Mathematical determination of the depth of drainage gallery in gravity dams

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Construction of drainage gallery in gravity dams has an important employment in minimizing uplift pressure acting on the dam base. In the present work, a theoretical study, based on hydrodynamic theory, is used to evaluate depth of the drainage gallery constructed in gravity dams. The mathematical solution is deduced considering the drainage gallery as a vertical line sink with image. The complex potential, the equipotential and stream functions are established, and from which depth of drainage gallery and quantity of seepage discharge are determined.

ان انشاء بئر تخفيف قوى الرفع المائى أسفل فرشة السد التثاقلى له دور حيوى فى تقليل هذه القوى المؤثرة أسفل الفرشة. هذا وقد تم فى هذا البحث عمل دراسة نظرية لتحديد عمق اختراق بئر التخفيف (h) فى طبقة التأسيس المقام عليها جسم السد ذات العمق (D). وهذه الدراسة النظرية تعتمد على النظرية الهيدروديناميكية و فيها تم اعتبار بئر تخفيف قوى الرفع المائى خط رأسى مجمع للمياه مع صورة له. فى هذا البحث تم استنتاج كل من دالة الجهد المركب (W) و دالة جهد السرعة (Φ) وكذلك دالة السريان (Ψ). ومن ثم تم ايجاد كمية المياه المتسربة أسفل السد التثاقلى الى داخل بئر التخفيف (q). و تم التوصل فى هذا البحث أنه عند تغير عمق الطبقة الغير منفذة النسبى أسفل السد التثاقلى (D/S) من ١ الى ٦ فان القيمة المتوسطة لعمق اختراق البئر النسبى (h/D) فى الطبقة الحاملة للسد التثاقلى والتى تسمح للمياه المتسربة أسفل السد بالدخول كلها الى البئر تكون مساوية ٠,٦٦.

Keywords: Drainage gallery, Gravity dams, Complex function

1. Introduction

The main aim of the drainage gallery, constructed at the floor of the gravity dam, is to receive seepage discharge beneath the dam and minimize uplift pressure acting on the floor of the dam. Abd El-Razek and Abo Elela [1] studied experimentally the optimum position of the drainage gallery underneath the gravity dam. They concluded that the optimum position could be achieved at middle of the dam floor at which the discharge entering the drainage gallery and the reduction in uplift force acting on the dam floor is maximum. Abd El-Razek and Abo Elela [2] also studied effect of the drainage gallery dimensions on the uplift force acting on the floor of the gravity dam.

Hathoot [3] solved the problem of seepage through an earth dam with a trapezoidal toe filter using the complex function. He deduced the complex potential, the velocity potential, and the stream function of the line sink.

In the present work, a mathematical solution based on the complex function is used to find depth of the drainage gallery, which collects all seepage discharge escaped beneath the dam. Quantity of seepage is also calculated according to the variation of depth of the impervious base.

2. Mathematical model

A theoretical solution based on hydrodynamic theory is used to find depth of the drainage gallery constructed at floor of the gravity dam. The problem is treated in the vertical direction, to find depth of drainage gallery, considering vertical line sink (ab) with image (cd) as shown in fig. 1.

2.1. Mathematical determination penetration depth of the drainage gallery

The complex potential of the vertical line sink (ab) is given as [3]:

$$dw = \frac{m}{h}\int_0^h ln(z-ih)dh$$
$$= \frac{-m}{ih}\int_0^h ln(z-ih)d(z-ih),$$

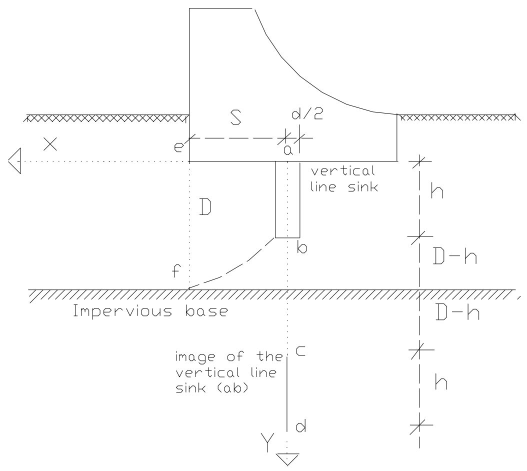


Fig.1. Section of the gravity dam with the drainage gallery under study.

$$W = \frac{im}{h}\left[(z-ih)\ln(z-ih)-(z-ih)\right]_0^h$$

$$= \frac{im}{h}\left[(z-ih)\ln(z-ih)-(z-ih)-z\ln z+z\right] .$$

Putting; $z = z$, $z\ln z = (x+iy)(\ln r + i\theta)$, and

$(z-ih)\ln(z-ih) = (x_1+iy_1)(\ln r_1 + i\theta_1)$.

Then, the complex potential of the vertical line sink (ab) can be expressed by eqs. (1) and (2):

$$W = \frac{im}{h}\begin{bmatrix} x_1 \ln r_1 + ix_1\theta_1 + iy_1 \ln r_1 - y_1\theta_1 - x\ln r \\ -ix\theta - iy\ln r + y\theta + ih \end{bmatrix}. \quad (1)$$

$$W = \phi + i\psi \ . \quad (2)$$

Equating real parts on both sides of eqs. (1) & (2), the velocity potential of the vertical line sink can be expressed by:

$$\phi_{sink} = \frac{m}{h}\left[-x_1\theta_1 - y_1\ln r_1 + x\theta + y\ln r - h\right]. \quad (3)$$

Where:

m = strength,

h = penetration depth of the drainage gallery,

$$r = \sqrt{x^2+y^2} , \quad (4)$$

$$\theta = \tan^{-1} y/x , \quad (5)$$

$$x_1 = x , \quad (6)$$

$$y_1 = y - h , \quad (7)$$

$$r_1 = \sqrt{x_1^2 + y_1^2} \text{ , and} \quad (8)$$

$$\theta_1 = \tan^{-1} y_1/x_1 \quad . \quad (9)$$

Also, the complex potential of the image of the vertical line sink (cd) is given as [3]:

$$dw = \frac{m}{h}\int_0^h \ln(z+ih)dh$$
$$= \frac{-m}{ih}\int_0^h \ln(z+ih)d(z+ih) ,$$

$$W = \frac{im}{h}\left[(z+ih)\ln(z+ih)-(z+ih)\right]_0^h$$
$$= \frac{im}{h}\left[(z+ih)\ln(z+ih)-(z+ih)-z\ln z+z\right]$$

Putting; $z = z - i(2D-h)$, and

$(z+ih)\ln(z+ih) = (x_6+iy_6)(\ln r_6 + i\theta_6)$.

Then, the complex potential of the image of the vertical line sink (cd) can be expressed by eq. (10):

$$W = \frac{im}{h}\begin{bmatrix} x_6 \ln r_6 + ix_6\theta_6 + iy_6 \ln r_6 - y_6\theta_6 \\ -x\ln r - ix\theta - iy\ln r + y\theta - ih \end{bmatrix}. \quad (10)$$

Equating real parts on both sides of eqs. (2) and (10), the velocity potential of image of the vertical line sink can be expressed by:

$$\phi_{image} = \frac{m}{h}\left[-x_6\theta_6 - y_6\ln r_6 + x\theta + y\ln r + h\right]. \quad (11)$$

Where:

$$x_6 = x, \quad (12)$$

$$y_6 = y - 2D + h, \quad (13)$$

$$r_6 = \sqrt{x_6^2 + y_6^2}, \text{ and} \quad (14)$$

$$\theta_6 = tan^{-1} y_6 / x_6 . \quad (15)$$

The velocity potential of the system (vertical line sink with image) can be expressed by:

$$\phi_{system} = \phi_{sink} + \phi_{image}$$

$$= \frac{m}{h}\begin{bmatrix} -x_1\theta_1 - y_1 \ln r_1 - x_6\theta_6 - y_6 \ln r_6 \\ +2x\theta + 2y \ln r \end{bmatrix}. \quad (16)$$

2.2. Boundary conditions

The velocity potential can be given as:

$$\phi = -k(\frac{p}{\rho g} + y) . \quad (17)$$

Where:
ϕ is the velocity potential,
k is the soil conductivity,
p is the gauge pressure,
ρ is the water density, and
g is the acceleration due to gravity.

Eqs. (16) and (17) can be applied coordinates $x = d/2$ and $y = h$, to find the strength "m" of the system.

2.3. Determination the free water surface

It is assumed that all seepage discharge beneath the dam should enter the drainage gallery (ab), therefore along the free water surface, the pressure is atmospheric, and the velocity potential can be given as:

$$\phi = -ky . \quad (18)$$

Applying eqs. (16) and (18), the free water surface can be obtained using the following equation:

$$y = \frac{-m}{kh}\begin{bmatrix} -x_1\theta_1 - y_1 \ln r_1 - x_6\theta_6 - y_6 \ln r_6 \\ +2x\theta + 2y \ln r \end{bmatrix}. \quad (19)$$

The free water surface is plotted as shown in fig. 2 according to eq. (19) for different depths of the impervious layer (D) to determine the required depth of the drainage gallery (h), which collects all seepage discharge beneath the dam. Results of eq. (19) were concluded in table 1.

From table 1 and fig. 2, it is clear that depth of the drainage gallery (h) increases with increasing depth of impervious layer (D) measured beneath heal of the dam. The average relative depth of the drainage gallery (h/D) may be considered equal 0.66 for the relative depth of the impervious layer (D/S) ranging from 1 to 6. The results shown in table 1 based on the assumption that all seepage discharge passing beneath the dam is received by

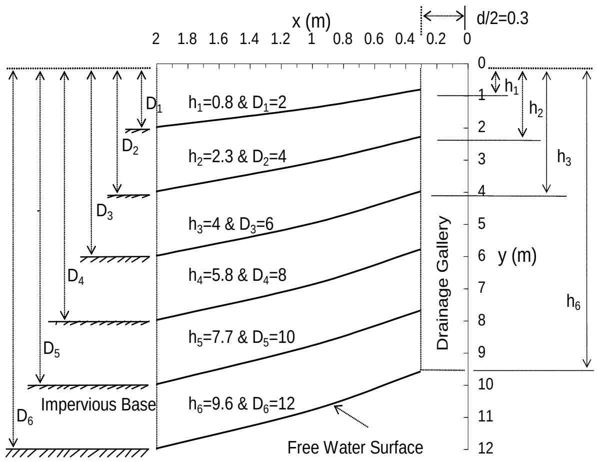


Fig. 2. Depth of drainage gallery (h) corresponding to depth of impervious layer (D).

Table 1
Calculated values of the drainage gallery depths (h) corresponding to the impervious layer depths (D), and the seepage discharge (q) [d = 0.6 m, S = 2 m & k = 0.00016 m/sec]

D (m)	*h (m)*	*q ($m^3/s/m$)*	*h/D*	*D/S*	*q/(k.D)*
2	0.80	0.00016	0.40	1	0.500
4	2.30	0.00040	0.58	2	0.625
6	4.00	0.00067	0.67	3	0.698
8	5.80	0.00095	0.73	4	0.742
10	7.70	0.00124	0.77	5	0.775
12	9.60	0.00148	0.80	6	0.771

the drainage gallery of the relative depth h/D = 0.66. A computer program using “Microsoft Visual Basic” is prepared to solve the deduced equations.

2.4. Quantity of seepage discharge beneath the dam

Equating imaginary parts on both sides of eqs. (1) and (2), the stream function of the vertical line sink (ab) can be represented by:

$$\psi_{sink} = \frac{m}{h}\left[x_1 \ln r_1 - y_1\theta_1 - x \ln r + y\theta\right]. \quad (20)$$

Equating imaginary parts on both sides of eqs. (2) and (10), the stream function of the image of the vertical line sink can be represented by:

$$\psi_{image} = \frac{m}{h}\left[x_6 \ln r_6 - y_6\theta_6 - x \ln r + y\theta\right]. \quad (21)$$

The stream function of the system (vertical line sink with image) is expressed by:

$$\psi_{system} = \psi_{sink} + \psi_{image}$$

$$= \frac{m}{h}\left[x_1 \ln r_1 - y_1\theta_1 + x_6 \ln r_6 - y_6\theta_6 - 2x \ln r + 2y\theta\right]. \quad (22)$$

The seepage discharge beneath the dam can be given as:

$$q = \psi_e - \psi_f\,. \quad (23)$$

The results obtained from eq. (23) are also shown in table 1. Fig. 3-a shows the relationship between the relative penetration depth of the drainage gallery (h/D) and relative seepage discharge ($q/k.D$) for d/S = 0.3. The relationship between the relative penetration depth of the drainage gallery (h/D) and relative depth of the impervious layer (D/S) is also illustrated in fig. 3-b. From the results shown in table 1 and fig. 3, it is clear that seepage discharge increases with increasing depth of the impervious layer beneath the dam.

The penetration depth of the drainage gallery constructed at the floor of the gravity dam (h) and the seepage discharge beneath the dam (q) can be obtained for a known depth of impervious layer (D) using fig. 3 as a design chart. This procedure is illustrated in the following example.

Example:

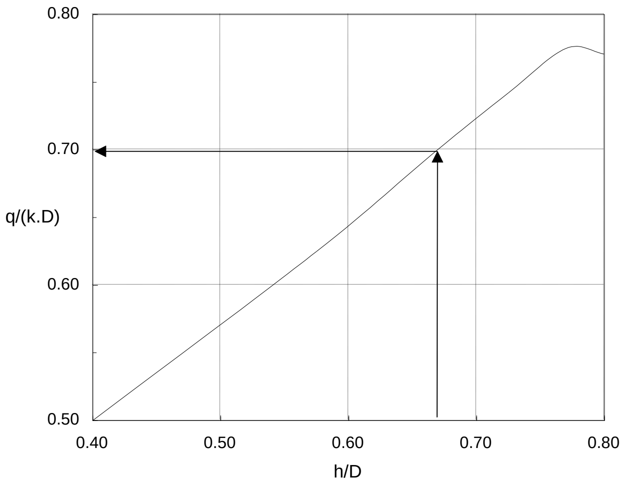
It is required to determine the penetration depth of the drainage gallery constructed beneath the gravity dam and the seepage discharge beneath the dam. The following data are available:
Depth of the impervious layer (D) = 6 m,
Distance of the drainage gallery measured from heel of the dam (S) = 2 m,
Sand conductivity beneath the dam floor (k) = $1.6*10^{-2}$ cm/sec.

Solution:

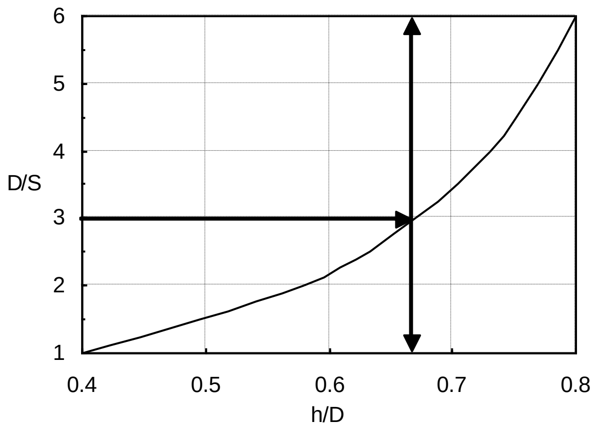
The relative depth of the impervious layer is calculated (D/S = 6/2 =3). Using fig. 3-b, the vertical axis is entered with the value of 3, then vertically down, and the relative penetration depth of the drainage gallery is determined (h/D = 0.67). Using fig. 3-a, the horizontal axis is entered with the value of 0.67, then vertically up, and the relative seepage discharge is determined ($q/k.D$ = 0.7). In this case, h = 6*0.67 = 4 m and q = 0.7* (0.00016*6) 0.00067 m^3/s/m.

3. Conclusions and recommendations

In the present study a mathematical solution is established based on the complex functions to determine depth of the drainage gallery constructed beneath gravity dams. The following conclusions can be made:
1. The complex potential, the velocity potential and the stream function for the suggested system are established.
2. For the relative depth of the impervious layer (D/S) ranging from 1 to 6, the depth of the drainage gallery (h) may be taken 0.66 the depth of the impervious layer (D), which allow all seepage discharge entering the drainage gallery.



(a)



(b)

Fig. 3. Relative penetration depth of the drainage gallery (*h/D*) versus relative depth of the impervious layer *(D/S)* and relative seepage discharge (*q/k.D*) for *d/S* = 0.3.

3. All seepage discharge entering the drainage gallery increases with increasing depth of the impervious layer.
4. It is recommended to determine the horizontal distance of the drainage gallery constructed beneath the gravity dam using the complex functions.

Nomenclature

ab is the length of the vertical line sink, fig.1,
cd is the length of image of the vertical line sink, fig.1,
d is the diameter of the drainage gallery,
D is the depth of the impervious layer,
h is the penetration depth of the drainage gallery,
m is the strength,
q is the seepage discharge,
S is the distance of the drainage gallery measured from heal of the dam,
x is the horizontal coordinate,
y is the vertical coordinate,
z = x + iy,
W is the complex potential,
Φ is the equipotential function,
Ψ is the stream function, and
i is an imaginary number = $\sqrt{-1}$.

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Received 22 September, 2003
Accepted November 19, 2003