Economical analysis of ship structure repair alternatives based on reliability

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After a new ship is delivered, the ship's hull structure must be monitored by a series of internal and external inspections to assess the integrity of the ship structures. During an inspection, several types of structural failures can be found. Fatigue cracks, corrosion and buckling are the most common failures. Different types of repairs namely; crack repair, steel renewal and steel reinforcement are considered. When a structural failure is discovered, a decision must be made as to the most effective repair. Due to the random nature of applicable load and strength characteristics, probabilistic analysis is proposed to assess the economics of each repair type to make a rational decision on the selection of the most economical repair alternative for the type of failure in question. In this paper, repair life of a structural connection is estimated based on the probability of structural failure, i.e., the probability of the applicable load exceeding the strength of the repaired joint. Different models of repair alternatives are analyzed to demonstrate the proposed approach.

إصلاح الوصلات من المهام الصعبة التي تواجه مشاكل بناء السفن .الكشف على بدن السفينة في المراحل الأولية قد يؤدي إلى اكتشاف وجود عطب في الوصلات الإنشائية. تحديد الطريقة المثلى للإصلاح من أصعب القرارات وذلك نتيجة وجود عوامل كثيرة ومتداخله لتحديد أنسب طرق الإصلاح. ثلاثة أنواع من إصلاح الوصلات سوف نقوم بدراستها على سبيل المثال تنظيف الوصلة ولحامها و استبدال اللوح أو تقوية الوصلة. نتيجة للطبيعه العشوائيه للأحمال ومقاومة تلك الوصلات، يتم استخدام الطرق الإحتماليه لتحديد أنسب الاختيارات على اساس إقتصادى مع الأخذ في الاعتبار مقاومة التعب. تم إستخدام المعوليه الإنشائيه لحساب عمر كل وصله بعد الإصلاح.

Keywords: Reliability analysis, Ship structural failures, Repair alternatives, Fatigue strength, Ship structural connections

1. Introduction

The decision on selecting the proper repair type for failed joint is difficult due to the vast array of engineering, construction and repair knowledge. Three subjects are considered when selecting the most appropriate repair alternative namely; technical, economical and environmental objectives [01].

The technical objective is to develop a practical tool for crack repair alternatives to help improve the durability of existing ships.

The economical objective is to establish a rational method for the tradeoff between initial and running costs. In the same way that a more durable ship has lower maintenance costs, more durable repairs will have lower future repair costs.

The environmental objective is to ensure structural reliability in order to minimize the risk of marine pollution into an acceptable societal risk for the environment according to national and international regulations [1].

In this paper, repair life for cracked longitudinal–transverse intersection representing a structural connection in an existing ship is studied. Both applicable load and strength are assumed to follow a normal distribution for simplification. The uncertainty characteristics associated with the strength are based on the selected S-N curve. Also, the uncertainty associated with the acting load is considered. Different repair alternatives for the structural connection are discussed [1]. Different models of repair alternatives are analyzed using Finite Element Method (FEM) to determine variation of Stress Reduction Factor (SRF). Repair life is estimated based on the probability of failure, i.e., the probability of the applicable load exceeding the strength of the repaired joint. The required man-hour and cost of each repair type-alternative is then estimated based on actual data provided by the Egyptian Shipbuilding and Repair Company. The total cost (initial and cost of repair time) is assessed for the remaining life of the ship based on the continuous repetition of each repair type, which in turn has been based on the probabilistic repair life of each type/alternative.

2. Repair alternatives

The general strategies for crack repair of critical structural connections can be classified in the following way [1];

1. Grind out crack and re-weld: Re-welding is an easy and common way of repair. However, the strength of re-welded cracks maybe less than the original one.
2. Re-weld the cracks plus post welding improvements: This repair is basically the same as the previous one, except that the weld is ground into smooth surface to improve its fatigue strength.
3. Replace the cracked plate: The inserted new plate has a new fatigue life. If the loading history and material properties are identical to those of the failed plate, its fatigue life should be about the same as the failed time of the crack.
4. Modify designs by adding bracket, lug or collar plate: The more robust way of repair is to modify the local geometry to reduce the stress concentration. Improving the structural design can reduce the stress concentration and therefore increase the working life.
5. Enhance scantling in size or thickness: Increasing the size of the structural detail like a bracket is a good practice. However increasing plate thickness may lead to discontinuity in the connection. Depending on the economic goals of the owner, a different repair alternative could be selected. For example, if the ship has only two more years in service, the cheapest alternative with an expected life of greater than two years will be selected.

There is a variety of designs for longitudinal web intersections. In this paper, a longitudinal transverse intersection representing a typical structural connection as shown in fig. 1, after ref. [1] in an existing ship is considered. The model contains one span of transverse web and longitudinal. The dimension and radius of cutout were taken from an existing ship. Different models representing proper repairs of failures for the structural connections are investigated as shown in table 1 quoted from [1]. These models were analyzed using FEM to determine the variation of Stress Reduction Factor (SRF). After that the repair life maybe estimated as shown in sec. 3 below. In these analyses we calculate the stress reduction factors at radius of cutout for the given structural detail under the effect of a hydrostatic pressure.

Table1
Repair alternatives for a ship structural cutout [1]

Type of repair	S-N curve	SRF
1- Welding the crack	E-class	1.0
2- Welding + post welding	D-class	1.0
3- Insert plate	C-class	1.0
4- Welding + added Lug	E-class	0.76
5- Post welding + added Lug	D-class	0.76
6- Insert plate + added Lug	C-class	0.76
7- Welding + added T. bracket	E-class	0.66
8- Post welding + T. bracket	D-class	0.66
9- Insert plate + T. bracket	C-class	0.66
10-Welding +Lug +T. bracket	E-class	0.52
11-Post Welding +Lug+T. bracket	D-class	0.52
12- Insert plate +Lug +T. bracket	C-class	0.52

In order to reduce the stress concentration in the cutout radius a lug or a bracket is added. For a model with lug the stress concentration is reduced to 76% of its original value. For a model with a bracket, the stress concentration is reduced to 66% of its original value. The stress concentration is reduced to 52% of its original value when lug and bracket are added to the model. Then, the repair life is estimated for the chosen repair alternative on the basis of a selected S-N curve.

3. Repair life estimation

In this paper the cumulative fatigue damage model is applied when estimating repair life. Miner's damage factor link between the constant load and the variable load by using a damage concept. In a stress history of several stress ranges, S_i each with a number of cycles n_i, the damage sum may be given as follows [3, 4]:

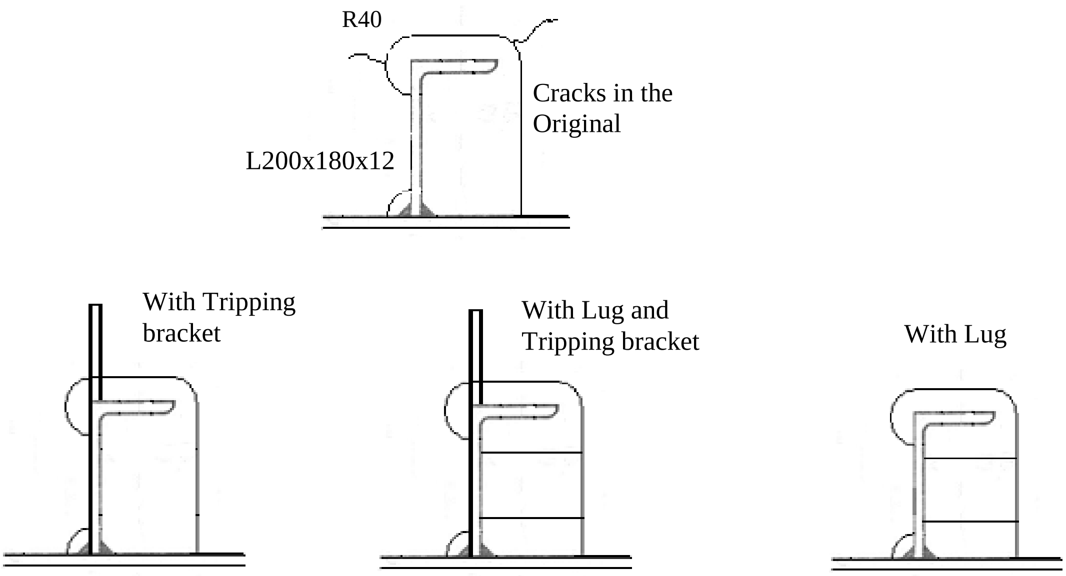

Fig. 1. Crack in original structural connection and different repair alternatives [1].

$$D = \sum_i \left(\frac{n_i}{N_i} \right) < \Delta \, . \tag{1}$$

Where:

Δ is the acceptable cumulative damage, Δ must be less than 1,

n_i is the number of cycles corresponding to S_i., and

N_i is the number of cycles to failure at S_i.

The most common way of representing irregular load histories for fatigue is by stress spectral analysis. In many cases, the stress spectrum can be approximated by a Weibull distribution function. Hence, the repair life, T, may be estimated as follows;

$$T = \frac{\Delta C}{B^m} \Omega \, , \tag{2}$$

$$\Omega = \frac{S_{max}^m}{(ln(N_o))^m} \Gamma(1+m) \, , \tag{3}$$

where B is an uncertainty factor and, Γ is the Gamma function.

Using the Miner' rule and the S-N curve, the maximum stress range S_{max} is given by:

$$S_{max} = \theta \left[\frac{\Delta . C}{[N_o \Gamma(1+m)]^{1/m}} \right] ln(N_o) \, , \tag{4}$$

where, N_o is the total number of fatigue cycles, θ is a factor that takes into account the effect of initial imperfections.

When a repair is made, the following procedure is carried out:

1. Assume N_o, the life of joint at inspection when a crack is discovered.
2. Calculate the acting stress S_{max1}, which causes failure using eq. (4).
3. Calculate new acting stress, S_{max2} after repair alternative using:

$$KS = S_{max2} / S_{max1} \, , \tag{5}$$

where, S_{max1} is the hot spot stress range before repair, and S_{max2} is the hot spot stress range after repair.

4. Calculate the fatigue life, T, which corresponds to S_{max2} using eq. (2).

This process is illustrated using *S-N* curve as shown in fig. 2 after ref. [1], where, t_f = life at first failure of the original joint, and t_R = repair life of the repaired joint.

4. Reliability analysis

Reliability-based analysis starts with the definition of the performance functions that correspond to limit states for significant failure modes. In general, the problem can be considered as one of supply and demand. A failure occurs when the supply (i.e., strength of the joint) is less than the demand (i.e., acting loading on the joint). A generalized form for the performance function for the structural joint is given by [5].

$$g = S - L . \tag{6}$$

Where g is the performance function, S is the strength and L is the load. The failure occurs when g is less than zero or S is less than L, that is;

$$G < 0.0 \text{ or } S < L. \tag{7}$$

If both strength and load are treated as random variables, the reliability analysis can be approached using probabilistic methods. In order to perform a reliability analysis, a mathematical model that relates the strength and load needs to be derived. Furthermore, the statistical characteristics (means and standard deviations or variances) of the strength and load must be quantified.

The failure occurs when $S - L < 0$, and the probability of failure when S & L are statistically independent becomes [5].

$$P_f = p(S < L) = \int_{-\infty}^{\infty} F_S(x) f_L(x) dx, \tag{8}$$

where F_S is the cumulative distribution function of S, and f_S is the probability density function of load, L, and f_S is the probability density function of strength, S. Commonly used reliability methods utilize the mean and variance (first and second moments) of basic random variables in calculating a reliability measure according to a specified performance function.

The advanced second-moment method has the advantage of allowing us to deal with non-linear performance functions and with non-normal random variables. For this purpose, the performance function can be defined in terms of the following reduced variables [5].

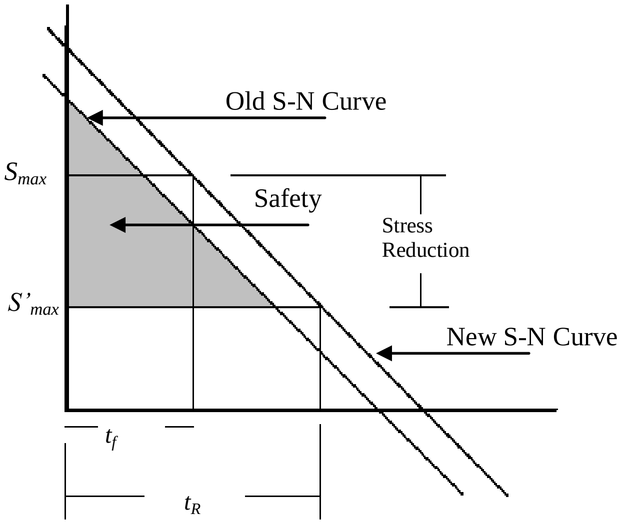

Fig. 2. Life estimation of the repaired joint, t_R, [1].

$$u_i = \frac{X_i - \overline{X}_i}{\sigma_{X_i}} . \tag{9}$$

Where u_i is the reduced variable for X_i (S is the strength, and L= load), and the limit state g' in the reduced space, which represents the failure surface that separate the failure region from the safe region is given by:

$$g' = S - L = 0 . \tag{10}$$

The safety index, β, is defined as the minimum distance from the origin of the reduced coordinates of the basic random variables to the limit state (failure surface) as shown in fig. 3 for two variables $S \, and \, L$. The safety index, β, is determined by iteratively solving the following set of eqs. [5].

$$\alpha_S = \frac{\left(\frac{dg}{dS}\right)\sigma_S}{\left[\sum_{i=1}^{n}\left(\frac{dg}{dS}\right)^2 \sigma_S^2\right]^{\frac{1}{2}}} , \tag{11}$$

$$S^* = \bar{S} - \alpha_S \beta \sigma_S \,, \tag{12}$$

$$g\left(S^*, L^*\right) = 0 \,, \tag{13}$$

where the derivatives dg / dS are evaluated at the design point or the most probable failure point $\left(S^*, L^*\right)$, and α_S = the directional cosine of the variable S, and β = reliability index. As shown in fig. 3, point S^* is called the Most Probable Failure Point (MPFP) and corresponds to the shortest distance on the limit state (i.e. failure surface).

This method deals with non-normal probability distributions for basic random variables by determining equivalent normal distributions at the design point in each iteration in the solution of eqs. (11) through (13). The mean value and the standard deviation of an equivalent normal distribution of the strength is given respectively by [5]:

$$\mu_S^N = S^* - \Phi^{-1}\left(F_S\left(S^*\right)\right)\sigma_S^N \,, \tag{14}$$

and

$$\sigma_S^N = \phi\left[\frac{\left(\Phi^{-1}\left(F_S\left(S^*\right)\right)\right)}{f_S\left(S\right)}\right], \tag{15}$$

where Φ is the cumulative distribution for the standard normal; ϕ = density function of the standard normal; F = cumulative distribution of the random variable; f is the density function of the random variable; and μ_S^N, and σ_S^N is the mean value and standard deviation of the equivalent normal distribution of the strength, S, respectively.

After solving the previous set of equations for the safety index, β, the probability of failure of the joint is given by:

$$P_f = 1 - \Phi(\beta), \tag{16}$$

where Φ is the cumulative distribution function of the standard normal variant β.

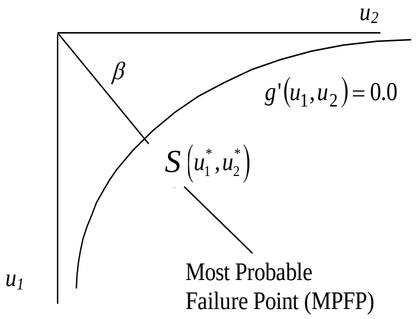

Fig. 3. Limit state in reduced coordinates.

5. Case study

The case study handles the comparison between different repair alternatives for a cracked longitudinal–transverse intersection representing a structural connection in an existing ship. This case study is aimed to answer the following questions: What is the most economical repair type/alternative that should be selected for this failure? What is the expected repair life for this type/ alternative? Is that type/alternative economical for the remaining life of the ship?

To answer these questions, the following steps are followed:

1. The repair life of each repair type/ alternative is calculated based on the old and new joint using the S-N curve as shown in fig. 2.
2. Uncertainty characteristics of load and strength are assigned as shown in table 2 below.
3. Probabilistic analysis is utilized to assess the probability of failure of the repaired joint based on the load-strength interference theory.
4. The expected number of repair repetitions in the remaining life of the ship is calculated using:

$$\textit{No. of repair cycles} = \textit{1}^{st}\ \textit{Repair} + \frac{\textit{Remaining life of Ship after 1}^{st}\ \textit{Failure}}{\textit{(Reliability of new joint) x Repair life}} \,. \tag{17}$$

$$N_R = 1 + \frac{T_R}{\left(1 - P_f\right) x\, t_R} = 1 + \frac{T_{ship} - t_{f1}}{\left(1 - P_f\right) x\, t_R} \,. \tag{18}$$

5. Cost of each repair type/alternative are calculated using:

Table 2
Uncertainty characteristics for load and strength

Random variable	Mean	COV	Distribution type
Strength and load for the modified joint	$S'_{max} = ln(f_o t_R)\left[\dfrac{\Delta A'}{f_o t_R \Gamma(m'+1)}\right]$ t_R = Repair life of modified joint A' = shape factor of modified joint from S-N curve m' = slope factor of modified joint from S-N curve f_o = number of cycles per years	0.15 (assumed values based on literature) [6, 7]	Normal / lognormal
Strength and load for the original joint	$S_{max} = ln(f_o t_{f1})\left[\dfrac{\Delta A}{f_o t_{f1} \Gamma(m+1)}\right]$ A = shape factor of original joint from S-N curve m = slope factor of original joint from S-N curve t_{f1} = time of first failure of original joint	calculated from S-N curve for each type as shown in table 3 below.	Normal / Extreme Value Distribution (EVD) type I

Table 3
S-N curve characteristics for each class

Class	A (MPa)	m	COV(A)
C	4.27E+13	3.5	0.50
D	1.51E+12	3	0.51
E	1.05E+12	3	0.63
F	6.31E+11	3	0.54

$$C_T = N_R . C_R . \qquad (19)$$

Where: T_{ship} is the designed life of the ship, T_R is the remaining life of ship after 1st failure, P_f is the probability of structural failure of repaired joint, t_{f1} is the life of ship at 1st failure, CT is the total repair cost, C_R is the cost of repaired joint, and t_R is the life of new joint after repair.

It is to be noted that in this study, the impact of the redistribution of load acting on the failing connection into adjacent intact connection has been ignored. The redistribution of loads will result in an increase in the load acting on the adjacent connections as shown in fig. 4, hence, a decrease in the calculated life of those connections. Analysis of the dependent failure of adjacent connections is a complicated problem and beyond the scope of this study.

5.1. Program development for calculations

The Advanced Second Moment method was selected to calculate the probability of failure due to the existence of non-normal basic random variables in the corresponding limit states for the structural connections. The generalized form of the limit state function can be put in the following generalized form [8]:

$$g(X_1,...,X_4) = C_1 X_1^{n1} X_2^{n2} + C_2 X_3^{n3} X_4^{n4} . \qquad (20)$$

Where C_i is the deterministic coefficient, X_i is the probabilistic basic random variables, ni is the real-valued power. The input data sheet for an example alternative is shown in table 4, which shows the normalized mean and standard deviations for the strength and the load.

The program runs eqs. (11) through (16) for 10 iterations to solve for the index, β. Figs. 5-a,b show the relationship between the total cost in US Dollars of repaired joints and the life at first failure. The cost of repair of each type is based on the common rates used in the invoice charged to ship owners at the "Egyptian Shuipbuilding and Repair Company", Alexandria, Egypt. As shown in these figures, the highest cost is for alternative "4", "welding and added lug" if it is used continuously on the remaining life of the ship. This is because the initial cost is high ($1000) and the number of repair repetition is also high (repair life is ranging from 2.3 to 3.6 years multiplied by a reliability of 48% to 52% respectively). While the lowest cost is for

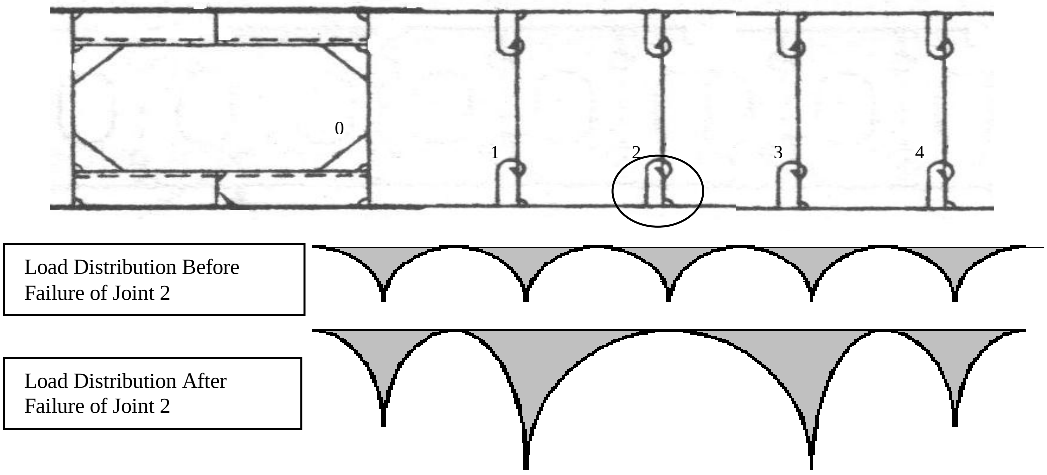

Fig. 4. Redistribution of load on intact adjacent connections after failure.

Table 4
Sample input data sheet for the developed program

Input information:		$C1$ =	1	$C2$ =	-1
$X_1 = \mu_S =$	732 N/mm^2	$_\Omega X1 = COV(S) =$	0.50	$_\sigma X1 = {}_\sigma S =$	366 N/mm^2
$X_2 =$	1	$COV(X_2) =$	0	$_\sigma X2 =$	0
$X_3 = \mu_L =$	418 N/mm^2	$_\Omega X3 = COV(L) =$	0.15	$_\sigma X3 = {}_\sigma L =$	62.6 N/mm^2
$X_4 =$	1	$COV(X_4) =$	0	$_\sigma X4 =$	0

alternative “9” “Insert plate + Tripping Bracket” due to the moderate life of repaired joint (37 to 61 years) multiplied by a reliability ranging from 64% to 71%, leading to a one time repair in the remaining life of the ship at a cost of ($1200), as shown in table 5.

As shown in tables 6-a, b, the cost of repair of the non-normally distributed strength and load is generally higher than that of normally distributed values for alternatives “4” and “9”. This is explained by the higher probability of failure in case of non-normally distributed strength-load interference. This indicates the impact of uncertainty characteristics in the strength and load values on the estimation of repaired joints life.

It is noted that the ratio of repair cost is almost constant for three types (6, 9 &12); this is explained by the very expected long life of these connections (from 20 to 169 years), which covers the remaining life of the ship (15 years) multiple times, i.e., once this repair type is done, it will never fail again and the cost of repair ($1050, $1200 & $1600, respectively) will not be repeated again.

Finally, it could be concluded that the choice should be a constant-cost alternative, i.e., one of (6, 9 or 12) since they have the least cost, the longest expected life, and the least number of failures (only 1) during the remaining life of the ship after the first failure of the original connection. The low number of failures of these connections helps improve the structural safety of the ship and decreases the required labor hours required for repair considering the high number (thousands) of these connections in the hull.

6. Conclusions and recommendations

The repair life of longitudinal- transverse intersection representing structural connections in an existing ship has been studied. Different alternatives for structural repair are discussed. A comparative analysis is proposed based on structural reliability of each alternative to estimate the cost of repair. From this study, the following are concluded:
1. The proposed probabilistic analysis is very useful in calculating repair life of structural joints based on more realistic understanding

of variability and uncertainty of nature of strength and load.

2. Economic considerations play a dominant role in repair decisions. If a longer life continuance is expected for the ship, it is possible to define which repair alternative is the most reliable and cost effective for this crack type.

Table 5
Estimated life (years) and cost of each repair type for normally distributed strength and load

Type of repair	S-N curve	Life before 1st Failure t_f					Cost of repair US$	SRF
		6	7	8	9	10		
		Expected life of new joint t_R						
1- Welding the crack	E-class	0.84	0.95	1.1	1.2	1.3	$ 350	1.0
2- Welding + post welding	D-class	1.3	1.52	1.7	1.9	2	$ 450	1.0
3- Insert plate	C-class	6.0	7	8	9	10	$ 650	1.0
4- Welding + added Lug	E-class	2.3	2.6	3	3.3	3.6	$ 1,000	0.76
5- Post welding + added Lug	D-class	3.7	4.2	4.7	5.2	5.6	$ 1,100	0.76
6- Insert plate + added Lug	C-class	20	23.3	26	30	33	$ 1,050	0.76
7- Welding + added Tripping bracket	E-class	4.0	4.5	5	5.5	6	$ 900	0.66
8- Post welding + Tripping bracket	D-class	6.2	7.05	7.8	8.7	9.5	$ 1,000	0.66
9- Insert plate + Tripping bracket	C-class	37.0	43.0	49	55	61	$ 1,200	0.66
10-Welding +Lug + Tripping bracket	E-class	9.5	10.7	12	13	14.4	$ 1,300	0.52
11- Post Welding +Lug + Tripping bracket	D-class	14.8	16.9	18.7	20	22.5	$ 1,400	0.52
12- Insert plate +Lug + Tripping bracket	C-class	103	119	136	152	169	$ 1,600	0.52

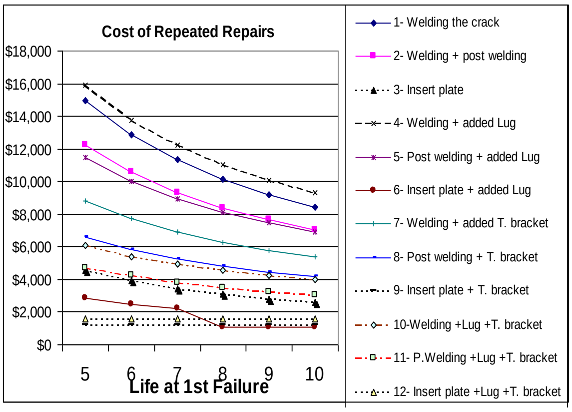

Fig. 5-a. Cost in US Dollars of repair for a joint if load & strength are normally distributed.

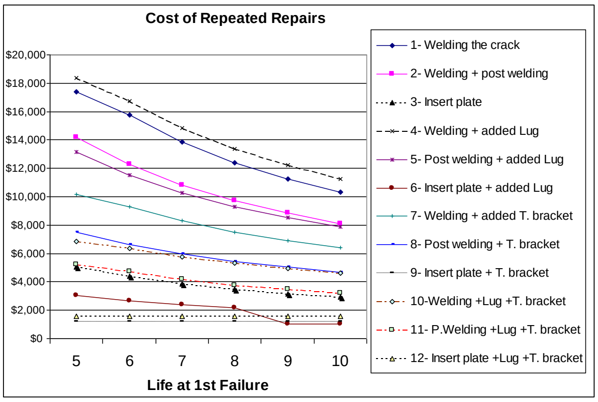

Fig. 5-b. Cost of repair US Dollars of repair for a joint if load & strength are normally distributed.

Table 6-a
Estimated life (years) and cost of types “4” and “9” for normally distributed strength and load

Type of repair	Life before 1st failure t_f					Cost of repair US$
	6	7	8	9	10	
	Expected life of new joint t_R					
4- Welding + added lug	2.3	2.6	3	3.3	3.6	Cost / repair = $1,000
Reliability	0.48	0.48	0.49	0.51	0.52	
Expected life of alternative “4” = estimated life x reliability	1.11	1.26	1.48	1.68	1.87	
	Total cost US$					
	15913	12905	11135	9928	9021	
9- Insert plate + tripping bracket	37.0	43.0	49	55	61	Cost / repair = $1,200
Reliability	0.64	0.66	0.69	0.71	0.71	
Expected life of alternative “9” = estimated life x reliability	23.7	28.3	34	39	43.3	Total cost = $1,200

Table 6-b
Estimated life (years) and cost of types “4” and “9” for non-normally distributed strength and load

Type of repair	Life before 1st failure t_f					cost of repair us$
	6	7	8	9	10	
	Expected Life of new Joint t_R					
4- Welding + added Lug	2.3	2.6	3	3.3	3.6	cost / repair = $1,000
Reliability	0.4	0.41	0.41	0.42	0.43	
expected life of alternative “4” = estimated life x reliability	0.92	1.06	1.26	1.42	1.55	total cost
	Total cost					
	18304	15371	12905	11571	10690	
9- Insert plate + tripping bracket	37.0	43.0	49	55	61	cost / repair = $1,000
Reliability	0.6	0.63	0.65	0.67	0.69	
Expected life of alternative “9” = estimated life x reliability	22.2	28.3	34	39	43.3	total cost = $1,200

3. Statistical analysis of uncertainty characteristics of strength and load of structural connections is needed to use the real distribution type since it has been proven to have a major impact on the cost of failure as shown in this study.

4. Mathematical optimization is needed to investigate the economics of using different repair alternatives for the same joint to cover the exact remaining life of the ship instead of repeating the same more costly alternative that may last longer than needed.

5. Further work is needed and recommended to analyze load redistribution on adjacent intact connections and recalculate life estimation of these connections. Dependent failure theory should be implemented in the structural reliability of these connections.

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