Design load for conveyors set between two poisson manufacturing processes

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This paper discusses is the loading of carrying conveyors when set to transfer an arbitrary type of materials between two random manufacturing processes. Controlling of the conveyor maximum allowable loading represents a constrained level of material waiting which adversely reduces the two processes rated output. Thus, an optimum conveyor capacity is sought to minimize such inevitable output drop. Queue theory is implemented to define a comparison measure for the conveyor expected loads, enabling the process designer to assess the output drop below the process anticipated requirements. Two criteria are then put forth towards refinement of capacity selection; namely, optimization at a satisfactory gain level and the optimization at the gain drop retardation threshold.

تنتشر السيور الناقلة فى مختلف التطبيقات الصناعية ٠ ولتحديد المواصفات الهندسية لهذه النواقل يجب أن نتأكد من تـوافر الأمان و الحماية لتحقيق المهمة المصمم لها. يقوم البحث بدراسة عمليات التحميل لمنظومة سيور ناقلة لمادة ما بين نقطتـين عشوائيتين خلال خط تصنيع فى مؤسسة معينة٠ التحكم فى أقصى حمولة مسموح بها يمثل مستوى القيود المـسلطة علـى المادة المراد نقلها للحصول على أقصى معدلات ممكنة لنقل للمادة٠ ولتحقيق ذلك تمت دراسة السعة التحميلية المثالية التـى عندها تكون معدلات المخرجات المفقودة أقل ما يمكن باستخدام نظرية خطوط الانتظار تمت المقارنة بـين نتـائج القيـاس للأحمال المتوقعة والتي تم تصميم النظام عليها لتحديد الفاقد من الخرج النهائى الذى يتسبب فى عدم تحقيق المعدل المصمم عليه ٠ وقد تم اتباع نظامين يقدمان اختيارات لتحديد سعة النقل للسير :الظروف المثالية لتحقيق مستوى النقـل المطلـوب بشكل مرض . والظروف المثالية عند وجود مفاقيد فى مستوى النقل٠

Keywords: Queue lines, Poisson distribution, Conveyor's load selection, Materials flow

1. Background and problem outlines

Conveyors are found in a wide range of industrial applications. Selection of engineering specifications for a conveyor should meet two types of conflicting requirements; namely, conveyor protection and operation output [1]. On one hand, safety and protection of conveyors are achieved by including load reducing factors such as monitored maximum tension, running speed, and slack between successive supporting wheels. On the other hand, operations output and reliability are ensured by including capacity augmenting factors such as monitor-red average size and weight of transported material, duration of loading, service life, temperature, and mutual surface interaction [2].

Conveyor pulling tension depends on motion resistances in terms of slack weight, weight of transported materials, and mechanical friction at the dragged surfaces. A design strategy of the conveyor capacity commonly starts with the transported weight notion implied beforehand by the process schemers. Other selection refinement parameters are always of secondary importance depending on the application in question.

Nevertheless, most manufacturing plants set conveyors between production processes of random nature [3]. The conveyor receives goods from the upstream process at a random arrival rate, and it discharges these goods to the downstream process where their manufacturing stay is random as well. Randomness of both the arrival rate and the stay time results in intermittent and unsteady conveyor feeding and discharging. There may be moments of conveyor emptiness and others of crowdedness, exceeding the conveyor capacity being controlled by a special safety system.

Conveyor nominal load is then viewed as a limiting constraint negatively affecting the performance of both up- and down-stream processes. Upstream process will cause the system worst effects whenever its feeding rate goes beyond the conveyor capacity [4].

Since conveyor control prevents any extra amount to pass to the conveyor, four alternatives appear for the system watchers to respond. First, to slow down the upstream rate. Second, to speed up the downstream process. Third, to let the extra amount bypass the downstream servicing without processing. Fourth, to allocate the extra amount towards a large waiting area (reservoir) until a downstream vacancy becomes available. As a result, selection of conveyor capacity will rely on the trade-off or sacrifice afforded along the four alternatives. Such a view calls out for establishing a detailed criterion to assess alternative sacrifices.

The present study is based on the implementation of queue theory as applied between two Poisson random processes without violating their implied rates. Therefore the possibility of either the slowing down of upstream rate or the speeding up of downstream rate is excluded. As a result, the queue length is theoretically unlimited with a well estimated free mean value which is a convenient measure for the process antici-pated output. Practically, however conveyor allowable load capacity can not be exceeded, thereby only a portion of the unlimited queue will be admitted to pass on the conveyor. A much less mean transferred output is thus expected for a constrained process than the theoretical mean of the same process when unconstrained. The difference between the two stated means is a quantitative measure to assess the sacrifice in the system output.

2. Conveyors between two poisson processes

Assume that we have a conveyor whose maximum tension not to exceed q =4 tons capacity. This conveyor is set to transfer the goods between two Poisson random manufacturing processes. Thus the upst-ream process feeds the conveyor with goods following a Poisson distribution whose mean arrival rate is λ = 8 ton/hr. In the downstream process, manufactu-ring time of a serviced ton also follows a Poisson distribution, leading to a mean departure μ= 10 ton/hr. As a short notation, the two processes are described by a relative combined index ;

$$Z = \lambda/\mu = 0.8 .$$

Based on the queue theory, received tons are expected to wait on the conveyor before being manufactured in the downstream process. Since the conveyor maximum capacity is controlled at q=4 tons, possible weights on the conveyor and in the servicing process sweep up a discrete lumped range, being n=0,1,2,3,4 tons. For instance, if $n = q$ the conveyor is visualized as fully loaded. The probab-ility P_n here expresses the percentage of total working life where n tons are being incorporated in the system. Pn is given by the conditional probability;

$$P_n = (q,z) = \frac{Z_n(1-z)}{1-z}, n = 0,1,2,3,4 . \quad (1)$$

Eq. (1) represents the terms of a geometric series whose sum is one, thereby it expresses a probability density function. In our illustrating example the conveyor is empty (unloaded) for P_o = 29.75 % and is variably loaded for 70.25 % of the total utilization life.

Following the queue theory terminology, the practical Mean Transferred Tonnage (MTT) is the mean in the system $W(q,z)$, and the mean tonnage on the conveyor is the average queue length $C(q,z)$. Mathematically;

$$MTT = W_{(q,z)} = \sum_{n=0}^{q} np_n , \quad (2)$$

and

$$C(q,z) = \frac{[1-z^{(q+1)}]}{1-(z)^q} W_{(q-z)} - z . \quad (3)$$

Although eq. (3) accura-tely expresses the conveyor mean load, eq. (2) will be preferred here aiming at a more comprehensive analysis. It should be recalled that such circumstances involve two main features. First, a randomly arriving lumped ton will be passed onto the conveyor as long as the waiting load on conveyor is less than four. Second, the conveyor load is automatically controlled below maxim-um capacity of allowable tonnage, q=4 tons. Thus, if the conveyor is full, an arriving lumped ton is either process-denied or delayed in a reservoir until replacing a departing serviced ton.

Both rejection and delay of the excess of four tons reflect a negative drop or sacrifice in the conveyor and the system anticipated efficiencies, being roughly estimated as η = (MTT / q)=39%. In addition, the process losses result in inconvenient parametric indicators such as high percentage of unloaded life P_o =29.75 % and low percentage of full load life P_4 =11%. In other critic words, our good capacity conveyor is mostly partially loaded, and only fully loaded for a limited fraction of its total utilization life. In terms of a numerical assessment, loading conditions of a conveyor with a q-ton capacity are specified by three parameters; namely, mean transferred tonnage $W(q,z)$, distribution of individual loads contrib-ution $P_n(q,z)$, and percentage loaded life. The present report is devoted to analyze the parameter $W(q,z)$. The other two parameters will be discussed in a forthcoming separate study.

3. Factors affecting the controlled mean transferred tonnage

Suppose that we have a conveyor with a given q-ton load capacity and look for its mating process index z which may own the largest MTT. This situation can be handled by holding q at a specific constant and plotting eq. (2) over the full range of process index 0<z<1. Fig. 1 shows the variation of MTT for some selected conveyor capacities. Based on fig. 1 and observations of real applications [5, 6], one can notice that as whether the process index increases due to accidental acceleration of upstream rate, or the conveyor capacity is becom-ing large due to its exaggerated size, the MTT generally increases.

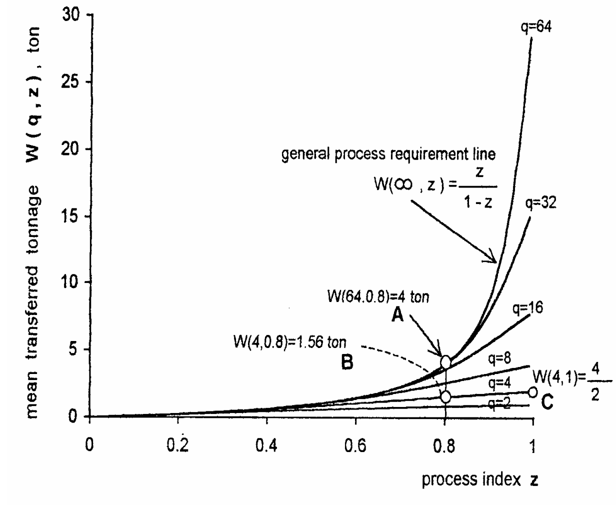

Fig. 1. Variation of mean transferred tonnage for different process indices. Notice that $W(q,1) = \{q/2\}$ ton.

3.1. Bounds of mean transferred output tonnage

A possible practical situation is infrequently encountered where the upstream rate approaches the downstream rate, i.e. the process index approaches an integer one. Referring to eq. (2) and fig. 1 as z approaches a whole value of one, it appears for any q-capacity conveyor that (MTT) tends to the Maximum Mean of the Conveyor Output (MMCO), i.e. mathematically;

$$MMCO(q) = \lim_{z \to 1} W(q,z) = \lim_{z \to 1} \sum_{n=0}^{q} n\, p_n(q,z) = q/2. \quad (4)$$

This is the largest MTT that a q-ton selected conveyor can transfer.

On the other hand, process schemers anticipate a hypothetically full efficiency if any large content of the reservoir can entirely be transferred, where the unlimited queue theory is valid. Under such circumstances, the mean in the whole system corresponds to the Mean Requirement of Process Output MRPO. This whole system consists of the tonnage being serviced in the downstream process, the tonnage waiting on the conveyor, and the extra amount displayed to the reservoir. Mathematically;

$$MRPO(z) = \lim_{q \to \infty} W(q,z)$$
$$= \lim_{q \to \infty} \sum_{n=0}^{q} n\, p_n(q,z)$$
$$= \frac{z}{1-z} \quad . \qquad (5)$$

Eq. (5) is traced as the ceiling line close to that of q=64 ton in fig. 1. For instance, MRPO(0.4) = 0.666 ton and MRPO(0.8) =4 ton. Going rightwards to the critical process z = 1, eq. (5) reveals that the MRPO is theoretically unbounded if tonnage transfer is unconstrained, but the transferred tonnage will be bounded to MMCO(q)={$q/2$} if the tonnage transfer is constrained by a q-ton conveyor controlled capacity.

3.2. The practical mean transferred tonnage

For an arbitrary system, the control of maximum transferred tonnage within the conveyor capacity results in a MTT less than the two upper bounds derived above. Thus, for arbitrary q, provided $z << 1$:

$$W(q,z) \;\; MMCO(q) = \frac{q}{2} \quad . \qquad (6.a)$$

And for arbitrary z, provided $q << \infty$:

$$W(q,z) < MRPO(z) = \frac{z}{1-z} . \qquad (6.b)$$

Finally, the selected conveyor will then cause an inevitable drop or sacrifice in the process anticipated requirement. The drop is estimated as:

$$S(q,z) = MRPO(z) - W(q,z) \quad . \qquad (7)$$

4. Standardized conveyors

4.1. Operations with controlled capacity conveyors

Now, the steps to use fig. 1 will be summarized by an example. The plant schemers have a process data at z =0.8, being fully efficient if tonnage transfer as an unlimited queue is reached. They then anticipate that MRPO is at point A. Such data when passed to the conveyor designer however, he has to recommend a certain form of limited transfer, selecting e.g. a 4-ton conveyor capacity. Thus, the practical MTT= w (4,0.8) drops to point B. The difference between points A and B is a measure of the output sacrifice. Later during operational conditions with the same selected conv-eyor, suppose that the arrival rate accidentally jumps from the current process z = 0.8 to the critical process, i.e. z = 1. Accordingly, MRPO of such a new critical process shifts to infinity, and the practical MTT shifts from B to C where it reaches MMCO ={4 /2} = 2 ton.

4.2. Conflicting trends upon selection of conveyor capacity

It was mentioned in article 2 that the supply of the two random processes goes naturally beyond full loading of conveyor capacity. Controlling the conveyor loads ensures its safe operation and leads to a shorter duration of its full loading. The system scheme involves two conflicting interests. Conveyor designer seeks minimum external load-ing to be achieved with low conveyor capacity. Process designer seeks maxim-um transferred tonnage to be achieved with high conveyor capacity. Fig. 1 shows that for an arbitrary process z, any q-controlled capacity conveyor safely and satisfactorily works. Thus, preference of a specific q value depends on more additional requirements put upon MTT.

4.3. Selected conveyor capacity versus mean transferred tonnage

The designer here is given the process index e.g. Z = 0.8 and seeks the enhancement of the MTT without conveyor exaggerated sizing. He is thus recommended to use eq. (2) along with fig. 1 to monitor the trend of $W(q,z)$ for available standard sizes of conveyor capacities, e.g. q=2,4,...., 64. Optimum selection is readily achieved by starting with minimum q and then gradually raising its value so that MTT either approaches the process requirements MRPO, or until any further increment in q leads to relatively insignificant effect. Numerically speaking, selecting a 2-ton controlled conveyor capacity provides $W(2,0.8)=0.85$ ton. Similarly, selecting a 64-ton controlled conveyor capacity provides $W(64,0.8)=4$ tons. Since 3.15-ton gain in MTT is too little compared to a 62-ton conveyor capacity jump, optimum capacity selection has to be reviewed through a detailed investigation of the functional behavior of $W(q,z)$.

5. Functional analysis of the mean transferred tonnage

Although conveyor capacities are standardized and discrete, we will handle them as a real variable in the load function $W(q,z)$. For arbitrary process indices, e.g. z = 0.4, 0.6, 0.8, and 1.0, the plot in fig. 2 shows a general ascending behavior of the function $W(q,z)$. Interestingly, for critical processes z = 1, the function $W(q,z)=\{q/2\}$ is the ceiling straight line. Specially for the process Z=0.8, the three marked points A, B, and C match the same comment made above for fig. 1. In order to investigate its behavior in detail, the function and its first partial derivative are plotted in figs. 3, 4.

It is obvious that in the low-capacity range, MTT progressively increases. In other words, the gain function defined as the rate by:

$$G(q,z) = \frac{\partial\, w(q,z)}{\partial\, q}$$
$$= z^{(q+1)}\,[1-z^{(q+1)}]^{-2}\; x$$
$$[z^{(q+1)} - 1 - (q+1)Log\,(z)]. \tag{8}$$

is considerably high, about 0.45-0.25 ton/ton for Z=0.8. Thus a slight shift to $q+\Delta q$ leads to significant positive gain ΔG to be added to MTT, reflecting a sensitive selection range. Such an approach is followed in most real applications.

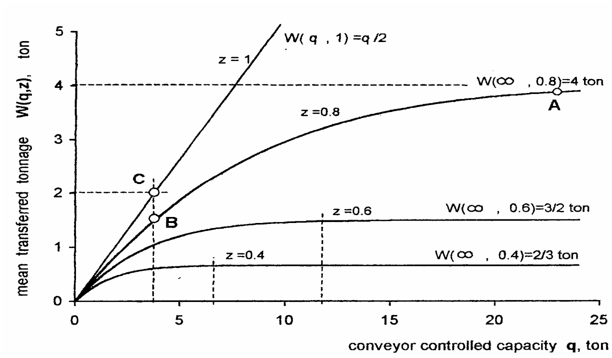

Fig. 2. Variation of mean transferred tonnage for conveyor different capacities. Notice that $W(\infty,z) = Z/\,(1\text{-}z)$.

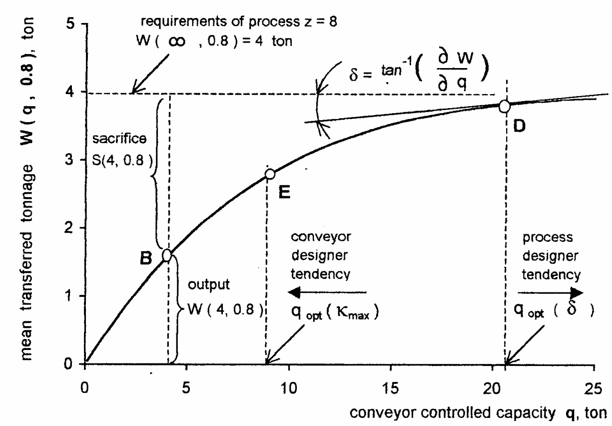

Fig. 3. Variation of the defined system sacrifice for different conveyor controlled capacity. The case of process index z = 0.8.

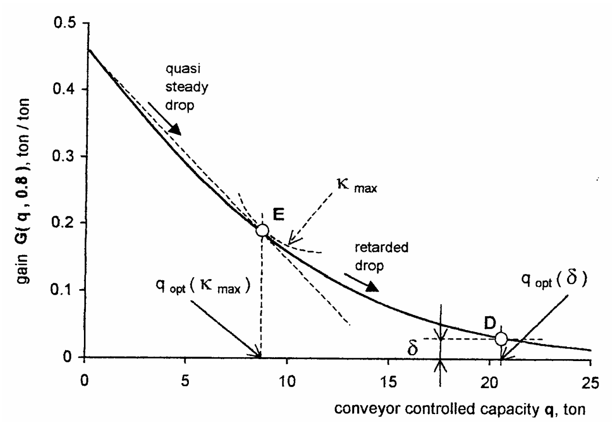

Fig. 4. Gain variation for conveyor different capacity. The case of process index z = 0.8.

Nevertheless, in the high-capacity range, MTT slowly increases. As the selected q tends theoretically to infinity, MTT approaches the horizontal ceiling of the process at MRPO(z)= $W(\infty,z)$= z / (1-z) =4 ton for z = 0.8. Thus for such a range, a major increment in the selected q value leads to a modest gain in MTT. In other words, capacity exaggeration within the high capacity range has insignificant effect on MTT.

6. Discussion

At this stage, naming the low-capacity range and the high-capacity range is still an improvised implementation. An accepted criterion has to be proposed to pinpoint where the parting limit lies to guide the designer while selection. In order to search an optimum capacity value, two criteria are next put forth.

6.1. The criterion of the satisfactory gain

Referring to figs. 3,4, q_{opt} is found from the condition,

$$q_{opt}.\ \delta = max(q)\ //\ \frac{\partial\ w(q,z)}{\partial\ q} > \delta. \qquad (9)$$

Where δ is an accepted level of satisfactory gain achieved in MTT. For instance, if we acknowledge a gain level δ = 0.05 ton/ton for our process z = 0.8, conveyor optimum capacity is $qopt\delta$ = 21 ton, indicated by point D in figs. 3,4. This reveals that the MTT of our 21-ton selection leads the MTT of a smaller-capacity predecessor by about 0.05 ton/ton and lags the MTT of a higher-capacity successor by about 0.05 ton/ton. Due to our current selection, a sacrifice displaced to the reservoir can be estimated according to the definition of eq. (7) and fig. 3, given by:

$$S\ (q_{opt}\delta,\ z\ ) = W(\infty,z) - W(q_{opt}\delta,z)$$

$$= \frac{z}{1-z} - W\ (q_{opt}\ \delta,,z). \qquad (10)$$

6.2. the criterion of the threshold of gain drop retardation

Observation of the gain function in fig. 4 prompts to an early distinguished range where the gain maintains an almost constant fast descending rate. At conveyor higher capacities, the gain drops with a slower rate. Accordingly, the gain should pass a capacity value which marks the transition from fast to retarded drop. Looking for such a threshold, we make use of the curvature of the gain function defined by:

$$K(q) = \left[\frac{\partial^2\ G}{\partial\ q2}\right] \left\{1 + \left[\frac{\partial\ G}{\partial\ q}\right]^2\right\}^{3/2}, \qquad (11)$$

which is plotted in fig. 5. From the figure, it is clear that the gain curvature undergoes a maximum at point E where the gain drop starts a retarded trend. At this characterized point E in figs. 3 to 5, the gain in MTT is extremely low compared to conveyor high selected capacity. Therefore, it is suggested to consider our design threshold to correspond to $Kmax(q_{opt})$. The threshold capacity expresses a recommended value where rightward selection provides insignificant gain in MMT, whereas a leftward selection prompts to an overlooked significant gain compared to conveyor capacity. Based on such a criterion, the graph of fig. 6 is plotted to provide recommended conveyor capacities for different process indices in the range 0<z<1. Although the curvature criterion provides lower transferred tonnage than the satisfactory gain level does, its use is more preferred since the values of conveyor capacity are more conservative.

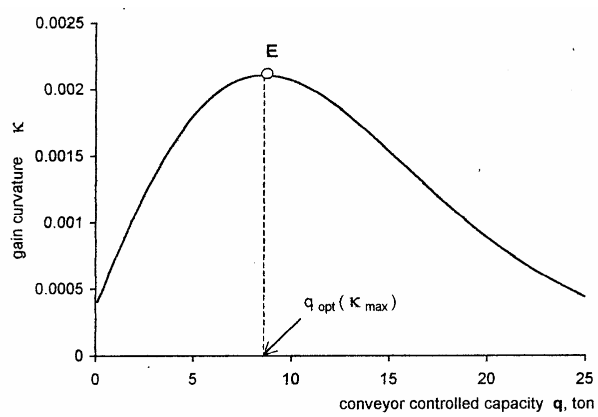

Fig. 5. Variation of gain curvature for conveyor different capacities. The case of process index x = 0.8.

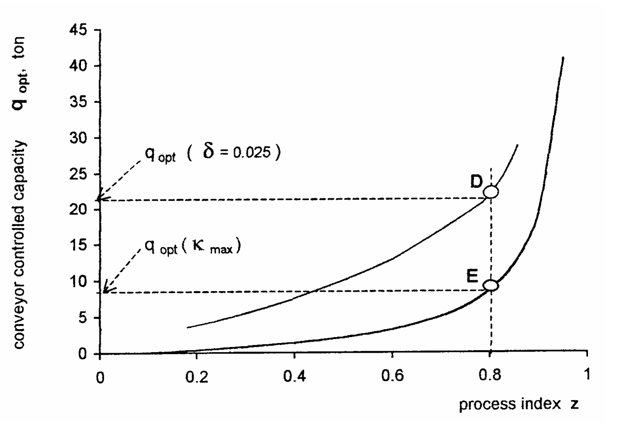

Fig. 6. Recommended conveyor controlled capacity for diff erent process indices based on the two suggested criteria.

Furthermore, since the curvature thresh-old is based on analytical considerations, a unique capacity value becomes always well determined. Capacity values estimated by satisfactory gain are various and are left to the discretion of system designers.

7. Conclusions

The main endeavor of the present study is to optimize the conveyor capacity based on the mean transferred tonnage. The technique first focuses on the bounds of mean transferred tonnage for the unconstrained and constrained processes in terms of unlimited and limited length of the queue theory. Application of unlimited queue analysis sets a process intended requirement at $W(\infty,z) = z / \{1-z\}$ for any feasible process in the range $0<z<1$. However, for critical processes with $z = 1$, the mean transferred tonnage has no bound, i.e. beyond practical estimations. On the other hand, to analyze conveyor controlled capacity, application of limited queue reveals that the process mean tonnage becomes far below the process requirement bound at $W(\infty,z)$. This implies that a certain efficiency drop or process sacrifice has to be accepted. In addition, the mean tonnage of the constrained process will be governed by the conveyor capacity at a bound of mere $(q/2)$. Depending on the process index, the sacrifice is considerably large for critical processes with $z = 1$, and reasonable otherwise.

For an arbitrary process, two recommendations are proposed regarding the conveyor capacity selection. First, when selecting a conveyor in the low-capacity range, a reasonable sacrifice is acknowledged. Upper capacity values of this range are thus preferred towards maximization of a sensitive behavior of the mean tonnage. Second, when selecting a conveyor in the high-capacity range, the mean tonnage undergoes insignificant change and becomes close to the process bound with negligible sacrifice. Therefore lower values of this range are preferred to avoid conveyor worthless size exaggeration. To make the technique more specific for process designers, a capacity threshold is suggested based on the gain curvature to split the low-range and the high-range.

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