A methodology for design pipe networks on the basis of random search technique

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Analysis and design of pipe networks are one of the more complex mathematical problems that engineers are called upon to solve, particularly if the network is large as occurs in the water distribution systems. The design problem leads to a significant fraction of entire set of equations consists of nonlinear equations and large number of these equations must be solved simultaneously. The solution process endeavors to determine the size of pipes, the discharge in every pipe, and the pressure at every junction in the network. Economically, the most costly single item in the construction of networks system generally is the distribution network. Therefore, the pipe sizes should be carefully selected on the basis of adequate service and overall economic considerations. Presented herein a methodology for design water distribution systems based on random search technique for estimating the unknown pipe diameters and discharges (quantity and directions), while satisfying the demand requirements, the theoretical hydraulic constraint and the working practical conditions. The technique considers the minimum and the maximum limitations for velocity in pipes as 0.7 m/s to 2.0 m/s, as a constraint in the solution to obtain the most economical pipes diameters. The Darcy-Weisbach equation was used to estimate the head losses in the pipes as a function of the discharges that passing in pipes and the friction coefficient. The friction coefficient was defined by the Colebrook-White equation. The method overcomes the limitations of previous methods such that the initial flow distribution does not remain constant until the end of the solution but it is correctly balanced. It is not necessary to assume a pressure surface profile since it is computed from the network analysis. The presented method eliminates the task that the designer must propose the pipes sizes before using the ready-made packages, especially if his experience in this field is not quite enough.

يعتبر تحليل وتصميم شبكات المواسير من أعقد المسائل الرياضيه التي يمكن أن يواجها المهندس ويستطيع القيام بحلها كما يحدث فى شبكات توزيع المياه بالمدن. الطرق التقليديه لتصميم شبكات المواسير تعطى في النهايه مجموعه من المعادلات الرياضيه اللاخطية التي يجب حلها بطريقة التتابع. حل هذه المعادلات يعطي في النهاية أحجام المواسير والتصرف المار في كل ماسورة و الضغط عند كل وصلة من وصلات الشبكه. من الناحية الاقتصادية يمكن القول أن المواسير كمواد انشاء تعتبر هي الاكثر تكلفة عند تقدير تكلفة مكونات الشبكه. ولذلك يجب إختيار أحجام وأقطار المواسير بحرص لتكون في النهاية هي الافضل في الأداء والاقل في التكلفة من الناحية الاقتصادية. نقدم في هذا البحث طريقة تفصيلية لتصميم شبكات توزيع المياه وهذه الطريقة تعتمد في أساسها على نظرية البحث العشوائي لتعطي في النهاية أقطار خطوط المواسير المناسبة والتصرفات الماره في كل خط (كميات وإتجاهات) ويتم الإختيار على أسس هيدروليكية وعملية. روعي في هذه الطريقه حدود كل من السرعات القصوى والدنيا بحيث تقع بين ٠,٧ متر/ ث الي ٢,٠٠ متر/ ث. وقد وجد أن هذه الحدود تعطي أفضل أقطار لخطوط المواسيرمن الناحية الاقتصادية. تم إستخدام فرض دارسي. فايسبخ لحساب الفواقد في خطوط المواسير و أيضا معادلة كوليبروك– هويت لحساب معامل الاحتكاك لكل ماسورة. هذه الطريقة تغلبت على القصور في الطرق الاخرى في فرض تصرفات مبدئية و يظل هذا الفرض ثابت الى نهاية الحل حيث يتم في هذه الطريقة تغير هذا الفرض باستمرار للحصول على الاتزان الصحيح للتصرفات عند نقط إتصال الشبكة. أيضا ليس من الضروري في هذه الطريقة فرض ضغوط مبدئية عند وصلات الشبكة حيث يتم حساب الضغط عند كل وصلة بعد حساب التصرف و القطر المناسب لكل خط من الخطوط. من مميزات هذه الطريقة أيضا انه ليس من الضروري فرض أقطار مبدئية للشبكة كخطوة مسبقة لحساب التصرفات المارة في كل خط وبخاصة إذا كانت خبرة من يقوم بفرض هذه الاقطار ليست على القدر الكافي.

Keywords: Pipes networks, Closed-loop water distribution, Random search technique

1. Introduction

Closed-loop water distribution pipe networks are widely used for public water supply systems. In the last half century, a tremendous number of research works have tackled the design of such conveyance systems with regard to complicated networks of large municipalities. The problem of pipe sizing still not been completely solved yet. This is in fact because most of the proposals are applicable for simple networks and valid only under certain conditions.

The basic hydraulic equations that link the flows to the peizometric heads are the linear continuity equations and the nonlinear head loss equations in which the flow resistance relates pipe head loss to discharge. These form an indeterminate and partially nonlinear set of equations. In order to get the optimal solution among hundreds of solutions, a number of conditions or constraints should be introduced. Present design practice is based on more or less arbitrary selection of pipe sizes and pressures in the network. Then the hydraulics is evaluated to determine if given requirements or constraints with regard to discharges and pressures at various points are met. If not then some of the pipe diameters are changed, then pressures and discharges of the network are re-evaluated. Much elaborated work is needed by repeating this process until a hydraulically acceptable network is found. Because the computational burden involved usually allows a possibility that a more sizes exists, consequently, an optimization technique for determining the combination of pipe sizes and node pressures with minimum tolerance rate of errors would be of value.

The problem has already been investigated with several techniques. Mainly, four techniques among others were widely used for dealing with the subject: Hardy-Cross, Newton- Raphson, mathematical programming methods and statistical iterative techniques. The Hardy-Cross method [1] is the first one, which provides a systematic solution of a pipe network. It is considered a check method rather than design from which the pipe diameters are required to be fixed initially. Digital computers were used the Hardy-Cross analysis by Hoag and Weinberg [2], Graves and Branscome [3], Adams [4] and Dillingham [5]. The method depends on initial estimate of flows and it suffers from slow convergence. Other studies on the analysis of a hydraulic network have concentrated on finding the heads at the nodes from known pipe sizes. Other studies concentrated on solving pipe sizes from known nodal values of heads. Tong et al. [6] presented the method of balancing equivalent pipe length in a network to arrive at the proper sizes of pipes from known pressure surface profiles. This approach is not arrived at mathematically, but is based on observation and experience. Raman and Raman [7] modified the equivalent length approach to be fit mathematically. Shamir and Howard [8] have considered pipes and hydraulic elements in a network by using Newton-Raphson method. It depends on adjusting the flow or heads simultaneously along all loops. Donachie [9] added modifications to the Newton-Raphson technique to improve computation efficiency and to improve program stability under low flow conditions. Epp and Flower [10] presented the simultaneous loop (path) method, while Shamir and Howard [11] presented the simultaneous node method. All of those algorithms are generally formulated to yield flow rates and pressures for specific network characteristics and thus do not yield design information directly.

The work on mathematical programming techniques in pipe network analysis has been started four decades ago. The various techniques employed have included linear programming, nonlinear programming and dynamic programming. Karmeli, et al. [12] presented a method of design of branched water distribution networks using the theory of linear programming. Jacoby [13] proposed a nonlinear programming method with continuous variables, thus obtaining a solution with theoretical diameters to be rounded off to commercial values. Any of nonlinear and dynamic programming techniques does not appear to be a method applicable to large closed loop networks. Cembrowicz and Harrington [14] determined the theoretical diameters using the Graph Theory. Rajiv Gupta and T. D. Presad [15] extended the work of linear graph theory for analyzing the pipe networks.

Watanada [16] used a gradient technique with taking into account the constraint equations by means of penalty function to solve a network with two loops, with no reference to more complex systems. Basha and Kassab [17] applied the perturbation method for a set of nonlinear equations representing flow and heads in the network.

This paper describes a methodology that allows network balancing and new pipe sizing to be accomplished without need for repeated trials. The method offers a basis for optimum hydraulic design in the sense that the design can be carried out to just meet specified hydraulics conditions. The method overcomes the limitations of previous methods in that the initial flow distribution does not remain constant until the end of the solution but it is correctly balanced. It is not necessary to assume a pressure surface profile since this is computed from the network analysis. Opposed to equivalent diameter or length, actual lengths are used in the analysis and obtained diameters are finally transformed into actual commercial sizes. The technique considers the minimum and the maximum limitations for velocity in pipes, 0.7 m/s to 2.0 m/s, as a constraint in the solution to obtain the most economical pipes diameters as recommended by David Stephenson [18].

2. Statement of the problem

Analysis and design of pipe networks can be one of the most complex tasks that engineers are called upon to solve, particularly if the network is large as occurs in the water distribution systems. The pipe network shown in fig. 1-a consists of 60 pipes; 30 pipes in the h-direction and 30 pipes in the v-direction. The initial flow directions are assumed as shown in fig. 1-b. Considering the assumed flow discharge $Qh_{(3,1)}$, $Qv_{(2,2)}$, $Qh_{(2,1)}$, and $Qv_{(2,1)}$ passing through pipes $Ph_{(3,1)}$, $Pv_{(2,2)}$, $Ph_{(2,1)}$, and $Pv_{(2,1)}$ respectively and according to the assumption that in a closed loop the summation of head losses are equal to zero [1], and using the *Darcy-Weisbach* principal, loop equation for the closed loop 1 may stated as:

$$Kh_{(3,1)}Qh_{(3,1)}^2 + ch_{(3,1)}\frac{Qh_{(3,1)}^2}{2g\cdot Ah_{(3,1)}^2} + Kv_{(2,2)}Qv_{(2,2)}^2 + cv_{(2,2)}\frac{Qv_{(2,2)}^2}{2g\cdot Av_{(2,2)}^2} - Kh_{(2,1)}Qh_{(2,1)}^2 - ch_{(2,1)}\frac{Qh_{(2,1)}^2}{2g\cdot Ah_{(2,1)}^2} - Kv_{(2,1)}Qv_{(2,1)}^2 - cv_{(2,1)}\frac{Qv_{(2,1)}^2}{2g\cdot Av_{(2,1)}^2} = 0, \tag{1}$$

in which:

$$Kh_{(3,1)} = fh_{(3,1)}\frac{Lh_{(3,1)}}{2Dh_{(3,1)}\cdot g\cdot Ah_{(3,1)}^2}, \tag{2-a}$$

$$Kv_{(2,2)} = fv_{(2,2)}\frac{Lv_{(2,2)}}{2Dv_{(2,2)}\cdot g\cdot Av_{(2,2)}^2}, \tag{2-b}$$

$$Kh_{(2,1)} = fh_{(2,1)}\frac{Lh_{(2,1)}}{2Dh_{(2,1)}\cdot g\cdot Ah_{(2,1)}^2}, \tag{2-c}$$

and

$$Kv_{(2,1)} = fv_{(2,1)}\frac{Lv_{(2,1)}}{2Dv_{(2,1)}\cdot g\cdot Av_{(2,1)}^2}. \tag{2-d}$$

in which $Lh_{(3,1)}$, $Lv_{(2,2)}$, $Lh_{(2,1)}$, and $Lv_{(2,1)}$ are the lengths of pipes $Ph_{(3,1)}$, $Pv_{(2,2)}$, $Ph_{(2,1)}$, and $Pv_{(2,1)}$, respectively, g is the gravitational acceleration, $fh_{(3,1)}$, $fv_{(2,2)}$, $fh_{(2,1)}$, and $fv_{(2,1)}$ are the friction coefficients which can be evaluated from the Colebrook-White equation as follows:

$$\frac{1}{\sqrt{fh_{(3,1)}}} = 1.14 - 2\log_{10}\left[\frac{e}{Dh_{(3,1)}} + \frac{9.35}{Rh_{(3,1)}\sqrt{fh_{(3,1)}}}\right], \tag{3-a}$$

$$\frac{1}{\sqrt{fv_{(2,2)}}} = 1.14 - 2\log_{10}\left[\frac{e}{Dv_{(2,2)}} + \frac{9.35}{Rv_{(2,2)}\sqrt{fv_{(2,2)}}}\right], \tag{3-b}$$

$$\frac{1}{\sqrt{fh_{(2,1)}}} = 1.14 - 2\log_{10}\left[\frac{e}{Dh_{(2,1)}} + \frac{9.35}{Rh_{(2,1)}\sqrt{fh_{(2,1)}}}\right], \tag{3-c}$$

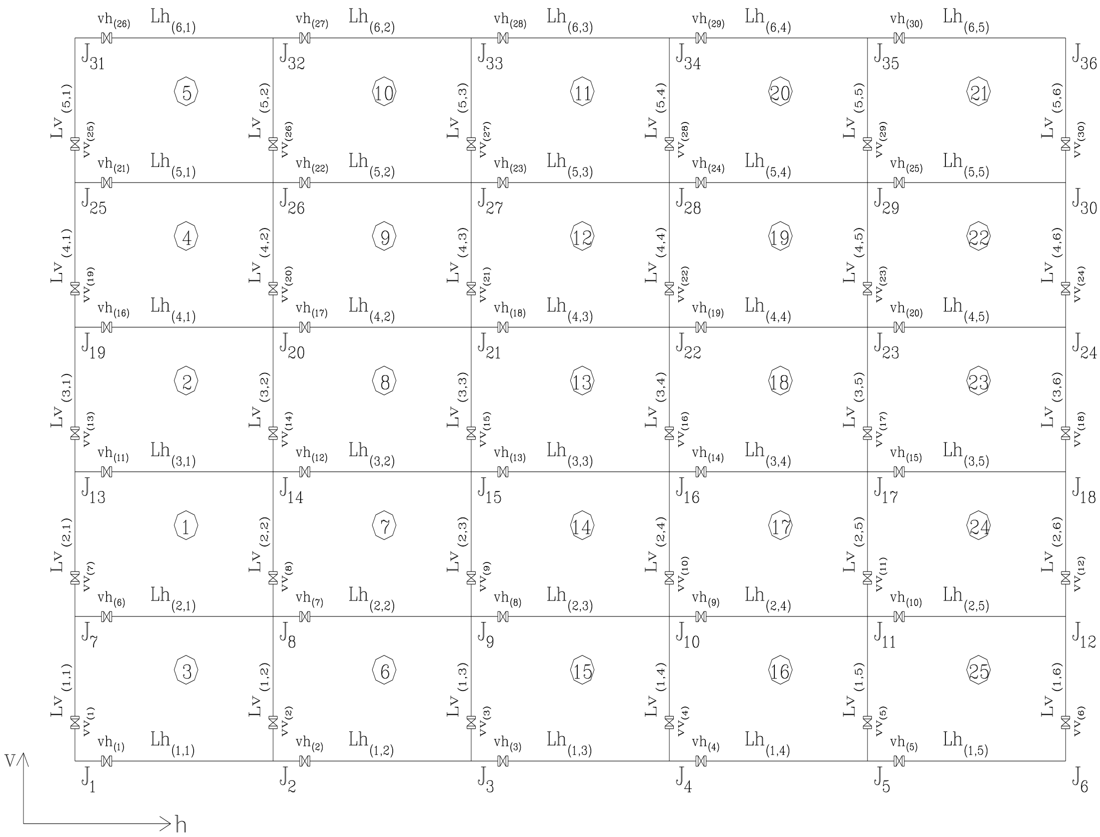

Fig. 1-a. Pipes network layout.

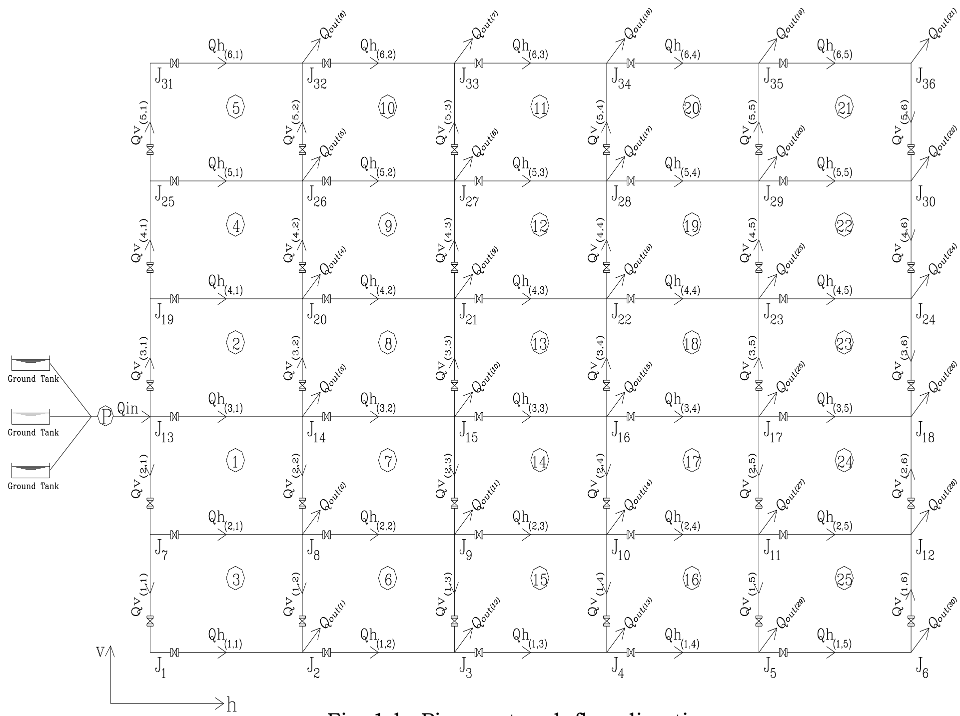

Fig. 1-b. Pipes network flow directions.

and

$$\frac{1}{\sqrt{fv_{(2,1)}}} = 1.14 - 2\log_{10}\left[\frac{e}{Dv_{(3,1)}} + \frac{9.35}{Rv_{(2,1)}\sqrt{fv_{(2,1)}}}\right], \quad (3\text{-d})$$

in which $Rh_{(3,1)}$, $Rv_{(2,2)}$, $Rh_{(2,1)}$, and $Rv_{(2,1)}$ are the Reynolds numbers and $e/Dh_{(3,1)}$, $e/Dv_{(2,2)}$, $e/Dh_{(2,1)}$, and $e/Dv_{(3,1)}$ are the relative roughness values of pipes $Ph_{(3,1)}$, $Pv_{(2,2)}$, $Ph_{(2,1)}$, and $Pv_{(2,1}$ with diameters of $Dh_{(3,1)}$, $Dv_{(2,2)}$, $Dh_{(2,1)}$, and $Dv_{(2,1)}$ and cross sectional areas $Ah_{(3,1)}$, $Av_{(2,2)}$, $Ah_{(2,1)}$, and $Av_{(2,1)}$, respectively. Also, $ch_{(3,1)}$, $cv_{(2,2)}$, $ch_{(2,1)}$, and $cv_{(2,1)}$ are the coefficients of local losses due to the existing valves erected on pipes $Ph_{(3,1)}$, $Pv_{(2,2)}$, $Ph_{(2,1)}$, and $Pv_{(2,1)}$, respectively.

Similarly, set of equations for the closed loops 2, 3, 4 and 5 could be writen as:

$$Kh_{(3,1)}Qh_{(3,1)}^2 + ch_{(3,1)}\frac{Qh_{(3,1)}^2}{2g\cdot Ah_{(3,1)}^2} + Kv_{(3,2)}Qv_{(3,2)}^2 + cv_{(3,2)}\frac{Qv_{(3,2)}^2}{2g\cdot Av_{(3,2)}^2} - Kh_{(4,1)}Qh_{(4,1)}^2 - ch_{(4,1)}\frac{Qh_{(4,1)}^2}{2g\cdot Ah_{(4,1)}^2} - Kv_{(3,1)}Qv_{(3,1)}^2 - cv_{(3,1)}\frac{Qv_{(3,1)}^2}{2g\cdot Av_{(3,1)}^2} = 0, \quad (4)$$

$$Kh_{(2,1)}Qh_{(2,1)}^2 + ch_{(2,1)}\frac{Qh_{(2,1)}^2}{2g\cdot Ah_{(2,1)}^2} + Kv_{(1,2)}Qv_{(1,2)}^2 + cv_{(1,2)}\frac{Qv_{(1,2)}^2}{2g\cdot Av_{(1,2)}^2} - Kh_{(1,1)}Qh_{(1,1)}^2 - ch_{(1,1)}\frac{Qh_{(1,1)}^2}{2g\cdot Ah_{(1,1)}^2} - Kv_{(1,1)}Qv_{(1,1)}^2 - cv_{(1,1)}\frac{Qv_{(1,1)}^2}{2g\cdot Av_{(1,1)}^2} = 0, \quad (5)$$

$$Kh_{(4,1)}Qh_{(4,1)}^2 + ch_{(4,1)}\frac{Qh_{(4,1)}^2}{2g\cdot Ah_{(4,1)}^2} + Kv_{(4,2)}Qv_{(4,2)}^2 + cv_{(4,2)}\frac{Qv_{(4,2)}^2}{2g\cdot Av_{(4,2)}^2} - Kh_{(5,1)}Qh_{(5,1)}^2 - ch_{(5,1)}\frac{Qh_{(5,1)}^2}{2g\cdot Ah_{(5,1)}^2} - Kv_{(4,1)}Qv_{(4,1)}^2 - cv_{(4,1)}\frac{Qv_{(4,1)}^2}{2g\cdot Av_{(4,1)}^2} = 0, \quad (6)$$

and

$$Kh_{(5,1)}Qh_{(5,1)}^2 + ch_{(5,1)}\frac{Qh_{(5,1)}^2}{2g\cdot Ah_{(5,1)}^2} + Kv_{(5,2)}Qv_{(5,2)}^2 + cv_{(5,2)}\frac{Qv_{(5,2)}^2}{2g\cdot Av_{(5,2)}^2} - Kh_{(6,1)}Qh_{(6,1)}^2 - ch_{(6,1)}\frac{Qh_{(6,1)}^2}{2g\cdot Ah_{(6,1)}^2} - Kv_{(5,1)}Qv_{(5,1)}^2 - cv_{(5,1)}\frac{Qv_{(5,1)}^2}{2g\cdot Av_{(5,1)}^2} = 0. \quad (7)$$

3. Formulation of the problem

The problem of pipe network shown in fig. 1-a could be formulated as an optimization problem by the help of eqs. (1), (4), (5), (6) and (7) as follows:

For loops 1 and 2:

Minimize;

$$f_1(\boldsymbol{X}) = Kh_{(3,1)}Qh_{(3,1)}^2 + ch_{(3,1)}\frac{Qh_{(3,1)}^2}{2g\cdot Ah_{(3,1)}^2} + Kv_{(2,2)}Qv_{(2,2)}^2 + cv_{(2,2)}\frac{Qv_{(2,2)}^2}{2g\cdot Av_{(2,2)}^2} - Kh_{(2,1)}Qh_{(2,1)}^2 - ch_{(2,1)}\frac{Qh_{(2,1)}^2}{2g\cdot Ah_{(2,1)}^2} - Kv_{(2,1)}Qv_{(2,1)}^2 - cv_{(2,1)}\frac{Qv_{(2,1)}^2}{2g\cdot Av_{(2,1)}^2} = 0. \quad (8)$$

Subjected to the constraints, $g_1(\boldsymbol{x})$, which describes the continuity equation at junction, J_{13}, and the velocities constraints $g_2(\boldsymbol{x})$, $g_3(\boldsymbol{x})$, $g_4(\boldsymbol{x})$ and $g_5(\boldsymbol{x})$ as the following:

$$g_1(\boldsymbol{x})=Q_{\text{in}}-Qh_{(3,1)}-Qv_{(3,1)}-Qv_{(2,1)}=0, \quad (9)$$

$$g_2(\boldsymbol{x}) \qquad 0.7 \leq Vh_{(3,1)} \leq 2.0, \quad (10\text{-a})$$

$$g_3(\boldsymbol{x}) \qquad 0.7 \leq Vv_{(2,2)} \leq 2.0, \quad (10\text{-b})$$

$$g_4(\boldsymbol{x}) \qquad 0.7 \leq Vv_{(2,1)} \leq 2.0, \quad (10\text{-c})$$

and

$$g_5(\boldsymbol{x}) \qquad 0.7 \leq Vh_{(2,1)} \leq 2.0, \quad (10\text{-d})$$

in which $Vh_{(3,1)}$, $Vv_{(2,2)}$, $Vv_{(2,1)}$ and $Vh_{(2,1)}$ are the flow velocities in the pipes $Ph_{(3,1)}$, $Pv_{(2,2)}$, $Pv_{(2,1)}$ and $Ph_{(2,1)}$, respectively.

Also, minimize;

$$f_2(\boldsymbol{X}) = Kh_{(3,1)}Qh_{(3,1)}^2 + ch_{(3,1)}\frac{Qh_{(3,1)}^2}{2g \cdot Ah_{(3,1)}^2} + Kv_{(3,2)}Qv_{(3,2)}^2 + cv_{(3,2)}\frac{Qv_{(3,2)}^2}{2g \cdot Av_{(3,2)}^2} - Kh_{(4,1)}Qh_{(4,1)}^2 - ch_{(4,1)}\frac{Qh_{(4,1)}^2}{2g \cdot Ah_{(4,1)}^2} - Kv_{(3,1)}Qv_{(3,1)}^2 - cv_{(3,1)}\frac{Qv_{(3,1)}^2}{2g \cdot Av_{(3,1)}^2} = 0. \quad (11)$$

Subjected to the constraint, $g_1(\boldsymbol{x})$, and the velocities constraints $g_6(\boldsymbol{x})$, $g_7(\boldsymbol{x})$, $g_8(\boldsymbol{x})$ and $g_9(\boldsymbol{x})$ as the following:

$$g_6(\boldsymbol{x}) \qquad 0.7 \leq Vh_{(3,1)} \leq 2.0, \quad (12\text{-a})$$

$$g_7(\boldsymbol{x}) \qquad 0.7 \leq Vv_{(3,2)} \leq 2.0, \quad (12\text{-b})$$

$$g_8(\boldsymbol{x}) \qquad 0.7 \leq Vv_{(3,1)} \leq 2.0, \quad (12\text{-c})$$

and

$$g_9(\boldsymbol{x}) \qquad 0.7 \leq Vh_{(4,1)} \leq 2.0, \quad (12\text{-d})$$

in which $Vh_{(3,1)}$, $Vv_{(3,2)}$, $Vv_{(3,1)}$ and $Vh_{(4,1)}$ are the flow velocities in the pipes $Ph_{(3,1)}$, $Pv_{(3,2)}$, $Pv_{(3,1)}$ and $Ph_{(4,1)}$ respectively.

For loop 3:

Minimize;

$$f_3(\boldsymbol{X}) = Kh_{(2,1)}Qh_{(2,1)}^2 + ch_{(2,1)}\frac{Qh_{(2,1)}^2}{2g \cdot Ah_{(2,1)}^2} + Kv_{(1,2)}Qv_{(1,2)}^2 + cv_{(1,2)}\frac{Qv_{(1,2)}^2}{2g \cdot Av_{(1,2)}^2} - Kh_{(1,1)}Qh_{(1,1)}^2 - ch_{(1,1)}\frac{Qh_{(1,1)}^2}{2g \cdot Ah_{(1,1)}^2} - Kv_{(1,1)}Qv_{(1,1)}^2 - cv_{(1,1)}\frac{Qv_{(1,1)}^2}{2g \cdot Av_{(1,1)}^2} = 0. \quad (13)$$

Subjected to the constraint, $g_{10}(\boldsymbol{x})$, which describes the continuity equation at junction , J_7, and the velocities constraints $g_{11}(\boldsymbol{x})$, $g_{12}(\boldsymbol{x})$, $g_{13}(\boldsymbol{x})$ and $g_{14}(\boldsymbol{x})$ as the following:

$$g_{10}(\boldsymbol{x}) = Qv_{(2,1)} - Qh_{(2,1)} - Qv_{(1,1)} = 0, \quad (14)$$

$$g_{11}(\boldsymbol{x}) \qquad 0.7 \leq Vh_{(2,1)} \leq 2.0, \quad (15\text{-a})$$

$$g_{12}(\boldsymbol{x}) \qquad 0.7 \leq Vv_{(1,2)} \leq 2.0, \quad (15\text{-b})$$

$$g_{13}(\boldsymbol{x}) \qquad 0.7 \leq Vv_{(1,1)} \leq 2.0, \quad (15\text{-c})$$

and

$$g_{14}(\boldsymbol{x}) \qquad 0.7 \leq Vh_{(1,1)} \leq 2.0, \quad (15\text{-d})$$

in which $Vh_{(2,1)}$, $Vv_{(1,2)}$, $Vv_{(1,1)}$ and $Vh_{(1,1)}$ are the flow velocities in the pipes $Ph_{(2,1)}$, $Pv_{(1,2)}$, $Pv_{(1,1)}$ and $Ph_{(1,1)}$, respectively.

For loop 4

Minimize;

$$f_4(\boldsymbol{X}) = Kh_{(4,1)}Qh_{(4,1)}^2 + ch_{(4,1)}\frac{Qh_{(4,1)}^2}{2g \cdot Ah_{(4,1)}^2} + Kv_{(4,2)}Qv_{(4,2)}^2 + cv_{(4,2)}\frac{Qv_{(4,2)}^2}{2g \cdot Av_{(4,2)}^2} - Kh_{(5,1)}Qh_{(5,1)}^2 - ch_{(5,1)}\frac{Qh_{(5,1)}^2}{2g \cdot Ah_{(5,1)}^2} - Kv_{(4,1)}Qv_{(4,1)}^2 - cv_{(4,1)}\frac{Qv_{(4,1)}^2}{2g \cdot Av_{(4,1)}^2} = 0. \quad (16)$$

Subjected to the following constraint, $g_{15}(\boldsymbol{x})$, which describes the continuity equation at junction J_{19} and the velocities constraints $g_{16}(\boldsymbol{x})$, $g_{17}(\boldsymbol{x})$, $g_{18}(\boldsymbol{x})$ and $g_{19}(\boldsymbol{x})$ as the following:

$$g_{15}(\boldsymbol{x}) = Qv_{(3,1)} - Qv_{(4,1)} - Qh_{(4,1)} = 0, \quad (17)$$

$$g_{16}(\boldsymbol{x}) \qquad 0.7 \le Vh_{(4,1)} \le 2.0, \quad (18\text{-a})$$

$$g_{17}(\boldsymbol{x}) \qquad 0.7 \le Vv_{(4,2)} \le 2.0, \quad (18\text{-b})$$

$$g_{18}(\boldsymbol{x}) \qquad 0.7 \le Vv_{(4,1)} \le 2.0, \quad (18\text{-c})$$

and

$$g_{19}(\boldsymbol{x}) \qquad 0.7 \le Vh_{(5,1)} \le 2.0, \quad (18\text{-d})$$

in which $Vh_{(4,1)}$, $Vv_{(4,2)}$, $Vv_{(4,1)}$ and $Vh_{(5,1)}$ are the flow velocities in the pipes $Ph_{(4,1)}$, $Pv_{(4,2)}$, $Pv_{(4,1)}$ and $Ph_{(5,1)}$, respectively.

For loop 5

Minimize;

$$f_5(\boldsymbol{X}) = Kh_{(5,1)} Qh_{(5,1)}^2 + ch_{(5,1)} \frac{Qh_{(5,1)}^2}{2g \cdot Ah_{(5,1)}^2} + Kv_{(5,2)} Qv_{(5,2)}^2 + cv_{(5,2)} \frac{Qv_{(5,2)}^2}{2g \cdot Av_{(5,2)}^2} - Kh_{(6,1)} Qh_{(6,1)}^2 - ch_{(6,1)} \frac{Qh_{(6,1)}^2}{2g \cdot Ah_{(6,1)}^2} - Kv_{(5,1)} Qv_{(5,1)}^2 - cv_{(5,1)} \frac{Qv_{(5,1)}^2}{2g \cdot Av_{(5,1)}^2} = 0. \quad (19)$$

Subjected to the constraint, $g_{20}(\boldsymbol{x})$, which describes the continuity equation at junction J_{25} and the velocities constraints $g_{21}(\boldsymbol{x})$, $g_{22}(\boldsymbol{x})$, $g_{23}(\boldsymbol{x})$ and $g_{24}(\boldsymbol{x})$ as the following:

$$g_{20}(\boldsymbol{x}) = Qv_{(4,1)} - Qv_{(5,1)} - Qh_{(5,1)} = 0, \quad (20)$$

$$g_{21}(\boldsymbol{x}) \qquad 0.7 \le Vh_{(5,1)} \le 2.0, \quad (21\text{-a})$$

$$g_{22}(\boldsymbol{x}) \qquad 0.7 \le Vv_{(5,2)} \le 2.0, \quad (21\text{-b})$$

$$g_{23}(\boldsymbol{x}) \qquad 0.7 \le Vv_{(5,1)} \le 2.0, \quad (21\text{-c})$$

and

$$g_{24}(\boldsymbol{x}) \qquad 0.7 \le Vh_{(6,1)} \le 2.0, \quad (21\text{-d})$$

in which $Vh_{(5,1)}$, $Vv_{(5,2)}$, $Vv_{(5,1)}$ and $Vh_{(6,1)}$ are the flow velocities in the pipes $Ph_{(5,1)}$, $Pv_{(5,2)}$, $Pv_{(5,1)}$ and $Ph_{(6,1)}$, respectively.

4. Design algorithm

A glance to eqs. (8), (11), (13), (16) and (19), leads to conclude that, this problem is a non-linear optimization problem subjected to linear constraints stated by eqs. (9), (10-a to10-d), (12-a to12-d), (14), (15-a to15-d), (17), (18-a to18-d) (20) and (21-a to 21-d). To solve this problem an optimization algorithm was prepared on the basis of Random Search Technique [19]. The design variables V_1, V_2, V_3,..., V_n, {[$Qh_{(3,1)}$, $Qv_{(2,2)}$, $Qh_{(2,1)}$ and $Qv_{(2,1)}$] for loop (1), [$Qh_{(3,1)}$, $Qv_{(3,2)}$,$Qh_{(4,1)}$ and $Qv_{(3,1)}$] for loop (2), [$Qh_{(2,1)}$, $Qv_{(1,2)}$, $Qh_{(1,1)}$ and $Qv_{(1,1)}$] for loop (3), [$Qh_{(4,1)}$, $Qv_{(4,2)}$, $Qh_{(5,1)}$ and $Qv_{(4,1)}$]for loop (4), [$Qh_{(5,1)}$, $Qv_{(5,2)}$, $Qh_{(6,1)}$, and $Qv_{(5,1)}$] for loop (5)} constitute the design vector $\boldsymbol{V}$. An initial random design vector was chosen by:

$$\boldsymbol{V}^{(o)} = \boldsymbol{V}_L^{(o)} + [\boldsymbol{V}_U^{(o)} - \boldsymbol{V}_L^{(o)}] R_n \quad . \quad (22)$$

In which $\boldsymbol{V}_L$ and $\boldsymbol{V}_U$ are the lower and the upper limits of $\boldsymbol{V}$, respectively, R_n is a uniformly distributed random number lying between 0 and 1; and the superscript o denotes the initial value. The initial random design was checked for all the constraints. If any of the constraints is violated, a new random design was considered. The process was repeated till all the constraints were satisfied. The process was repeated for another feasible design. If the present objective function $f(\boldsymbol{X})$ {error in eqs. (8), (11), (13), (16) and (19)} was less than the previously obtained feasible design, the present design vector was retained by naming it as $\boldsymbol{V}_s$. The process was repeated for a large number of times to get the least error. Reducing the range of, V, as shown below was refined the search

$$\boldsymbol{V}_L^{(r+1)} = \boldsymbol{V}_s^{(r)} - 0.45[\boldsymbol{V}_U^{(r)} - \boldsymbol{V}_L^{(r)}], \quad (23)$$

$$\boldsymbol{V}_U^{(r+1)} = \boldsymbol{V}_s^{(r)} + 0.45[\boldsymbol{V}_U^{(r)} - \boldsymbol{V}_L^{(r)}]. \quad (24)$$

In which, *r*, is the number of cycles. The process was repeated for several cycles till the error of two successive cycles has a tolerance of 1.0^{-4}.

5. Solution of the problem

The solution procedures start by optimizing the objective functions described by eqs. (8), (11), (13), (16) and (19), on the other hand, the constraints described by eqs. (9), (10-a to10-d), (12-a to12-d), (14), (15-a to15-d), (17), (18-a to18-d) (20) and (21-a to 21-d) should be satisfied, otherwise, the process will be repeated. By the end if these steps, the flow discharge passing through pipes $Ph_{(3,1)}$, $Pv_{(2,2)}$, $Ph_{(2,1)}$, $Pv_{(2,1)}$, $Pv_{(3,2)}$, $Ph_{(4,1)}$, $Pv_{(3,1)}$, $Pv_{(1,2)}$, $Ph_{(1,1)}$, $Pv_{(1,1)}$, $Pv_{(4,2)}$, $Ph_{(5,1)}$, $Pv_{(4,1)}$, $Pv_{(5,2)}$, $Ph_{(6,1)}$, and $Pv_{(5,1)}$ will be obtained and the convenient diameters of these pipes, which guaranty flow velocity varying between 0.7 m/s to 2.0 m/s, is obtained. Also, head loss (friction losses) in these pipes is obtained using *Darcy-Weisbach* principal. The initial pressure (static head of the pump station) at joint, J_{13}, is known and elevation of each joints is known, so, the pressure at junctions J_7, J_{19}, ... 1, 25, 31, 14, 20, 8, 2, 26, and 32 could be obtained. For loop 6, since the pressures at junctions, J_8, and J_2, are known the flow discharge $Qv_{(1,2)}$ will be obtained. If the total energy at joint J_8 is greater than the total energy at joint, J_2, the flow direction of $Qv_{(1,2)}$ is correct; otherwise, the flow direction will be reversed. Then the continuity equation at joint J_2 will be applied and the flow discharge $Qh_{(1,2)}$ is obtained. Then the head loss between joint, J_2, and joint, J_3, will be obtained, then, the pressure at joint, J_3, will be obtained. The process will be repeated simultaneously for loops 7,8,9, 10... till loop 25.

6. Numerical example

The presented methodology has been tested using the pipes network shown in fig. 1-a, which consists of 60 pipes, 30 pipes in the, *h*, direction and 30 pipes in the, *v*, direction. According to the flow demand at every junction as shown in table 2, the inflow discharge coming from the main source (ground water tanks) to the network, at J_{13}, is 2000.0 liter/sec and the static head of pumps station at J_{13} is assumed to be 100.0 meter. The data required to solve the proposed network have been tabulated in table 1 and table 2.

7. Conclusions

In this paper a methodology for design water distribution systems is presented. This method is based on random search technique for estimating the unknown pipes sizes and discharges, while satisfying the demand requirements, the theoretical hydraulic constraint and the working practical conditions. The technique considers the minimum and the maximum limitations for velocity in pipes as 0.7 m/s to 2.0 m/s, as a constraint in the solution to obtain the most economical pipes diameters. The method overcomes the limitations of the previous methods in that the initial flow distribution does not remain constant until the end of the solution but it is correctly balanced. Also, the presented method eliminates the task that the designer must propose the network diameters before using the ready-made packages, especially if his experience in this field is not quite enough.

The presented method is applied for the given data and the results are shown in the Appendix.

Nomenclature

The following symbols have been used in this paper:

$Ah_{(i,j)}$ is the cross sectional area of pipe number (i,j) in the *h*-direction,
$Av_{(i,j)}$ is the cross sectional area of pipe number (i,j) in the *v*-direction,
$ch_{(i,j)}$ is the valve coefficient erected on pipe number (i,j) in the *h*-direction,
$cv_{(i,j)}$ is the valve coefficient erected on pipe number (i,j) in the *v*-direction,
$Dh_{(i,j)}$ is the diameter of pipe number (i,j) in the *h*-direction,
$Dv_{(i,j)}$ is the diameter of pipe number (i,j) in the *v*-direction,
e is the absolute roughness,
$f(\mathbf{X})$ is the objective function,

$fh_{(i,j)}$ is the friction factor of pipe number (i,j) in the h-direction,
$f\upsilon_{(i,j)}$ is the friction factor of pipe number (i,j) in the υ-direction,
g is the gravitational acceleration,
$g(\boldsymbol{X})$ is the constraint,
$J_{(i)}$ is the junction number I,

Table 1
Pipes lengths, materials and valves coefficients

Pipe No.	Length (m)	Material	Valve coefficient	Pipe No.	Length (m)	Material	Valve coefficient
$Ph_{(1,1)}$	1500.0	Cast-iron	5.0	$P\upsilon_{(1,1)}$	1000.0	Cast-iron	5.0
$Ph_{(1,2)}$	1500.0	Cast-iron	5.0	$P\upsilon_{(1,2)}$	1000.0	Cast-iron	8.0
$Ph_{(1,3)}$	1500.0	Cast-iron	6.0	$P\upsilon_{(1,3)}$	1000.0	Cast-iron	8.0
$Ph_{(1,4)}$	1500.0	Cast-iron	7.0	$P\upsilon_{(1,4)}$	1000.0	Cast-iron	9.0
$Ph_{(1,5)}$	1500.0	Cast-iron	8.0	$P\upsilon_{(1,5)}$	1000.0	Cast-iron	9.0
$Ph_{(2,1)}$	1500.0	Cast-iron	6.0	$P\upsilon_{(1,6)}$	1000.0	Cast-iron	7.0
$Ph_{(2,2)}$	1500.0	Cast-iron	5.0	$P\upsilon_{(2,1)}$	1000.0	Cast-iron	4.0
$Ph_{(2,3)}$	1500.0	Cast-iron	6.0	$P\upsilon_{(2,2)}$	1000.0	Cast-iron	8.0
$Ph_{(2,4)}$	1500.0	Cast-iron	7.0	$P\upsilon_{(2,3)}$	1000.0	Cast-iron	8.0
$Ph_{(2,5)}$	1500.0	Cast-iron	8.0	$P\upsilon_{(2,4)}$	1000.0	Cast-iron	9.0
$Ph_{(3,1)}$	1500.0	Cast-iron	6.0	$P\upsilon_{(2,5)}$	1000.0	Cast-iron	9.0
$Ph_{(3,2)}$	1500.0	Cast-iron	5.0	$P\upsilon_{(2,6)}$	1000.0	Cast-iron	7.0
$Ph_{(3,3)}$	1500.0	Cast-iron	6.0	$P\upsilon_{(3,1)}$	1000.0	Cast-iron	6.0
$Ph_{(3,4)}$	1500.0	Cast-iron	7.0	$P\upsilon_{(3,2)}$	1000.0	Cast-iron	8.0
$Ph_{(3,5)}$	1500.0	Cast-iron	8.0	$P\upsilon_{(3,3)}$	1000.0	Cast-iron	8.0
$Ph_{(4,1)}$	1500.0	Cast-iron	5.0	$P\upsilon_{(3,4)}$	1000.0	Cast-iron	9.0
$Ph_{(4,2)}$	1500.0	Cast-iron	5.0	$P\upsilon_{(3,5)}$	1000.0	Cast-iron	9.0
$Ph_{(4,3)}$	1500.0	Cast-iron	6.0	$P\upsilon_{(3,6)}$	1000.0	Cast-iron	7.0
$Ph_{(4,4)}$	1500.0	Cast-iron	7.0	$P\upsilon_{(4,1)}$	1000.0	Cast-iron	5.0
$Ph_{(4,5)}$	1500.0	Cast-iron	8.0	$P\upsilon_{(4,2)}$	1000.0	Cast-iron	8.0
$Ph_{(5,1)}$	1500.0	Cast-iron	5.0	$P\upsilon_{(4,3)}$	1000.0	Cast-iron	8.0
$Ph_{(5,2)}$	1500.0	Cast-iron	5.0	$P\upsilon_{(4,4)}$	1000.0	Cast-iron	9.0
$Ph_{(5,3)}$	1500.0	Cast-iron	6.0	$P\upsilon_{(4,5)}$	1000.0	Cast-iron	9.0
$Ph_{(5,4)}$	1500.0	Cast-iron	7.0	$P\upsilon_{(4,6)}$	1000.0	Cast-iron	7.0
$Ph_{(5,5)}$	1500.0	Cast-iron	8.0	$P\upsilon_{(5,1)}$	1000.0	Cast-iron	7.0
$Ph_{(6,1)}$	1500.0	Cast-iron	6.0	$P\upsilon_{(5,2)}$	1000.0	Cast-iron	8.0
$Ph_{(6,2)}$	1500.0	Cast-iron	5.0	$P\upsilon_{(5,3)}$	1000.0	Cast-iron	8.0
$Ph_{(6,3)}$	1500.0	Cast-iron	6.0	$P\upsilon_{(5,4)}$	1000.0	Cast-iron	9.0
$Ph_{(6,4)}$	1500.0	Cast-iron	7.0	$P\upsilon_{(5,5)}$	1000.0	Cast-iron	9.0
$Ph_{(6,5)}$	1500.0	Cast-iron	8.0	$P\upsilon_{(5,6)}$	1000.0	Cast-iron	7.0

Table 2
Elevation and outflow demand at each junction

Junction No.	Elevation (m)	Outflow demand (Liter/sec)
J_1	12.0	-
J_2	12.0	100.0
J_3	11.9	100.0
J_4	11.75	50.0
J_5	11.9	100.0
J_6	11.8	50.0
J_7	12.2	-
J_8	12.1	100.0
J_9	12.2	50.0
J_{10}	11.8	100.0
J_{11}	12.0	50.0
J_{12}	11.8	50.0
J_{13}	12.1	-
J_{14}	12.0	50.0
J_{15}	12.2	50.0
J_{16}	11.9	100.0
J_{17}	11.9	50.0
J_{18}	11.9	100.0
J_{19}	12.0	-
J_{20}	12.1	50.0
J_{21}	12.1	50.0
J_{22}	11.9	50.0
J_{23}	12.0	50.0
J_{24}	11.9	50.0
J_{25}	12.2	-
J_{26}	12.0	100.0
J_{27}	12.0	50.0
J_{28}	11.9	50.0
J_{29}	11.9	50.0
J_{30}	11.8	100.0
J_{31}	12.2	-
J_{32}	12.0	100.0
J_{33}	12.0	50.0
J_{34}	11.9	50.0
J_{35}	11.9	50.0
J_{36}	11.8	50.0

$Lh_{(i,j)}$ is the length of pipe number (i,j) in the *h*-direction,
$Lv_{(i,j)}$ is the length of pipe number (i,j) in the *v*-direction,
$Kh_{(i,j)}$ is the friction coefficient of pipe number (i,j) in the *h*-direction,
$Kv_{(i,j)}$ is the friction coefficient of pipe number (i,j) in the *v*-direction,
$Ph_{(i,j)}$ is the pipe number(i,j) in the *h*-direction,
$Pv_{(i,j)}$ is the pipe number(i,j) in the *v*-direction,
$Qh_{(i,j)}$ is the discharge in pipe number (i,j) in the *h*-direction,
$Qv_{(i,j)}$ is the discharge in pipe number (i,j) in the *v*-direction,
Rn is the random number,
$Rh_{(i,j)}$ is the Reynolds number for flow passing through pipe number (i,j) in *h*-direction,
$Rv_{(i,j)}$ is the Reynolds number for flow passing through pipe number (i,j) in *v*-direction,
$\boldsymbol{V}$ is the design variables vector,
$\boldsymbol{V}_L$ is the lower limits of the design variables,
$\boldsymbol{V}_U$ is the upper limits of the design variables,
$Vh_{(i,j)}$ is the velocity of flow through pipe $Ph_{(i,j)}$ in the *h*-direction, and
$Vv_{(i,j)}$ is the velocity of flow through pipe $Pv_{(i,j)}$ in the *v*-direction.

Appendix

For the given data shown in tables 1 and 2, the pipes network has been solved using the described methodology considering the proposed flow directions shown in fig. 1-b. At the end of the process, a message tells the user of the program which was written in FORTRAN language that the direction of flow in pipes $Ph_{(3,4)}$, $Ph_{(6,4)}$, $Ph_{(6,5)}$, and $Ph_{(3,5)}$ should be reversed. The diameter, discharge, velocity and friction coefficient for each pipe have been obtained and tabulated in table A-1. Pressure, hydraulic gradient level, total energy level and discharge balance at each joint have been tabulated in table A-2.

References

[1] Cross Hardy, "Analysis of Flow in Network of Conduits or Conductors," Bulletin (286), University of Illinois Experimental Station, Urbana, Ill., (1936).

[2] L.N. Hong, and G. Weinberg, "Pipeline Network Analysis for Electronic Digital Computer," Journal of the American Water Works Association, Vol. 49, pp. 517-526 (1957).

[3] Q.B. Graves, and D. Branscome, "Digital Computers for Pipe line Network Analysis," Journal of the Sanitary Engineering Division, ASCE, Vol. 84, (SA2), Proc. Paper 1608, pp. 1608-1615 (1958).

[4] R.W. Adams, "Distribution Analysis by Electronic Computer," Journal of the Institute of Water Engineering, Vol. 15, pp. 415-423 (1961).

[5] J.H. Dillingham, "Computer Analysis of Water Distribution System, Part II," Journal of Water and Sewage works, pp. 43-52 (1967).

Table A-1
Diameter, discharge, velocity and friction factor of each pipe

No.	Diameter (mm.)	Discharge (Liter/sec)	Velocity (m/sec)	Friction factor	No.	Diameter (mm.)	Discharge (Liter/sec)	Velocity (m/sec)	Friction factor
$Ph_{(1,1)}$	500	276.5248	1.40833	.0175215	$Pv_{(1,1)}$	500	276.5248	1.40833	.0175215
$Ph_{(1,2)}$	550	344.7269	1.45097	.0171315	$Pv_{(1,2)}$	400	168.2021	1.33851	.0184731
$Ph_{(1,3)}$	550	329.9073	1.38860	.0171594	$Pv_{(1,3)}$	300	85.18041	1.20506	.0200000
$Ph_{(1,4)}$	550	303.0898	1.27572	.0172145	$Pv_{(1,4)}$	150	23.18259	1.31187	.0235278
$Ph_{(1,5)}$	450	197.0326	1.23886	.0180350	$Pv_{(1,5)}$	100	6.05721	0.77123	.0269370
$Ph_{(2,1)}$	450	192.0925	1.20780	.0180529	$Pv_{(1,6)}$	400	147.0327	1.17005	.0185736
$Ph_{(2,2)}$	400	152.4898	1.21348	.0185457	$Pv_{(2,1)}$	650	468.6173	1.41222	.0165258
$Ph_{(2,3)}$	350	123.1224	1.27971	.0190917	$Pv_{(2,2)}$	450	228.5993	1.43734	.0179328
$Ph_{(2,4)}$	350	128.2038	1.33252	.0190610	$Pv_{(2,3)}$	300	105.8129	1.49695	.0200000
$Ph_{(2,5)}$	400	157.5818	1.25400	.0185202	$Pv_{(2,4)}$	350	128.264	1.33315	.0190609
$Ph_{(3,1)}$	1000	1102.248	1.40343	.0150719	$Pv_{(2,5)}$	250	73.32082	1.49368	.0205779
$Ph_{(3,2)}$	800	593.9055	1.18154	.0200000	$Pv_{(2,6)}$	500	254.6147	1.29674	.0175762
$Ph_{(3,3)}$	600	324.8968	1.14909	.0200000	$Pv_{(3,1)}$	650	429.1346	1.29323	.0165805
$Ph_{(3,4)}$	200	30.88384	0.98306	.0221369	$Pv_{(3,2)}$	450	229.743	1.44453	.0179287
$Ph_{(3,5)}$	450	228.2768	1.43531	.0179328	$Pv_{(3,3)}$	350	113.1958	1.17653	.0191575
$Ph_{(4,1)}$	400	147.9399	1.17727	.0185676	$Pv_{(3,4)}$	350	127.5166	1.32538	.0190650
$Ph_{(4,2)}$	400	166.3167	1.32351	.0184812	$Pv_{(3,5)}$	300	74.07213	1.04791	.0199755
$Ph_{(4,3)}$	400	153.036	1.21782	.0200000	$Pv_{(3,6)}$	300	73.66212	1.04211	.0199797
$Ph_{(4,4)}$	400	188.3026	1.49846	.0183972	$Pv_{(4,1)}$	500	281.1947	1.43211	.0175108
$Ph_{(4,5)}$	400	162.3631	1.29204	.0184984	$Pv_{(4,2)}$	400	161.3661	1.28411	.0185034
$Ph_{(5,1)}$	350	108.4334	1.12703	.0191936	$Pv_{(4,3)}$	300	76.4764	1.08192	.0200000
$Ph_{(5,2)}$	300	75.42616	1.06706	.0199588	$Pv_{(4,4)}$	200	42.2501	1.34486	.0200000
$Ph_{(5,3)}$	250	64.14384	1.30673	.0206805	$Pv_{(4,5)}$	250	50.0116	1.01883	.0209062
$Ph_{(5,4)}$	200	37.48494	1.19318	.0219394	$Pv_{(4,6)}$	200	38.70097	1.23189	.0219099
$Ph_{(5,5)}$	250	49.40409	1.00645	.0209178	$Pv_{(5,1)}$	400	172.7614	1.37479	.0184541
$Ph_{(6,1)}$	400	172.7614	1.37479	.0184541	$Pv_{(5,2)}$	300	94.37337	1.33511	.0197697
$Ph_{(6,2)}$	400	167.1348	1.33002	.0184776	$Pv_{(5,3)}$	200	37.75873	1.20190	.0200000
$Ph_{(6,3)}$	400	154.8935	1.23260	.0185340	$Pv_{(5,4)}$	150	18.90899	1.07003	.0234524
$Ph_{(6,4)}$	350	123.8025	1.28678	.0190874	$Pv_{(5,5)}$	150	11.90755	0.71383	.0243624
$Ph_{(6,5)}$	250	61.89494	1.26091	.0207095	$Pv_{(5,6)}$	150	11.89494	0.71312	.0243641

Table A-2
Pressure, hydraulic gradient level, total energy level and discharge balance at each joint

Joint No.	Pressure (m)	Hydraulic gradient level (m)	Total energy level (m)	Summation of inflow and outflow
J_1	93.05951	105.06	105.161	0.0000
J_2	87.24031	99.24	99.341	0.0000
J_3	81.78402	93.684	93.791	0.0000
J_4	76.75415	88.504	88.602	0.0000
J_5	72.14446	84.044	84.127	0.0000
J_6	66.92073	78.721	78.799	0.0000
J_7	96.90690	109.107	109.209	0.0000
J_8	92.11385	104.214	104.288	0.0000
J_9	86.41830	98.618	98.693	0.0000
J_{10}	79.47955	91.28	91.363	0.0000
J_{11}	71.24603	83.246	83.337	0.0000
J_{12}	65.24886	77.049	77.129	0.0000
J_{13}	100.0000	112.10	112.20	0.0000
J_{14}	97.22726	109.227	109.328	0.0000
J_{15}	94.03247	106.232	106.304	0.0000
J_{16}	90.56760	102.468	102.535	0.0000
J_{17}	99.10835	111.008	111.058	0.0000
J_{18}	106.1692	118.069	118.174	0.0000
J_{19}	97.42841	109.428	109.514	0.0000
J_{20}	92.07124	104.171	104.242	0.0000
J_{21}	85.41872	97.519	97.608	0.0000
J_{22}	79.50955	91.410	91.485	0.0000
J_{23}	70.67416	82.674	82.789	0.0000
J_{24}	64.22053	76.121	76.206	0.0000
J_{25}	93.02553	105.226	105.33	0.0000
J_{26}	87.61618	99.616	99.681	0.0000
J_{27}	81.54131	93.541	93.599	0.0000
J_{28}	70.29113	82.191	82.278	0.0000
J_{29}	57.85772	69.758	69.83	0.0000
J_{30}	51.08596	62.886	62.938	0.0000
J_{31}	87.91505	100.115	100.211	0.0000
J_{32}	80.87054	92.871	92.967	0.0000
J_{33}	74.17863	86.179	86.269	0.0000
J_{34}	68.44468	80.345	80.422	0.0000
J_{35}	60.94332	72.843	72.928	0.0000
J_{36}	50.32925	62.129	62.210	0.0000

[6] A.L. Tong, et al., "Analysis of Distribution Networks by Balancing Equivalent Pipe Lengths," Journal of the American Water Works Association, Vol. 53 (2), pp. 192-201 (1961).

[7] V. Raman, and S. Raman, "New method of Solving Distribution System Networks Based on Equivalent Pipe Lengths," Journal of American works Association, Vol. 58 (5), pp. 615-625 (1966).

[8] U. Shamir and C.C.D. Howard, "Water Distribution System Analysis," Journal of the Hydraulic Division, ASCE, Vol. 94 (1), pp. 219-234 (1968).

[9] R.P. Donachie, "Digital Program for Water Network Analysis," Journal of the Hydraulic Division, ASCE, Vol. 100 (3), pp. 393-403 (1974).

[10] EPP and A.G. Flower, "Efficient Code for Steady State Flows in Networks," Journal of the Hydraulic Division, ASCE, Vol. 96 (1), pp. 43-56 (1970).

[11] U. Shamir and C.D.D. Howard, "Engineering Analysis of Water-Distribution Systems," Journal of American Water Works Association, Vol. 69 (9), pp. 355-368 (1977).

[12] D. Karmeli, Y. Gadish, and S. Meyers, "Design of optimal Water Distribution Networks," Journal of the Pipeline Division, ASCE, Vol. 94, (PL1) Proc. paper 6130, pp.1-10 (1968).

[13] S.S.L. Jacoby, "Design of Optimal Networks," Journal of Hydraulics Division, ASCE, Vol. 94, (HY3) Proc. Paper 5930, pp.641-661 (1968).

[14] R.G. Cembrowicz, J.J. Harrington, "Capital-Cost Minimization of Hydraulic Network," Journal of Hydraulics Division, ASCE, Vol. 99, (HY3) Proc. Paper 9606 pp.431-440 (1973).
[15] Rajiv Gupta and T.D. Prasad, "Extended Use of Linear Graph Theory for Analysis of Pipe Networks", Journal of Hydraulic Engineering, Vol. 126 (1), pp.56-62 (2000).
[16] T. Watanada, "Least-Cost Design of Water Distribution Systems," Journal of Hydraulics Division, ASCE, Vol. 99, (HY9) Proc. Paper 9974, pp. 1497-1513 (1973).
[17] H.A. Basha and B. G. Kassab, "Analysis of Water Distribution Systems using a Perturbation Method", Journal of Applied Mathematical Modelling, Vol. 20, pp. 290-297 (1996).
[18] David Stephenson, "Pipeline Design for Water Engineers", third revised and updated edition, Elsevier, Amsterdam-Oxford-New York-Tokyo (1989).
[19] P.K. Swame, G.C. Mishra and A.S. Salem, Adel "Optimal Design of Sloping Weir", Journal of Irrigation and Drainage Engineering, ASCE, 122 (4) (1996).

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