An experimental study on finite amplitude oscillations in ducts: the effect of adding variable area part to the open end of constant-area resonant tube

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The influence of adding variable area portions to the open end of a constant-area tube on finite amplitude wave deformation and radiation, under resonant conditions, is studied experimentally. Three different variable area portions are considered; namely: convergent, divergent and convergent-divergent area portions. The pressure histories are recorded using pressure transducers connected with a computer-built in data acquisition card through Lab-View software. Results of pressure histories recorded at different locations along the constant area part show that the geometry of variable area portion has a significant effect on wave evolution regarding the wave amplitude, waveform, reflected and radiated wave energy. The power spectrum of the measured data confirmed the aforementioned results. Higher harmonics are noticed when the wave steepness is significant. The noticed differences between the results of open end tube and that when tube is provided with variable area portions are presumably attributed to the deviation of the natural frequency due to adding variable area portions at tube open end.

تم في هذا البحث إجراء دراسة معملية لإيجاد تأثير إضافة جزء ذومساحة مقطع متناقص أو اتساعي أو متناقص–متزايد إلي النهاية المفتوحة لأنبوبة رنين ذات مساحة مقطع ثابت على تغير شكل وانبعاث الموجات الناشئة في الأنبوبة من مكبس يتردد في النهاية الأخرى للأنبوبة بقيمة مساوية للتردد الطبيعي للهواء في الأنبوبة المفتوحة ذات مساحة المقطع الثابت(حالة ال Resonance). كان تردد المكبس ٨٠هرتز وتم حساب طول أنبوبة الرنين فكان طولها ١٠٨ سم وأختير قطرها ٥,٠٨سم و كان طول مشوار المكبس ٥,٨ سم. تم قياس تغير الضغط عند نقطة قرب نهاية أنبوبة الرنين وأخرى قرب المكبس باستخدامPressure Transducers موصلة بكارت تجميع بيانات بجهاز كمبيوتروباستخدام برنامجLab-View تم تسجيل القراءات. أظهرت النتائج أنه فى حالة أنبوبة الرنين فقط تتزايد سعة موجة الضغط مع الزمن بصورة خطية تقريبا حتى تصل لقيمة ثابتة تقريبا تأخذ شكل Waveform . تتناقص سعة الموجة كثيرا قرب النهاية المفتوحة للأنبوبة ويتغيرالضغط بقيم أقل من الضغط الجوى. يتغير شكل الموجة كلما ابتعدنا عن المكبس ليكون التغير أكثر حدة أثناء الانضغاط وأكثر انبساطا أثناء التمدد. عند اضافة جزء ذو مساحة مقطع متناقص تقل سعة موجة الضغط عن مثيلتها في الحالة السابقة و يزداد التناقص بتناقص مساحة مقطع المخرج ولكن قرب النهاية المفتوحة للأنبوبة يتغيرضغط الموجة حول الضغط الجوى. عند اضافة جزء ذو مساحة مقطع اتساعي تزداد سعة موجة الضغط عن مثيلتها في الحالة الأولي وتزداد سعتها بتناقص مساحة مقطع المخرج. يتغيرضغط الموجة قرب النهاية المفتوحة للأنبوبة بقيم أقل من الضغط الجوى وتزداد حدة التغير في شكل الموجة فتأخذ شكل N-wave. عند اضافة جزء ذو مساحة مقطع متناقص–متزايد لوحظ تزايد سعة الموجة إلى قيمة عظمى يليها تناقص تدريجي حتى تصل إلى شكل Waveformولكن سعة الموجة عامة أقل عن مثيلتها في الحالة الأولي خاصة قرب المكبس. تتناقص سعة الموجه قرب المكبس بتناقص مساحة مقطع العنق أما قرب نهاية الأنبوبة فالتناقص محدود. أوضحت حسابات Power spectrum density ظهور قيم أعلي للتردد عن التردد الطبيعي خاصة قرب نهاية الأنبوبة وذلك في حالات التغير الحاد في شكل الموجة. ومعنى ذلك أن هناك ارتباط بين ظهور الترددات العالية والتغير الحاد للموجه. أوضحت الحسابات أيضا تناقص طاقة الموجه كلما ابتعدنا عن مصدر الذبذبات.

Keywords: Open end resonant tube, Finite amplitude waves, Shock waves, Area variation

1. Introduction

In the present paper an experimental study is performed for finite amplitude oscillations in open constant area duct and constant area duct provided with variable area portions; namely convergent, divergent or convergent-divergent sections under resonant condition. The acoustic wave is excited by an oscillating piston whose frequency equals the natural frequency of the open end constant area duct. The present work is inspired by

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problems of acoustic instability in solid rocket motors and combustion chambers, the noise pollution resulting from the exhaust system of internal combustion engines and noise associated with jet flow.

In the acoustic linear theory, the amplitude of the generated acoustic waves in ducts under resonant condition is infinite. When viscous and thermal effects are incorporated, the resulting amplitude becomes finite because of the energy dissipation in the boundary layer and in the wave itself. For larger amplitude excitation, nonlinear effects have to be included.

The acoustic oscillations in closed end tubes have been studied intensively experimentally, theoretically and numerically, [1-6]. The shock wave formation was found to be inevitable at or near resonance frequencies. In open end ducts under resonance frequency, the nonlinear effects cause shock wave formation during the traverse of compression wave from the oscillating source to the duct open end and it disappears when the wave is reflected as expansion wave from the tube open end, Wang and Kassoy [4] and Sileem [5].

Zapirov and Ilhamov [1] studied experimentally and theoretically the gas oscillations in a closed duct under resonant situation. Their results show that when the piston oscillates in a frequency below the natural frequency, the gas oscillates in accordance with the sine law. When the piston frequency approaches the natural frequency of the gas column, the pressure wave takes an N-type shape.

Erickson et al. [2] studied theoretically the behavior of forced finite amplitude oscillations in cylindrical constant and variable area ducts whose ends are closed. The objective of their study was to determine the effect of duct shape on the resulting oscillation amplitude, the wave waveform, the higher harmonics energy content and to identify duct shapes that produce large amplitude oscillations for a given energy input. Galarkin method is used to find the solution of the governing equations. Shock waves are observed in the form of saw-tooth like waveforms in the case of constant area duct when the duct oscillates at resonant frequency. This leads to energy dissipation due to shock wave formation. In this case energy is transferred from the fundamental mode to its higher harmonic oscillations due to nonlinearities resulting in shock wave formation and decrease in wave amplitude. Here, the natural resonant frequencies of the system are integer multiples of the fundamental mode. For horn-type ducts the natural resonant frequencies of the system are not integer multiples of the fundamental mode. Consequently, energy transfer from the driven fundamental mode to its harmonics is not as efficient as in straight ducts. The input energy is concentrated in the fundamental mode resulting in high wave amplitude.

Merkli and Thomann [3] studied the thermoacoustic effects in resonant closed tube. Their experimental results near or at resonant conditions showed the steepness of the wave and shock wave formation. They referred also to the possibility of temperature variation along the closed tube.

Wang and Kassoy [4] studied and discussed using perturbation methods the nonlinear oscillations in a resonant gas column. They studied both closed and ideally open (isobaric exit) tubes. Isentropic relations are used to relate the thermodynamic variables. For closed tube and after a period of linear wave amplitude growth, shock wave appears with small limiting amplitude as a result of nonlinearity. They attributed the wave limiting amplitude to the energy dissipation in the shock wave. For ideally open end, they showed that shock wave appears intermittently; i.e. appears and disappears. They attributed that to the effect of rarefaction waves from the open end.

Sileem [5] studied nonlinear evolution of wave in ducts ended with variable area portions (convergent and divergent) assuming isobaric exit under resonance conditions. Unlike the isentropic relations assumed by Wang and Kassoy, he solved continuity, momentum and energy equations numerically in one-dimensional domain. For ducts ended with convergent part of small area ratio, shock wave appears and sustains. In constant area ideally open duct a decrease in wave amplitude in the limit cycle is noticed. For ducts ended with divergent area, the wave amplitude becomes very small because of wave propagation through increasing area portion.

Seymour and Mortell [7], studied the acoustic wave damping under resonance conditions. They considered the case of gas oscillations in a tube driven by a piston oscillating at one end while the other end is open, partially open or closed. For open end tubes, they attributed the wave damping inside the tube to the energy radiation into the adjacent medium. In case of closed end tubes no energy radiates out side, so shock wave is formed accompanied by energy dissipation. In case of partially open end tubes, part of acoustic energy is radiated which prevents shock wave formation when the exit to tube area ratio is relatively high.

Jimenez [8] concluded from his theoretical results for case of open end duct under resonant condition, that shock wave appears during compression and then it is cancelled due to the opposite expansion wave. However, shock wave appears and sustains in case of closed duct.

Disselhorst and Wijngarden [9] studied experimentally and theoretically the effects of open tube edge on wave amplitude and acoustic energy. They referred to their previous study on acoustic waves in open end tubes with sharp edge. In that work, they stated that jet is formed at the open end during out flow and boundary-layer separation takes place during inflow associated with vortex formation in the pipe near the open end. This causes energy dissipation during inflow. Vortex shedding occurs during the out flow. They demonstrated their concept by the sketch shown in fig. 1.

For open rounded edge tube, they showed the disappearance of vortex and boundary layer-separation. They stated also that, for open end tube of rounded edge during out flow, jet formation occurs and the pressure approximately equals the ambient pressure.

$$p = p_o \quad . \tag{1}$$

During the inflow no separation takes place and Bernoulli's theorem gives the following exit condition;

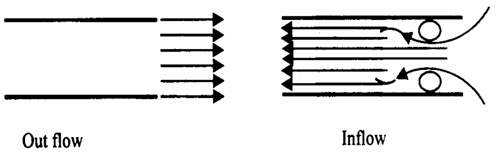

Fig. 1. Out and inflow boundary condition [9].

$$p - p_o = -\frac{1}{2}\rho_o u^2 \, . \tag{2}$$

For open end tubes of sharp edges, the exit condition during out flow is the same as given in eq. (1). During the inflow the exit condition is:

$$p - p_o = -\rho_o u^2 \, , \tag{3}$$

where p_o and ρ_o are the ambient pressure and density while, p an u are the pressure and velocity at tube exit. In their study about longitudinal acoustic instability in solid rocket motors Garcia and Linan [11] stated that when the nozzle throat area is small compared to the tube area, the attenuation of acoustic oscillations that may be exited will be weak because the tube is nearly closed.

From the previous literature review it is found that experimental works in cases of open end constant area ducts and ducts ended with open variable area portions under resonance frequency are very few, even though such cases are important in a variety of engineering applications. The acoustic instability in rocket motor chambers, the exhaust system of internal combustion engines and noise generation and propagation from air jets are just few examples.

The important engineering applications cited above and the need to understand the wave evolutions more deeply were the motivations of the present work, particularly under resonant conditions which represent the worst case. In addition, the case of constant area duct provided with open end variable area portion at its end is not previously addressed.

In the present work an experimental study on finite amplitude oscillations in constant area open ducts and constant area ducts provided with variable area portions at its open end are conducted. The acoustic energy is provided by an oscillating piston at one end of the duct, while the other end is open to the atmosphere. Three variable area portions are considered, namely: convergent, divergent and convergent-divergent portions. The piston oscillates with a frequency equal to the natural frequency of the open end duct. We focus here on the effect of geometry at the duct end on the wave amplitude evolution inside the duct and also the waveform amplitude and frequency in the limit cycle.

2. Experimental set up

The considered variable area test section parts are shown schematically in fig. 2, while the experimental setup layout is shown in fig. 3. It consists of the following parts: an oscillating piston (1) {motor cycle piston-cylinder system of 5.08 cm diameter and 5.8 cm stroke} driven by an electric motor (2) of 3 kW through a belt (3) and pulley (4). The piston oscillates at one end of a tube (5) of diameter D=5.08 cm and its length is 108 cm. Convergent, divergent or convergent-divergent test section parts are added to the open end of the tube. The detailed dimensions of the considered test sections are given in table 1. Pressure histories are recorded at two locations along the tube, the lower one is 13 cm apart from top dead center and the upper one is 5.5 cm apart from the constant area tube end are measured using capacitive pressure transducers (6) {Model SA, Data Instruments, Action MA01720, USA 0-100PSIS, accuracy ± 1%} connected directly to computer (7) through a lab view soft ware. The data are recorded in files and plotted as will be shown later. The uncertainties of the measured pressure shown herein are estimated according to the procedures given by Hugh and Glenn [13] and found to be within ± 2.5% for a confidence interval of 95.45%.

3. Results and discussions

Before considering the results, one should recognize that according to the linear theory, resonance condition for a constant area open end tube is obtained as follows: the front of the compression wave delivered by the oscilating piston at the beginning of compression stroke will be reflected from the open end as a rarefaction wave. The latter wave will arrive back to the piston at the beginning of the piston backward motion. This rarefaction wave will be reflected from the piston (closed end) as a rarefaction wave, as well. When the latter wave and arrives to the piston at the beginning of its compression stroke. Then a second cycle starts. Therefore, during one complete cycle of the piston, the acoustic wave completes two round trips across the tube length. This is the condition of resonance in this case. However, for closed end tube, the acoustic wave completes only one round trip during one complete cycle of the piston motion, in case of resonance.

In the present experiments, the piston oscillates at a constant frequency of 80 HZ. For resonant condition to be prevailed, the open end constant area duct length is 1.08 m (based on sound speed calculated at 300K).

At early times of wave evolution, the results indicate generally, that the wave amplitude grows almost linearly until it reaches a maximum value. Then, a quasi - steady

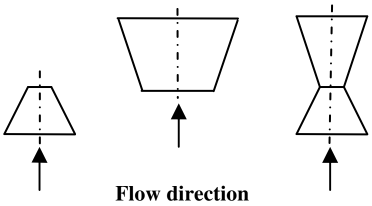

Fig. 2. Schematic configuration of the variable area portions.

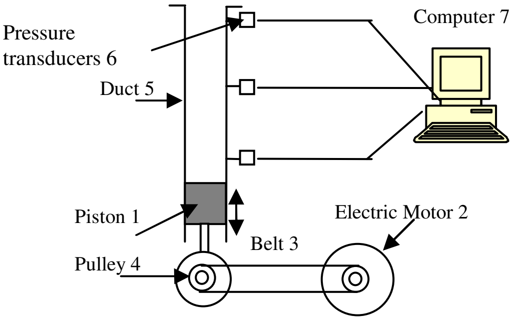

Fig. 3. Schematic diagram of the experimental set-up.

Table 1
The detailed dimensions of the experimental test sections

Part	Type	Length (cm)	Exit diameter (cm)	Throat diameter (cm)
Tube	Constat area	108	5.08	-----
Convergent	Variable area	10	2.54,1.905 ,1.27	-----
Divergent	Variable area	10	3.05, 4.12, 5.26	-----
Convergent –Divergent	Variable area	10+ 18	17.45, 11.56, 6	2.54,1.905, 1.27

waveform of almost constant amplitude is obtained. In convergent-divergent area case, the wave amplitude declines to a certain value before reaching the quasi-steady waveform. However, the maximum wave amplitude and the waveform amplitude differ from case to another. In addition, the waveform amplitude decreases along the duct. It is anticipated that the waveform is established when a balance occurs between the input energy, viscous and heat transfer energy losses, energy dissipation loss (in case of shock wave formation) and the acoustic energy radiation to the atmosphere. The power spectrum calculations show that the fundamental frequency of gas oscillation is equal the piston oscillating frequency. Higher harmonics appear in some cases near the end of constant area duct. The energy content of theses harmonics is subtracted from that of the fundamental one. The appearance of higher harmonics is an indication of wave steepness. A detailed discussion of the results will be presented in the following sections.

3.1. Constant area duct ended with convergent area portion

In this case, the converging area part is a cone of length L_c=10cm and its exit diameter takes three values; namely d_e= 2.54, 1.905 and 1.27 cm.

The recorded pressure histories at a point 5cm from the end of constant area duct (upper) for the three cases in addition to that of constant area duct with open end are given in figs. 4-a to 4-d. The results show that for constant area duct, the recorded pressure is mainly below the atmospheric pressure. This is because all of the wave energy that reaches the open end is transmitted off the duct during the piston compression stroke while in the suction stroke air is sucked into the tube with the highest negative pressure close to the piston (see fig. 5-a) and the smallest negative pressure near the open end. For cases of tubes ended with convergent area portion, the wave energy is partially reflected from the convergent part wall into the tube and the rest is transmitted off the duct. Hegab et al. [12] has noticed similar behavior in their work about unsteady side-wall injection of gas into a solid rocket motor chamber / nozzle model.

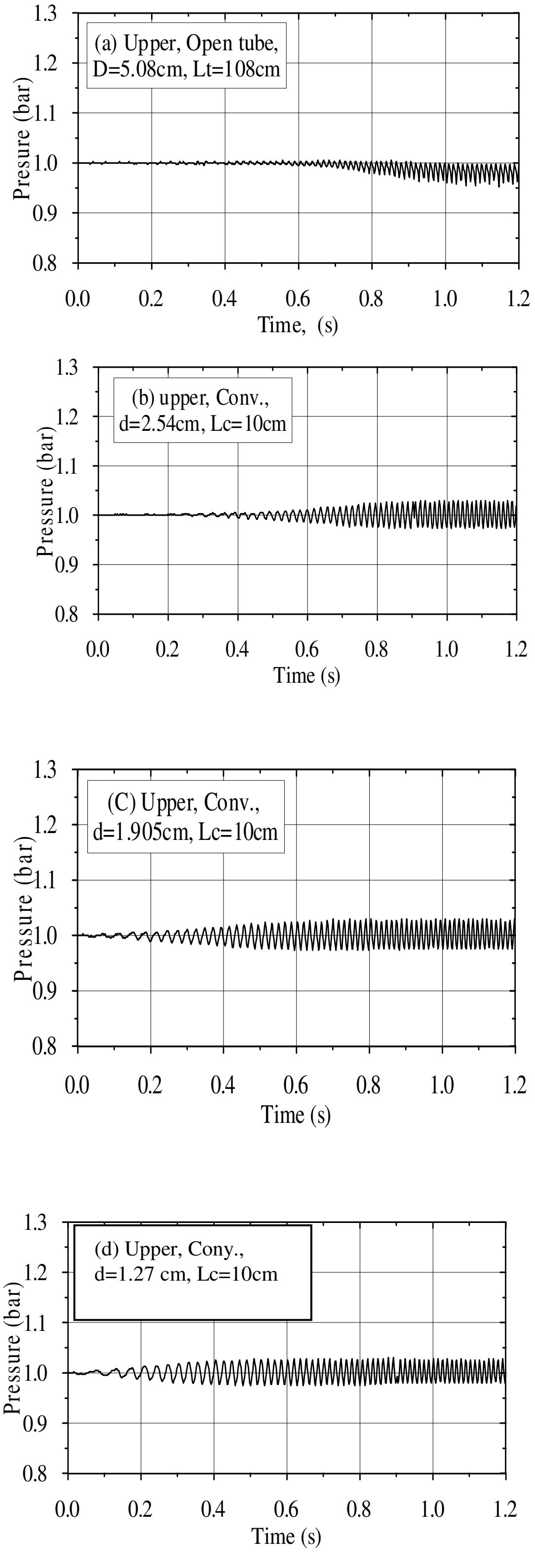

Fig. 4. Pressure histories at the upper point.

These energy portions depend on the exit area of the convergent part. That explains the noticed wave oscillation around the atmospheric pressure unlike the wave oscillation below the atmospheric pressure in the open end tube case. One can also note that the quasi-steady waveform amplitude decreases when the exit area decreases. However, the positive part of the wave increases with the decrease of exit area. The presumably causes of this behavior may be explained as follows: First, the partial reflection of wave from the converging portion wall causes an increase in the positive part of the wave. Consequently, it results in a reduction in energy transmitted off the duct as the exit area decreases. Second, the reflected compression waves from the convergent portion wall takes place as series of compression waves of different phase shifts followed by an expansion wave in the suction stroke of the piston. The complex interaction of these waves results in a deviation of the natural frequency from that of constant area duct. So, the system becomes out of resonance. This results in a reduction of the waveform amplitude as mentioned in [1].

The recorded pressure histories at a point 13cm (lower) from the top dead center of the piston, given in figs. 5-a to 5-d, show greater wave amplitude for all cases, compared with that recorded near the open end, shown in figs. 4-a to 4-d. The reduction in waveform amplitude associated with the reduction in convergent part exit area is explained earlier.

The maximum wave amplitude belongs to the open end tube case and the smallest belongs to the smallest exit area. The decrease in wave amplitude downstream of the energy source (piston) has been noticed by other researches including Disselhorst and Vijngarden [9] and this reduction is most conceivable to be caused by viscous and heat transfer losses. It could apparently also due to the wave steepness associated with a reduction of wave amplitude. Fig. 6-a shows clearly that the waveform amplitude at the upper point decreases slightly with the decrease in exit diameter, while at the lower point the reduction is significant. It is of importance to note also that both upper and lower amplitudes are equal at a certain exit diameter. It is presumed that at this value the waveform does not change along the duct. Below this value of exit diameter, the waveform amplitude becomes smaller at lower point. This is believed to be due to some increase of the reflected part of the wave relative to the radiated part.

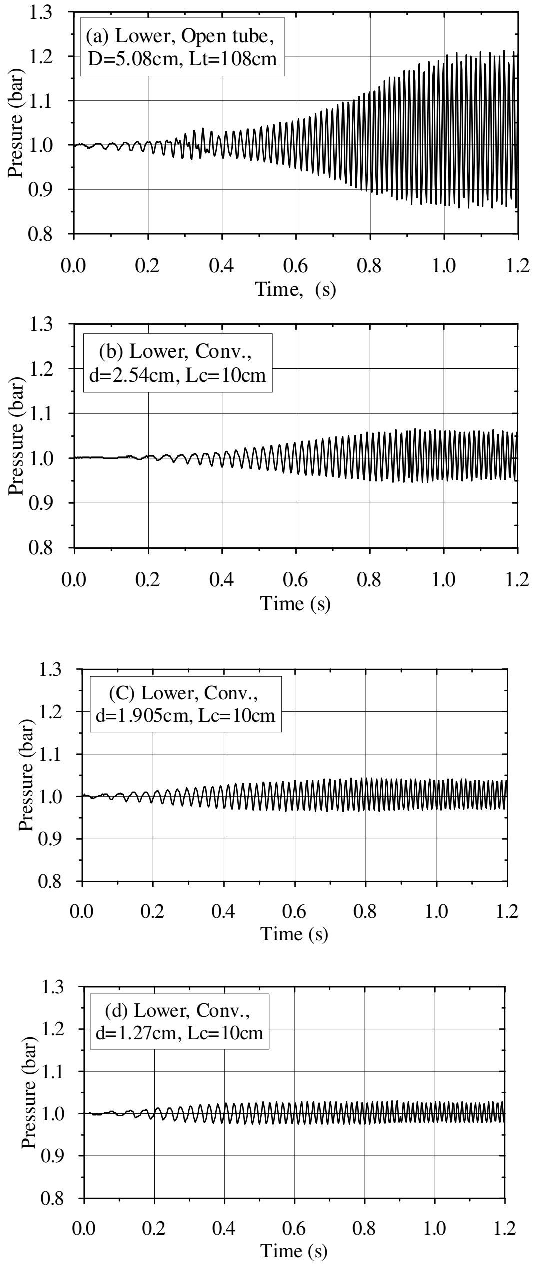

Fig. 5. Pressure histories at the lower point.

It may be understood that the addition of convergent portion has two important effects, a reduction in radiated wave energy to the atmosphere and a change in the natural frequency of the gas. The latter has a substantial influence on wave amplitude because off-resonance situation prevents the acoustic wave from growing up. It is expected that the natural frequency would increase as the exit area decreases because the natural frequency of closed end tube of length 1.08 m is 160HZ which is twice as much as that of open end duct of the same length.

It is important to state the well known fact that the variation of the tube length or piston frequency from the resonance condition will decrease the wave amplitude. Fig. 6-b shows a sample of experimental results for the effect of the tube length increase on the wave amplitude reduction. However, the aim of the present work is to investigate the effect of adding variable area part to the open end of resonance tube on wave deformation.

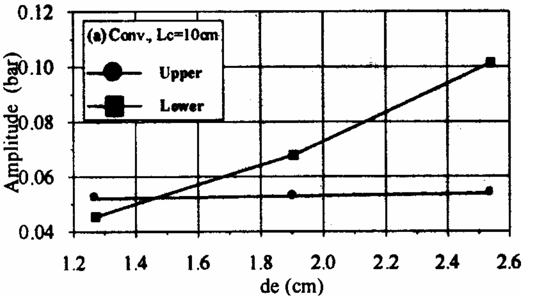

Fig. 6-a. Waveform amplitude variation with convergent portion exit diameter.

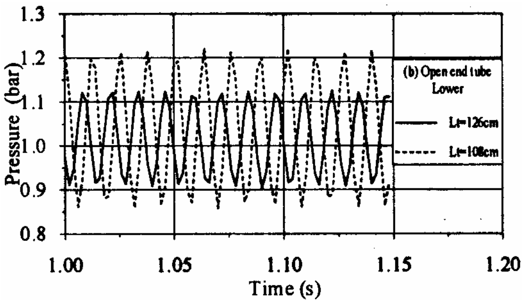

Fig. 6-b. Pressure histories in open end constant area tube of different lengths.

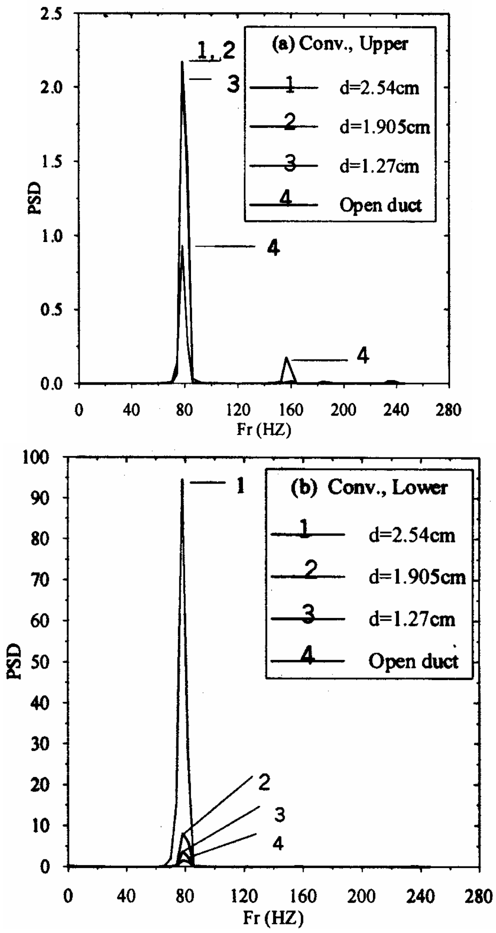

Fig. 7. Power spectrum density at (a) upper point, (b) lower point.

The power spectrum is calculated from the experimental data and is given in figs. 7-a (upper) and 7-b (lower). It shows that the gas oscillates at the same frequency of the piston. Higher harmonics of very low energy content could be noticed, particularly at the upper point. This is an indication of small wave deformation. This confirms the aforementioned understanding of the mechanism of wave amplitude reduction, which is mainly due to off-resonant situation caused by exit area reduction. The authors believe that shock wave will not appear even if the tube is closed and still have the present length while piston oscillates at the present frequency because the system will be out of resonance. One notices also that higher harmonics appear only at the upper point for the open flanged duct only. This indicates the wave steepness behavior; formation of a shock wave accompanied by energy dissipation and transfer of energy from the fundamental mode to its higher harmonics. The power spectrum illustrates also that near the duct exit, the energy content of different cases differs slightly, whereas that of open constant area duct is almost negligible because almost all the wave energy is transmitted out (radiated to the atmosphere). In contrast, near the piston the energy content decreases as the exit area decreases.

3.2. Constant area duct ended with divergent area portion

Here, conical divergent area portion is attached to the end of the constant area duct. Three divergent area portions of the same length L_d = 10cm and different exit diameters namely; d_e = 3.05, 4.12 and 5.26 cm are considered. The recorded pressure histories at a point 5 cm from the end of constant area duct (upper) depicted in figs. 8-a to 8-d, show that as the piston begins oscillating, the air pressure oscillates as well with almost linear growing amplitude below the atmospheric pressure (vacuum). Then the wave amplitude growth stops and waveform of certain amplitude, depending on the divergence angle of the conical area portion, is formed. The wave oscillation below the atmospheric pressure means that almost all the available wave energy at the end of the duct is radiated to the atmosphere during the piston compression stroke.

The recorded pressure histories at that location show that the waveform amplitude and shape varies with the divergence angle. Generally, the negative amplitude increases as the divergence angle decreases and the compression part of the wave becomes steeper. In particular, at exit diameter of 3.05 cm, the negative wave amplitude is largest. and its shape takes a saw-tooth like form as could be noticed. The compression part of the wave is much steeper compared with the expansion part. Therefore, a shock wave is almost formed. The open flanged duct shows the smallest wave amplitude (smallest negative pressure) and steepness. In order to see the wave steepness more clearly at the upper point, figs. 9-a to 9-d is plotted for a small period of time. The shock wave with the strongest shock is seen to belong to the smallest divergent angle. In all cases, all the wave energy is radiated to the atmosphere during compression stroke because the end is open. In other words, jet outflow occurs during the compression stroke as explained in [9] and [10].

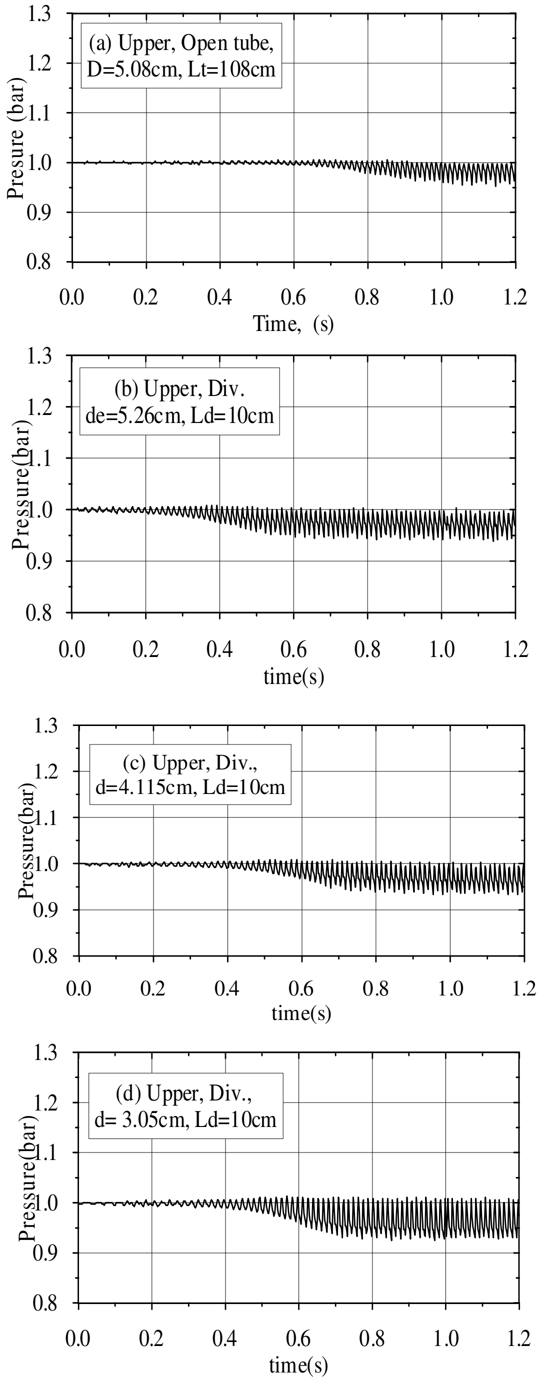

Fig. 8. Pressure histories at the upper point .

During the expansion stroke, air is sucked into the tube. That explains the negative pressure inside the duct with a maximum negative pressure in the neighborhood of the piston. However, the magnitude of negative pressure decreases with the increase in divergence angle with minimum value corresponding to the flanged duct. This means that a certain kind of resistance takes place and resists air flow into the duct as the divergence angle decreases during the expansion stroke. As mentioned earlier, Disselhorst and Van Wijngarden [9] attributed the resistance in case of open duct with sharp edge to the vortex formation and flow separation near the duct exit. It should be recognized that the present ducts have sharp edges.

Following the concepts given in [9], it could be concluded that as the divergence angle decreases the inflow separation increases and is accompanied with a stronger vortex. This in turn increases the resistance to the inflow and more pressure loss occurs near the entrance edge. These reasons explain the increase in the negative pressure as the divergence angle decreases. In addition, the pressure decreases more because of the friction losses at the duct wall during the inflow.

The pressure histories recorded in the constant area duct 13 cm above the top dead center of the piston (lower), as depicted in figs. 10-a to 10-d, show similar trends with greater wave amplitude and less steepness. This means that wave deformation occurs as the wave propagates towards the duct end. The deformation is accompanied by energy transfer from the fundamental mode to its higher harmonics. The waveform amplitude variation at lower and upper points with the exit diameter of the divergent part is shown in fig. 11. It is clearly seen that there is a slight variation of the waveform amplitude with the exit diameter.

The power spectrum discerned in fig. 12-a for the lower point, and fig. 12-b for the upper point elucidates clearly the appearance of higher harmonics at the upper point which confirms the intimate relation between the appearance of higher harmonics and the wave steepness. However, at the lower point no signal of higher harmonics is shown. One can also notice that a significant high energy content of the wave at the lower point, compared with that of the upper point. This is likely to occur because the lower point is closer to the acoustic energy source.

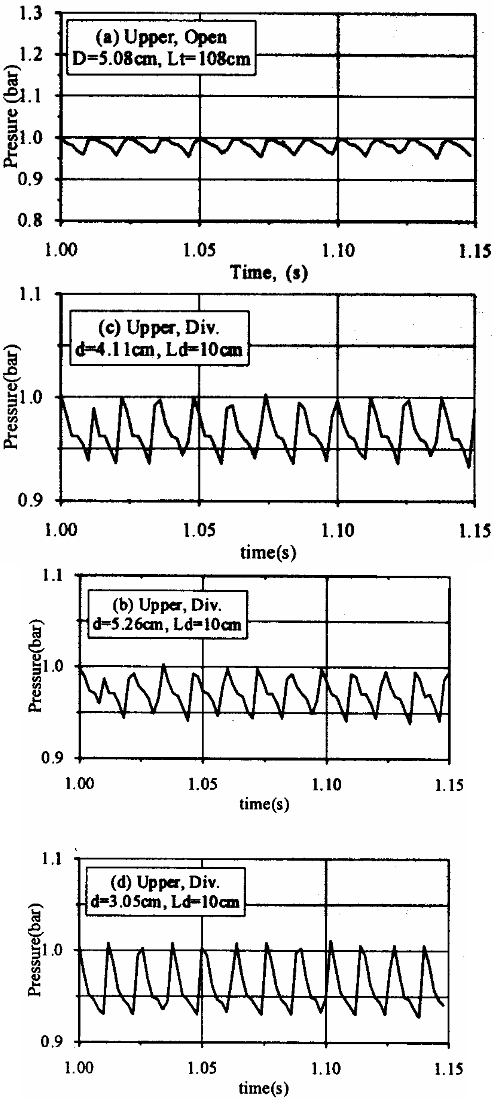

Fig. 9. Waveform configuration in a stretched time coordinate.

3.3. *Constant area duct ended with convergent-divergent area portion*

Here, the lengths of convergent and divergent cones remain constant at L_c=10cm and L_d = 18 cm. The throat diameter takes values of d^*= 2.54, 1.905 and 1.27 cm.

The results given in figs. 13-a to 13-d for the pressure histories at the upper point including the results for open constant area flanged duct case, show that the wave amplitude grows at early times of wave evolution. The wave amplitude reaches a

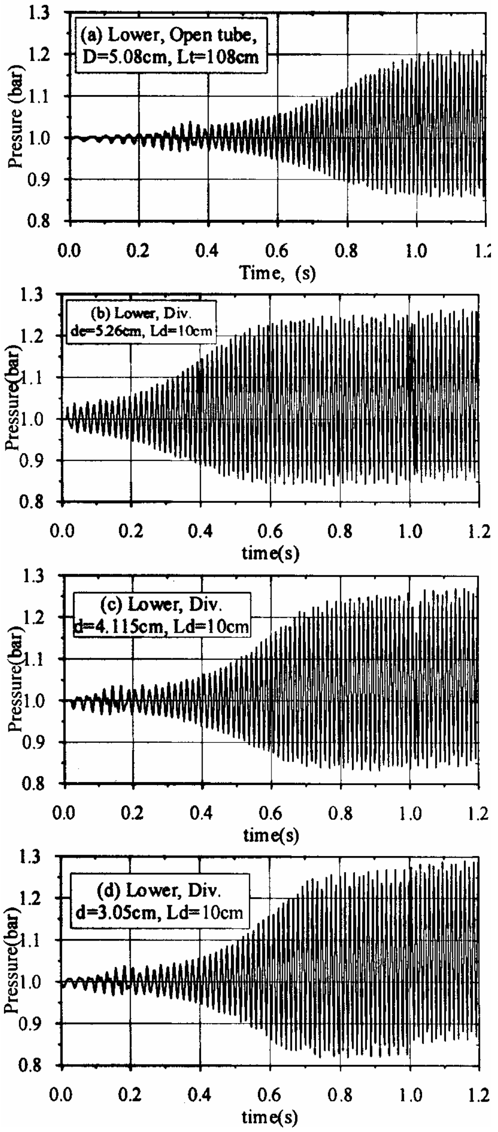

Fig. 10. Pressure histories at the lower point.

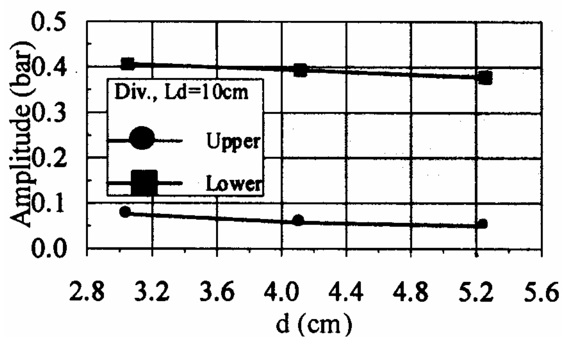

Fig. 11. Waveform amplitude variation with divergent portion exit diameter.

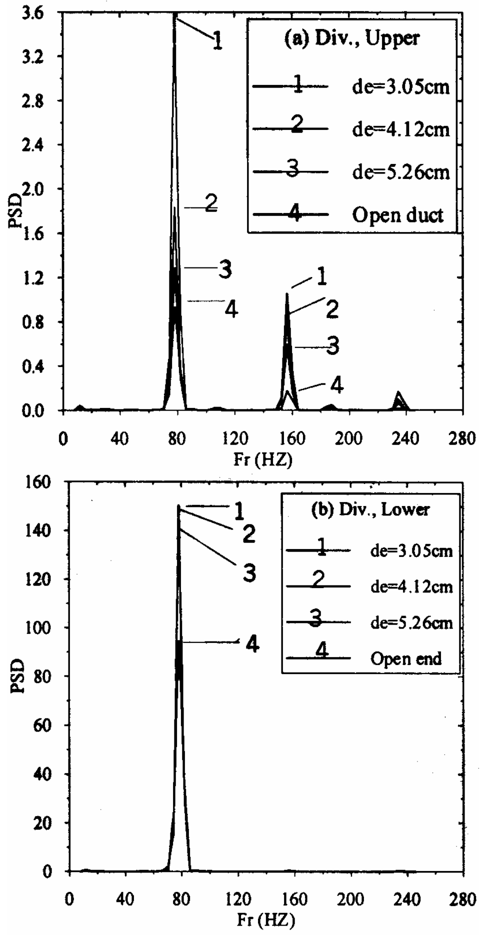

Fig. 12. Power spectrum density at (a) upper point, (b) lower point.

certain limit, depending on the variable area portion configuration, and it then decays to almost constant value when the waveform is reached. It should be emphasized that unlike the previous cases of convergent and divergent portion in which the wave amplitude remains almost constant as soon as it reaches the maximum value, here it increases and then decreases to a constant waveform amplitude. The results show also that the wave evolution at early times becomes smoother and the waveform amplitude decreases as the throat area decreases, as shown in figs. 13 and 14. A point to notice, also, is that the waveform amplitude is greater than that observed in the convergent cases when the exit diameter equals the throat diameter in the present case. These observations at a measuring point near the end of the constant area duct means that the addition of a divergent part at the end of convergent part not only increases the wave amplitude but also makes it steeper. The power spectrum shown in fig. 15-a confirms this conclusion in which higher harmonics are clearly seen, compared with that of the convergent part only. From the energy point of view, less sound energy is radiated. In this case energy dissipation is effectively due to wave steepness, and wave energy content is higher because the wave amplitude is greater. It is of importance to recognize that, both of the complex area variation and the increase in duct length lead to off-resonance situation.

The pressure histories measured at the lower point, depicted in figs. 16-a to 16-d, show similar trend with greater wave amplitude and less steepness. The power spectrum calculated at this location, shown in fig. 15-b, shows no indication of energy transfer from the fundamental mode to higher harmonics. This means that there is no wave deformation at this location. These figures illustrate also that the energy content decreases as the throat diameter decreases and this is in agreement with the conclusions cited earlier. The comparison with the results of constant diameter open end case shows that the maximum deformation occurs in the case of open end. Comparing between figs. 15-a and 15-b, it is clear that that the energy content decreases sharply near the exit.

4. Conclusions

The following conclusions could be inferred from experimental measurements:

1. In the case of constant-area open end tube and that of added divergent-area portion, the total energy delivered by the piston, except a part lost in viscous and heat transfer, is completely radiated out to the atmosphere during the compression stroke of the piston. At a measuring point close to tube end, the wave steepness increases with decrease in exit area, and N-type waveform is noticed at the smallest exit diameter. Power spectrum calculations show the appearance of higher harmonics at this point.
2. When the added part is of convergent-area type, the wave is partially radiated and the reflected part is a compression wave. The

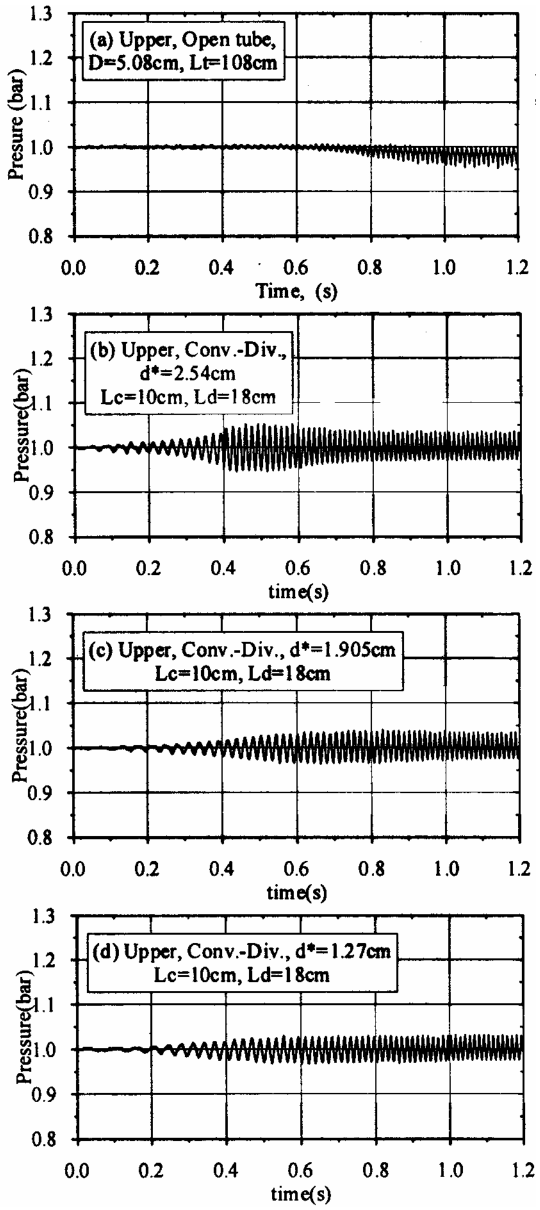

Fig. 13. Pressure histories at the upper point.

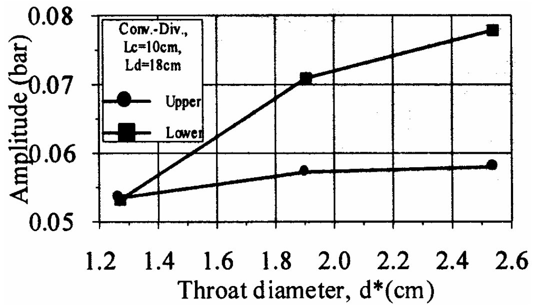

Fig. 14. Waveform amplitude variation with convergent-divergent portion throat diameter.

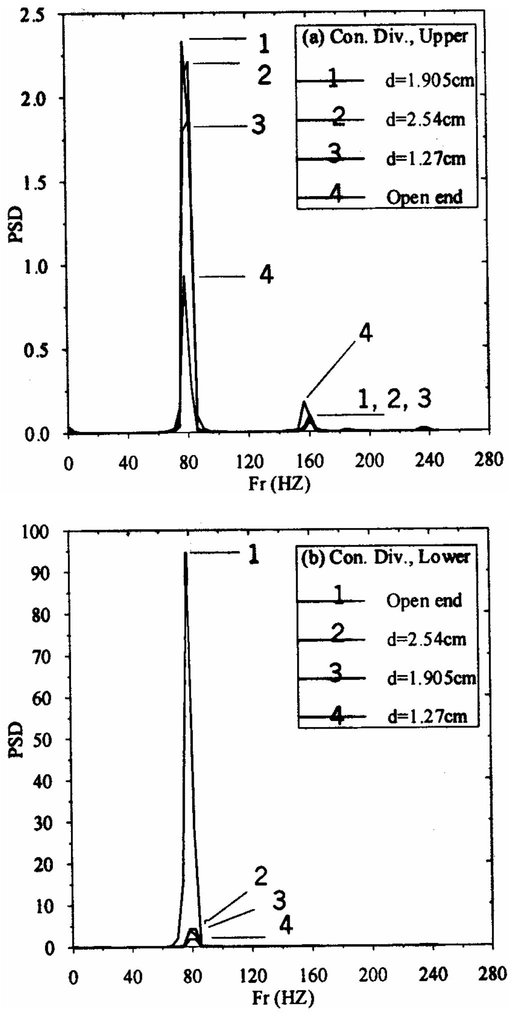

Fig. 15. Power spectrum density at (a) upper point (b) lower point.

latter decreases when the exit diameter decreases. There is a certain value of exit diameter at which the wave amplitude does not change along the duct. Below this value, the wave amplitude near the piston becomes smaller. Power spectrum calculations do not show any higher harmonics.

3. When the added part is of convergent-divergent-area type, the wave amplitude near the end of constant-area tube is greater than that of convergent-area part, only when the convergent part exit diameter is equal to the throat diameter. The wave energy content decreases with the decrease in throat diameter. Power spectrum calculations show the appearance of higher harmonics at the upper measuring point which is an indication of wave steepness.

Generally, the radiated part, of energy delivered by the piston to the atmosphere, depends on the tube end configuration. The noticed differences between the results of open end tube and that when variable area portion is added, presumably attributed to the deviation of the natural frequency of the variable area cases from that of the open end tube case.

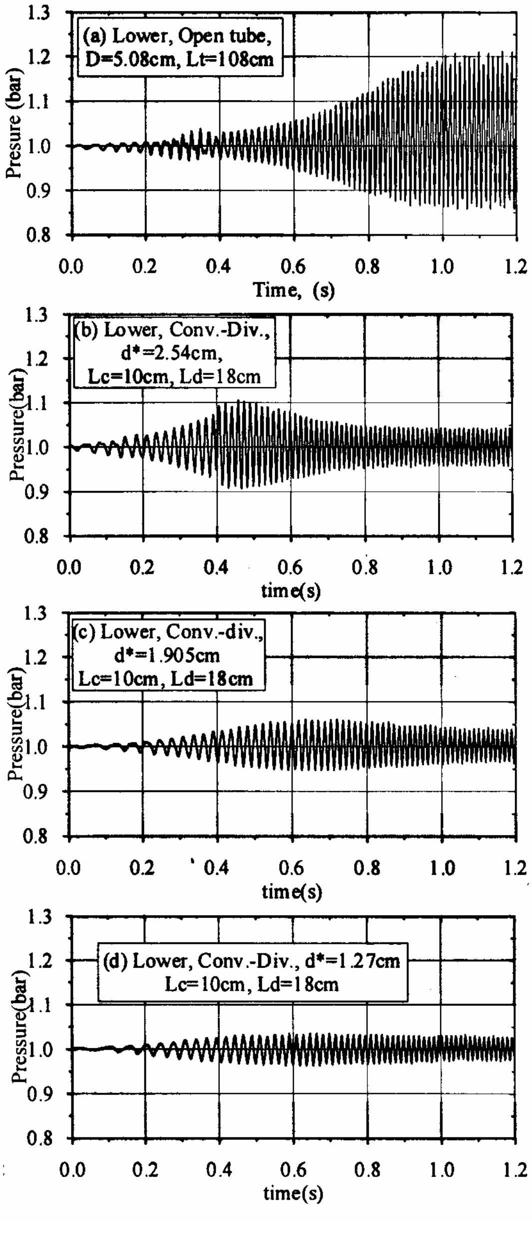

Fig. 16. Pressure histories at the lower point.

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