Drainage of semi-pervious layer above an artesian aquifer by a combined system of pipe and mole drains

Helmi M. Hathoot
Dept. of Mathematics, College of Science, King Saudi University, P.O. Box 2455, Riyadh 11451, Saudi Arabia

The problem analyzed in this paper is that of draining a semi-pervious layer overlying an artesian aquifer of high piezometric pressure. A combined system of pipe and mole drains is proposed to lower the water table to the required height to make a convenient depth of soil free from ground water. The complex functions and the theory of images are used in establishing the main equations. Examination of the velocity component equations shows that boundary conditions in the flow field are satisfied. A computer program is presented to carry out the necessary calculations contained in the trial and error procedure used in pipe drain spacing design. A numerical example is provided to show the results of applying the design procedure to a different practical case. It is proved that application of the combined system of pipe and mole drains provides economical designs. It is also found that application of the combined system is specially needed in the case of a thin semi-pervious layer subject to high piezometric pressure where application of the traditional system of pipe drains alone is expensive and impractical.

في هذا البحث تمت معالجة مسألة صرف طبقة غير منفذة تعلو طبقة ارتوازية ذات ضاغط بيزومتري مرتفع. افترض المؤلف في هذا البحث استخدام أنابيب صرف مغطى تساعدها أنابيب محفورة وذلك بغرض خفض سطح المياه الأرضية بصورة تسمح بوجود سمك مناسب من التربة العلوية خال من المياه الجوفية. استخدمت في التحليل الرياضي الدوال المركبة و نظرية الصور في استنباط المعادلات الأساسية. لقد تم التحقق من استيفاء الشروط الحدية و ذلك باختبار معادلات مركبات السرعة. تم تقديم برنامج حاسوب و ذلك لإجراء الحسابات اللازمة – خاصة دورات التجربة و الخطأ– بهدف التوصل لتصميم المسافات بين المصارف. كذلك تم إعطاء مثال عددي لبيان نتائج تطبيق طريقة التصميم و ذلك باعتبار إحدى الحالات العملية. لقد وجد أن استخدام النظام الثنائي للمصارف العادية و المحفورة يؤدي إلى الحصول على تصميمات اكثر اقتصاداً. كما اتضح أن استخدام النظام الثنائي ذو أهمية خاصة في حالة صرف طبقة علوية قليلة السمك معرضة لضاغط بيزومتري مرتفع و ذلك لان استخدام النظام المعتاد لأنابيب الصرف مكلف و غير عملي.

Keywords: Pipe drains, Mole drains, Complex function

1. Introduction

In the last century pipe drainage has been introduced to millions of hectares of agricultural land all over the world. It has proved to be the ideal drainage system in most drainage conditions. Mole drainage is much cheaper than pipe drainage, however its application is limited to heavy and organic soils Schilfgaarde [1] and Balchyunass [2].

The problem of the design of subsurface drainage has been investigated by researchers on different lines of approach Schilfgaarde et al [3], Kikham [4], Schilfgaarde [1], and Youngs [5]. The problem of draining a semi pervious layer over an aquifer of high piezometric pressure was investigated by Hinesly [6], Luthin, [7], Najamii and Kirkham [8], Hathoot [9, 10], Wesseling and Wesseling [11], Bazaraa et al. [12] and Others. When the piezometric head in the aquifer is higher than the soil surface, fig. 1, water will move vertically upwards and may cause water logging or even ponding Abdel Dayem et al. [13]. To lower the water level to the required depth below soil surface a pipe drainage system is to be introduced with the proper spacing between pipes. However for the case of semi-pervious layer of small thickness overlying an aquifer of high piezometric head rational design formulas provide small pipe spacing. This is uneconomical and sometimes impractical. It is suggested that a combined system of pipe and mole drains is the proper solution for such a problem Hathoot [14].

2. Theory

Pipe drains can be represented by point sinks of strength m whereas mole drains by point sinks of strength m_1. To simulate the flow pattern, fictitious point sources are assumed to act on the other side of the upper surface of the aquifer as shown in fig. 2.

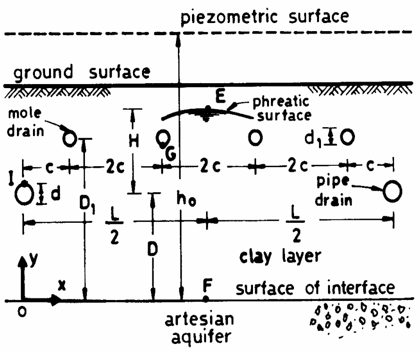

Fig. 1. Geological section.

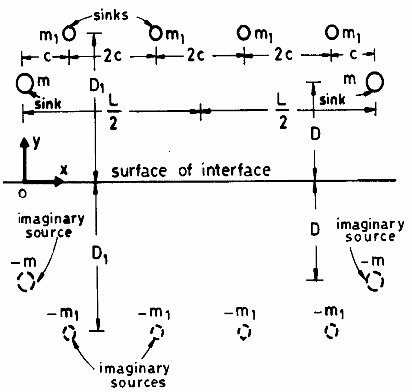

Fig. 2. Mathematical model.

The complex potential for the sinks representing pipe drains and the corresponding imaginary sources is given by Liggett [15]:

$$w_1 = mLn\, sin\, \frac{\pi(z - iD)}{L} - mLn\, sin \frac{\pi(z + iD)}{L} + C_1 , \qquad (1)$$

in which m is the strength of a point sink representing a pipe drain, D is the height of pipe drains above an artesian aquifer, L is the pipe drain spacing, $z = x + iy$, $i = \sqrt{-1}$ and C_1 = real constant.

The complex potential for the sinks representing mole drains and the corresponding imaginary sources is given by:

$$w_2 = m_1\, Ln\, sin\, \frac{\pi(z - c - iD_1)}{2c} - m_1\, Ln\, sin \frac{\pi(z - c + iD_1)}{2c} + C_2 , \qquad (2)$$

in which m_1 is the strength of a point sink representing a mole drain, $2c$ is the spacing between mole drains, D_1 is the height of mole drains above the artesian aquifer, and C_2 is a real constant.

The complex potential of the system is obtained by simply adding the two complex potentials:

$$w = m \quad Ln\, sin\, \frac{\pi(z - iD)}{L} - m \quad Ln\, sin \frac{\pi(z + iD)}{L} + m_1\, Ln\, sin \frac{(z - c - iD_1)}{2c} - m_1\, Ln\, sin\, \frac{\pi(z - c + iD_1)}{2c} + C , \qquad (3)$$

in which C is a real constant. Substituting $z = x + iy$, simplifying and rearranging produces:

$$w = m \left\{ Ln \left[sin \frac{\pi x}{L} cosh \frac{\pi}{L}(y - D) + i\, cos \frac{\pi x}{L} sinh \frac{\pi}{L}(y - D) \right] - Ln \left[sin \frac{\pi x}{L} cosh \frac{\pi}{L}(y + D) + i\, cos \frac{\pi x}{L} sinh \frac{\pi}{L}(y + D) \right] \right\}$$
$$+ m_1 \left\{ Ln \left[sin \frac{\pi(x - c)}{2c} cosh \frac{\pi(y - D_1)}{2c} + i\, cos \frac{\pi(x - c)}{2c} sinh \frac{\pi(y - D_1)}{2c} \right] - Ln \left[sin \frac{\pi(x - c)}{2c} cosh \frac{\pi(y + D_1)}{2c} + i\, cos \frac{\pi(x - c)}{2c} sinh \frac{\pi(y + D_1)}{2c} \right] \right\} + C . \qquad (4)$$

Setting $w = \phi + i\,\psi$, in which ϕ is the velocity potential and ψ is the stream function and rearranging yields:

$$\phi + i\psi = \frac{m}{2} Ln\left[sin^2 \frac{\pi x}{L} + sinh^2 \frac{\pi(y-D)}{L} \right]$$
$$+ i\, m tan^{-1}\left[cot \frac{\pi x}{L} tanh \frac{\pi(y-D)}{L} \right]$$
$$- \frac{m}{2} Ln\left[sin^2 \frac{\pi x}{L} + sinh^2 \frac{\pi(y+D)}{L} \right]$$
$$- i\, m tan^{-1}\left[cot \frac{\pi x}{L} tanh \frac{\pi(y+D)}{L} \right]$$
$$- \frac{m_1}{2} Ln\left[sin^2 \frac{\pi(x-c)}{2c} + sinh^2 \frac{\pi(y-D_1)}{2c} \right]$$
$$+ i\, m_1 tan^{-1}\left[cot \frac{\pi(x-c)}{2c} tanh \frac{\pi(y-D_1)}{2c} \right]$$
$$- \frac{m_1}{2} Ln\left[sin^2 \frac{\pi(x-c)}{2c} + sinh^2 \frac{\pi(y+D_1)}{2c} \right]$$
$$- i\, m_1 tan^{-1}\left[cot \frac{\pi(x-c)}{2c} tanh \frac{\pi(y+D_1)}{2c} \right] + C. \quad (5)$$

Equating real to real and imaginary to imaginary on both sides of eq. (5) and rearranging:

$$\phi = \frac{m}{2} Ln\left[\frac{sin^2 \frac{\pi x}{L} + sinh^2 \frac{\pi(y-D)}{L}}{sin^2 \frac{\pi x}{L} + sinh^2 \frac{\pi(y+D)}{L}} \right] + \frac{m_1}{2} Ln\left[\frac{sin^2 \frac{\pi(x-c)}{2c} + sinh^2 \frac{\pi(y-D_1)}{2c}}{sin^2 \frac{\pi(x-c)}{2c} + sinh^2 \frac{\pi(y+D_1)}{2c}} \right] + C. \quad (6)$$

and

$$\Psi = m\left\{ tan^{-1}\left[cot \frac{\pi x}{L} tanh \frac{\pi(y-D)}{L} \right] - tan^{-1}\left[cot \frac{\pi x}{L} tanh \frac{\pi(y+D)}{L} \right] \right\}$$
$$+ m_1\left\{ tan^{-1}\left[cot \frac{\pi(x-c)}{2c} tanh \frac{\pi(y-D_1)}{2c} \right] - tan^{-1}\left[cot \frac{\pi(x-c)}{2c} tanh \frac{\pi(y+D_1)}{2c} \right] \right\}. \quad (7)$$

It is evident that the pipe drain spacing, L, and spacing between mole drains, $2c$, are interrelated by:

$$2c = \frac{L}{n}, \quad (8)$$

in which n is the number of mole drains installed between two pipe drains.

Substituting eq. (8) into eqs. (6) and (7) yields:

$$\phi = \frac{m}{2} Ln\left[\frac{sin^2 \frac{\pi x}{L} + sinh^2 \frac{\pi(y-D)}{L}}{sin^2 \frac{\pi x}{L} + sinh^2 \frac{\pi(y+D)}{L}} \right] + \frac{m_1}{2} Ln\left[\frac{sin^2 \frac{\pi n}{L}\left(x - \frac{L}{2n}\right) + sinh^2 \frac{\pi n}{L}(y-D_1)}{sin^2 \frac{\pi n}{L}\left(x - \frac{L}{2n}\right) + sinh^2 \frac{\pi n}{L}(y+D_1)} \right] + C. \quad (9)$$

and

$$\psi = m\left\{ tan^{-1}\left[cot \frac{\pi x}{L} tanh \frac{\pi(y-D)}{L} \right] - tan^{-1}\left[cot \frac{\pi x}{L} tanh \frac{\pi(y+D)}{L} \right] \right\}$$
$$+ m_1\left\{ tan^{-1}\left[cot \frac{\pi n}{L}\left(x - \frac{L}{2n}\right) tanh \frac{\pi n}{L}(y-D_1) \right] - tan^{-1}\left[cot \frac{\pi n}{L}\left(x - \frac{L}{2n}\right) tanh \frac{\pi n}{L}(y+D_1) \right] \right\}. \quad (10)$$

3. Boundary conditions

The velocity components at a point (x,y) in the semi pervious layer are given by Liggett [15] as,

$$u = -\frac{\partial \phi}{\partial x}, \quad (11)$$

and

$$v = -\frac{\partial \phi}{\partial y}, \quad (12)$$

in which u is the horizontal velocity component and v is the vertical velocity component at the point. Differentiating (9) partially with respect to x and rearranging yields:

$$u=-\frac{m\pi}{2L}\left\{\frac{\left(sin\frac{2\pi x}{L}\right).\left[sinh^2\frac{\pi(y+D)}{L}-sinh^2\frac{\pi(y-D)}{L}\right]}{\left[sin^2\frac{\pi x}{L}+sinh^2\frac{\pi(y-D)}{L}\right].\left[sin^2\frac{\pi x}{L}+sinh^2\frac{\pi(y+D)}{L}\right].}\right\}$$
$$-\frac{m_1\pi n}{2L}\left\{\frac{\left[sin\frac{2\pi n}{L}\left(x-\frac{L}{2n}\right)\right].\left[sinh^2\frac{\pi n}{L}(y+D_1)-sinh^2\frac{\pi n}{L}(y-D_1)\right]}{\left[sin^2\frac{\pi n}{L}\left(x-\frac{L}{2n}\right)+sinh^2\frac{\pi n}{L}(y-D_1)\right].\left[sin^2\frac{\pi n}{L}\left(x-\frac{L}{2n}\right)+sinh^2\frac{\pi n}{L}(y+D_1)\right]}\right\}. \quad (13)$$

Differentiating eq. (9) partially with respect to y and rearranging yields:

$$v=-\frac{m\pi}{2L}\left\{\frac{sinh\frac{2\pi(y-D)}{L}\left[sin^2\frac{\pi x}{L}+sinh^2\frac{\pi(y+D)}{L}\right]-sinh\frac{2\pi(y+D)}{L}\left[sin^2\frac{\pi x}{L}+sinh^2\frac{\pi(y-D)}{L}\right]}{\left[sin^2\frac{\pi x}{L}+sinh^2\frac{\pi(y-D)}{L}\right].\left[sin^2\frac{\pi x}{L}+sinh^2\frac{\pi(y+D)}{L}\right]}\right\}$$
$$-\frac{m_1\pi n}{2L}\left\{\frac{sinh\frac{2\pi n}{L}(y-D_1)\left[sin^2\frac{\pi n}{L}\left(x-\frac{L}{2n}\right)+sinh^2\frac{\pi n}{L}(y+D_1)\right]-sinh\frac{2\pi n}{L}(y+D_1)\left[sin^2\frac{\pi n}{L}\left(x-\frac{L}{2n}\right)+sinh^2\frac{\pi n}{L}(y-D_1)\right]}{\left[sin^2\frac{\pi n}{L}\left(x-\frac{L}{2n}\right)+sinh^2\frac{\pi n}{L}(y-D_1)\right].\left[sin^2\frac{\pi n}{L}\left(x-\frac{L}{2n}\right)+sinh^2\frac{\pi n}{L}(y+D_1)\right]}\right\}. \quad (14)$$

It is evident from eq. (13) that the horizontal velocity component, u, vanishes at,

$$x=\pm s\frac{L}{2}, \quad (15)$$

in which

$$s = 0, 1, 2, \ldots \quad (16)$$

This satisfies the boundary condition that the vertical lines passing through pipe drains and midway between them are lines of symmetry along which velocities are purely vertical. In addition eq. (13) shows that the horizontal velocity component, u, is zero at $y = 0$. This satisfies another boundary condition that stream-lines intersect the upper surface of the aquifer at right angles.

4. Spacing design formula

The velocity potential, ϕ, at a point is dependent on some soil properties, the pressure and the elevation of the point above a datum, Harr [16] and Polubarinova-Kochina [17], and is given by,

$$\phi=K\left(\frac{p}{\rho g}+y\right), \quad (17)$$

in which K is the hydraulic conductivity of soil, p is the gauge pressure at the point, ρ is the density of water, g is the acceleration due to gravity, and y is the height of point above the artesian aquifer. Combination of eqs. (9) and (17) yields:

$$K\left(\frac{p}{\rho g}+y\right)=\frac{m}{2}Ln\left[\frac{sin^2\frac{\pi x}{L}+sinh^2\frac{\pi(y-D)}{L}}{sin^2\frac{\pi x}{L}+sinh^2\frac{\pi(y+D)}{L}}\right]$$
$$+\frac{m_1}{2}Ln\left[\frac{sin^2\frac{\pi n}{L}\left(x-\frac{L}{2n}\right)+sinh^2\frac{\pi n}{L}(y-D_1)}{sin^2\frac{\pi n}{L}\left(x-\frac{L}{2n}\right)+sinh^2\frac{\pi n}{L}(y+D_1)}\right]+C. \quad (18)$$

At point F ($\frac{L}{2}$, 0), fig. 1, the pressure head is h_o. Applying eq. (18) to point F and simplifying:

$$C = K\, h_o, \tag{19}$$

A pipe drain is usually running partially full and hence the pressure at the top point of drain is atmospheric Hathoot [18] and Hathoot and Rezk [19].

Applying eq. (18) to point I(0,D+d/2), fig. 1, noting that $C = K\, h_o$ and simplifying:

$$K\left(D+\frac{d}{2}\right) = mLn\left[\frac{\sinh\frac{\pi d}{2L}}{\sinh\frac{\pi\left(2D+\frac{d}{2}\right)}{L}}\right] + m_1 Ln\left[\frac{\cosh\frac{\pi n}{L}\left(D+\frac{d}{2}-D_1\right)}{\cosh\frac{\pi n}{L}\left(D+\frac{d}{2}+D_1\right)}\right] + Kh_o, \tag{20}$$

in which d is the pipe drain diameter.

At the bottom of a mole drain if the water depth is neglected the pressure may be considered atmospheric. Applying eq. (18) to point G (c, D_1 - $d_1/2$), fig. 1, considering eq. (19) and simplifying:

$$K\left(D_1-\frac{d_1}{2}\right) = \frac{m}{2}Ln\left[\frac{\cos^2\frac{\pi}{2n}+\sinh^2\frac{\pi\left(D_1-\frac{d_1}{2}-D\right)}{L}}{\cos^2\frac{\pi}{2n}+\sinh^2\frac{\pi\left(D_1-\frac{d_1}{2}+D\right)}{L}}\right] + m_1 Ln\left[\frac{\sinh\frac{\pi n d_1}{2L}}{\sinh\frac{\pi n}{L}\left(2D_1-\frac{d_1}{2}\right)}\right] + Kh_o, \tag{21}$$

in which d_1 is the mole drain diameter.

Solving eqs. (20) and (21), simultaneously for m and m_1:

$$m = \frac{K\left[\frac{\left(D+\frac{d}{2}\right)}{B_1} - \frac{\left(D_1-\frac{d_1}{2}\right)}{B_2} - A_1 h_o\right]}{A_2 - A_3}, \tag{22}$$

and

$$m_1 = K\left\{\frac{\left(D+\frac{d}{2}\right)}{B_1} - \frac{A_2\left[\frac{\left(D+\frac{d}{2}\right)}{B_1} - \frac{\left(D_1-\frac{d_1}{2}\right)}{B_2} - A_1 h_o\right]}{A_2 - A_3} - \frac{h_o}{B_1}\right\}, \tag{23}$$

in which

$$B_1 = Ln\left[\frac{\cosh\frac{\pi n}{L}\left(D+\frac{d}{2}-D_1\right)}{\cosh\frac{\pi n}{L}\left(D+\frac{d}{2}+D_1\right)}\right], \tag{24}$$

$$B_2 = Ln\left[\frac{\sinh\frac{\pi n d_1}{2L}}{\sinh\frac{\pi n}{L}\left(2D_1+\frac{d_1}{2}\right)}\right], \tag{25}$$

$$A_1 = \frac{1}{B_1} - \frac{1}{B_2}, \tag{26}$$

$$A_2 = \frac{Ln\left[\frac{\sinh\frac{\pi d}{2L}}{\sinh\frac{\pi\left(2D+\frac{d}{2}\right)}{L}}\right]}{B_1}, \tag{27}$$

and

$$A_3 = \frac{Ln\left[\dfrac{\cos^2\dfrac{\pi}{2n} + \sinh^2\dfrac{\pi\left(D_1 - \dfrac{d_1}{2} - D\right)}{L}}{\cos^2\dfrac{\pi}{2n} + \sinh^2\dfrac{\pi\left(D_1 - \dfrac{d_1}{2} + D\right)}{L}}\right]}{2B_2}. \quad (28)$$

It should be remembered that the function of the combined system of drains is to lower the water table from h_o to $D+H$ above the upper surface of the artesian aquifer. At the phreatic surface the pressure is atmospheric, applying eq. (18) to point E ($L/2$, $D+H$), fig. 1, and simplifying:

$$K(D+H) = mLn\left[\frac{\cosh\dfrac{\pi H}{L}}{\cosh\dfrac{\pi(2D+H)}{L}}\right] + \frac{m_1}{2}Ln\left[\frac{\cos^2\dfrac{\pi n}{2} + \sinh^2\dfrac{\pi n}{L}(D+H-D_1)}{\cos^2\dfrac{\pi n}{2} + \sinh^2\dfrac{\pi n}{L}(D+H+D_1)}\right] + Kh_o. \quad (29)$$

For convenience eq. (29) is written as:

$$K\,(D + H) = m\,H_1 + \frac{m_1}{2}\,H_2 + K\,h_o\,, \quad (30)$$

in which

$$H_1 = Ln\left[\frac{\cosh\dfrac{\pi H}{L}}{\cosh\dfrac{\pi(2D+H)}{L}}\right], \quad (31)$$

and

$$H_2 = Ln\left[\frac{\cos^2\dfrac{\pi n}{2} + \sinh^2\dfrac{\pi n}{L}(D+H-D_1)}{\cos^2\dfrac{\pi n}{2} + \sinh^2\dfrac{\pi n}{L}(D+H+D_1)}\right]. \quad (32)$$

Eq. (30) is the design equation for the spacing L between pipe drains with n mole drains installed between them. Substitution of the proper values of L and n in the right hand side of eq. (30) should yield equal values on both sides of this equation. It is obvious that L appears implicitly in eq. (30) and hence L is to be estimated through a trial and error procedure.

5. Computer program

Eq. (30) contains the variables m and m_1, each of which depends upon B_1, B_2, A_1,..., etc. and hence each trial cycle needs several calculations. For that reason a computer program is designed for estimating the proper spacing, L, taking into account the behaviour of eq. (30). For convenience eq. (30) is put in the form:

$$m = N, \quad (33)$$

in which,

$$N = \frac{1}{H_1}\left[K(D+H) - \frac{m_1}{2}H_2 - Kh_o\right]. \quad (34)$$

The required spacing, L, is that for which eq. (33) is satisfied. A detailed flow-chart of the suggested computer program is shown in fig. 3. The quantities D, D_1, d, d_1, n, K, h_o, and H are input of the program. The following steps describe this program.

1. A first trial value of the spacing, L_1, is reasonably assumed (tens of meters).
2. The corresponding N_1 is estimated by application of eq. (22) through eq. (34). If $m = N_1$, the assumed spacing, L_1 is the required design spacing, otherwise a second trial value of the spacing, L_2, is suggested. A second trial value L_2 may be estimated from:

$$L_2 \cong L_1\frac{m}{N_1}. \quad (35)$$

3. The value of N_2 corresponding to L_2 is then estimated and compared with the last m value and if they are not sufficiently close to each other, a third trial value, L_3, can be estimated by using linear interpolation/extrapolation through the application of the following equation.

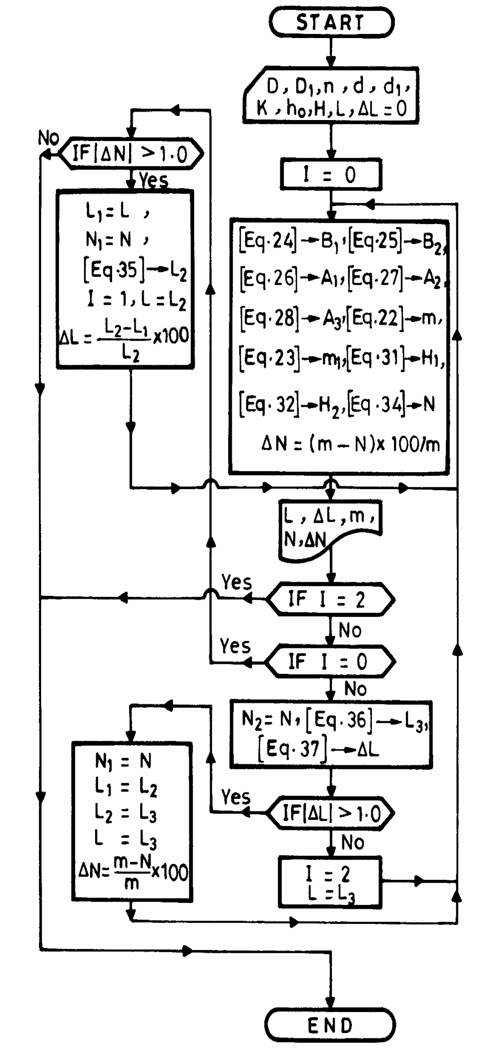

Fig. 3. Flow chart for the computer program.

$$L_3 = L_1 - \left[\frac{N_1 - m}{N_1 - N_2} \left(L_1 - L_2 \right) \right] \quad . \tag{36}$$

4. The value of N_3 corresponding to L_3 is estimated and the percentage difference ΔL given by the following equation is estimated.

$$\Delta L = \frac{L_3 - L_2}{L_3} \times 100 \, . \tag{37}$$

5. If ΔL is practically small (in the order of ± 1%) L_3 is the design spacing, otherwise linear interpolation/extrapolation is used to get a new spacing considering the last estimated value of m and the last two estimated values of both L and N.

6. Application of the trial and error procedure continues till the required design spacing L is reached.

It should be noted that in computing some variables containing sinh and/or cosh terms double precision should be used since these quantities contain very large and very small numbers, therefore they are sensitive to round-off error. In the following example drainage of a thin semi-pervious layer subject to high piezometric pressure is considered.

6. Numerical example 1

A semi-pervious layer, K = 0.09 m/day overlies an artesian aquifer 2.3m below ground surface. The piezometric head of the aquifer is h_o = 3.3m. It is required to design a combined system of pipe and mole drains to maintain the top 0.3 m of the soil free from ground water. Pipe drains are 0.1 m diameter tubes installed 1.8 m below ground surface. Mole drains are 0.076 m diameter holes formed at a depth of 0.6 m.

7. Solution

In this example it is evident that D = 0.5 m, D_1 = 1.7 m, H = 1.5 m, d = 0.1 m and d_1 = 0.076 m. It is assumed that n = 10 mole drains between two pipe drains. A first trial spacing 10.0 m is assumed and the successive trial cycles are listed in table 1.

The practical tube spacing in this example is L = 14.00 m and the spacing between moles is $2c$ = 1.4 m. It is worthy to note that if only drain pipes are used, applying rational spacing formulas Hathoot [9, 10] the resulting spacing is L = 2.0 m. This spacing is neither economical nor practical. It is evident that the combined system yields a pipe spacing seven times that of the single system.

8. Conclusions

The equations presented in this paper are found to satisfy boundary conditions of the flow pattern. The combination of the cheapest type of subsurface drains (mole drains) with

Table 1
Results of the trial cycles of example 1

L(m) (1)	ΔL (%) (2)	m (m²/day) (3)	N (m²/day) (4)	ΔN (%) (5)
10.0000	0.0000	0.06792	-0.01994	129.3581
34.0581	70.6384	0.07359	2.90113	-3842.2884
10.7704	-216.2194	0.06806	-0.01545	122.7006
11.4372	5.8300	0.06819	-0.00871	112.7731
19.0524	39.9698	0.07003	0.32149	-359.0747
13.2533	-43.7559	0.06860	0.02489	63.7172
14.1076	6.0556	0.06879	0.04934	28.2745
14.7874	4.5972	0.06896	0.07312	-6.0325
14.6683	-0.8120	0.06893	0.06867	0.3772

traditional pipe drains provides economical designs. In the case of thin semi-pervious soil subject to high piezometric pressure using pipe drains alone yields uneconomical and sometimes impractical spacing between pipes. A numerical example shows that using the combined system in this case has the effect of increasing the pipe spacing seven times.

Notations

The following symbols are used in this paper:

A_1 quantity defined by eq. (26),
A_2 quantity defined by eq. (27),
A_3 quantity defined by eq. (28),
B_1 quantity defined by eq. (24),
B_2 quantity defined by eq. (25),
c half spacing between mole drains,
$C_1, C_2, \ldots$ real constants,
D height of pipe drains above the artesian aquifer,
d pipe drain diameter,
D_1 height of mole drains above the artesian aquifer,
d_1 mole drain diameter,
g acceleration due to gravity,
H height of water table above pipe drains midway between two pipe drains,
H_1 quantity defined by eq. (31),
H_2 quantity defined by eq. (32),
h_o piezometric head of the artesian aquifer,
i $\sqrt{-1}$,
K hydraulic conductivity of the top semi-pervious layer,
L spacing between pipe drains,
$L_1, L_2, \ldots$ successive trial values of the spacing between pipe drains in the computer program,
ΔL quantity defined by eq. (37),
m strength of point sink representing pipe drain,
m_1 strength of point sink representing mole drain,
N quantity defined by eq. (34),
$N_1, N_2, \ldots$ successive trial N values,
N number of mole drains installed between pipe drains,
p pressure,
s 0, 1, 2, ... ,
u horizontal velocity component,
v vertical velocity component,
w complex potential $(= \phi + i\,\psi)$,
x horizontal coordinate of a point,
y vertical coordinate of a point,
z complex coordinate $(= x + i\,y)$,
ρ density of water,
ϕ velocity potential, and
ψ stream function.

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