Gradually varied flow along a wide channel connecting two reservoirs

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Open channel connecting two reservoirs, is still attracting civil engineers. When the channel is long, uniform flow prevails through most of the channel except a distance close to the end where flow escapes to the downstream reservoir in which the condition of water level determines the type of gradually varied flow curve. However, when the channel is short, gradually varied flow may prevail throughout the whole channel length. Long or short channel depends on an accurate estimate of gradually varied flow distance. Gradually Varied Flow (GVF) is well presented in textbooks. However, the approximate methods; direct step method, for example, are preferred to the accurate methods. The solution of gradually varied equation in the general form is complicated. The paper, herein, represents an accurate solution to a special form of gradually varied flow in a new dimensionless function. This form is called Bresse solution. It is valid only for the wide cross section. The method is applied to the channel connecting two reservoirs. Family curves of mild and steep slopes are presented in a new dimensionless form. The case study presented herein is “Toshka Spillway”, a wide channel connecting between Aswan Reservoir and Toshka Depression. Two cases are considered. In the first one, the flow escapes at the end to a low water level in the depression and an actual gradually varied flow distance of M_2 is estimated. While, In the second case, a weir is considered at end of the spillway and an actual gradually varied flow distance of M_1 is estimated. In both cases, several water levels in Aswan reservoir are considered. Moreover, the direct step method is represented in a new dimensionless form to be valid for a wide cross section. The new form is compared with Bresse solution.

البحث يعيد تقديم حلا لمشكلة حساب مسافة الجريان المتدرج خلال القنوات المكشوفة العريضة المقطع . فمن المعروف أن حل المعادلة العامة للجريان المتدرج يعتبر معقدا جدا والبحث يعيد تقديم أحد الحلول المبنية على فرض أن القناه المكشوفة عريضة مع إستخدام معادلة شيزى. والحل يسمى بإسم "برس" ويصلح للقنوات العريضة ذات الميول المنبسطة والحادة ولكن لا يتم تطبيقه خلال القنوات ذات الميول الطولية الأفقية أو الحرجة. ويشير البحث إلى أن الحل فى هاتين الحالتين قدمه الباحث فى بحث سابق. فى هذا البحث يتم تقديم طريقة "برس" فى صورة غير بعدية جديدة يتم تطبيقها على القناه المكشوفة التى تربط بين خزانين. تم تقديم عائلة المنحنيات المشهورة التى تحدث خلال الميل الطولى المنبسط والحاد فى صورة غير بعدية جديدة. البحث أيضا يتناول نموذجا للقناه التى تربط بين خزانين وهى مفيض توشكى التى تربط بين خزان أسوان ومنخفض توشكا. وتم تطبيق طريقة برس على مفيض توشكى لحالتين مختلفتين: الحالة الأولى يتم فيها إعتبار وجود هدار نهاية وحساب طول منحنى الرمو M_1 والحالة الثانية يتم فيها أيضا إهمال وجود هدار النهاية و حساب طول منحنى الرمو M_2 ويتضح أن وجود الهدار يطيل من طول منحنى الرمو ومع ذلك كان طول المنحنيين أقل من طول المفيض وبالتالى تعتبر القناه "طويلة". يتم أيضا فى هذا البحث تقديم طريقة تقريبية مشهورة هى طريقة الخطوة المباشرة فى صورة غير بعدية جديدة يصلح تطبيقها فى القنوات المكشوفة العريضة المقطع مع إستخدام معادلة شيزى. وتم مقارنتها بطريقة برس وإتضح أن طريقة الخطوة المباشرة تزيد دقتها بتقليل العمق التزايدى المستخدم فى الحسابات ولكن ذلك يزيد من كمية الحسابات ويتطلب معه عمل برنامج لهذه الحسابات. على النقيض من ذلك فإن طريقة برس يتم تطبيقها فى خطوة واحدة وبدقة عالية.

Keywords: Gradually varied flow, Wide channel, Toshka spillway, Bresse solution, Direct step method

1. Introduction

It is well known that the general equation of Gradually Varied Flow (GVF) is given by:

$$dy/dx = (S_o - S_e)/[1 - F_n^2], \tag{1}$$

in which dy/dy is the water slope, S_o is the longitudinal bed slope, S_e is the energy slope and F_n is the Froude number. It is also well known that when Manning's equation is used and wide cross-section is considered, eq. (1) is written as:

$$dy/dx = S_o\,[1 - (y_o/y)^{(10/3)}]/[1-(y_c/y)^3]. \quad (2)$$

Alternatively, when Chezy's equation is used and wide cross-section is considered, eq. (2) can be written as:

$$dy/dx = S_o\,[1 - (y_o/y)^3]/[1-(y_c/y)^3], \quad (3)$$

where y_o is the normal water depth, y_c is the critical depth.

Eq. (3) is valid for only wide open channel. The analytical solution of eq. (3) was first obtained by Bresse for very wide rectangular channels. The solution approach was extended by Bakhmeteff and finally fully developed by Chow [1] into a complicated method called the "hydraulic exponent method". It is a numerical method in the form developed by Chow, but a very tedious one that no longer is in use. For very wide cross section, explanation to Bresse solution was done by Davidian [2]. The solution is only valid and can be applied only along channel of mild and steep slopes. However, for channel of critical and horizontal bed slopes, the solution can not be applied. In these two types of slopes, Mostafa [3] gave the exact solution. For critical slope channel, he gave the following equation:

$$X_c = (y_d - 2\,y_u/3)\,(2y_u/3)^{(1/3)}/(g\,n^2), \quad (4)$$

in which X_c is the distance of gradually varied flow along critical slope, y_d is the water depth at the channel end, y_u is the difference between the reservoir water level and channel bed level, g is the gravitational acceleration, n is Manning's coefficient, providing that $y_d > y_c$ or $y_d > (2/3)y_u$. On the other hand, for horizontal slope, Mostafa [3] gave the following equation:

$$X_h = (c^2/g)[(y_d - y_1) + \{1/[(8y_1^2)(y_u - y_1)]\}\{(y_1^4 - y_d^4)\}], \quad (5)$$

where X_h is the distance of gradually varied flow along horizontal slope, y_d , y_u, g are defined before, c is Chezy's coefficient, y_1 is the water depth at the channel entrance.

Bresse solution is an accurate solution to eq. (3). It is written as:

$$X = (1/S_o)\,[\,y - y_o[1 - (y_c/y_o)^3]\,\phi\,(y/y_o)]. \quad (6)$$

where X is the distance of gradually varied flow along mild or steep slopes, y is the general water depth of gradually varied flow, y_c is the critical water depth, y_o is the normal water depth. The function, $\phi\,(y/y_o)$, can be defined as $\phi\,(u)$ and given by:

$$\phi\,(u) = (1/6)\,LN[(u^2+u+1)/(u-1)^2] - (3)^{-0.5}\ \text{Arc Tan}\,[3^{0.5}/(2u+1)] + A, \quad (7)$$

in which $u = y/y_o$ and A is an arbitrary constant. The value of the constant is immaterial because the function is evaluated between two points located a distance $(x_2\text{-}x_1)$ apart, and so the constant A cancels. The Bresse varied flow function is shown in fig. 1-a for subcritical and supercritical flow. The paper suggests that eq. (6) is nondimensionally defined as:

$$(X\,S_o\,/y_o) = y/y_o - [1 - (yc/y_o)^3]\,\phi\,(y/y_o). \quad (8)$$

In case of M_1 and M_2 for mild slope as well as S_2 and S_3 for Steep slope, the approach to normal depth is asymptotic as shown in fig. 1-a and fig. 1-b. The determination of the upstream or downstream boundary condition for a subcritical or supercritical profiles in a natural channel requires an asymptotic method. The computation is started further downstream or upstream than the reach of interest depending on the type of curve.

2. Application of Bresse solution

The paper represents Bresse solution for GVF. Eq. (6) in a dimensionless form of eq. (8). The well known family GVF curves of M_1, M_2, M_3 in mild slope as well as S_1, S_2, S_3 in steep slope as shown in fig. 1-b are accurately estimated and represented in a dimensionless form. Two initial values of water depth are used in the calculation. In the estimation of M_1 and S_2, initial water depths are y = 1.01 y_o and y = 1.05 y_o. However, for the estimation of M_2 and S_3, the intial water depths are y = 0.99

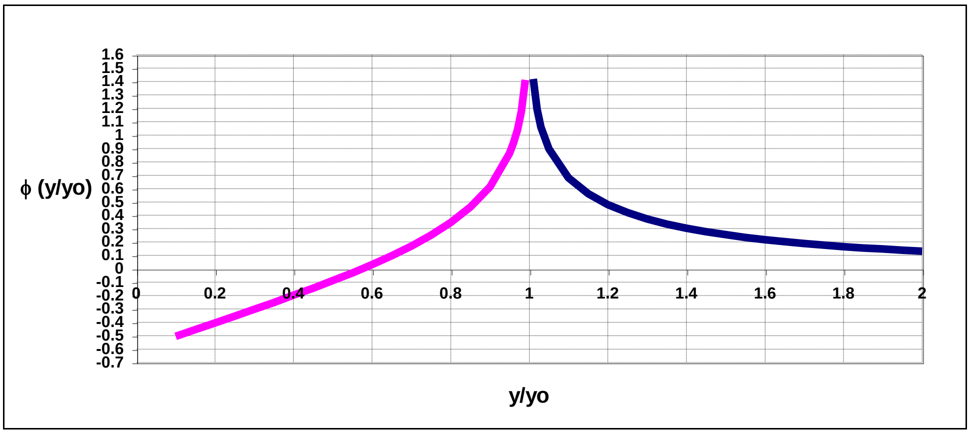

Fig. 1-a. Function $\phi\,(y/y_o)$ versus y/y_o ($y/y_o > 1$ subcritical, $y/y_o < 1$ supercritical).

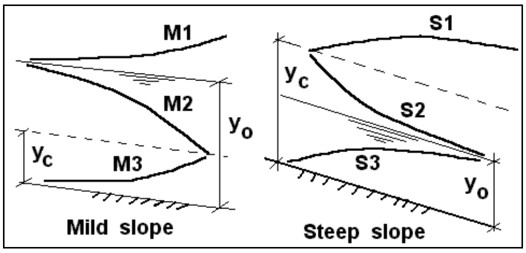

Fig. 1-b. GVF curves for mild and steep channel slopes.

y_o and y = 0.95 y_o. A comparison is done to show the effect of the chosen intial water depth value. The solution is applied to a case study; Toshka Spillway which is considered a wide channel connecting between Aswan Reservoir and Toshka Depression.

3. Steps of calculating the dimensionless gradually varied flow distance

The estimation of gradually varied flow distance, *X*, along a wide channel of mild or steep slopes, can be done by applying eq. (8) of Bresse solution. Steps of calculation are summarized as:

1. Determine the initial and final flow depths, y_1 and y_2, respectively. For gradually varied flow of the type, M_1 and S_2, let the initial water depth, y_1 = 1.01 y_o and for those of M_2 and S_3, let the initial depth, y_1 = 0.99 y_o.
2. Estimate the critical water depth, y_c, and normal water depth, y_o.
3. For values of y_1, y_2, y_o, apply eq. (7) to estimate the value $[\phi\,(y_2/y_o) - \phi\,(y_1/y_o)]$ where the constant *A* cancels.
4. Apply eq. (8) to estimate the dimensionless quantity ($S_o\,X/y_o$).
5. For a given slope, S_o, the absolute value of GVF distance, *X* is estimated.

4. Family curves of gradually varied flow along mild slope, M_1, M_2, M_3

Dimensionless relationships for relative distance of GVF, XS_o/y_o, versus y/y_o are shown in fig. 2 to 7. In the first three relationships, the initial water depth is y_1 = 1.01 y_o for M_1 and y_1 = 0.99 y_o for M_2. For the second three, the initial water depth is y_1 = 1.05 y_o for $M_1$and y_1 = 0.95 y_o for M_2. Samples of these data are given in table 1. In fig. 2 and fig. 5, y_c/y_o = 0.2 while in fig. 3 and fig. 6, y_c/y_o = 0.5. On the other hand, y_c/y_o = 0.8 in fig. 4 and fig. 7. For each relationship, GVF distance of M_1 is greater than that of M_2 which is more greater than distance of M_3 or in other words $M_1 > M_2 > M_3$. The comparison between fig. 2 and fig. 5, is given in table 2. It is shown that the less is the value of initial water depth y_1, the more is the distance of GVF. However, the value of y_1 does not affect the relative distance of GVF for M_3. Similar trend is obtained when fig. 3 and fig. 6 are compared as well as fig. 4 and fig. 7 are compared.

Table 1
Values of relative distance of GVF (Xs_o/y_o) versus y/y_o as well as y_c/y_o
(the initial water depth, $y1$ = 1.01 y_o for M_1 and y_1 =0.99 y_o for M_2)

y/y_o	y_c/y_o = 0.2			y_c/y_o = 0.5			y_c/y_o = 0.8		
	M_2 curve	M_1 curve	M_3 curve	M_2 curve	M_1 curve	M_3 curve	M_2 curve	M_1 curve	M_3 curve
0.1			0			0			0
0.15			0.000299			0.0062			0.0256
0.2	1.0121		0.000426			0.0121			0.0510
0.25	1.012					0.0179			0.0763
0.3	1.0113					0.0232			0.1014
0.35	1.0099					0.0279			0.1261
0.4	1.0075					0.0317			0.1503
0.45	1.0038					0.0343			0.1739
0.5	0.9982			0.8227		0.0353			0.1966
0.55	0.9902			0.8216					0.2180
0.6	0.9789			0.8175					0.2379
0.65	0.9632			0.8096					0.2556
0.7	0.9415			0.7963					0.2703
0.75	0.9112			0.7754					0.2807
0.8	0.8681			0.7433			0.3305		0.2849
0.85	0.8043			0.6929			0.3246		
0.9	0.7022			0.6088			0.2997		
0.95	0.5056			0.4413			0.2284		
0.96	0.4384			0.3832			0.2005		
0.97	0.3499			0.3063			0.1620		
0.98	0.2225			0.1951			0.1044		
0.99	0			0			0		
1.01		0			0			0	
1.02		0.2359			0.2093			0.1211	
1.03		0.3767			0.3347			0.1954	
1.05		0.5592			0.4979			0.2954	
1.1		0.8226			0.7363			0.4504	
1.15		0.9915			0.8911			0.5588	
1.2		1.1218			1.0119			0.6484	
1.25		1.2313			1.1144			0.7276	
1.3		1.3277			1.2054			0.8005	
1.35		1.4152			1.288			0.8689	
1.4		1.4963			1.3658			0.9342	
1.45		1.5725			1.4389			0.9971	
1.5		1.6450			1.5088			1.0582	
1.55		1.7145			1.5760			1.1178	
1.6		1.7816			1.6411			1.1762	
1.65		1.8467			1.7044			1.2336	
1.7		1.9101			1.7662			1.2902	
1.75		1.9721			1.8268			1.3461	
1.8		2.0329			1.8864			1.4014	
1.85		2.0927			1.9449			1.4562	
1.9		2.1516			2.0028			1.5106	
1.95		2.2096			2.0599			1.5646	
2		2.2670			2.1164			1.6182	

Note: at y/y_o = 0.9, y_c/y_o = 0.2 the dimensionless value (Xs_o/y_o) of M_2 = 0.7022

5. Family curves of gradually varied flow along steep slope, S_1, S_2, S_3

Dimensionless relationships for relative distance of GVF, Xs_o/y_o, versus y/y_o are shown in fig. 8 to 13. In the first three relationships, the initial water depth is $y_1 = 1.01\ y_o$ for S_2 and $y_1 = 0.99\ y_o$ for S_3. For the second three relationships, the initial water depth is $y_1 = 1.05\ y_o$ for S_2 and $y_1 = 0.95\ y_o$ for S_3. In fig. 8 and fig. 11, $y_c/y_o = 1.2$ while in fig. 9 and fig. 12, $y_c/y_o = 1.5$. On the other hand, $y_c/y_o = 2.0$ for fig. 10 and fig. 13. Samples of these data are given in table 3. Except in fig. 8, 11, GVF distance of S_1 is less than that of S_2 which is less than distance of S_3. ($S_1 < S_2 < S_3$) The comparison between fig. 8 and fig. 11, shown that the less is the value of initial water depth y_1, the more is the distance of GVF. However, the value of y_1 does not affect the distance of GVF for S_1. Similar trend is obtained when fig. 9 & fig. 12 as well as fig. 10 & fig. 13 are compared.

6. Case study; Toshka spillway

Fig. 14 shows a map describing Aswan reservoir and Toshka depression. The spillway is a wide channel connecting between Aswan reservoir and Toshka depression. Also, fig. 15 shows a longitudinal cross-section in Toshka canal. The channel length is almost 22 km, and its longitudinal bed slope is 15 cm/km. The bed level at the entrance is 178 m and 175 m at the exit. The bed width varies at the entrance from 500 to 750 m in a length of bout 5 km and then becomes constant at width 350 m. The channel is provided by an end weir with a sill level 176.0 m. The channel is designed to accommodate a maximum discharge of 250 million m^3/day when the water level in the reservoir is at about 182.6 m.

The channel is considered wide since the bed width is greater than the water depth several tens times. Bresse solution is applied to estimate gradually varied flow along Toshka channel. Two cases are considered. In the first one, weir of height 1.0 m is considered at the channel end. In the second, the flow is free fall in the depression.

Table 2
Comparison between the relative distance of GVF in fig. 2 and fig. 5

Relative distance of GVF, Xs_o/y_o	$y_1 = 1.01\ y_o$ for M_1 & $y_1 = 0.99\ y_o$ for M_2	$Y_1 = 1.05\ y_o$ for M_1 & $Y_1 = 0.95\ y_o$ for M_2
M_1 curve	2.3 (fig. 2)	1.7 (fig. 5)
M_2 curve	1.0 (fig. 2)	0.5 (fig. 5)
M_3 curve	No change	

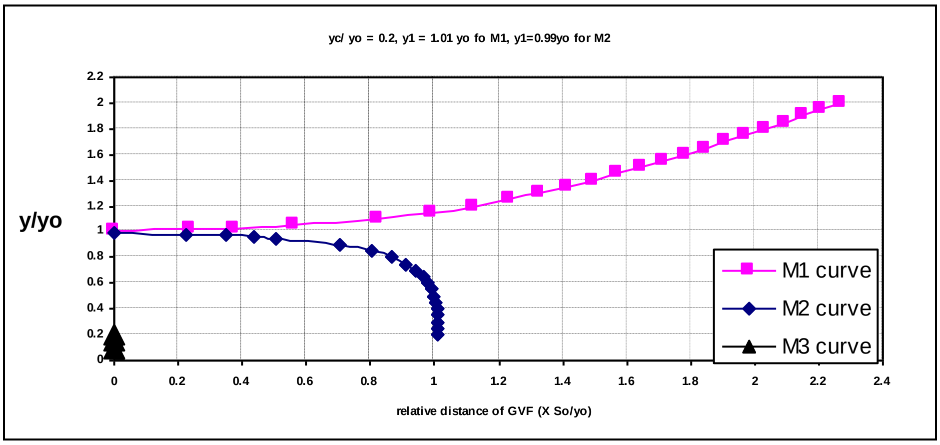

Fig. 2. (Xs_o/y_o) versus y/y_o ($y_c/\ y_o$ =0.2, y_1 =1.01 for M_1 and y_o , y_1 =0.99 y_o for M_2).

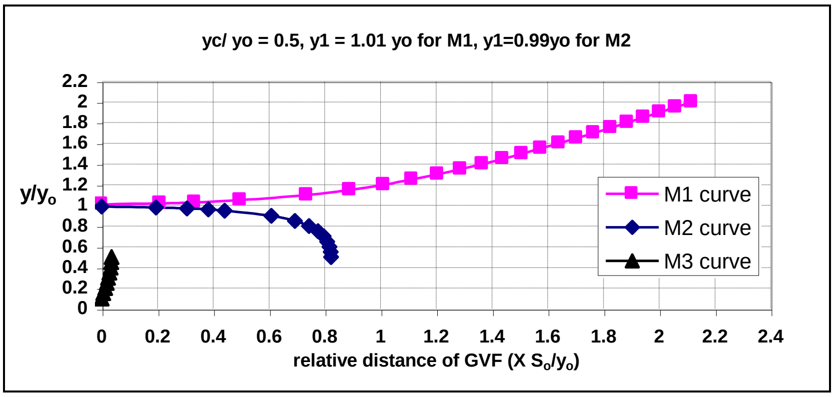

Fig. 3. (XS_o/y_o) versus y/y_o ($y_c/\ y_o$ =0.5, y_1 =1.01 for M_1 and y_o , y_1 =0.99 y_o for M_2).

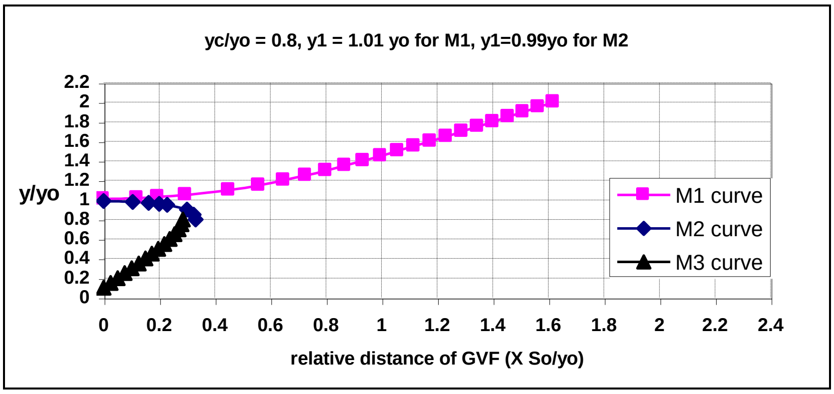

Fig. 4. (XS_o/y_o) versus y/y_o ($y_c/\ y_o$ =0.8, y_1 =1.01 for M_1 and y_o , y_1 =0.99 y_o for M_2).

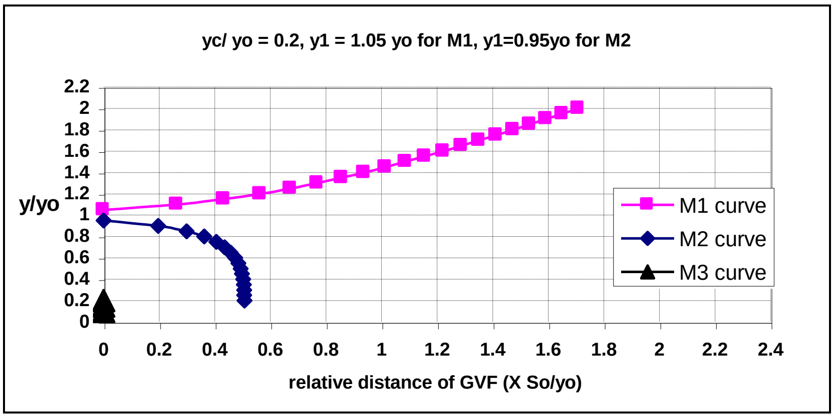

Fig. 5. (XS_o/y_o) versus y/y_o ($y_c/\ y_o$=0.2, y_1=1.05 y_o for M_1 and $y1$ = 0.95 y_o for M_2).

7. Case one: toshka spillway with end weir

In the first case, a weir is considered at the channel end, as shown in fig. 15. When water level is higher than 178 m, water flows into the channel. Uniform flow prevails after certain period. At the end of channel, Flow goes over the weir to fall into the depression. Flow is considered uniform up to certain location where the flow is affected by the existing weir. Through this distance, the flow is nonuniform and gradually varied (GVF). The relative distance of GVF (XS_o/y_o) is affected by y/y_o and y_c/y_o.

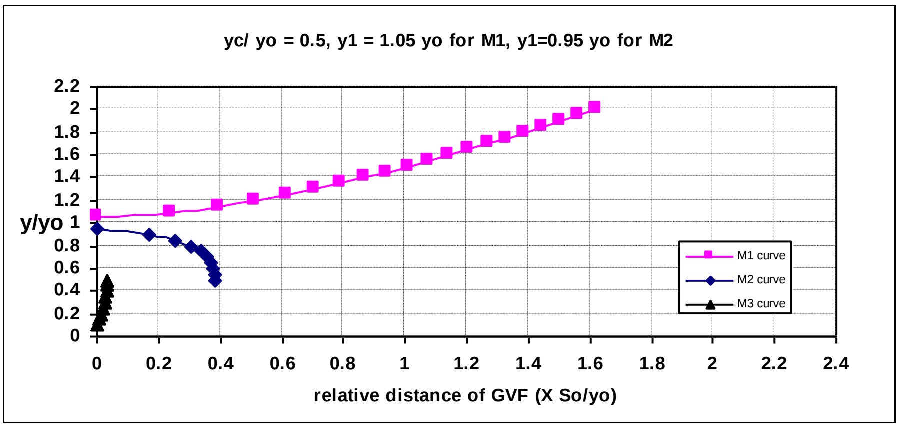

Fig. 6. (Xs_o/y_o) versus y/y_o (y_c/ y_o=0.5, y_1=1.05 y_o for M1 and $y1$ = 0.95 y_o for M_2).

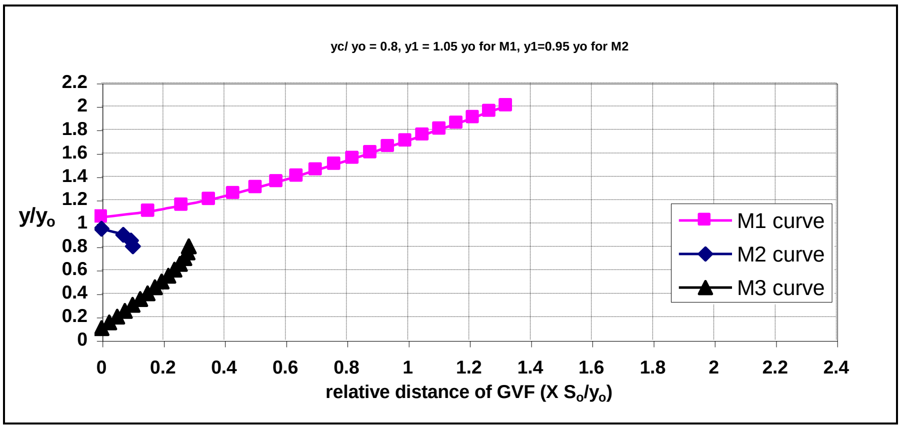

Fig. 7. (Xs_o/y_o) versus y/y_o (y_c/ y_o=0.8, y_1=1.05 y_o for M1 and y1 = 0.95 y_o for M_2)

Fig. 16 shows four cases for the water levels in Aswan reservoir, 178.9, 179.7, 180.94 and 182.38 m. For USWL = 178.9, the passing discharge is estimated and normal water depth is y_o = 0.9 m. The head of water above weir is estimated to be 0.5 m. Water depth just upstream the weir is 1.5 m (weir sill height is y_w=1.0 m). GVF distance of M_1 curve, X, with initial water depth, y_1 = 1.01 y_o, and final water depth, y_2 = 1.5 m is estimated as shown in fig. 16 to be about 9750 m. The other three levels of USWL are given in table 5. It is shown that the GVF curve is of type M_1 in the first two rows. In the third, the flow is uniform flow (UF) even with the weir existence. In the fourth, the GVF curve is of the type M_2. Generally, X increases as y_o increases and the distance of GVF for different USWL is less than 22000 m, channel total length. So the channel is considered long.

8. Case two: Toshka spillway without end weir

In this case, the channel is assumed to be free fall at the end. When water level is higher than 178 m, water flows into the channel. Uniform flow prevails after certain period of time. At the end of channel, Flow is free fall into the depression. Flow is considered uniform up to certain location where the flow is affected by the free fall end. Through this distance, the flow is nonuniform and GVF of the type M_2. Bresse solution is applied to estimate GVF distance, X. Fig. 17 and table 6 shows three cases. It is shown that as the UpStream Water Level, (USWL), increases in the reservoir, the normal water depth increases and the distance of GVF increases.

Table 3
Values of relative distance of GVF (Xs_o/y_o) versus y/y_o as well as y_c/y_o
(the initial water depth = 1.01 y_o for S_2 and 0.99 y_o for S_3)

y/y_o	y_c/y_o = 1.2			y_c/y_o = 1.5			y_c/y_o = 2		
	S_2 curve	S_1 curve	S_3 curve	S_2 curve	S_1 curve	S_3 curve	S_2 curve	S_1 curve	S_3 curve
0.100			0.000			0.000			0.000
0.150			0.086			0.169			0.401
0.200			0.173			0.338			0.803
0.250			0.260			0.509			1.207
0.300			0.347			0.680			1.614
0.350			0.435			0.853			2.027
0.400			0.523			1.028			2.446
0.450			0.613			1.207			2.876
0.500			0.703			1.390			3.318
0.550			0.796			1.579			3.777
0.600			0.891			1.776			4.260
0.650			0.989			1.983			4.773
0.700			1.092			2.205			5.329
0.750			1.201			2.447			5.946
0.800			1.319			2.720			6.653
0.850			1.453			3.042			7.506
0.900			1.614			3.456			8.629
0.950			1.845			4.097			10.419
0.990			2.286			5.443			14.310
1.010	0.494			2.275			8.0218		
1.050	0.153			1.072			4.398		
1.100	0.046			0.611			2.942		
1.150	0.009			0.377			2.153		
1.200	0.000	0.000		0.234			1.636		
1.250		0.006		0.142			1.266		
1.300		0.022		0.081			0.989		
1.350		0.045		0.041			0.774		
1.400		0.072		0.017			0.605		
1.450		0.103		0.004			0.470		
1.500		0.136		0.000	0.000		0.361		
1.550		0.172			0.003		0.274		
1.600		0.209			0.012		0.203		
1.650		0.248			0.026		0.147		
1.700		0.288			0.044		0.102		
1.750		0.330			0.065		0.067		
1.800		0.372			0.090		0.041		
1.850		0.415			0.116		0.022		
1.900		0.458			0.145		0.009		
1.950		0.502			0.176		0.002		
2.000		0.547			0.208		0.000	0.000	
2.250		0.775			0.388			0.045	
2.500		1.011			0.590			0.152	

Table 4
Comparison between the relative distance of GVF in fig. 8 and fig. 11

Relative distance of GVF, Xs_o/y_o	y_1 = 1.01 y_o for S_2 & y_1 = 0.99 y_o for S_3		y_1 = 1.05 y_o for S_2 & y_1 = 0.95 y_o for S_3
S_1 curve	1.0	No change	1.0
S_2 curve	0.5 (fig. 8)		0.15 (fig. 11)
S_3 curve	2.3 (fig. 8)		1.85 (fig. 11)

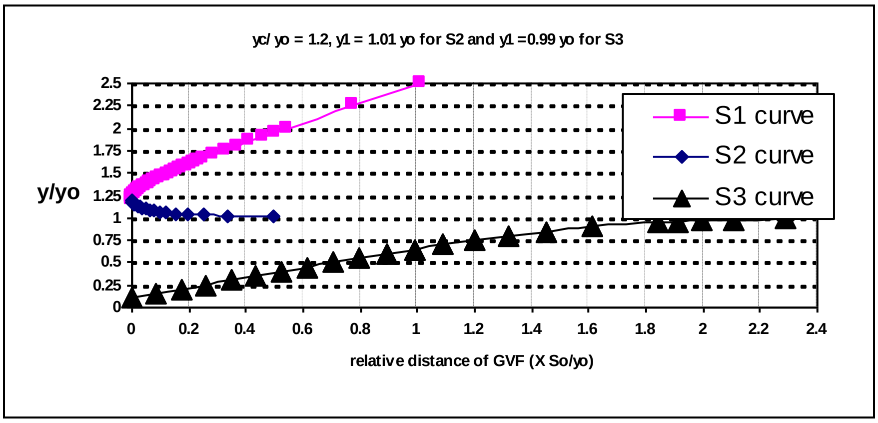

Fig. 8. (Xs_o/y_o) versus y/y_o (y_c/ y_o=1.2, y_1=0.99 y_o for S_2 and y_1 = 1.01 y_o for S_1).

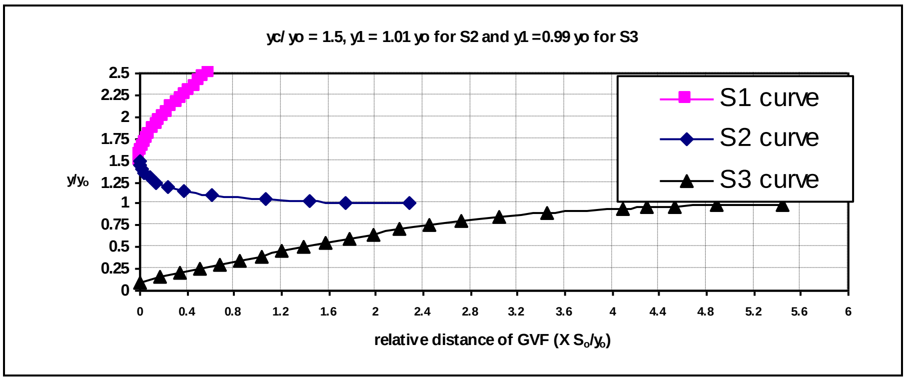

Fig. 9. (Xs_o/y_o) versus y/y_o (y_c/ y_o=1.5, y_1=0.99 y_o for S_2 *and* y_1 = *1.01* y_o for S_1).

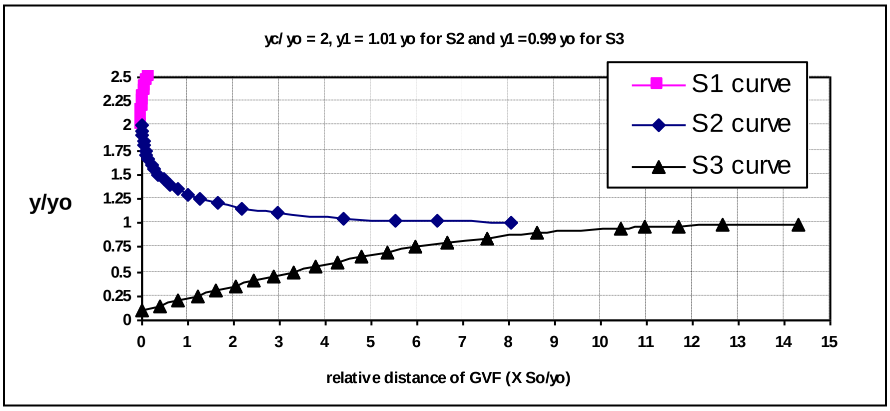

Fig. 10. (Xs_o/y_o) versus y/y_o (y_c/ y_o=2.0, y_1=0.99 y_o for S_2 and y_1 = 1.01 y_o for S_1).

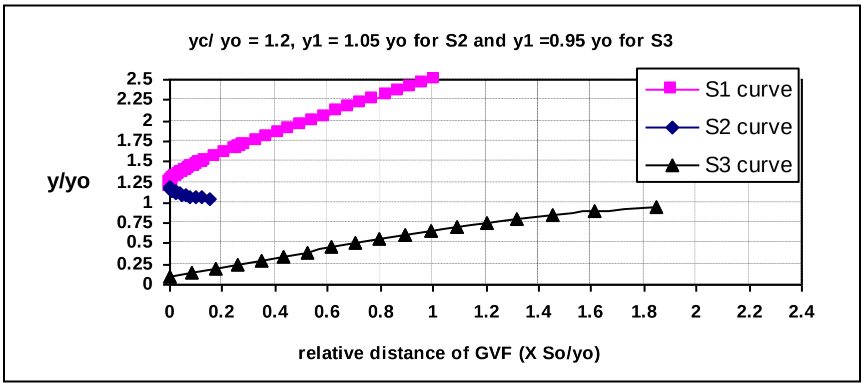

Fig. 11. (Xs_o/y_o) versus y/y_o ($y_c/\ y_o$=1.2, y_1=0.95 y_o for S_2 and y_1 = 1.05 y_o for S_1).

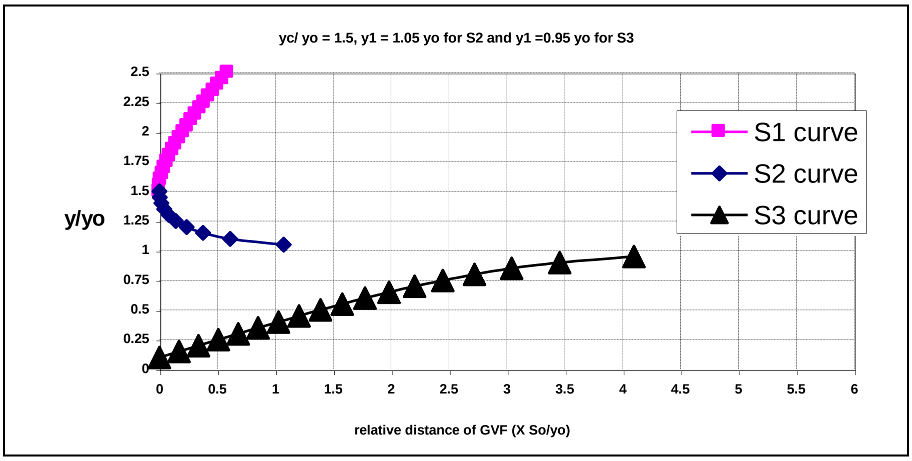

Fig. 12. (Xs_o/y_o) versus y/y_o ($y_c/\ y_o$=1.5, y_1=0.95 y_o for S_2 and y_1 = 1.05 y_o for S_1).

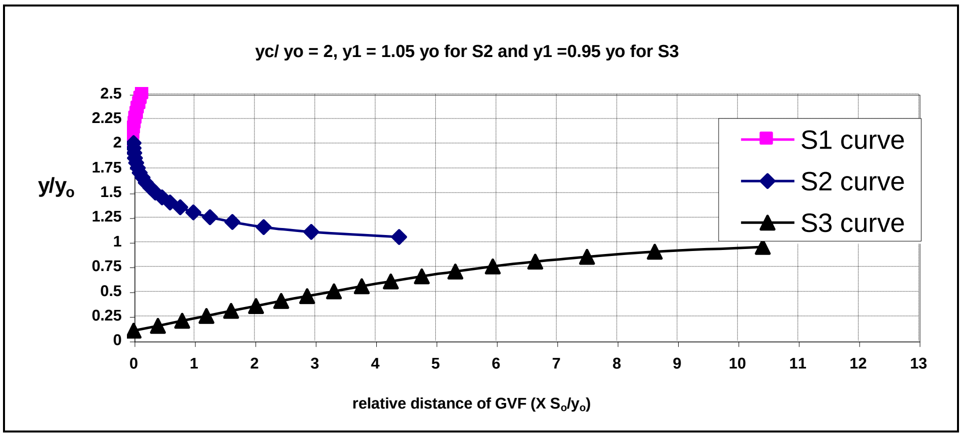

Fig. 13. (Xs_o/y_o) versus y/y_o ($y_c/\ y_o$=2.0, y_1=0.95 y_o for S_2 and y_1 = 1.05 y_o for S_1).

9. Comparison between case one and two

When tables 5 and 6 are compared, several remarks are taken into consideration. The value of GVF distance, X for y_o = 0.9, table 5 is 9750 m, while it is 6200 m as given in table 6. Also, for yo = 1.7 m, GVF distance, X with weir equals 11000 m, while X without weir equals 9900 m. This means that for the same discharge and normal water depth, the existence of weir increases the distance of GVF almost 1.5 times that without weir.

10. Dimensionless form of direct step method for wide cross section

It is well known that the Direct Step Method (DSM) is an approximate method can be applied in open channel. The formula is written as:

$$dX = (E_2 - E_1)/\ (S_e - S_o). \tag{9}$$

Where dX is the small distance between two locations of two successive water depths along the curve of GVF. E_1 is the energy at the first location of water depth, y_1. E_2 is the energy at the second location of water depth, y_2. S_e is the average energy slope at both locations corresponding to y_{av} and S_o is the longitudinal bed slope. It is well known that:

$$E_1 = y_1 + V_1^2/(2g), \tag{10}$$

$$E_2 = y_2 + V_2^2/(2g)\ . \tag{11}$$

For propose of presenting the direct step method in a dimensionless form, eq. (9) is multiplied by S_o/y_o. The new dimensionless form is

$$XS_o/y_o = (E_2/y_o - E_1/y_o)/\ (S_e/S_o - 1). \tag{12}$$

By applying the energy equation and critical depth equation,

$$(E_2/y_o - E_1/y_o) = [(y_2/y_o) + 0.5(y_c/y_o)^3(y_o/y_2)^2] - [(y_1/y_o) + 0.5(y_c/y_o)^3(y_o/y_1)^2]. \tag{13}$$

By applying Chezy equation for wide cross-section,

$$S_e/S_o = (y_o/y_{av})^3\ . \tag{14}$$

Where:

$$y_{av} = (y_1+y_2)/2\ . \tag{15}$$

Eq. (9) can be written as:

$$XS_o/y_o = [F(2)-F(1)]/\ F(3). \tag{16}$$

Where

$$F(1) = [(y_1/y_o) + 0.5\ (y_c/y_o)^3\ (y_o/y_1)^2], \tag{17}$$

$$F(2) = [(y_2/y_o) + 0.5\ (y_c/y_o)^3\ (y_o/y_2)^2], \tag{18}$$

$$F(3) = (y_o/y_{av})^3 - 1. \tag{19}$$

The accuracy of applying eq. (16) depends on the value of dy. The smaller is the value of dy, the greater is the accuracy of calculating X. The dimensionless form of DSM can be compared with the accurate method; Bresse solution.

Table 5
Values of GVF distance X estimated by applying Bresse solution, fig. 16

USWL	y_o	y_1= 1.01 y_o	y_2	GVF curve	GVF distance, X
179.0	0.9	0.91	1.5	M_1	9750
179.7	1.7	1.72	2.0	M_1	11000
180.94	2.8	2.83	2.8	UF (no curve)	15000
182.38	4.2	4.24	3.8	M_2	16300

Note: All dimensions are in meter

Table 6
Values of GVF distance X estimated by applying Bresse solution, fig. 17

USWL	y_o	y_1= 0.99 y_o	$y_2 = y_c$	GVF curve	GVF distance, X
178.5	0.5	0.51	0.18	M_2	3100
179.0	0.9	0.91	0.4	M_2	6200
179.7	1.7	1.72	0.7	M_2	9900

Note: All dimensions are in meter

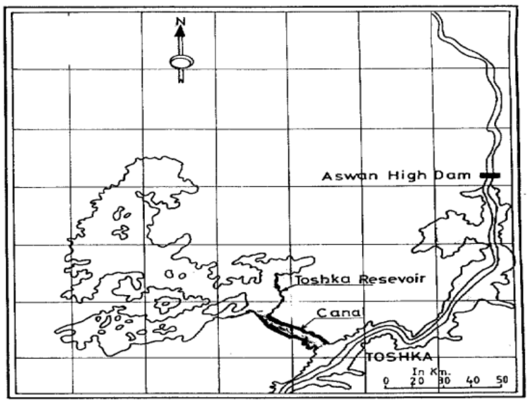

Fig. 14. Toshka canal (spillway).

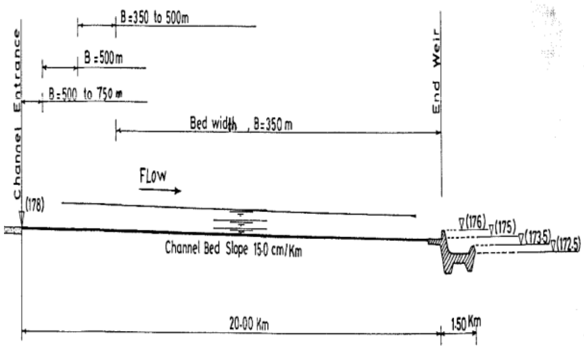

Fig. 15. Longitudinal section in toshka canal.

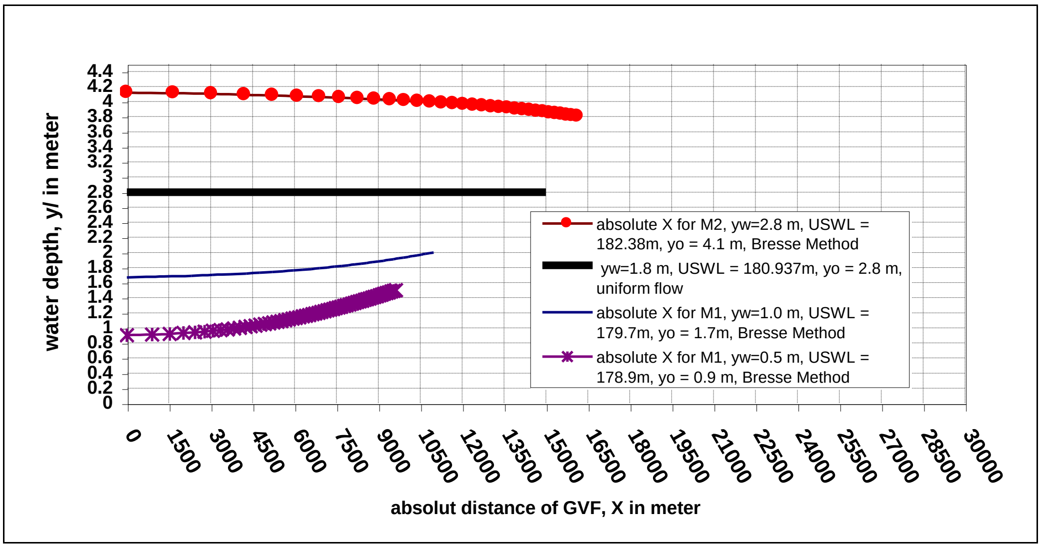

Fig. 16. GVF distance versus water depth for toshka spillway (case of weir).

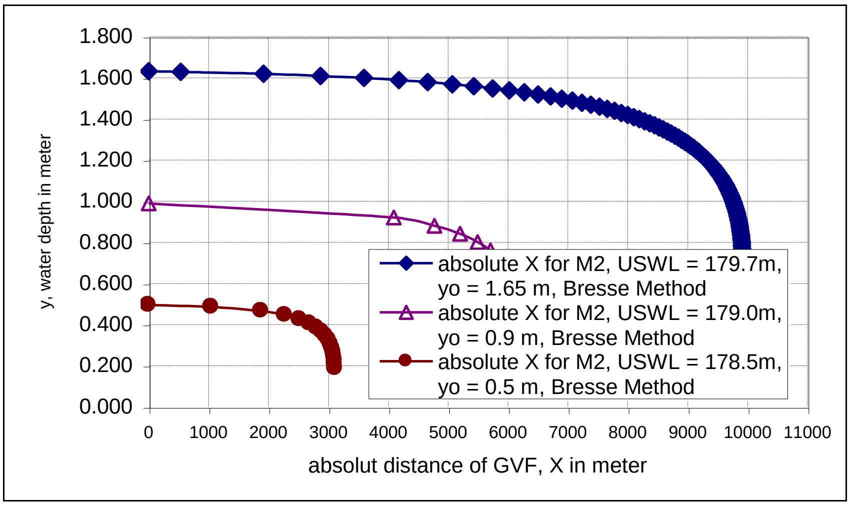

Fig. 17. GVF distance, *X*, versus water depth, M_2 for Toshka spillway (free fall noweir).

11. Comparison between bresse solution and direct step method

For selected cases, the approximate solution, direct step method, eq. (16) is applied to estimate the GVF distance, *X*. The value of *X* is compared with that estimated by applying the accurate method of Bresse Solution. fig. 18 shows the comparison for USWL = 179.7 m with end weir where M_1 curve occurs. Fig. 19 shows the comparison for the same USWL = 179.7 m without end weir where M_2 curve occurs.

For M2 curve, fig. 18, the comparison between Bresse solution and the direct step method, shows that the average error is 0.5% when dy = 0.01 m and increases to about 1.5 % when dy = 0.02 m, while increases to 3.0 % when dy = 0.04 m.

For M1 curve, fig. 19, the comparison between Bresse solution and the direct step method, shows that the average error is 0.4% when *dy* = 0.01 m and increases to about 0.9 % when dy = 0.02 m, while increases to 2.3 % when dy = 0.04 m.

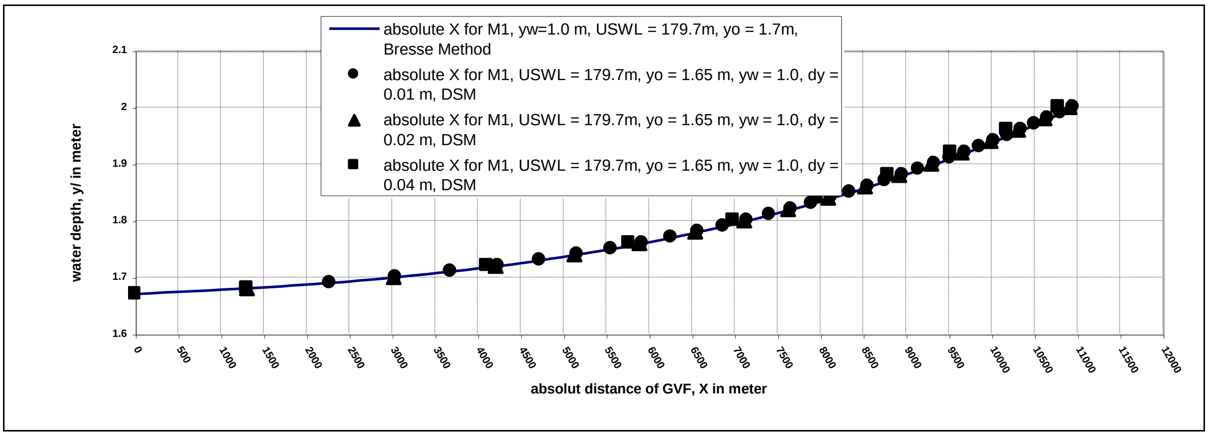

Fig. 18. Comparison between Bresse solution and direct step method (DSM), case of M_1 curve.

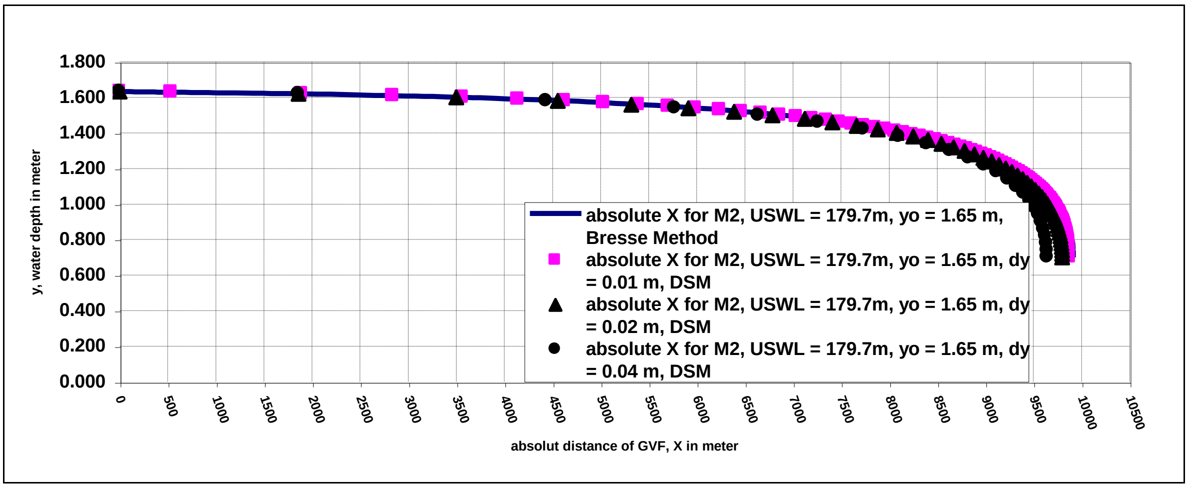

Fig. 19. Comparison between Bresse solution and direct step method (DSM), case of M_2 curve.

The comparison shows that Bresse solution is more accurate. Moreover, it is applied one step. On the other hand, direct step method can accurately be applied if *dy* is very small. In this case, steps of calculation increases and data is to presented in a tabular form.

12. Conclusions

The paper, herein, represents an accurate formula, Bresse solution, to a special form of Gradually Varied Flow (GVF) equation. The solution is represented in a new dimensionless function, eq. (8). The form is valid only for a wide cross section open channels. The formula is applied to the channel connecting two reservoirs. Family curves of GVF along mild slope, M_1, M_2, M_3 are presented in a new dimensionless form. Similarly, family curves of S_1, S_2, S_3 along steep slope, are presented.

A case study, "Toshka Spillway", is presented. It is a wide channel connecting between Aswan Reservoir and Toshka depression. Two cases are considered. In the first one, a weir is considered at end of the spillway and an actual gradually varied flow distance of M_1 is estimated. While, In the second case, the flow escapes at the end to a low water level in the depression and an actual gradually flow distance of M_2 is estimated. In both cases, several water levels in Aswan reservoir are considered to conclude that the weir existence increases the length of GVF distance. However, the length of the channel is always greater than the estimated GVF length. Therefore, the channel is considered long.

The paper represents the Direct Step Method (DSM) in a new dimensionless form, eq. (16) to be valid for a wide cross section. The new form of DSM is easier than the common form and is function of only several water depth ratios. The form becomes similar to that of Bresse solution. Both methods are applied to estimate the distance of GVF in Toshka spillway. Both cases of M_1 and M_2 are considered. For M_2 curve, the comparison between Bresse solution and the DSM, shows that the average error is 0.5% when *dy* = 0.01 m and increases to 3.0 % when *dy* = 0.04 m. For M_1 curve, the comparison shows that the average error is 0.4% when *dy* = 0.01 m and increases 2.3 % when *dy* = 0.04 m.

Generally, Bresse solution is more accurate. Moreover, it is applied one step. On the other hand, the accuracy of direct step method increases if *dy* is very small. In this case, steps of calculation increases and data is to presented in a tabulated form.

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