{6}$$

In the pervious equation, S is the submergence ratio (y_3/y_1), Y_o is the relative water depth (y_4/y_1), and F_1 is the Froude number at the supercritical flow depth (y_1).

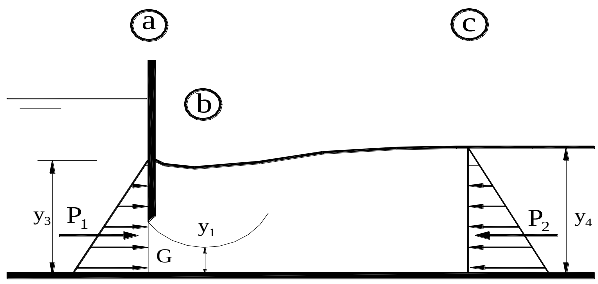

Fig. 1. Definition sketch for rectangular submerged hydraulic jump.

Applying the energy equation at control volume b-c

$$E_1 = y_3 + \frac{V_1^2}{2g}, \tag{7}$$

$$E_2 = y_4 + \frac{V_2^2}{2g}, \tag{8}$$

and

$$\frac{E_L}{E_1} = 1 - \frac{Y_o^3 + 0.5F_1^2}{Y_o^2\left(S + 0.5F_1^2\right)}. \tag{9}$$

In which, E_1 is the specific energy at the end of control volume, E_2 is the specific energy at the end of the submerged jump, and E_L is the energy loss.

It should be mentioned that applaying the momentum equation based on control volume a-c, El-Azizy [4] developed the following equation:

$$S^2 - Y_o^2 = 2F_G^2\left(\frac{1}{y_o} - 1\right), \tag{10}$$

In which S is the submergence ratio (y_3/G), G is the gate opening, Y_o is the relative water depth (y_4/G), and F_G is the under-gate Froude number. Also, she developed the following equations for the energy loss:

$$\frac{E_L}{E_1} = 1 - \frac{Y_o^3 + 0.5F_G^2}{Y_o^2\left(S + 0.5F_G^2\right)}. \tag{11}$$

4. Verification of the developed equation and comparisons

4.1. Relative depth ratio

The relative water depth either for $Y_o = y_4/G$ (El-Azizy [4]) or $Y_o = (y_4/y_1$ the present study) were drawn versus the prediction from eqs. (10) and (6) respectively as shown in figs. 2-a and 2-b. It was found that eq. (10) gives prediction values underestimated (The predicted values less than the measured one), on contrary, eq. (6) fits well the experimental data. The correlation coefficient and mean relative error for eq. (10) and eq. (6) were (97%, 10%) and (99%, 2%), respectively. The residuals for both equations were clarified figs. 3-a and 3-b, the residual for the present eq. (6) seems to be random and distributed around the line of zero error (fig. 3-b).

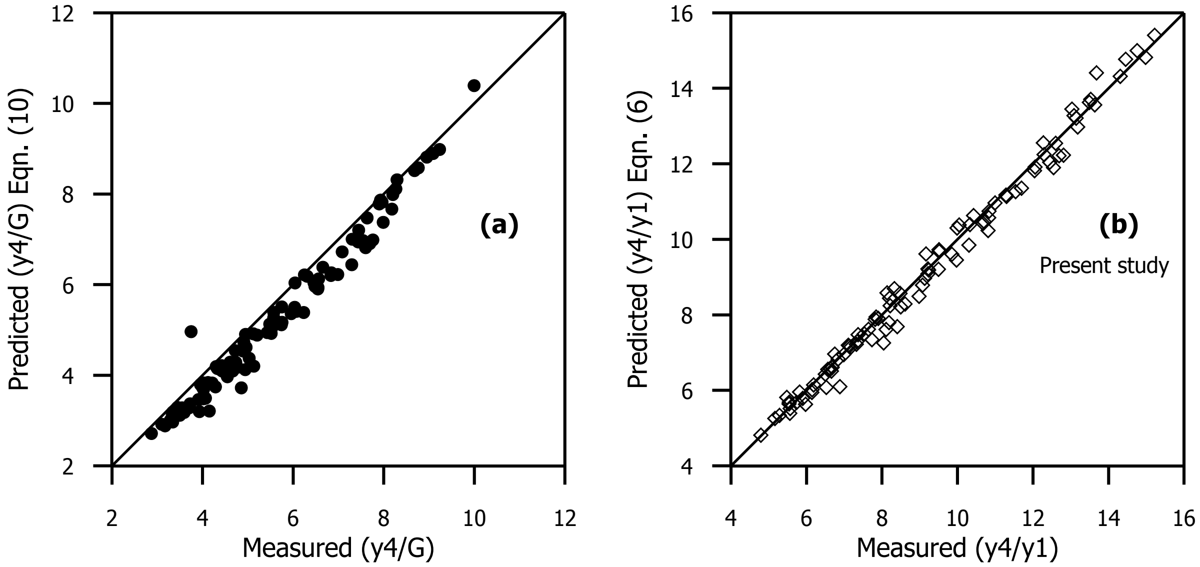

Fig. 2. The measured relative water depth compared with the predicted from (a) eq. (10), and (b) eq. (6) (present study).

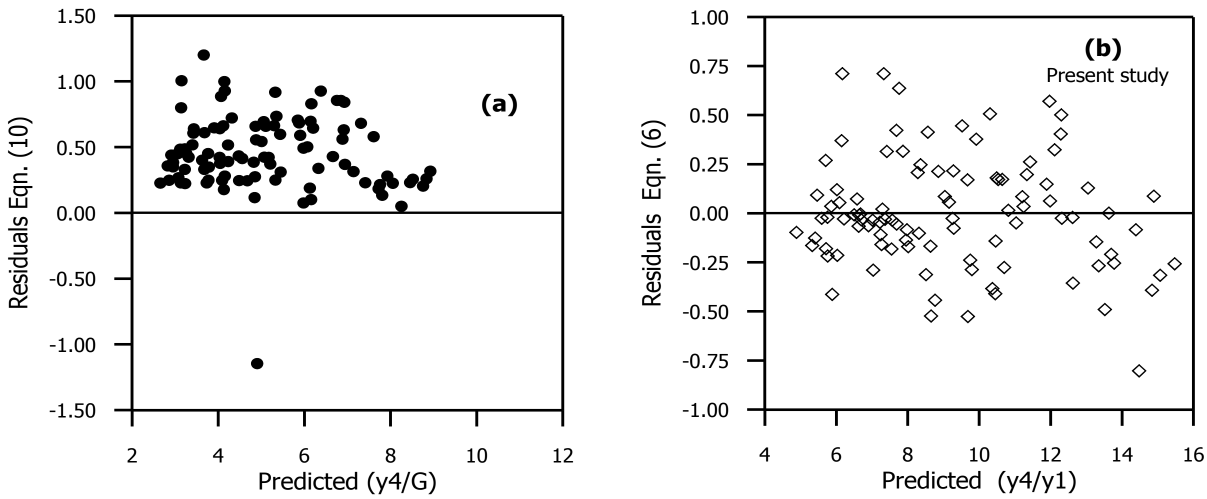

Fig. 3. The predicted (Y_o either for y_4/G or y_4/y_1) versus residuals for (a) eq. (10), and (b) eq. (6) (present study).

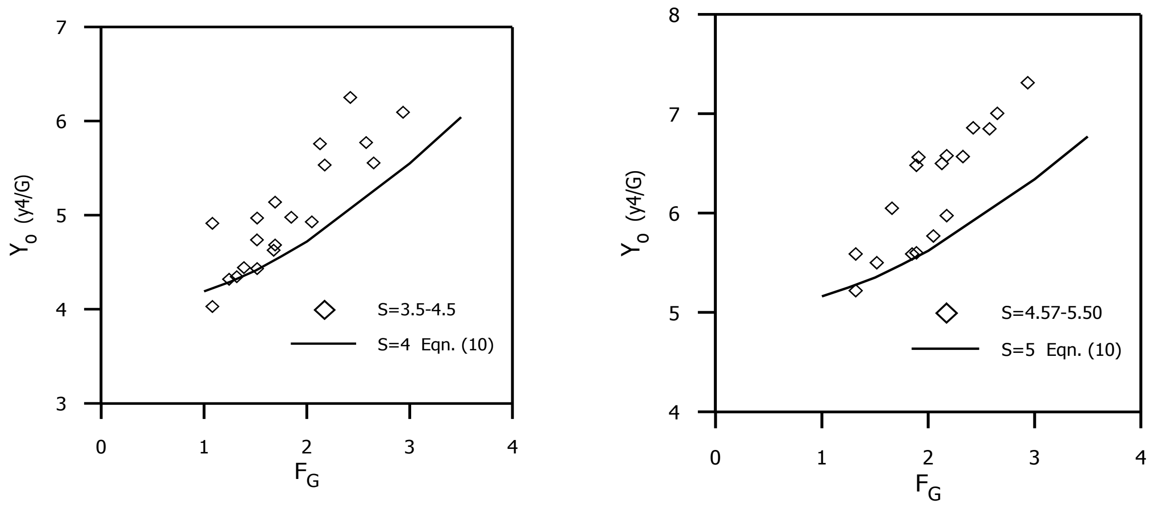

Fig. 4. The relationship between F_G and Y_o for S =3.5-4.5 and S = 4.57-5.50 according to eq. (10).

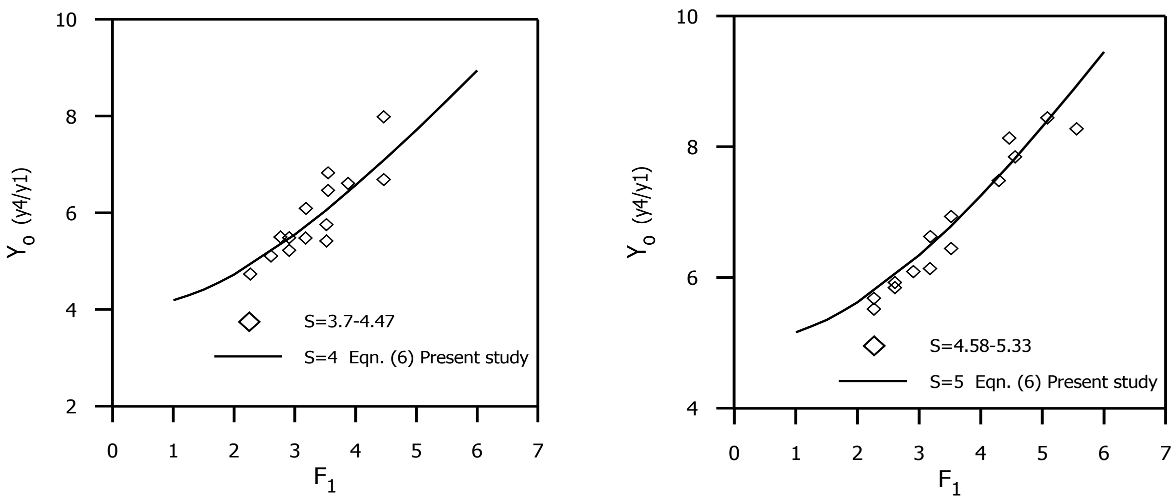

Fig. 5. The relationship between F_1 and Y_o for S =3.7-4.47 and S = 4.56-5.53 according to eq. (6).

A typical case for the relationship between F_G and y_4/G for S=3.5-4.5 and 4.57-5.5 was shown in fig. 4, also, the theoretical equation was plotted with S=4.0, and S=5 fig. 4. The correlation coefficient and mean relative error for both submergences (S) were (92%, 9.2%) and (96%, 9.3%) respectively. Also, A typical case for the relationship between F_1 and y_4/y_1 for S=3.7-4.47 and 4.58-5.53 was shown in fig. 5. Also, the theoretical equation was plotted with S=4.0 and S=5 fig. 5. The correlation coefficients and mean relative error for both relationships of fig. 5 were (92%, 3%) and (97%, 1%) respectively.

4.2. Relative energy loss

The measured relative energy loss was plotted versus the prediction from eq. (9) (present study) and (11) (El-Azizy [4]) as shown in figs. 6. It was found that eq. (11) gives predicted values higher than the actual measured data, on contrary, eq. (9) fits well the experimental data. The correlation coefficient and mean relative error for eq. (9) and eq. (11) were (97%, 4%) and (61%, 65%) respectively. The residuals for both equations were clarified figs. 7-a and 7-b, the residual for the present equation (eq. (9)) seems to be random and distributed around the line of zero error fig. 7-b.

A typical case for the relationship between F_G and E_L/E_1 eq. (11) for S=3.5-4.5 and 4.57-5.5 was shown in fig. 8, also, the theoretical equation was plotted with S=4.0 and S=5 fig. 8. The correlation coefficient and men relative error for both (S) were (85%, 79%) and (93%, 82%) respectively. Also, A typical case for the relationship between F_1 and E_L/E_1 eq. (9) for S=3.7-4.47 and 4.58-5.53 was shown in fig. 9, also, the theoretical equation was plotted with S=4.0 and S=5 fig. 9. The correlation coefficient and men relative error for both figs. were (91%, 5%) and (99%, 1%), respectively.

4.3. Comparison between present eqs. (6) & (9) and Govinda Rao and Rajaratnam eqs. (1) & (2)

Eqs. (1) and (2) for Govinda Rao and Rajaratnam [1] were solved by trial and error to predict the depth ratio (y_4/y_1) and the relative energy loss (E_L/E_1). These predicted values were compared to the predicted values from eqs. (6) and (9), respectively, as shown in table 2.

It was found that, the same theoretical values for both depth ratio and relative energy loss for eqs. (6) and (9) and eqs. (1) and (2) were obtained.

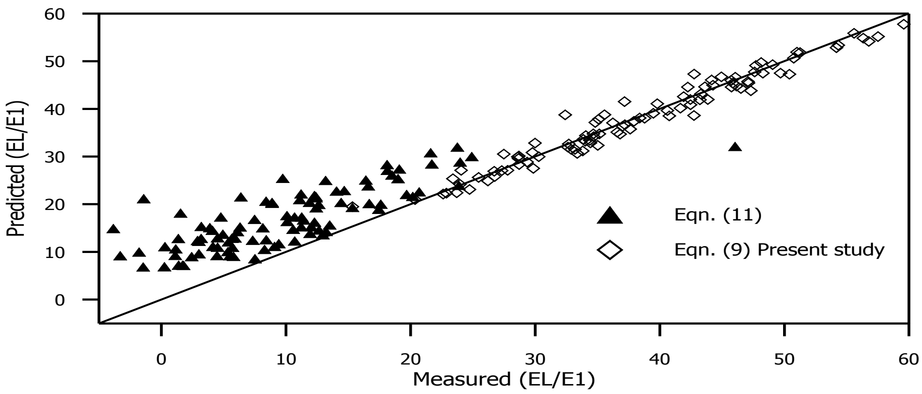

Fig. 6. The measured relative Energy loss compared with the predicted from eq. (11), and Eq. (9). (present study).

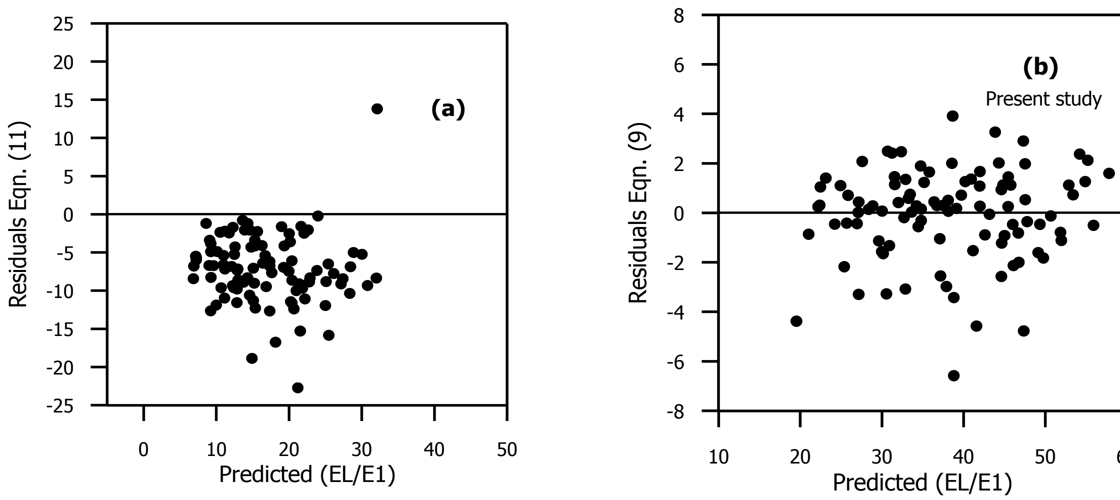

Fig. 7. The predicted (E_L/E_1) versus residuals for (a) eq. (11), and (b) eq. (9) (Present study).

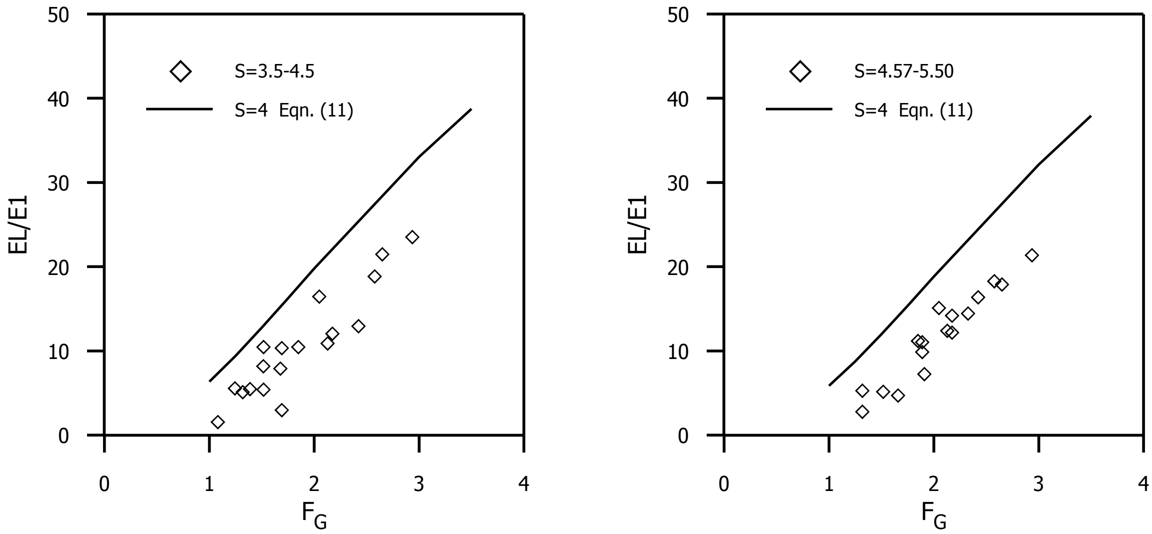

Fig. 8. The relationship between F_G and E_L/E_1 for S =3.5-4.5 and S = 4.57-5.50 according to eq. (11).

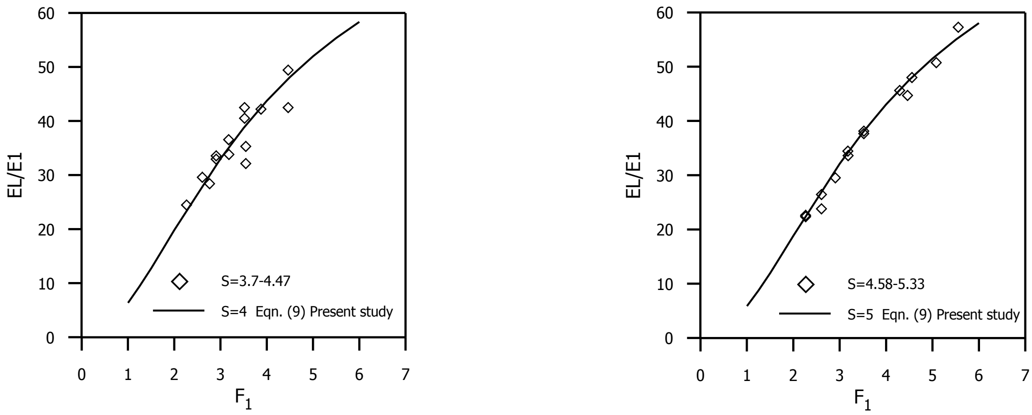

Fig. 9. The relationship between F_1 and E_L/E_1 for S =3.70-4.47 and S = 4.56-5.53 according to eq. (9).

Table 2
Comparison between the present eqs. (6) and (9) and eqs. (1) and (2)

y_1	F_1	S	Y_4/Y_1 Exp.	EL/Ey_1 Exp.	Y_4/Y_1 Theo. eq.(6)	E_L/E_1 Theo. eq. (9)	Y_4/Y_1 Theo. eq. (1)	E_L/E_1 Theo. eq. (2)
2.400	3.497	3.730	5.439	42.655	5.847	38.790	5.847	38.790
1.800	4.442	3.885	6.710	49.606	6.994	47.671	6.994	47.671
2.400	2.885	3.906	5.246	33.102	5.367	31.689	5.367	31.689
2.400	3.497	3.934	5.779	24.628	5.988	23.270	5.988	23.270
2.400	2.245	3.934	4.754	40.674	4.846	38.718	4.846	38.718
2.400	3.155	3.975	5.500	36.728	5.675	34.886	5.675	34.886
2.100	3.525	4.070	6.487	35.477	6.112	38.948	6.112	38.948
2.100	3.525	4.098	6.848	32.302	6.131	38.932	6.131	38.932
2.400	2.586	4.135	5.127	29.744	5.286	27.717	5.286	27.717
2.100	2.743	4.141	5.522	28.570	5.424	29.754	5.424	29.754
2.400	2.885	4.357	5.508	33.728	5.720	31.360	5.720	31.360
1.800	4.442	4.366	8.005	42.666	7.288	47.487	7.288	47.487
2.100	3.855	4.370	6.632	42.361	6.660	42.139	6.660	42.139
2.100	3.159	4.468	6.112	33.969	6.052	34.576	6.052	34.576

Table 3
Different statistical equations for rectangular submerged jump

Equation		R^2	MRE	$R^2_{residual}$
$\frac{Lj}{y_1} = -0.862 + 3.59S + 5.28F_1$	(12)	0.95	0.051	0.000007
$Y_o = \frac{y_4}{y_1} = 0.178 + 0.839S + 0.701F_1$	(13)	0.98	0.006	0.000006
$\frac{E_L}{E_1} = -5.026 - 1.225S + 19.448F_1 - 3.013F_1^{1.5}$	(14)	0.95	0.042	0.00

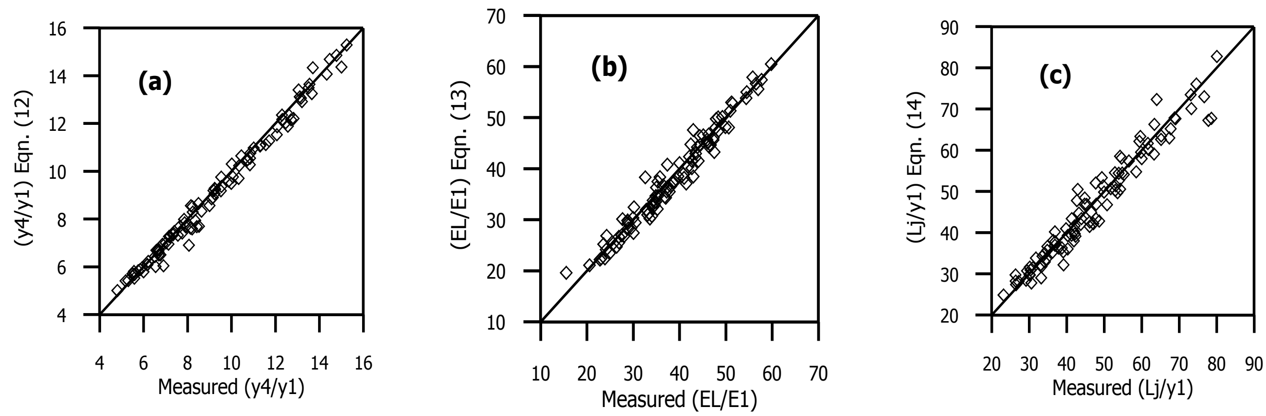

Fig. 10. Measured values of y_4/y_1, E_L/E_1, and L_j/y_1 versus predicted ones using (a) eq (12), (b) eq. (13), and (c) eq. (14).

5. Statistical models

The experimental data was treated using MLR (Multiple Linear Regression) to obtain different statistical equations for relative length of jump (L_j/y_1), depth ratio (y_4/y_1) and relative energy loss (E_L/E_1). These equation was fully described on table 3.

As shown in fig. 10, the statistical equations for depth ratio, relative energy loss, and relative length of jump were found to be in a good agreement with the experimental data. On the other hand, Govinda Rao and Rajaratnam [1] presented the following relationship between L_j/y_2 and S_r:

$$\frac{L_j}{y_2} = 4.9S_r + 6 \,. \tag{15}$$

6. Conclusions

Simplified theoretical equations for rectangular submerged hydraulic jump were developed based on the application of the 1-D momentum equation. The developed equation agreed well with the experimental results of the present study. Compared to the previously developed equations, the present equations are more simplified than that obtained by Govinda Roa and Rajaratnam [1] and more accurate than that obtained by El-Azizy [4]. Also, empirical models were developed for predicting the basic properties of the jump using multiple linear regression. The coefficients of these models were estimated based on the experimental data. The developed equations could be safely used in the design of the rectangular smooth stilling basin in the case of submerged hydraulic jump conditions.

Notations

B is the width of the channel,
E_1 is the total energy at the jump toe,

E_2 is the total energy at the jump heel,
E_L is the relative energy loss,
F_1 is the Froude's number at the vena contracta,
F_G is the Froude's number below the gate opening,
G is the gate opening,
L_j is the length of the hydraulic jump,
F_1 is the hydrostatic pressure before the jump,
F_2 is the hydrostatic pressure after the jump,
Q is the rate of flow,
R^2 is the coefficient of determination,
S is the degree of submergence, y_3/y_1 or y_3/G,
S_r is the Govinda Roa and Rajaratnam submergence ratio and equals $(y_4-y_2)/y_2$,
V_{in} is the average velocity at the vena contract (V_1), or below the gate opening (V_G),
V_2 is the average velocity at the sequent depth,
y_1 is the supercritical flow depth,
y_2 is the sequent depth for the free jump,
y_3 is the back up water depth just downstream the gate,
y_4 is the tail water depth at the end of the jump,
Y_o is the relative tail water depth, y_4/y_1 or y_4/G, and
γ is the specific weight.

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