Levelling for large areas using integrated GPS and global geopotential model: a case study over Alexandria city

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Global Positioning System (GPS) has today become an important geomatics tool. The geodetic vertical control is always considered as the most critical and time - consuming task in geodesy, especially if the area of interest is a large one. The recent tool for determining levels of a large area is GPS. Geographical coordinates or cartesian coordinates can be captured using GPS. Also, the ellipsoidal height can be obtained using GPS. An undulation geoidal chart was designed in this work. The geoid undulation can be obtained in terms of geographical or cartesian coordinates using this chart. Also, charts giving directly orthometric heights were designed using Ohaio State University model (OSU91A). Finally, the above mentioned product gives us the opportunity to determine ellipsoidal height, geoidal undulation, orthometric height and design an undulation chart for the desired area. That is through GPS observations and OSU91A geopotential model. Therefore, GPS technique can provide information, complementary to other geographical data of the earth's surface.

أصبح نظام التثبيت العالمى GPS من أهم الآليات المستخدمة فى علم المساحة والتقنية الأرضية. وتعتبر عمليات الضبط الرأسى (الميزانيات) من أهم موضوعات المساحة والتقنية الأرضية وخاصة الأراضى الواسعة والممتدة، حيث تستهلك اعمال الميزانيات الكثير من الوقت والجهد فضلا عما تتواجد بها من أخطاء. ويعتبر استخدام نظام التثبيت العالمى GPS من احدث الطرق استخداما فى أعمال الميزانيات وانتاج الخرائط الكنتورية للأراضى الشاسعة، حيث يتم عمل خريطة للفروق الجيوديسية لمجموعة من نقاط الربط الرأسى باستخدام بعض أرصاد GPS على مدينة الإسكندرية. نظام التثبيت العالمى يعطى لنا الارتفاع من القطع الناقص الممثل لشكل الأرض. وقد قمنا فى هذا البحث بتصميم أشكال عن طريقها يمكن الحصول على مناسيب النقط وذلك باستخراج الفارق بين الارتفاع عن القطع الناقص والارتفاع عن سطح البحر. وبمعلومية الارتفاع عن القطع الناقص من أرصاد GPS نحصل على الارتفاع عن سطح البحر وهو المنسوب المراد الحصول عليه. حصلنا أيضا على أشكال باستخدام نموذج جامعة أوهايو الأمريكية تعطى المناسيب مباشرة بدلالة الإحداثيات الجغرافية أو الإحداثيات الكارتيزية للنقط وذلك لمدينة الإسكندرية.

Keywords: GPS, Spirit levelling, Geodetic height, Orthometric height, Geoid undulation

1. Introduction

GPS has today become an important geomatics tool, with numerous applications in various fields, including mapping, GIS and land surveying [1]. The geodetic vertical control is always considered as the most critical and time-consuming task in geodesy. Precise {spirit} levelling is the most accurate method to establish such a vertical geodetic control. However, if the area of interest is a large one, the precise levelling technique is then extremely expensive and time-consuming. Other techniques such as trigonometric and barometric levelling do not offer, in most cases, the required high accuracy of levelling [2].

A precise alternative technique for levelling large areas can be achieved by using the Global Positioning System (GPS) technology. In this technique we compute the required orthometric height as the difference between the ellipsoidal height computed by the GPS technology and the geoid undulation computed by a geoid determination technique which will be presented in this paper.

2. Ellipsoid, geoid and height

Levelling large areas by using the precise levelling technique is a very expensive and time-consuming task. It also faces many practical and field problems, such as hard accessibility of levelling stations and long levelling lines. The error propagation of small errors inherent causes also a technical problem, which needs a special dealing to solve, especially the systematic errors due to vertical refraction. Corrections associated with precise levelling technique should apply in order to obtain sufficient accuracy adopted for precise leveling. All of these problems show the drawback of using the traditional precise levelling technique for levelling large areas.

A good alternative technique for levelling large areas is achieved by using the GPS technology. Let us first define the relationship between the orthometric and ellipsoidal heights, fig. 1. The orthometric height *H* is the distance between the ground point *P* and its projection *J* onto the geoid, measured along the slightly curved plumb line. Point *Q* is the projection of the geoidal point *J* onto the ellipsoid by means of a strait ellipsoidal normal. The distance *JQ* is the geoid undulation *N*. The ellipsoidal height *h* is the distance between the ground point *P* and its projection *Q* onto the ellipsoid, measured along the straight ellipsoidal normal. The relationship between the orthometric height H and the ellipsoidal height h can be written as follows [3,4,5,6,7,8].

3. Establishment of a network covering the area of interest

The network consists of 15 stations covering Alexandria City, which is the study area. First station was set in the faculty of engineering. Second station was set in Shatby district. Station 3 in Smouha district. Station 4 in Roshdy district. Station 5 in Mayamy district. Station 6 in Montaza district. Station 7 in Maamora district. Station 8 in Alras Alsoda district. Station 9 in Abees district. Station 10 in Meharam Bek district. Station 11 in Kabbary district. Station 12 in Elmax district. Station 13 in Agamy district. Station 14 in Kom Elbasal district. Station 15 in Gomrok district.

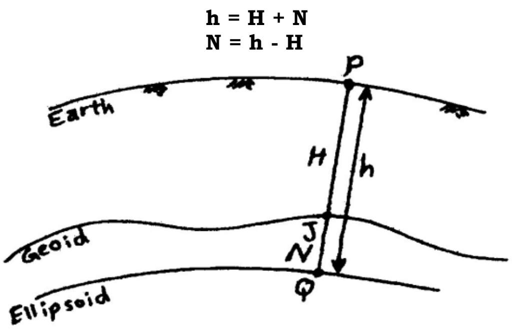

Fig. 1. The relationship between *H, N* and *h*..

4. Measuring the ellipsoidal height h using GPS

The geographic coordinates of fifteen bench-marks were observed (ϕ, λ ,h) which are shown in table 1. Coordinates were transformed to the Cartesian form (WGS84) and were tabulated in table 2. Some observations were performed using the z-xtreem receiver. The z-xtreem receiver begins with state of the art satellite electronics coupled with Ashtech's patented z tracking to deliver the highest level of GPS signal commercially available. Some sets of observations were considered to be simulated data to complete the work and show how to get the undulation geoidal chart for Alexandria City.

5. Determination of the geoid, geoidal undulation and global geopotential model

The determination of the geoid and geoid height above the reference ellipsoid *N* has attracted the attention of many geodesists and with the advent of GPS there is now an abundance of literature on its definition and practical competition.

The position of the geoid with respect to a predefined reference surface, usually the mean- Earth ellipsoid, can be determined using either geometrical or physical methods.

The physical geodetic approach to geoid determination uses a combination of satellite-derived and terrestrial gravity observations to determine the geometric geoid over a particular region.

The Ohaio State University model (OSU91A global geopotential model) [9] had been utilised as an existing gravimetric geoid solution. The geoid height above the reference ellipsoid *N* is computed from the spherical harmonic coefficients that define the OSU 91A global geopotential model [9] by:

$$N = \frac{GM}{r\gamma}\sum_{n=2}^{M\,max}\frac{a^n}{r^n}\sum_{m=0}^{n}(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)P_{nm}(\cos\theta),$$

where:

GM	is the geometric gravitational constant,
γ	is normal gravity on the reference ellipsoid,
(r, λ, θ)	are the spherical polar coordinate of the computed point,
a	is the equatorial radius,
$P_m(\cos\theta)$	are the fully normalized associated Legendre functions for degree n and order m, and
C_{nm}, S_{nm}	are the fully normalized OSU91A coefficients, reduced for the even zonal harmonics of the ellipsoid, and complete to degree and order M_{max} = 360°.

6. Design of the undulation geoidal chart

Ohaio State University model (OSU91A global geopotential model) was used to get the orthometric heights for stations as shown in table 1 and table 2. The golden software (surfer) was used to get charts shown in fig. 2

Table 1
Number of station, latitude, longitude and orthometric height computed using Ohaio state university model
(OSU91A)

#	ϕ			λ			H1(m)
1	31	12'	22,9"	29	55'	31.5"	18.016
2	31	12	44.8	29	55	01.3	6.555
3	31	12	52.0	29	56	43.0	1.237
4	31	14	01.0	29	56	55.0	3.348
5	31	15	36.0	29	58	53.0	3.256
6	31	16	46.0	30	00	33.0	4.176
7	31	17	01.0	30	02	21.0	5.114
8	31	15	19.0	30	00	45.0	4.208
9	31	12	13.0	29	59	32.0	6.118
10	31	10	53.0	29	55	38.0	3.412
11	31	10	03.0	29	53	23.0	5.510
12	31	09	14.0	29	50	49.0	6.595
13	31	09	15.0	29	50	45.0	6.595
14	31	11	08.0	29	53	15.0	5.498
15	31	11	59.0	29	53	42.0	6.474

Table 2
Number of station, x-coordinate, y-coordinate and orthometric height computed using Ohaio State University model (OSU91A)

#	X(m)	Y(m)	H1(m)
1	512430.171	944219.655	18.016
2	511637.267	944901.990	6.555
3	514331.601	945097.725	1.237
4	514669.477	947219.941	3.348
5	517819.467	950116.740	3.256
6	520484.686	952248.700	4.176
7	523345.646	952685.441	5.114
8	520778.151	949566.208	4.208
9	518794.327	943854.778	6.118
10	512575.395	941449.012	3.412
11	508985.024	939944.245	5.510
12	504890.625	938476.703	6.595
13	504784.991	938508.607	6.595
14	508793.271	941948.416	5.498
15	509523.918	943512.088	6.474

and fig. 3. Fig. 2 illustrates the relation between latitude, longitude versus orthometric heights computed using Ohaio state university model. Fig. 3 shows the relation between x-coordinate, y-coordinate and orthometric heights computed using Ohaio state university model. Table 3 shows number of station, geoidetical heights captured by GPS, orthometric heights and the separation height *N*. Table 4 shows number of station, latitude, longitude and geoid undulation *N*. Table 5 shows number of station, x-coordinate, y-coordinate and geoid undulation *N*. The golden software (surfer) was used to get charts shown in fig. 4 and fig. 5. Fig. 4 shows the relation between latitude, longitude and geoid undulation *N*. Fig. 5 shows the relation between x-coordinate, y-coordinate and geoid undulation *N*.

7. Conclusions

It can be concluded that Global Positioning System (GPS) can be performed to establish a precise relative three-dimensional positioning system. Ohaio State University model was used to get the orthometric heights in terms of geographical coordinates (ϕ, λ) as shown in fig. 2. Also, the orthometric heights can be easily derived in terms of cartesian coordinates (x,y) as shown in fig. 3.

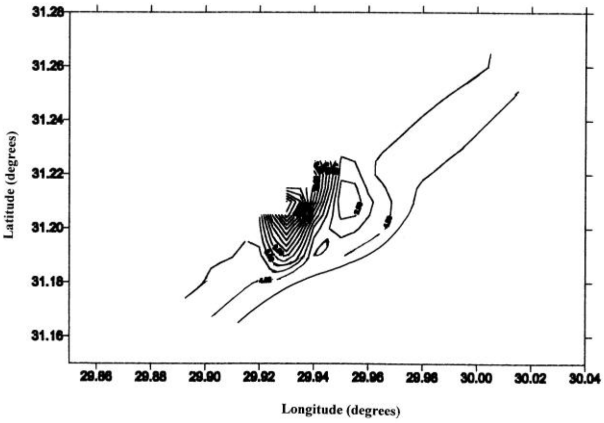

Fig. 2. The relationship between ϕ,λ and H based on WGS84.

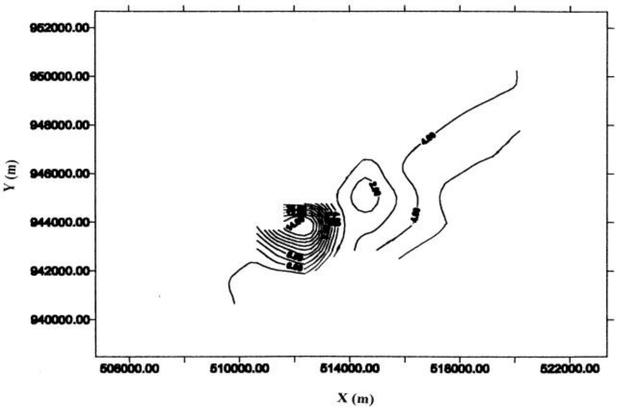

Fig. 3. The relationship between x, y and H based on WGS84.

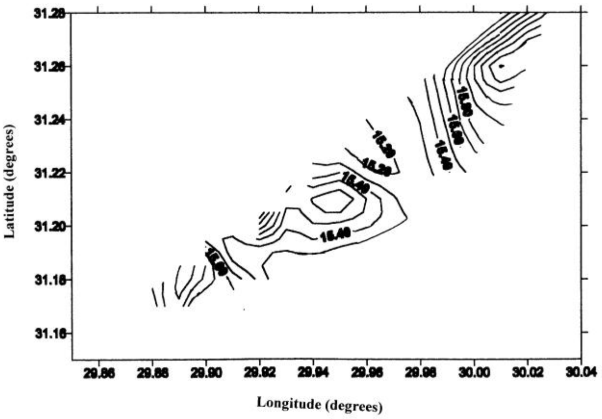

Fig. 4. The relationship between ϕ, λ and N based on WGS84.

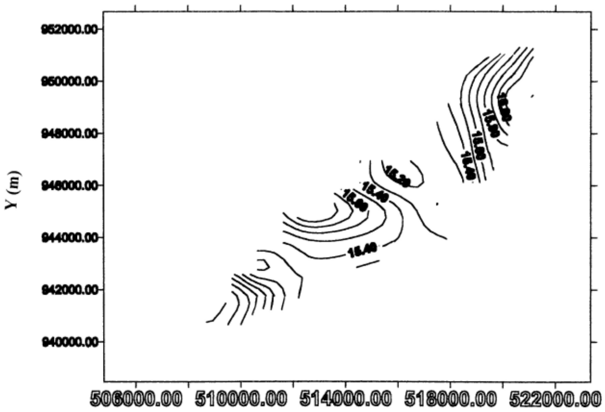

Fig. 5. The relationship between x, y and N based on WGS84.

Table 3
Number of station, geoidetical heights captured by GPS, orthometric heights and the separation height N

#	h(m)	H(m)	N(m)
1	34.34	18.7940	15.5460
2	22.13	5.9066	16.2234
3	16.86	1.1834	15.6766
4	19.12	4.0012	15.1188
5	19.23	3.8642	15.3658
6	20.14	4.6523	15.4877
7	21.24	4.8645	16.3755
8	20.31	4.0012	16.3088
9	21.98	6.7436	15.2364
10	18.97	3.8042	15.1658
11	21.11	5.2342	15.8758
12	22.21	7.4053	14.8047
13	22.14	7.3812	14.7588
14	20.89	4.836	16.0540
15	22.01	6.8483	15.1617

Table 4
Number of station, latitude, longitude and geoid undulation N

#	ϕ			λ			N(m)
1	31	12'	22,9"	29	55'	31.5"	15.5460
2	31	12	44.8	29	55	01.3	16.2234
3	31	12	52.0	29	56	43.0	15.6766
4	31	14	01.0	29	56	55.0	15.1188
5	31	15	36.0	29	58	53.0	15.3658
6	31	16	46.0	30	00	33.0	15.4877
7	31	17	01.0	30	02	21.0	16.3755
8	31	15	19.0	30	00	45.0	16.3088
9	31	12	13.0	29	59	32.0	15.2364
10	31	10	53.0	29	55	38.0	15.1658
11	31	10	03.0	29	53	23.0	15.8758
12	31	09	14.0	29	50	49.0	14.8047
13	31	09	15.0	29	50	45.0	14.7588
14	31	11	08.0	29	53	15.0	16.0540
15	31	11	59.0	29	53	42.0	15.1617

Table 5
Number of station, x-coordinate, y-coordinate and geoid undulation *N*

#	X(m)	Y(m)	N(m)
1	512430.171	944219.655	15.5460
2	511637.267	944901.990	16.2234
3	514331.601	945097.725	15.6766
4	514669.477	947219.941	15.1188
5	517819.467	950116.740	15.3658
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15	509523.918	943512.088	15.1617

Ellipsoidal height differences derived from GPS positioning can be converted to orthometric height differences using predicted geoidal undulation differences $\{H = h - N\}$. The geoid undulation N can be obtained in terms of geographical coordinates through the chart illustrated in fig. 4. Also, the geoidal undulation N can be obtained in terms of cartesian coordinates $(x.y)$ as shown in fig. 5.

This paper presents the application of GPS technique in leveling purpose for large areas. It can be concluded that the GPS has the potential for performing and establishing a precise relative three-dimensional positioning system instead of traditional leveling operation. Ohaio State Global Geopotential model (OSU91A) was used to get the orthometric heights in terms of geographical or cartisian coordinates.

However, GPS offers an additional and complement technique for use with other dependent or independent data to help the surveyors and geodesists to determine the earth's surface data through the three-dimen sional respect. The results of this paper resumed the practical phase of GPS observations that were performed in Alexandria district and these results can be successfully applied and implemented for different areas.

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