Minimum-cost design of pipelines transporting non-newtonian fluids

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This paper addresses the minimum-cost design of pipelines transporting non-Newtonian fluids and equipped with equally spaced pumping units. The author used a set of recently presented equations for flow of Herschel-Bulkley fluids in establishing minimum cost design equations. One of these equations is for laminar flow whereas the other is for turbulent flow. The equations are based on hydraulic and economic considerations. Since both equations are implicit, two procedures are suggested to show the steps to be followed to estimate the minimum-cost diameter. A practical numerical example is given and both procedures are followed in solving the example. Application of both procedures yields the same rational minimum-cost diameter.

في هذا البحث تمت معالجة التصميم الأمثل لخطوط الأنابيب التي تنقل سوائل لانيوتونية. لقد أخذ في الإعتبار حالة خطوط الأنابيب المركب عليها مضخات متماثلة على مسافات متساوية. تم استخدام مجموعة من المعادلات المستنبطة حديثا و الخاصة بتدفق سوائل هرشل-بكلي في استنباط معادلات تصميمية لخطوط الأنابيب. لقد تم تقديم معادلتين احداهما خاصة بالتدفق الطبقي و الأخرى خاصة بالتدفق الإضطرابي. كان الأساس في استنباط المعادلات الإعتبارات الهيدروليكية جنبا الى جنب مع الإعتبارات الإقتصادية. و لما كانت المعادلات التصميمية المستنبطة من النوع الضمني فقد تم اقتراح طريقتين لحلها و ذلك للتوصل للقطر الأمثل لخط الأنابيب. كذلك تم تقديم مثال عددي لحالة عملية و تم استخدام كلتا الطريقتين في حل المثال و كانت النتائج في الحالتين متطابقة و منطقية.

Keywords: Pipeline, Non-newtonian, Economic design

1. Introduction

The optimal design of pipelines transporting Newtonian fluids has been investigated on different lines of approach Cheremisinoff [1]. A preliminary optimal design was provided in which the coefficient of friction was considered as a constant depending on the relative pipe roughness Daugherty and Franzini [2]. Considering that the coefficient of friction depends on both Reynolds' number and the pipe relative roughness, optimal pipeline designs were presented in a series of papers Hathoot [3]; Hathoot [4]; Hathoot [5] and Hathoot et al. [6]. Establishment of pipeline optimal design equations was based on the well known analytical and experimental equations that describe the motion of Newtonian fluids in pipes. On the other hand, owing to the complexity and the diversity of behavior of non-Newtonian fluids, the development of "universal" equations for the prediction of head losses was not available. For this reason no attempts have been made to investigate optimal design in cases of non-Newtonian fluids. Recently (Chilton and Stainsby [7]) a coherent set of equations for the laminar and turbulent flow of non-Newtonian (Herschel-Bulkley) fluids has been presented. Since non-Newtonian fluids include fluids which are essential in industry, it is of practical importance to establish optimal design equations for pipelines transporting non-Newtonian fluids.

2. Non-newtonian fluids

The rheology of the fluid is described by the Herschel-Bulkley model [8]

$$\tau = \tau_y + j\left(\gamma\right)^n , \qquad (1)$$

in which τ = shear stress; γ = shear strain rate; τ_y = yield stress; and j and n= parameters for a particular fluid. The Herschel-Bulkley model has been applied to a wide variety of fluids including sewage sludges, kaolin slurries, and mine tailings. Chilton et al. [9] presented the following equation for pressure loss in the case of laminar flow.

$$\frac{\Delta p}{L}=\frac{4j}{D}\left(\frac{8V}{D}\right)^{n}\left(\frac{3n+1}{4n}\right)^{n}\left(\frac{1}{1-X}\right),$$

$$\left(\frac{1}{1-aX-bX^{2}-cX^{3}}\right)^{n}\ , \qquad (2)$$

where Δp = pressure loss; L = pipe length; D = pipe diameter; and V = mean pipe velocity. X is given by:

$$X=\frac{4L\,\tau_{y}}{D\,\Delta p}, \qquad (3)$$

$$a=\frac{1}{(2n+1)};\quad b=\frac{2n}{(n+1)(2n+1)};$$

and

$$c=\frac{2n^{2}}{(n+1)(2n+1)}. \qquad (4)$$

Given the mean velocity, or discharge, eq. (2) can be solved for the pressure loss by an iterative technique. Alternatively, eq. (2) can be rearranged to give a direct solution of the mean velocity, or discharge, for a given pressure loss. For turbulent flow an equation similar to eq. (2) can be obtained by combining the coefficient of friction, presented later on, and the Darcy-Weisbach equation Chilton and Stainsby [7].

3. Reynolds' numbers and coefficients of friction

For laminar flow the Metzner-Reed [10] Reynolds' number is given as

$$R_{MR}=\frac{\rho\,V\,D}{j\left(\frac{8V}{D}\right)^{(n-1)}\left(\frac{3n+1}{4n}\right)^{n}\left(\frac{1}{1-X}\right)\cdots\left(\frac{1}{1-aX-bX^{2}-cX^{3}}\right)^{n}}, \qquad (5)$$

where ρ = fluid density. For turbulent flow the special form of Reynolds' number for Herschel-Bulkley fluids is,

$$R=\frac{\rho\,V\,D}{\mu_{w}\left(\frac{3n+1}{4n}\right)\left(\frac{1}{1-aX-bX^{2}-cX^{3}}\right)}, \qquad (6)$$

where:

$$\mu_{w}=\tau_{w}^{(n-1)/n}\left(\frac{j}{1-X}\right)^{(1/n)}, \qquad (7)$$

and

$$\tau_{w}=\frac{D\,\Delta p}{4L}. \qquad (8)$$

The critical Reynolds' number was found to lie between 2,500 and 5,000.

Analogous to Newtonian fluids, the coefficient of friction in the case of laminar flow is given by:

$$f=\frac{64}{R_{MR}}\ . \qquad (9)$$

For turbulent flow (Chilton and Stainsby [7]) a coefficient of friction based on experimental data was given by:

$$f=0.316\left(\frac{R}{n^{2}\,(1-X)^{4}}\right)^{-0.25}. \qquad (10)$$

4. Minimum-cost design

4.1. Cost of pipe

The levelized net annual cost of pipe per unit pipe length (Hathoot et al. [6]) can be written as:

$$K_{p}=\pi\,D\,t\,\gamma_{p}\,C_{1}, \qquad (11)$$

in which t = pipe wall thickness; γ_p = specific weight of pipe material; and C_1 = levelized net annual cost of pipes per unit weight of pipe material (C_1 is constant for a suitable range of diameters). The pipe wall thickness t may be assumed to be roughly proportional to the pipe diameter Davis and Sorensen [11]; Russel [12] so that,

$$t = C\,D\,, \tag{12}$$

in which C = constant of proportionality that depends upon the expected pressure and diameter ranges of the pipe. Substitution of eq. (12) in eq. (11) yields:

$$K_p = C\,C_1\,\gamma_p\,\pi\,D^2 \quad . \tag{13}$$

4.2. Cost of energy

For a pipeline with equally spaced pumping units, fig. 1, the power required per pump is given by:

$$P = \frac{\gamma\,Q\,H_p}{E}\,, \tag{14}$$

in which γ = specific weight of the liquid to be pumped; Q = the discharge, H_p = head required per pump; and E = the overall efficiency of each pump. Neglecting the effect of pipe fittings the head per pump is given by

$$H_p = \frac{\Delta p}{\gamma} \quad , \tag{15}$$

in which Δp = the pressure loss between two pumping units. From eqs. (14) and (15) the power required per pump is,

$$P = \frac{Q\,\Delta p}{E}\,. \tag{16}$$

It follows that the power required per unit length of pipe is

$$W = \frac{Q\,\Delta p}{E\,L} \quad , \tag{17}$$

in which L = the spacing between pumps.

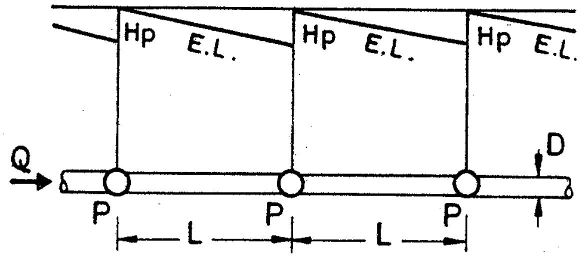

Fig. 1. Pipeline with equally spaced pumping units.

The levelized net annual cost of pumping energy per unit pipe length (Hathoot et al. [6]) is given by:

$$K_{en} = \frac{C_2 Q\,\Delta p}{E\,L}\,, \tag{18}$$

in which C_2 = the levelized net annual cost of pumping energy per watt.

5. Total cost of pipeline

The total cost of the pipeline is simply the sum of the annual costs of pipes and energy. From eqs. (11) and (18), the levelized total annual cost of the pipeline per unit pipe length is given by:

$$K_{tu} = C\,C_1\,\gamma_p\,\pi\,D^2 + \frac{C_2 Q\,\Delta p}{E\,L}\,. \tag{19}$$

6. Establishment of design equations

In general a pipe of large diameter produces a small friction head loss against which the pump should act. On the other hand, though a pipe of a smaller diameter is cheaper, it produces a greater friction head loss. Therefore, an optimal diameter exists for which the total annual cost of pipes and energy is a minimum. For minimum-cost design differentiation of K_{tu} with respect to the diameter D should equal zero. Differentiating both sides of eq. (19) and equating to zero yields,

$$2C\,C_1\,\gamma_p\,\pi\,D + \frac{C_2 Q}{E}\,\frac{d\left(\frac{\Delta p}{L}\right)}{d\,(D)} = 0 \quad , \tag{20}$$

solving for *D*,

$$D = -G \frac{d\left(\frac{\Delta p}{L}\right)}{d(D)}, \tag{21}$$

in which,

$$G = \frac{C_2 Q}{2\, C\, C_1\, \gamma_p\, \pi\, E} \; . \tag{22}$$

It is worthy to note that the right hand side of eq. (21) is a function of the diameter *D*. Since the pressure drop and the pipe diameter are interrelated by different equation in cases of laminar and turbulent flows, each case is to be considered separately.

5.1. Laminar flow

Substituting $V = 4Q / \pi D^2$ into eq. (2) we can write:

$$\frac{\Delta p}{L} = \phi\left(D, \frac{\Delta p}{L}\right), \tag{23}$$

in which,

$$\phi\left(D, \frac{\Delta p}{L}\right) = A\left(\frac{1}{D^{3n+1}}\right)\cdot\left(\frac{1}{1-X}\right) \cdot$$

$$\left(\frac{1}{1-aX-bX^2-cX^3}\right)^n, \tag{24}$$

where,

$$A = 4j\left[\frac{32\,Q}{\pi}\left(\frac{3n+1}{4n}\right)\right]^n \; . \tag{25}$$

For convenience eq. (23) is put in the form:

$$F\left(D, \frac{\Delta p}{L}\right) = \frac{\Delta p}{L} - \phi\left(D, \frac{\Delta p}{L}\right) \quad . \tag{26}$$

From calculus (Swokowski [13]) we have,

$$\frac{d\left(\frac{\Delta p}{L}\right)}{d(D)} = -\frac{F_D\left(D, \frac{\Delta p}{L}\right)}{F_{\Delta p/L}\left(D, \frac{\Delta p}{L}\right)}, \tag{27}$$

in which $F_D\,(D, \Delta p/L)$ and $F_{\Delta p/L}\,(D, \Delta p/L)$ are partial derivatives of the right hand side of eq. (26) with respect to *D* and $\Delta p/L$, respectively. Differentiating partially and rearranging:

$$F_D\left(D, \frac{\Delta p}{L}\right) = \frac{A_1}{D}\left(nX_2X_3 + X_1 + 3n\right), \tag{28}$$

in which,

$$A_1 = \frac{A\, X_1\, X_2^n}{D^{3n+1}}, \tag{29}$$

$$X_1 = \frac{1}{1-X}, \tag{30}$$

$$X_2 = \frac{1}{1-aX-bX^2-cX^3}, \tag{31}$$

and

$$X_3 = aX + 2bX^2 + 3cX^3. \tag{32}$$

Similarly:

$$F_{\Delta p/L}\left(D, \frac{\Delta p}{L}\right) = 1 + \frac{L\, A_1}{\Delta p} \cdot$$

$$\left(nX_2X_3 + XX_1\right). \tag{33}$$

From eqs. (21) and (27) the optimal diameter equation for laminar flow is,

$$D = G.\frac{F_D\left(D, \frac{\Delta p}{L}\right)}{F_{\Delta p/L}\left(D, \frac{\Delta p}{L}\right)}. \tag{34}$$

It is obvious that eq. (34) is implicit and the optimal diameter is to be solved through a trial and error procedure.

5.2. Turbulent flow

The Darcy-Weisbach equation can be put in the form:

$$\frac{\Delta p}{\gamma} = \frac{8 f\, L\, Q^2}{\pi^2 g\, D^5}\,, \tag{35}$$

in which g = acceleration due to gravity and f = coefficient of friction.

Rearranging eq. (35),

$$\frac{\Delta p}{L} = \frac{8 f\, \rho\, Q^2}{\pi^2\, D^5}\,. \tag{36}$$

Substitution $V = 4Q/\pi D^2$ into eq. (6) yields,

$$R = \frac{4\, \rho\, Q}{\pi\, D\, \mu_w \left(\frac{3n+1}{4n}\right)\left(\frac{1}{1 - aX - bX^2 - cX^3}\right)}\,. \tag{37}$$

Combining eqs. (35), (10) and (37), we get:

$$\frac{\Delta p}{L} = \frac{B_1}{D^5} = \left[\frac{\mu_w\, D\,(1-X)^4}{1 - aX - bX^2 - cX^3}\right]^{0.25}, \tag{38}$$

in which,

$$B_1 = \frac{2.528\, \rho\, Q^2 \sqrt{n}}{\pi^2\, B^{0.25}}\,, \tag{39}$$

and

$$B = \frac{4\, \rho\, Q}{\pi\left(\frac{3n+1}{4n}\right)}\,. \tag{40}$$

By means of eq. (38) the pressure loss for turbulent flow can be estimated. In fact eq. (38) is analogous to eq. (2) which is used to estimate the pressure loss in case of laminar flow. For convenience eq. (38) is put in the form:

$$\frac{\Delta p}{L} = \psi\left(D, \frac{\Delta p}{L}\right), \tag{41}$$

in which,

$$\psi\left(D, \frac{\Delta p}{L}\right) = \frac{B_1}{D^5}\left[\frac{\mu_w\, D\,(1-X)^4}{1 - aX - bX^2 - cX^3}\right]^{0.25}, \tag{42}$$

Again for convenience eq. (41) is put in the form:

$$H\left(D, \frac{\Delta p}{L}\right) = \frac{\Delta p}{L} - \psi\left(D, \frac{\Delta p}{L}\right). \tag{43}$$

Starting with eq. (21) and following the same procedure as in the case of laminar flow the optimal diameter in the case of turbulent flow is,

$$D = G.\frac{H_D\left(D, \frac{\Delta p}{L}\right)}{H_{\Delta p/L}\left(D, \frac{\Delta p}{L}\right)}\,, \tag{44}$$

in which $H_D\ (D, \Delta p/L)$ and $H_{\Delta p/L}\ (D, \Delta p/L)$ are partial derivatives of the right hand side of eq. (43) with respect to D and $\Delta p/L$, respectively. Differentiating partially and rearranging:

$$H_D\left(D, \frac{\Delta p}{L}\right) = \frac{A_2}{D}\left(X_4 + X_3 + \frac{19}{X_2}\right). \tag{45}$$

in which,

$$A_2 = 0.25\, B_1 X_2 \left[\frac{\mu_w\, X_2}{D^{19}\, X_1^4}\right]^{0.25}, \tag{46}$$

and

$$X_4 = \frac{\left(\frac{1}{n} - 4\right) X\, X_1 + \frac{1}{n} - 1}{X_2}\,. \tag{47}$$

Similarly,

$$H_{\Delta p/L}\left(D, \frac{\Delta p}{L}\right) = 1 + \frac{A_2 L}{\Delta p}\ (X_3 + X_4)\ . \qquad (48)$$

Since eq. (44) is implicit the optimal diameter in the case of turbulent flow is to be found through a trial and error procedure.

7. Procedure

For minimum-cost design of a pipeline with equally spaced pumping units the following procedure is recommended:

1. The fluid properties such as ρ, τ_y, j and n should be known in advance, also the required discharge, Q, should be given.
2. A rational value of the pipeline diameter, D, is to be assumed and the corresponding value of the pressure loss $\Delta p/L$ is to be evaluated by using eq. (2) in the case of laminar flow and eq. (38) for turbulent flow. $\Delta p/L$ is evaluated by trial and error using a simple iteration technique (Chilton and Stainsby [7]).
3. Another diameter is assumed and the corresponding $\Delta p/L$ value is obtained.
4. Step 3 is then repeated for other diameters till a suitable number of ($D, \Delta p/L$) pairs are available.
5. A graph of D versus $\Delta p/L$ is plotted which we call the pressure-loss curve.
6. A reasonable value of $\Delta p/L$ is assumed and eq. (34) or eq. (44) is used to estimate D for laminar or turbulent flow, respectively.
7. Step 6 is repeated for different $\Delta p/L$ values to obtain a suitable number of ($\Delta p/L, D$) pairs.
8. On the same aforementioned graph paper $\Delta p/L$ versus D are plotted according to eq. (34) or eq. (44) and the resulting curve is called the optimal diameter curve.
9. The point of intersection of the pressure-loss curve and the optimal diameter curve provides the minimum-cost diameter and the corresponding $\Delta p/L$.

8. Alternative procedure

The following alternative procedure is also recommended.

1. The pressure-loss curve is obtained as explained in the first procedure.
2. The total-cost K_{tu} is estimated by applying (19) and by inserting $\Delta p/L$ and corresponding D values obtained from the pressure-loss curve.
3. The pipe diameter D is plotted versus the total cost and the point of minimum K_{tu} on the graph corresponding to the required minimum-cost diameter. It is worthy to note that in all steps X should be less than unity, otherwise calculations will not be completed. This is because negative quantities raised to fractional powers will appear.

9. Numerical example

It is required to find the minimum-cost diameter of a pipeline delivering 0.02 m^3/s of Kaolin slurry (ρ = 1,105 kg/m^3, τ_y = 4.18 N/m^2, j = 0.035 $Pa.s^n$ and n = 0.719), provided that γ_p = 73575 N/m^3; C = 0.01, C_1 0.011, C_2 = 1.9 and E = 0.7.

10. Solution

The steps of the recommended procedure are followed and the results are shown plotted in fig. 2. Since laminar flow dominates eqs. (2) and (34) are used in performing calculations. As shown in fig. 2 the point of intersection of curves provides the minimum-cost diameter D = 0.288 m which corresponds to $\Delta p/L$ = 66.5 Pa/m with R_{mr} = 202. In addition the alternative procedure, mentioned above, is applied as shown in fig. 3 and the same minimum-cost diameter is obtained.

11. Conclusions

Two equations for estimating the minimum-cost diameter of pipelines transporting non-Newtonian fluids are presented. Although, the two equations, which are for laminar and turbulent flow, are implicit they can be solved by any simple iteration technique. A numerical example considering a practical case provides a rational minimum-cost diameter. The example is solved using an alternative procedure which yields the same diameter.

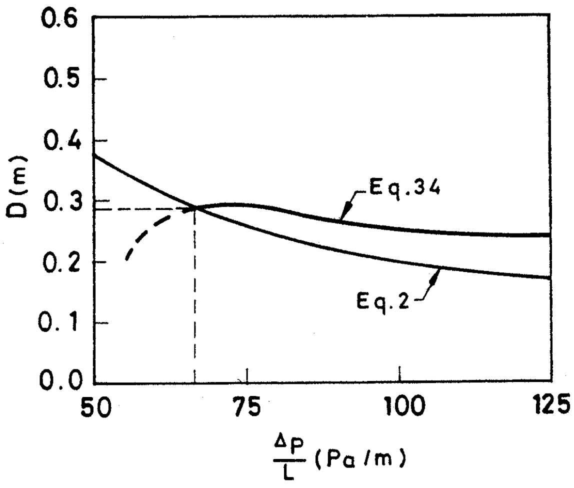

Fig. 2. Graphical evaluation of the optimal diameter.

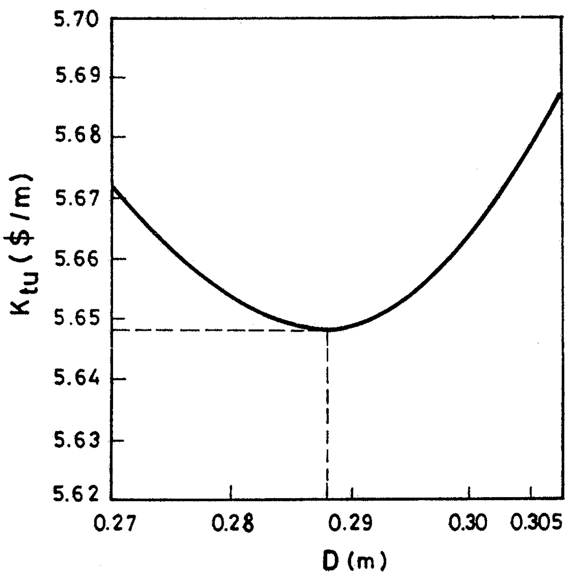

Fig. 3. Graphical evaluation of the optimal diameter by considering the minimum total cost.

Notations

The following symbols are used in this paper:

A is the quantity defined by eq. (25)
A_1 is the quantity defined by eq. (29),
A_2 is the quantity defined by eq. (46)
a is the quantity defined by eq. (4)0,
B is the quantity defined by eq. (40),
B_1 is the quantity defined by eq. (39),
b is the quantity defined by eq. (4),
C is the constant contained in eq. (12),
C_1 is the levelized net annual cost of pipes per unit weight of pipe material,
C_2 is the levelized net annual cost of pumping energy per watt;,
c is the quantity defined by eq. (4),
D is the pipe diameter,
E is the pump efficiency,
f is the coefficient of friction;
G is the quantity defined by eq. (22),
g is the acceleration due to gravity,
H_p is the head provided by pump,
j is the parameter of a particular fluid,
K_{en} is the levelized net annual cost of pumping energy per unit pipe length,
K_p is the levelized net annual cost of pipe per unit pipe length,
K_{tu} is the levelized total annual cost of the pipeline per unit pipe length,
L is the pipe length,
n is the parameter of a particular fluid,
P is the power provided per pump,
Δp is the pressure loss,
Q is the pipeline discharge,
R is the Reynolds number for turbulent flow,
R_{MR} is the Metzner-Reed Reynolds number for laminar flow,
t is the pipe wall thickness,
V is the pipe mean velocity,
W is the power required per unit length of pipe,
X is the quantity defined by eq. (3),
X_1 is the quantity defined by eq. (30),
X_2 is the quantity defined by eq. (31),
X_3 is the quantity defined by eq. (32),
X_4 is the quantity defined by eq. (47),
γ is the specific weight of fluid,
γ_p is the specific weight of pipe material,
μ_w is the apparent viscosity at the wall,
ρ is the fluid density,
τ is the shear stress, and
τ_y is the yield stress.

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