3-D groundwater seepage around a simple hydraulic structure

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Design of a hydraulic structure retaining an appreciable head of water such as pump stations, regulators and weirs must include the analysis of seepage flow beneath and around the structure. Induced uplift forces, hydraulic gradients at the exit faces of water and the expected total seepage discharge should be carefully estimated. Current design procedures of such structures ignore the effect of side seepage and assume two-dimensional (2-D) seepage flow only. In some cases this may lead to unsafe design that may cause failure of the structure. The present study aims at analyzing the three-dimensional (3-D) problem of seepage flow beneath and around a water-retaining structure having a simple horizontal floor, with neither sheetpiling walls nor cutoffs. The problem has been investigated numerically using a modified version of the three-dimensional finite element program SWICHA. The parameters involved in the problem include the relative width of the floor, the relative depth of the underneath pervious soil and its relative width. Results are presented in graphs. They show great differences between two and three-dimensional values of the piezometric head, the exit gradient and the total seepage discharge. Exit gradients determined by the 2-D model are significantly smaller than the 3-D values. The percentage error may reach (-100) %. It is concluded that the seepage problem for a simple floor can be treated as a 2-D problem when the floor width exceeds two times its length. Otherwise, 3-D analysis should be considered.

معظم منشآت الري و الصرف و التي تحجز ارتفاعا ملحوظا من المياه أمامها، مثل القناطر و الهدارات، أو خلفها، مثل محطات الرفع، تقام على تربة رسوبية منفذة و يجب أن تصمم بعناية لتقاوم جميع القوى التي تؤثر علي سلامتها و اتزانها. ومن أهم هذه القوى تلك التي تنشأ نتيجة حركة المياه تحت وحول المنشأ، وتشمل قوى الضغط أسفل الفرشة والقوى الناتجة عن الميل الهيدروليكي عند المخرج والتي تؤثرعلى اتزان حبيبات التربة في قاع وجوانب المجرى. كذلك فإن حساب مقدار التسرب المتوقع تحت وحول المنشأ يكون له أهمية كبيرة في بعض هذه المنشآت. والهدف من الدراسة الحالية هو تحليل مشكلة التسرب ثلاثي الأبعاد للمياه تحت وحول المنشأ، ومحاولة تحديد الحدود الفاصلة التي يمكن عندها تبسيط طريقة التحليل واعتبار أن التسرب ثنائي الأبعاد بدون خطأ يذكر. تم توضيح الفروض المستخدمة والشروط الحدية، وخطوات الحل باستعمال طريقة العناصر المحددة ، وأهم المؤثرات التي اعتبرت في هذا البحث، و تشمل : العرض النسبي لفرشة المنشأ، و القيم النسبية لسمك و عرض الطبقة المنفذة من التربة المقام عليها المنشأ ، ثم عرض مختصر لبرنامج الحاسب المستخدم والذي تم تحقيقه بمثالين أحدهما ثنائي الأبعاد و الآخر ثلاثي الأبعاد.تم عرض شامل للمنحنيات التفصيلية للنتائج وتحليلها ومناقشتها، وتحديد تأثير كل من المؤثرات السابقة علي : توزيع الضاغط البيزومتري أسفل الفرشة، الميل الهيدروليكي للسرعة عند المخرج ، سواء في القاع أو الجوانب، وقيم التصرف للمياه المتسربة من القاع والجوانب. وتم مقارنة نتائج النموذج ثلاثي الأبعاد مع النموذج ثنائي الأبعاد و تحديد نسب الخطأ الناتج عن إهمال تأثير التسرب الجانبي. وختمت الدراسة ببيان لأهم النتائج التي تم التوصل إليها و كذلك التوصيات المقترحة في التصميم لمثل هذه المنشآت.

Keywords: Hydraulic structures, Simple floor, 3-D seepage, Finite element model

1. Introduction

Hydraulic structures are mostly constructed on pervious soil through which seepage flow occurs. The hydraulic design of such structures should include the analysis of seepage flow below as well as around the structure to determine as accurately as possible the uplift pressure force, the hydraulic gradients at the exit faces and the total seepage discharge.

The two-Dimensional (2-D) analysis of the seepage beneath such structures has been successfully utilized for the design of many hydraulic structures. However, for some structures, such as irrigation and drainage pumping stations the side seepage of water can not be neglected and the three-Dimensional (3-D) analysis should be used.

Neglecting the effect of side seepage may result in unsafe design that may lead to complete failure of the structure. Two examples of field studies for pumping stations on EL-Nasr canal and on Edko drain, west of the Nile Delta, indicated that, among other causes, severe exit gradients at suction side have led to lining failure and contributed to the instability of side slopes.

Kimura and Ohne [1] studied the 3-D seepage around the abutment foundation of an earth dam with a cutoff wall using both model tests and numerical computations by the Finite-Element Method (FEM). They proposed some practical formulas to obtain the quantity of seepage and the shape of phreatic surface along the cutoff wall. They reported that the side seepage led to abnormal rise of the phreatic surface in the downstream (D/S) part of the dam, resulting in local slide due to increase of pore water pressure. Abourohim [2] used a sand model to investigate the effect of side seepage on uplift pressures acting on structures with: i) simple floor and ii) floor having an intermediate sheetpile. He concluded that such an effect becomes negligible when the canal width exceeds 2.6 times the length of the floor of the structure. Using the FEM, Rashwan [3] prepared a computer program to solve the 3-D steady seepage through a homogeneous isotropic earth dam founded on an impervious soil.

The present study aims at investigating the problem of 3-D seepage beneath and around a simple floor and abutments of a hydraulic structure. A comparison is done between the 2-D and the 3-D solutions of this problem to define the limits of validity of the 2-D solution.

2. Problem definition and assumptions

Seepage flow below the floor of the structure (bed seepage) is a confined flow. The piezometric head, h, at any point varies from H at the upstream (U/S) inlet to (0.0) at the downstream (D/S) exit. On the other hand, seepage flow around the wing walls and the abutments of the structure, (side seepage, known also as roundabout seepage or seepage past structure), represents an unconfined flow, with a phreatic surface extending between the U/S and the D/S water surfaces.

Fig. 1 shows the water-retaining structure considered in investigation. It has a simple floor of length L and width B. The floor is embedded in the pervious soil beneath and around the structure by a distance t in the vertical direction and s_1 in the horizontal direction. The pervious soil itself extends to a depth T below the bottom of the canal and a width S in the transversal direction. The lengths of U/S and D/S seepage faces are taken equal to three times the structure's length L. El-Fitiany and Nasr [4] showed that the results of a 2-D model with such dimensions fairly represent the case of infinite seepage faces.

Since the flow domain is geometrically and dynamically symmetrical about the vertical plane passing thought the centerline of the canal, the velocity component perpendicular to that plane vanishes and this plane can be treated as an impervious boundary. Hence, it is sufficient to study only one half of the flow domain.

The following assumptions are considered in the current study:

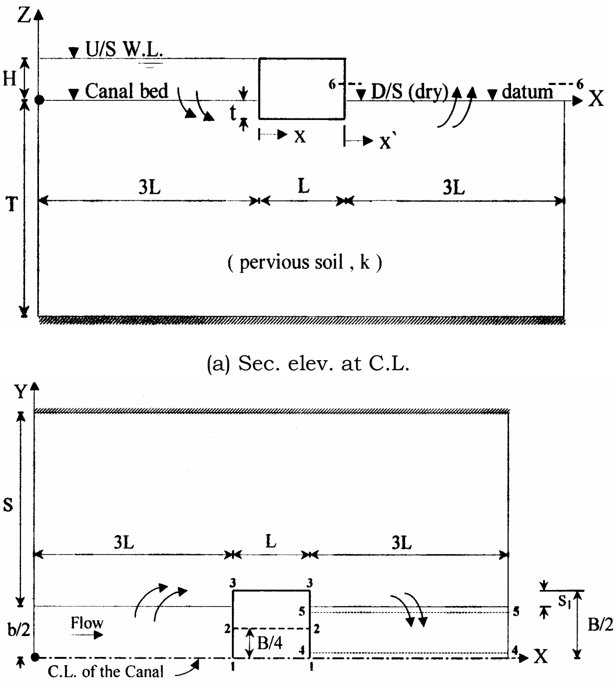
1. The pervious layer is homogenous, isotropic ($k_x = k_y = k_z = k$) and fully saturated.
2. U/S and D/S cross sections of the canal are rectangular.
3. U/S and D/S bed levels of the canal are the same.
4. Capillary rise is neglected. Hence, pressures along the phreatic surface are considered equal to the atmospheric pressure.

3. Governing equation and boundary conditions

3.1. Governing equation

The general governing equation for 3-D groundwater flow can be derived by applying the principle of conservation of mass and Darcy's law, Bear [5]. For homogeneous and isotropic soil and steady flow conditions this equation reduces to the well known Laplace's equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 , \tag{1}$$



(a) Sec. elev. at C.L.

(b) Half plan.

Fig. 1. Problem definition.

in which *h(x,y,z)* = piezometric (potential) head *L* at any point in the flow domain,

$$h = \frac{p}{\gamma} + Z\,, \tag{2}$$

where p/γ = pressure head *L* and *Z* is the position head *L*, measured upward from an arbitrary datum. The D/S water level in the canal is chosen here to be the datum. The effective head H is the difference between U/S and D/S water levels. In fig. 1 it is assumed that the D/S is dry. This, however, does not limit the generality of the analysis.

3.2. Boundary conditions

Boundary conditions for the problem are shown in figs. 2-a and 2-b. In the current study, no mixed type boundary exists. Prescribed head boundaries are represented by the U/S inlet surface, for which $h = H$, and the D/S exit surface, for which h = 0.0.

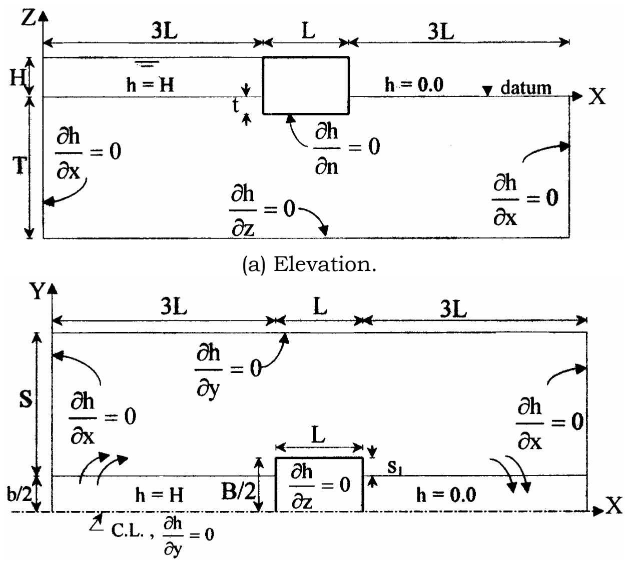
Impervious boundaries, for which $I = \partial h/\partial n = -q_n/k = 0$, are considered for the following items:

1. The floor of the structure,
2. Abutments, U/S and D/S wing walls,
3. The front boundary which coincides with the vertical plane of symmetry along the centerline of the canal,
4. U/S and D/S vertical planes in the Y-direction, arbitrarily chosen at distances equal 3L from the edges of the solid floor;
5. Vertical planes in the X-direction chosen at arbitrarily distances equal (S+0.5 b) from the centerline of the canal, and
6. An impervious horizontal surface at a depth (T) from the bottom of the canal.

In steady flow without accretion or abstraction, and when the capillary fringe is neglected, the phreatic surfaces on both sides of the structure are stream surfaces, with pressure equals to atmospheric allover, i.e. h = *z* for the phreatic surface. Dupuit's assumption of essentially horizontal flow (V_z = 0) in the phreatic aquifer is utilized to simplify the problem within some zones.

4. Main variables and parameters

The main variables investigated here are: the piezometric head (h) at any point along the



(a) Elevation.

(b) Plan.

Fig. 2. Boundary conditions.

surface of contact between the structure and the soil, the exit velocity gradient (I) at the D/S ($I = \frac{\partial h}{\partial z}$) and the seepage discharges below and around the structure.

These variables basically depend on the following parameters: the length L of the structure, its width B, the effective head H, the vertical depth T of the pervious soil, its horizontal (transversal) width S, the vertical embedded depth t of the structure, its horizontal (transversal) embedded width s_1, in addition to permeability coefficient k of the soil.

These variables are directly proportional to the effective head H. Seepage discharges are also proportional to the permeability k. On the other hand, the effect of both t and s_1 is expected to be rather small. Hence, the main variables can be generally represented by the following dimensionless functions:

$$h/H = f_1\left(\frac{B}{L}, \frac{T}{L}, \frac{S}{L}\right), \quad (3\text{-a})$$

$$I/(H/L) = f_2\left(\frac{B}{L}, \frac{T}{L}, \frac{S}{L}\right), \quad (3\text{-b})$$

and

$$Q/(kHL) = f_3\left(\frac{B}{L}, \frac{T}{L}, \frac{S}{L}\right). \quad (3\text{-c})$$

For the current study, values of the parameters (t/L) and (s_1/L) are kept constant and equal to 0.10. The ratio (H/L) is also taken equal to 0.10 Selected values for the remaining parameters are shown in table 1.

Table 1
Selected values of the parameters

	B/L	T/L	S/L
Effect of (B/L)	0.25, 0.50, 1.0, 2.0 and 4.0	3.0	3.0
Effect (T/L)	1.0	0.25, 0.50, 1.0, 2.0 and 3.0	3.0
Effect (S/L)	1.0	3.0	0.5, 1.0 2.0, 3.0 and 4.0

5. The finite element model

The objective of the finite element method is to transform the partial differential eqs. (1,2) into an integral equation which includes derivatives of the first order only. The flow domain is divided into small triangular or rectangular elements of arbitrary sizes. Numerical integration is then performed over these elements. More details of the F.E. technique can be found in Zienkiewicz and Taylor [6] and Bear [7].

Fig. 3 shows an example of the finite element mesh for the current problem. The mesh includes 15 to 20 horizontal slices (parallel to XY plane), each slice has 33 columns and 10 to 17 rows. Total number of elements ranges from 5472 to 9728 and total number of nodes ranges from 6600 to 11220.

A modified version of the 3-D computer program "SWICHA", EL-Dakak [8] and Huyakorn et al. [9], has been utilized for the current problem. The modifications include:

a) Introducing a successive elimination procedure to get the actual phreatic surface which satisfies the condition: $h = Z$.

b) Increasing the number of slices, elements and nodes to improve the accuracy of the results.

The input data includes system geometry, properties of the porous medium and boundary conditions. The output from the model includes nodal head values at various time levels and element centroidal values and directions of Darcy's velocity.

The computer program results have been verified using the results of the 2-D analytical solution derived by Pavlovsky [10] for the same problem and the results of the 3-D electric analogue model for seepage around a tail hydraulic structure, Nasr [11].

6. Results and analyses

Effects of the various parameters on the piezometric heads, the exit gradients and the quantity of seepage discharge are briefly presented and discussed below.

6.1. Piezometric head distribution

Piezometric heads acting on the floor are investigated for three longitudinal sections:

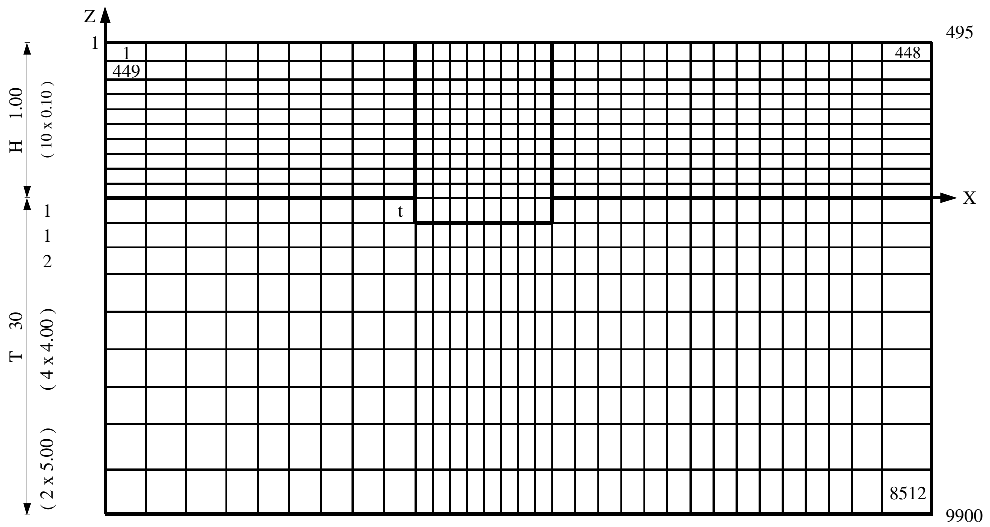
1-1, 2-2 and 3-3, fig. 1.

6.1.1. Effect of relative floor width (B/L)

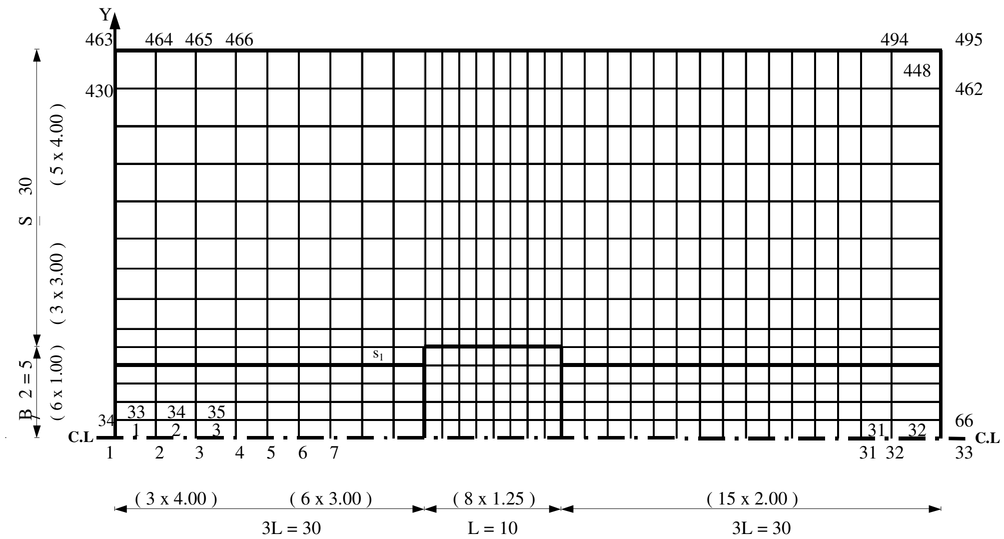
Effect of B/L on piezometric heads is illustrated by fig. 4, for constant depth and width ratios of the pervious soil; T/L = 3.0 and S/L = 3.0. The relative distance x/L is measured from the U/S edge of the floor.

For most of the U/S half of the floor, head values computed by Pavalovsky's solution are always greater than those of the current 3-D model while the opposite is true for the rest of the floor. Largest deviations occur at the two lateral edges of the floor.

As the ratio B/L increases from 0.25 to 1.0, differences between the two solutions decrease significantly for sections 1-1 and 2-2 while for section 3-3 appreciable differences still exist even with B/L = 4.0. This may be due to strong convective acceleration component in the Y-direction for the flow adjacent to that section. Such component, of course, does not exist in the case of 2-D analysis.



(a) Vertical section at t he C.L.



(b) Horizontal plan at U/S water level.

Fig. 3. Finite element mesh.

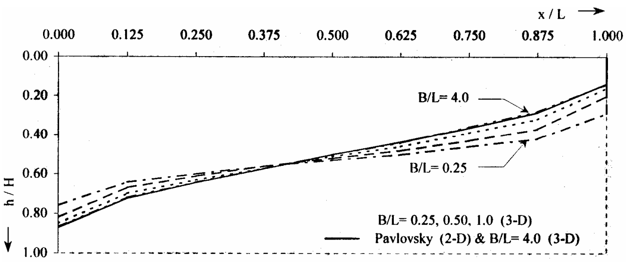
Percentage error *Rh* of piezometric head between 2-D and 3-D is defined as:

$$Rh = \frac{h_{2D} - h_{3D}}{h_{3D}} * 100.$$

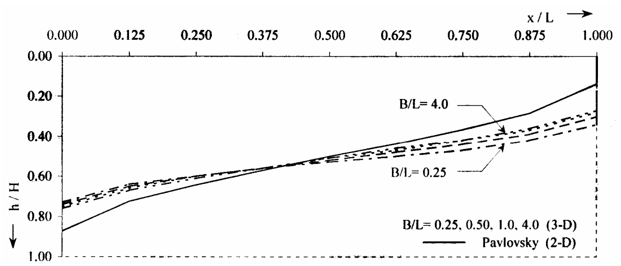
Fig. 5 shows the variation of Rh with the ratio *B/L* at different points along the floor, for sections 1-1 and 3-3. Considering section 1-1, values of Rh decrease significantly as *B/L* increases and become negligible for *B/L* > 2.0. Percentage errors for section 3-3 slightly decrease as *B/L* increases and become constant for *B/L* > 2.0. Maximum error always occurs at the end of the floor *x/L* = 1.0. Its value is, however, much exaggerated due to two reasons:

i. The piezometric head at that location is essentially small. This magnifies the effect of computational errors however small.

ii. For small values of *B/L*, computational errors at corner points are expected to increase.

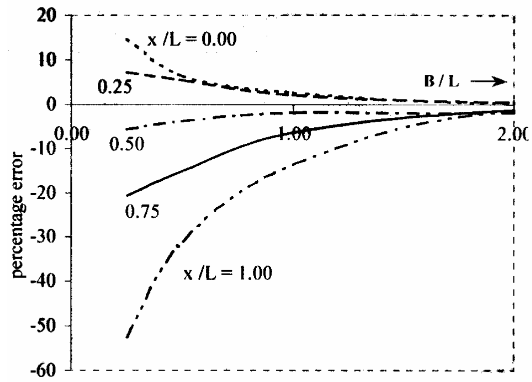


(a) At C.L section (1-1).

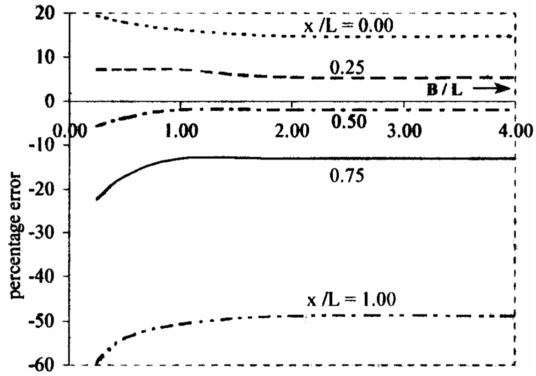


(b) At section (3-3).

Fig. 4. Effect of *B / L* on piezometric heads (For *S / L* = 3 and *T / L* = 3).



(a) At section (1-1).



(b) At section (3-3).

Fig. 5. Variation of head percentage error (*Rh*) with the relative width (*B / L*) (For *S / L* = 3 and *T / L* = 3).

6.1.2. Effect of the pervious layer depth ratio (T/L)

Figs. 6-a and 6-b show the effect of *T/L* on piezometric heads along sections (1-1) and (3-3), respectively, for constant values of *B/L*=1.0 and *S/L* = 3.0. As (*T/L*) increases, piezometric head values along the U/S half of the floor tend to decrease while those along the D/S half tend to increase.

6.1.3. Effect of the pervious layer width (S/L)

Compared to effects of *B/L* and *T/L*, the relative width *S/L* of the pervious layer has in general less influence on piezometric heads. Differences between 2-D and 3-D models are insignificant for section 1-1 while consistent errors are noticed for section 3-3, even with a small value of *S/L* = 0.5, fig. 7.

6.2. Exit velocity gradients

For 3-D flow, two kinds of exit seepage surfaces exist; bed and side surfaces. With regard to the bed exit surface, two longitudinal sections 4-4 and 5-5 at relative distances (b/16) and (7b/16) from the centerline are considered, fig.1. Only one section, section (6-6), is considered for the side exit surface.

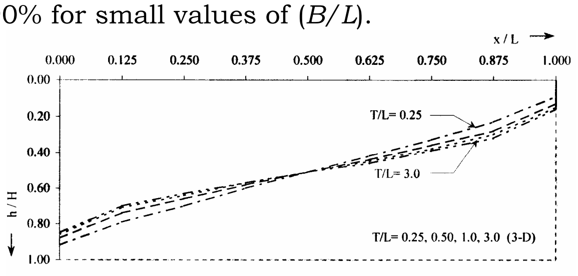
6.2.1. Effect of width ratio (B/L)

Fig. 8 indicates that for constant ratios of $T/L = S/L = 3.0$, exit gradients are strongly influenced by the width ratio (B/L). They become significantly higher for relatively narrow channels, i.e. for small values of (B/L). Exit gradients decrease when the ratio (B/L) is increased. At section 4-4 they become very close to the 2-D values whenever B/L exceeds 2.0. However, at sections 5-5 and 6-6 appreciable differences still exist even for $B/L = 4.0$.

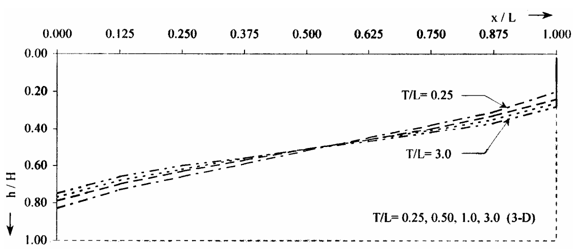
Percentage error *Rh* of bed gradient values obtained from 2-D solution is expressed as:

$$RI = \frac{I_{2D} - I_{3D}}{I_{3D}} * 100. \quad (5)$$

Fig. 9 shows that such error may exceed 90% for small values of (B/L).



(a) At C.L section 1-1.



(b) at section 3-3.

Fig. 6. Effect of (T/L) on piezometric heads (For $B / L = 1$ and $S / L = 3$).

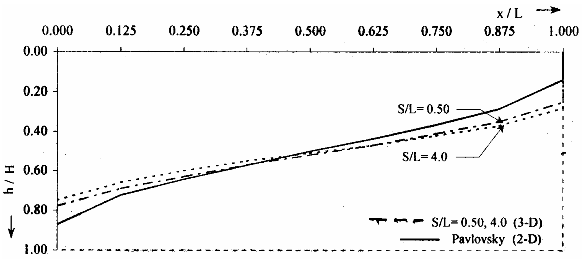


Fig. 7. Effect of (S/L) on pizometric heads at section 3-3 (For $B/L = 1$ and $T/L = 3$).

6.2.2. Effect of the pervious layer depth (T/L)

For constant values of $B/L = 1.0$ and $S/L = 3.0$, variation of (T/L) has a significant effect on bed and side exit gradients, fig.10. For all sections considered exit gradients appreciably increase as (T/L) increases, up to $T/L = 2.0$. Actual bed exit gradients are always underestimated by the 2-D model as manifested by percentage error (*Rh*) as shown in fig. 11.

6.2.3. Effect of the pervious layer width (S/L)

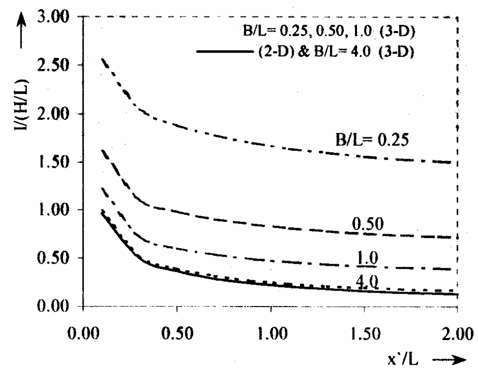
Variation of (S/L) has rather less influence on bed and side exit gradients, compared to that of relative width (B/L) of the floor or to that of relative depth (T/L) of the pervious layer, figs. 12-a, 12-b and 12-c. Bed exit gradients produced by the 2-D model are always smaller than those of the 3-D one.

6.3. Seepage discharge

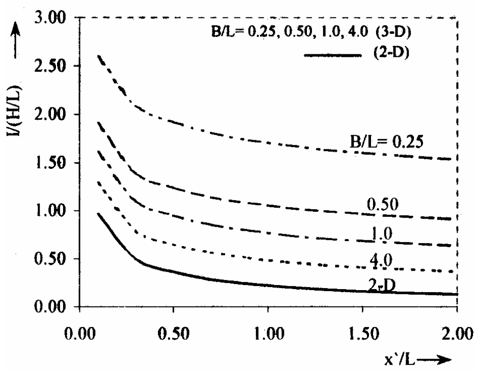
In the 3-D model the total seepage discharge (*Qt*) is the sum of bed discharge (*Qb*) and the side discharge (*Qs*). No side discharge exists in the 2-D model and, therefore, total seepage discharge is equal to bed discharge only.

A length of (*3L*) is considered for both U/S and D/S seepage surfaces. All discharges are expressed as dimensionless ratios in terms of the quantity (*kHL*).

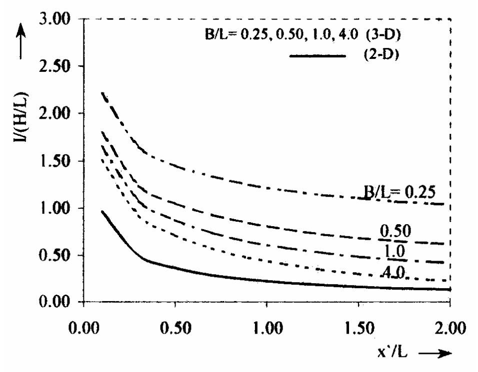
Fig. 13 shows that as the value of (B/L) increases from 0.0 to 1.0, the inlet bed discharge (*Qb*) increases but with a decreasing rate, then the rate of increase becomes constant. Meanwhile, the side discharge, *Qs*, slightly decreases and becomes nearly constant for $B/L > 2.0$. The 3-D total discharges are always greater than the 2-D ones. The difference between the two values decreases gradually as the relative width (B/L) increases.



(a) At sec. 4-4 (bed surface).



(b) At sec. 5-5 (bed surface).



(c) at sec. 6-6 (side surface).

Fig. 8. Effect of relative width (B / L) on relative exit gradient $[I/(H/L)]$ (For $S / L = T / L = 3$).

Variation of relative bed, side and total seepage discharges at inlet with the relative depth (T/L) of the pervious layer is shown in fig. 14. As (T/L) increases, these discharges increase too but with a decreasing rate. Compared to the effect of relative width (B/L), the relative depth has much less influence on the quantity of seepage discharges. Moreover, for the selected values of B/L = 1.0 and S/L = 3.0, total 2-D discharge is nearly half of that of the 3-D model.

Effect of relative width (S/L) of the pervious layer on seepage discharges is rather similar to that of (B/L) but less significant, fig. 15.

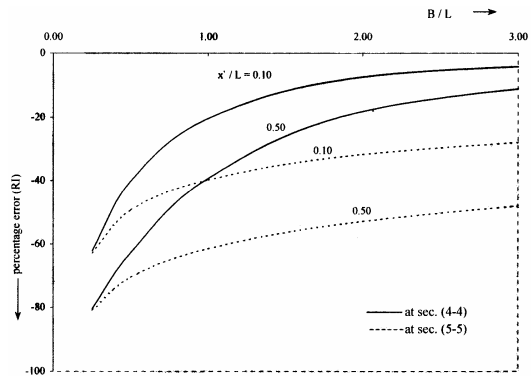
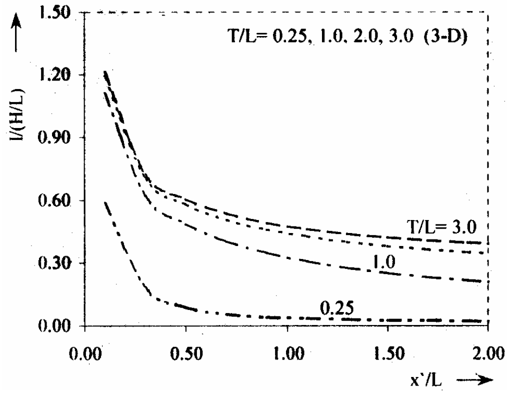


Fig. 9. Variation of percentage errors (Rh) of the exit gradients with the relative width (B / L) (For $SL=T / L = 3$).

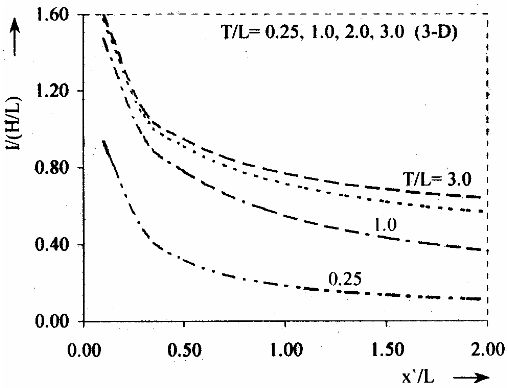
7. Conclusions

The main conclusions may be listed as follows:

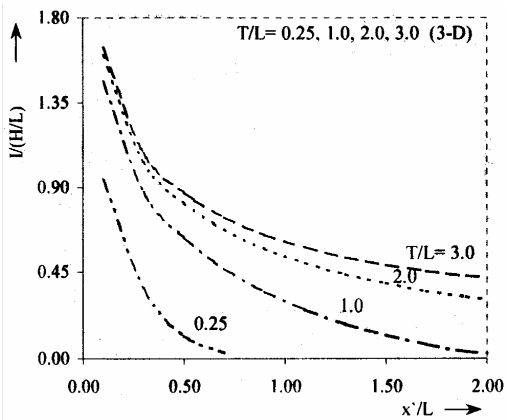
1. For the D/S part of the floor, the piezometric heads calculated by the 3-D model are always greater than those obtained from the 2-D model. As the relative width (B/L) of the floor increases differences gradually decrease and become negligible for $B/L \geq 2.0$.
2. Compared to the effect of the relative width (B/L), variation of the relative depth (T/L) and/or the relative width (S/L) of the pervious layer have less influence on the piezometric heads. Increasing (T/L) and (S/L) beyond 1.0 does not produce any change in the piezometric head values.
3. Exit gradients determined by the 2-D model are significantly smaller than 3-D values. The percentage error may reach (-100%). For B/L ≥



(a) At sec. 4-4 (bed surface).



(b) at sec. 5-5 (bed surface).



(c) At sec. 6-6 (side surface).

Fig. 10. Effect of relative depth of the pervious soil (T / L) on relative exit gradient [$I/(H/L)$](For B / L = 1 & S / L =3).

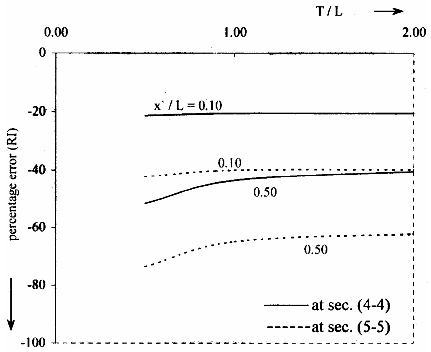
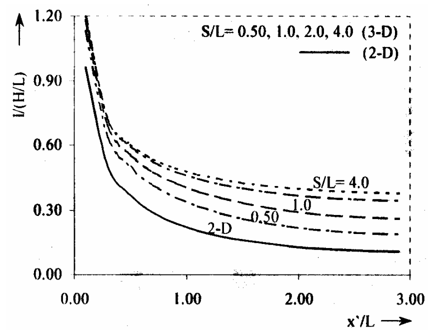
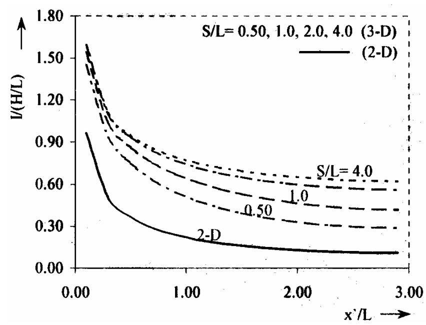


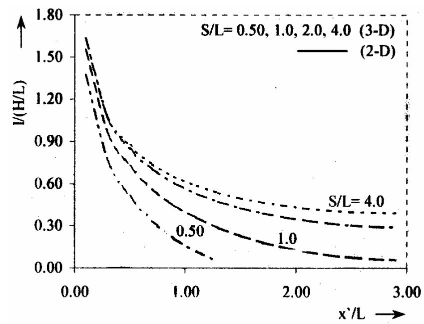
Fig. 11. Variation of percentage errors (Rh) of bed exit gradients with (T / L) (For B / L = 1 & S / L =3).



(a) At sec. 4-4 (bed surface).



(b) At sec. 5-5 (bed surface)



(c) At sec. 6-6 (side surface)

Fig. 12. Effect of relative width of the pervious soil (S / L) on relative exit gradient [$I/(H/L)$] (For B/L = 1 & T/L =3).

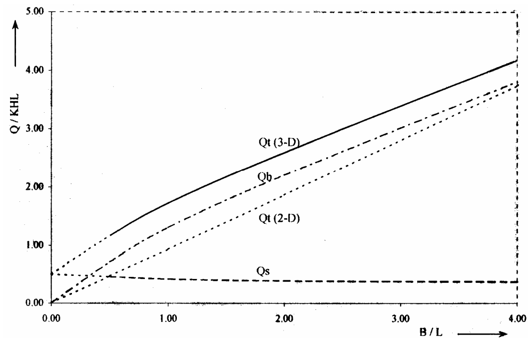


Fig. 13. Effect of relative width (B / L) on bed, side and total discharges (For S / L = 3 & T / L =3).

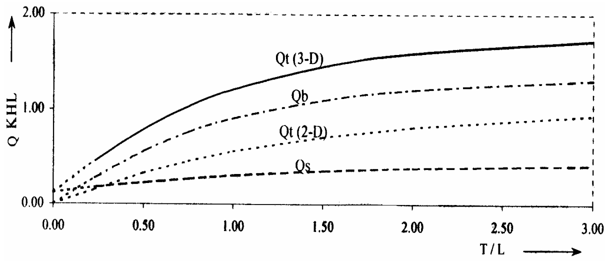


Fig. 14. Effect of relative depth of the pervious soil (T / L) on bed, side and total discharges (For B/L = 1 & S / L =3).

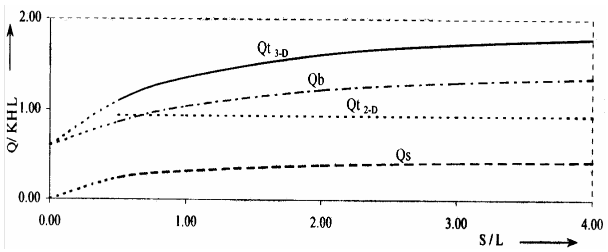


Fig. 15. Effect of relative width of the pervious soil (S / L) on bed, side and total discharges (For B/L = 1 & T / L =3).

2.0, results of the two models become very close.

4. Exit gradients at bed and side surfaces increase as the relative depth (T/L) and/or the relative width (S/L), of the pervious layer increase. For (T/L) and (S/L) > 1.5, no significant change in the exit gradients occurs.

5. The 3-D total seepage discharge is always greater than the 2-D one. The percentage error may reach 60 % for small values of (B/L).

6. Increasing the relative width (B/L), the relative depth (T/L) and/or the relative width (S/L) results in significant increase of the bed discharge (Qb) but has no appreciable effect on the side discharge (Qs).

It is, therefore, recommended that for seepage analysis for simple-floor, water-retaining structures of width to length ratio B/L < 2.0, results of the 3-D model presented herein should be considered. For structures with more complex floors, graphs presented in this study may be utilized for preliminary design.

Notations

The following symbols are used in the text:

b	Width of the canal bed, [L],
B	Width of the floor of the structure, [L],
h (x,y,z)	Piezometric head at any point (x,y,z) in the domain, [L],
H	Effective water head on the structure, [L],
I	Velocity gradient,
K	Hydraulic conductivity, [LT^{-1}],
L	Length of the floor of the structure, [L],
p	Water pressure, [FL^{-2}],
Qb	Bed seepage discharge, [L^3T^{-1}],
Qs	Side seepage, [L^3T^{-1}],
Qt_{2D}	Total discharge based on two dimensional model, [L^3T^{-1}],
Qt_{3D}	Total discharge based on three dimensional model, [L^3T^{-1}],
S	Width of the pervious soil, [L],
S_1	Horizontal penetration of the floor in the soil, [L],
t	Vertical penetration of the floor in the soil, [L],
T	Depth of the pervious soil, [L],
Z	Position head, [L], and
γ	Unit weight of water; [FL^{-3}].

Abbreviations

2-D	Two-dimensional model,
3-D	Three-dimensional model,
D/S	Down stream,
FEM	Finite element method,
G.W.T	Ground water table, and
U/S	Upstream.

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