

Formulation of a financial plan for crushing plant based on dynamic programming

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Due to the continuous development in the construction industry, crushing plants are studying to replace the used equipment to fulfill the construction projects requirement. The replacement objective is to increase the target quality. The capacity for the crushing plant is selected to produce a given rate of heaviness of output. The financial management of liquid money and cash like assets is an important consideration in the crushing plant management. The cash holdings are considered an inventory like the inventory of other commodities. It should be incorporated in a mathematical model to help in making financial management decisions. The main objective of this paper is to provide an understanding of capital effective use in crushing plants. The sources, types, and means of acquiring funds and capital are described. Examples illustrating the capital invested are given. A case study illustrating the accounts for a typical construction company is given.

نتيجة التطور المستمر في صناعة التشييد في مصر تقوم الآن محطات تكسير الركام بدراسة تجديد المعدات المستخدمة بما يتواءم مع احتياجات المشروعات المختلفة، وذلك لرفع مستوي الجودة المطلوب. إن قدرة محطات تكسير الركام يتم اختيارها لتحقيق معدل إنتاج مطلوب. إن الإدارة المالية للسيولة النقدية و الأصول الثابتة من الاعتبارات الهامة في محطات تكسير الركام، حيث يتم اعتبار النقد كما لو كان مخزون مثل باقي منتجات التكسير. و لهذا يجب دمج الإدارة المالية مع النمذجة الرياضية لاتخاذ القرار المناسب. إن الهدف الرئيسي من هذا البحث هو تفهم قيمة رأس المال في محطات تكسير الركام من حيث النوع، و المصدر، و وسيلة اكتسابه. و تم الاستعانة ببعض الأمثلة لتوضيح ذلك، بالإضافة إلي دراسة حالة حقيقية.

Keywords: Project management, Mathematical modeling, Dynamic programming, Financial management

1. Introduction

Linear and dynamic programming are mathematical techniques for determining optimal allocation of scarce resources. Linear Programming (LP) is a mathematical procedure that has found practical application in almost all facets of construction projects, from feasibility studying to project closing. Construction methods and financial planning problems are the most typical objects of LP analysis.

Most of the construction projects have been essentially only one-period problem. The formulations act as if decisions of this period were influenced by decisions in future periods. Commonly, the crushing plant produces more of a certain product in this period than a constraint requires. That extra production will not be worthless. However, it can probably be used in the next period. A common word with the same meaning for multi-period is dynamic.

Therefore, a multi-period LP may be referred to as a Dynamic Programming (DP) model. These interactions between periods can be represented within DP models.

In most of construction projects, the need to represent the multi-period aspects is obvious. One setting in which multi-period DP has been used for a number of years is crushing plants. Production decisions must be made yearly or even monthly. However, the production time for many aggregate types may be months depending upon the required grading.

The crusher takes the raw material and breaks it into small sizes. To do this the machine is set to the maximum size required. However, the rocks will fragment into a range of sizes down to dust. Then, it can be fed away for further reduction or screening and sorting. The machine is usually described in terms of its crushing ratio.

$$\text{Crushing} = \frac{\text{Size of feed opening}}{\text{Setting for crusher}} \quad (1)$$

The ranges of crushing ratios for various types of machine are shown in table 1.

Table 1
Crushing ratios [1]

Crusher machines	Smaller units	Larger units
Jaw	4-8	3-10
Gyrator	5-8	4-6
Impact	5-25	5-50
Cone	3-14	4-14
Twin-roller	1-4	2.8

There are several different types of crushing machine for handling different size feed and material. The production output from each varies between types for a given crushing ratio. Therefore, careful costing is necessary when several stages of reduction are required. For primary crushing, (i.e., breaking material down from large pieces) either a jaw or gyrator is preferred while secondary or further reduction, the cone, hammer mill or roll crusher is usually selected.

For example, fine sand may need to be stored as inventory for up to ten months. Coarse sand may take from two to four months, and for other grades of sand obtained it depends upon the number of weeks held in storage.

2. Multi-period aspects

Most multi-period (dynamic) models chop off the analysis at some finite time in the future. The manner in which this chopping off is done can be important. In general, project manager cares about the state in which things are left at the end of a planning model (e.g., inventory levels, capital investments, etc.). If the project manager arbitrarily make a planning model at the fifth year in the future, an optimal solution to our model may be an optimal solution to how to go out of business in five years.

Grinold [2], provides a comprehensive discussion of various methods for mitigating end-of-horizon effects. Some of these options for handling the end effect are discussed in

later (a) Truncation which simply drops from the model all periods beyond a chosen cutoff point (b) Primal limits that are the placed reasonable limits on things such as inventory level at the end of the initial period (c) Salvage values or dual prices are the placed reasonable salvage values on things such as inventory level at the end of the final period (d) The final-period represents (models) a period of infinite length for which the same decisions applies throughout the period. Net present value discounting is used in the objective function to make the final period comparable to the earlier finite periods.

This was the approach used by Carino et al. [3] and Schrage [4] in their mathematical models.

3. Financial management

Mathematical models for financial planning divide the project life cycle into a number of periods. The portion of the model corresponding to a single period might include some combination of crushers, screening process, etc. An inventory variable and material balance links these single periods or static models. The linking (inventory) variable represents the amount of commodity transferred from one period to the next.

3.1. Material balance

A material balance (or "sources = uses") constraint is applied for each commodity and period. The simplest form of this constraint is as follows.

$$\begin{aligned} &\text{Beginning Inventory} + \text{Aggregate Production} \\ &= \text{Ending inventory} + \text{Aggregate sold.} \end{aligned} \quad (2)$$

Multi-period models are usually used in a rolling or sliding format. In this format, the mathematical model is solved at the beginning of each period. The recommendations of the solution for the first period are implemented. As one period elapses and better data and forecasts become available, the model is slid forward one period. The period that had number two becomes number one, etc.

3.2. Types of capital

The small construction companies beginning operations will normally need to find the initial capital to start the business from private resources. Few others are willing to take the risks associated with an unknown enterprise. Gradually, though, as the firm prospers by ploughing back retained profits, the usual lenders of capital may then consider providing loans and investments.

Loan capital can be classified into short-term and long-term borrowings.

3.3. Capital sources

3.3.1. Long-term finance

Long-term finance is that capital required for up to 5 years, either to start the business or to carry out expansion programs. Broadly, the capital is used to purchase buildings, plant, and equipment and to carry stocks of materials.

3.3.2. Short-term finance

Short-term loans usually carry lower interest rates, by virtue of the fact that repayment periods are short of the order of months, and therefore open to much less risk than loans granted over a period of years.

The firm when established often needs short-term capital to overcome immediate cash flow problems. Materials have to be purchased, plant hired, labor and subcontractors paid and so on before payment is received for the finished product. Furthermore, capital may be required to smooth out the strains on cash flow resulting from rapid fluctuations in the market demand for the company's goods.

3.4. Period length

There is nothing sacred about having all periods of the same length. For example, when the crushing plant plans the production rates for the coming year, it is sensible to have the periods correspond to the seasons of the year.

Some companies, which based on mineral resources, plan as much as 50 years into the future. In such a case, one might have the first two periods be one year each. The next period is two years. The next two periods are

three years each. The next two periods are five years each. Moreover, the final three periods ten years each.

3.5. Inter-period interactions

Inter-period interactions are usually accounted for in DP by the introduction of inventory decision variables. These variables link adjacent periods. As an example, suppose a single explicit decision has to be made for each period, namely, how much to produce of a single product (i.e. coarse sand).

Let P_j be the amount of production for period j . Further, suppose we have contracts in period j to sell d_j (known amounts) of this product. Define the decision variable I_j as the amount of inventory left over at the end of period j . By this convention, the beginning inventory in period j is I_{j-1} .

The simplest form of this constraint is as follows.

$$I_{j-1} + P_j = I_j + d_j \quad (3)$$

In that case, the DP formulation will contain one constraint "sources of product = uses of product" for each period. For period #2, the sources of product are beginning inventory, I_1 , and production in the period, P_2 . The uses of product are demand, d_2 , and end-of-period inventory, I_2 . For example, if $d_2=60.00$ and $d_3=40.00$, the constraint for period #2 is $I_1 + P_2 = I_2 + 60.00$ or

$$1.00 I_1 + 1.00 P_2 - 1.00 I_2 = 60.00 \quad (4)$$

The constraint for period 3 can be expressed as:

$$1.00 I_2 + 1.00 P_3 - 1.00 I_3 = 40.00 \quad (5)$$

Notice how I_2 links appears in both the constraints for period #2 and period #3.

3.6. Cash flow stream

In some problems, the net outflow need not exactly equal the net inflow into the next period. For example, if the product is cash, one of the linking variables may be short-term borrowing or lending. For each dollar that is

carried over from period #2 by lending, we will enter period #3 with \$1.05 if the interest rate is 5.00% per period.

3.7. Predictable attrition rate

On the other hand, if the product is workforce and there is a predictable attrition rate of 10.00% per period, the above two constraints would be modified to the followings:

$$0.90 I_1 + 1.00 P_2 - 1.00 I_2 = 60.00, \tag{6}$$

$$0.90 I_2 + 1.00 P_3 - 1.00 I_3 = 40.00. \tag{7}$$

The following simple but realistic case study illustrates the formulation of a financial planning for a crushing plant based on DP.

4. Model formulation

Suppose that, because of a careful planning exercise the project manager has concluded that to meet certain commitments. Hence, he will need the following amounts of cash for the current plus next 14 years, table 2.

A common example where such a projection is made is in a person injury lawsuit. Both parties may reach an agreement that the injured party should receive a stream of payments such as above or its equivalent. Other examples where the above approach has been used are the design of bond portfolios to satisfy cash needs for a pension fund. Or for so-called balance sheet defeasance to satisfy cash needs for a pension fund, or for so-called balance sheet defeasance, where one kind of debt is replaced by another having the same cash flow stream. For administrative simplicity in the personnel injury example, both parties prefer an immediate single lump sum payment that is equivalent to the above stream of 15 payments. The party receiving the lump sum will argue that the lump sum payment should equal the present value of the stream using a low interest rate. Such as that obtained in a very low risk investment, for instance a government guaranteed savings account. For example, if an interest rate of 4.00% is used, the present value of the stream of payments is \$230,437.00. The party that

must pay the lump sum, however, would like to argue for a much higher interest rate. To be successful, such an argument must include evidence that such higher interest rate investments are available and are no riskier than savings accounts. The investments usually offered are government securities. Generally, broad spectrums of such investments are available on a given day. For simplicity, assume that there are just two such investments available with the specific features, table 3.

Table 2
Cash for the current plus next 14 years

Year	Cash (\$)
0	\$10,000.00
1	\$11,000.00
2	\$12,000.00
3	\$14,000.00
4	\$15,000.00
5	\$17,000.00
6	\$19,000.00
7	\$20,000.00
8	\$22,000.00
9	\$24,000.00
10	\$26,000.00
11	\$29,000.00
12	\$31,000.00
13	\$33,000.00
14	\$36,000.00
Total	\$319,000.00

Table 3
Features of available investments

Investment section	#1	#2
Current cost	\$980.00	\$965.00
Yearly return	\$60.00	\$65.00
Years to maturity	5	12
Principal repayment at maturity	\$1,000.00	\$1,000.00

The paying party will offer a lump sum now with a recommendation of how much should be invested in sections #1 and #2. In savings accounts, the yearly cash requirements are met with the minimum lump sum payment.

4.1. Methodology

To construct the mathematical model, the following three questions must be answered. Firstly, what does the model ask to determine?

In other words, what are the decision variables (unknowns) of the problem? Secondly, what constraints must be imposed on the variables to satisfy the limitations of the modeled system? Thirdly, what is the objective (goal) that needs to be achieved to determine the optimum solution from among all the feasible values of the variables?

4.2. Defining decision variables

The following decision variables are useful in solving this problem. B_1 is the amount of invested now into investment section #1, measured in face value amount. B_2 is the amount invested now into investment section #2, measured in face value amount. S_i is the amount invested into account in year I . L is the initial lump sum.

4.3. Model constraints

If we assume that idle cash is invested at 4.00% in, a savings account and all amounts are measured in \$1000s. The following set of constraints is required to achieve the financial strategy for the current plus next 14 years.

$$\text{Current year} \\ L - 0.980 B_1 - 0.965 B_2 - S_0 = 10. \quad (8)$$

$$\text{First year} \\ 0.060 B_1 + 0.065 B_2 + 1.04 S_0 - S_1 = 11. \quad (9)$$

$$\text{Second year} \\ 0.060 B_1 + 0.065 B_2 + 1.04 S_1 - S_2 = 12. \quad (10)$$

$$\text{Third year} \\ 0.060 B_1 + 0.065 B_2 + 1.04 S_2 - S_3 = 14. \quad (11)$$

$$\text{Fourth year} \\ 0.060 B_1 + 0.065 B_2 + 1.04 S_3 - S_4 = 15. \quad (12)$$

$$\text{Fifth year} \\ 1.060 B_1 + 0.065 B_2 + 1.04 S_4 - S_5 = 17. \quad (13)$$

$$\text{Sixth year} \\ 0.065 B_2 + 1.04 S_5 - S_6 = 19. \quad (14)$$

$$\text{Seventh year} \\ 0.065 B_2 + 1.04 S_6 - S_7 = 20. \quad (15)$$

$$\text{Eighth year} \\ 0.065 B_2 + 1.04 S_7 - S_8 = 22. \quad (16)$$

$$\text{Ninth year} \\ 0.065 B_2 + 1.04 S_8 - S_9 = 24. \quad (17)$$

$$\text{Tenth year} \\ 0.065 B_2 + 1.04 S_9 - S_{10} = 26. \quad (18)$$

$$\text{Eleventh year} \\ 0.065 B_2 + 1.04 S_{10} - S_{11} = 29. \quad (19)$$

$$\text{Twelfth year} \\ 1.065 B_2 + 1.04 S_{11} - S_{12} = 31. \quad (20)$$

$$\text{Thirteenth year} \\ 1.04 S_{12} - S_{13} = 33. \quad (21)$$

$$\text{Fourteenth year} \\ 1.04 S_{13} - S_{14} = 36. \quad (22)$$

The constraint coefficients give a better appreciation of the problem structure. In order to fit as much information on a page, table 4 represents numbers bigger than 10 and less than 100 by "B"; and numbers bigger than 1.00 but less than 10.00 by an "A." Numbers less than 1.00 but at least 0.10 are represented by a "T." Numbers less than 0.10 but at least 0.01 are represented by a "U."

Notice that in row #7, B_1 has a coefficient of 1.06. This represents the principal repayment of \$1000.00 plus the interest payment of \$60.00 measured in \$1000.00s.

Variable S_{14} (investment of funds in a savings account after the final payment is made) appears in the problem although at first you might think it is useless to allow such an option. S_{14} is effectively a surplus cash variable in the final period. Nevertheless, it is not usual for the solution that minimizes the lump sum payment to have cash left over at the end of the problem. This is because a bond may be the most economical way of delivering funds to intermediate periods. This may cause the big principal repayment at the end of a bond's life to "overpay" the most distant periods.

Table 4-a
Constraint coefficients

#	L	B ₁	B ₂	S ₀	S ₁	S ₂	S ₃	
1	1							min
2	1	-T	-T	-1				A
3		+U	+U	+A	-1			B
4		+U	+U		+A	-1		B
5		+U	+U			+A	-1	B

Table 4-b
Constraint coefficients

#	L	B ₁	B ₂	S ₃	S ₄	S ₅	S ₆	
6		+U	+U	+A	-1			B
7		+A	+U		+A	-1		B
8			+U			+A	-1	B

Table 4-c
Constraint coefficients

#	L	B ₁	B ₂	S ₆	S ₇	S ₈	S ₉	
9			+U	+A	-1			B
10			+U		+A	-1		B
11			+U			+A	-1	B

Table 4-d
Constraint coefficients

#	L	B ₁	B ₂	S ₉	S ₁₀	S ₁₁	S ₁₂	
12			+U	+A	-1			B
13			+U		+A	-1		B
14			+A			+A	-1	B

Table 4-e
Constraint coefficients

#	L	B ₁	B ₂	S ₁₂	S ₁₃	S ₁₄	
15				+A	-1		B
16					+A	-1	B

4.4. Objective function

The objective function will be to minimize the initial lump sum, *L*. There will be a constraint for each year, which forces the cash flows to net to zero. By assuming all amounts of cash are measured in \$1000s, then the objective function is minimizing *L*, as in eq. (23).

$$\text{MIN.} \quad Z = 1.00 L \quad (23)$$

5. Tools and techniques

The process of solving the proposed mathematical model requires a large number of iterations. Therefore, a computer program best performs it. LINDO for Windows is the computer program that was used. The acro-

nym stands for Linear, Interactive, and Discrete Optimizer (LINDO). The main purpose of LINDO is to allow an end user to quickly input a DP formulation, solve it. In addition, it assesses the correctness or appropriateness of the formulation based on the solution. Then, LINDO quickly make minor modifications to the formulation and repeat the process. LINDO features a wide range of commands, which may be invoked at any time. LINDO checks whether a particular command makes sense in a particular context.

6. Dynamic programming outputs

When you direct the computer to solve a DP, the possible outcomes are indicated in fig.1.

For a properly formulated DP, the solution procedure will first attempt to find a feasible solution. Feasible solution is a solution that simultaneously satisfies all constraints but does not necessarily optimize the objective function. If a feasible solution has been found, the procedure attempts to find an optimal solution.

If the unbounded solution termination occurs, it implies that the formulation admits the unrealistic result that an infinite amount of profit can be made. A more realistic conclusion is that an important constraint has been omitted or the formulation contains a critical typographical error.

No feasible solution will be found if the formulator has been too demanding. That is, two or more constraints are specified which cannot be simultaneously satisfied. In practice, the “No Feasible Solution” outcomes might occur in a large complicated problem.

6.1. Solution reports

When the proposed mathematical model is solved with the LINDO computer program, the following solution report is produced as shown in table 5.

Of the \$195,683.70 lump sum payment, \$10,000 goes to immediate requirements, \$4,804.50 goes into a savings account, and $0.980 * 95,795.77 + 0.965 * 90,154.74 = \$180,879.20$ goes into longer term securities. By considering a wide range of investments

rather than just savings accounts, the amount of the lump sum payment has been reduced by about \$34,750 or 15%.

6.2. Portfolio constraints

In actual solutions, one may find that a major portion of the lump sum is invested in a single investment section. For example, appending the constraint enforces limiting the amount invested in investment section #1 to half the initial lump sum.

$$+0.98 B_1 - 0.50 L \leq 0 \tag{24}$$

Such constraints are typically called portfolio constraints.

An additional complication may arise due to integrality requirements on the B_1 and B_2 investments. For example, bonds can be

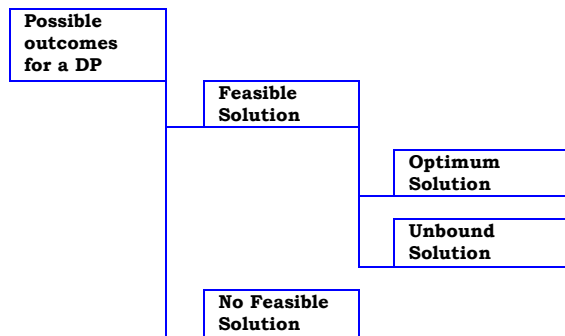


Fig. 1. Possible outcomes for a DP.

Table 5
Solution report

Variable	Value	Reduced costs
L	\$195.683700	0.000000
B_1	\$95.795770	0.000000
B_2	\$90.154735	0.000000
S_0	\$4.804497	0.000000
S_1	\$5.604481	0.000000
S_2	\$5.436464	0.000000
S_3	\$3.261727	0.000000
S_4	\$0.000000	0.106979
S_5	\$90.403574	0.000000
S_6	\$80.879776	0.000000
S_7	\$69.975025	0.000000
S_8	\$56.634084	0.000000
S_9	\$40.759506	0.000000
S_{10}	\$22.249944	0.000000
S_{11}	\$0.000000	0.141246
S_{12}	\$65.014792	0.000000
S_{13}	\$34.615385	0.000000
S_{14}	\$0.000000	0.379637

bought only in \$1000.00 increments. Generally, with a modest amount of judgment, the fractional values can be rounded to neighboring integer values with no great increase in lump sum payment. For example, if B_1 and B_2 are set to \$96.00 and \$90.00 in the previous example, the total cost increases to \$195,726.50 from \$195,683.70. When this is done, S_{14} becomes nonzero; specially, the last period is overpaid by \$40.00.

Useful computations are a number of opportunities or marginal cost figures that appears in the third column, listed in table 6. The interpretation of these “reduced costs” and “dual prices” is discussed in the next sections.

Table 6
Marginal cost

Constraint	Slack	Dual prices
# 1	0.000000	-1.000000
# 2	0.000000	-0.961538
# 3	0.000000	-0.924556
# 4	0.000000	-0.888996
# 5	0.000000	-0.854804
# 6	0.000000	-0.719063
# 7	0.000000	-0.691406
# 8	0.000000	-0.664814
# 9	0.000000	-0.639244
# 10	0.000000	-0.614658
# 11	0.000000	-0.591017
# 12	0.000000	-0.568286
# 13	0.000000	-0.410615
# 14	0.000000	-0.394822
# 15	0.000000	-0.379637

7. Sensitivity analysis

A substantial amount of interesting economic information can be gleaned from the solution report of a model. In addition, optional reports such as range analysis can provide further information. The usual use of this information is to do quickly “what if” analysis. The typical kinds of what if questions are as follows. Firstly, what would be the effect of increasing a capacity or demand? Secondly, what-if a new opportunity becomes available? Thirdly, is it a worthwhile opportunity?

Realistic mathematical models require large amount of data. Accurate data are expensive to collect and so we will generally be forced to use data in which we have less than complete confidence. A time-honored adage in

data processing circles is garbage in, garbage out. A user of a model should be concerned with how the recommendations of the model are altered by changes in the input data. Sensitivity analysis is the term applied to the process of answering this question. Fortunately, an LP solution report provides supplemental information that is useful in sensitivity analysis. This information falls under two headings, reduced costs, and dual prices.

Sensitivity analysis can reveal which process of information should be estimated most carefully. For example, if it is blatantly obvious that a certain product is unprofitable, little effort need be expended in accurately estimating its costs. Schrage [5] concluded that do not waste time accurately estimating a parameter if a modest error in the parameter has little effect on the recommended decision.

7.1. Reduced costs

The reduced cost is a quantity associated with each variable in any solution. If the units of the objective function are dollars and the units of the variable are tons, the units of the reduced cost are dollars per ton. The reduced cost of a variable is the amount by which the profit contribution of the variable must be improved before the variable in question would have a positive value in an optimal solution. Obviously, a variable that already appears in the optimal solution will have a zero reduced cost.

7.2. Profitability

Correct interpretation of the reduced cost is the rate at which the objective function value will be hurt if a variable currently at zero is arbitrarily forced to increase a small amount. Suppose the reduced cost of x is \$2.00/ton. This means that if the profitability of x could be increased by \$2.00/ton, increasing the amount produced of x by one unit (if 1 unit is a "small change") would not change the total profit. The profitability can be increased by some combination of raising its selling price/unit and decreasing its cost/unit. Clearly, if we did not alter the

original profit contribution of x , the total profit would be reduced by \$2.00 if x is arbitrary increased by 1.00.

7.3. Dual prices

The dual price is a quantity associated with each constraint. If the units of the objective function are dollars and the units of the constraint in question are tons, the units of the dual price are \$/ton. The dual price of constraint is the rate at which the objective function value will improve as the right hand side or constant term of the constraint is increased a small amount.

7.4. Sign conventions

Different LP packages may use different sign conventions with regard to the dual process. LINDO uses the convention that a positive dual price means that increasing the right-hand side in question will improve the objective function value, whereas a negative dual price means increasing the right-hand side will cause the objective value to deteriorate. A zero dual price means that changing the right-hand side a small amount will have no effect on the solution value.

It follows that under this convention, \leq constraints will have non-negative dual prices, \geq constraints will have non-positive dual prices, and $=$ constraints can have dual prices of any sign.

8. Concluding remarks

One of the fundamental assumptions of LP is that all input data are known with certainty. There are situations, however, where certain essential data are highly random. For example, when a crushing plant makes its grade size production decisions for the coming month, the demand for that aggregate is very much a random variable. If, however, the distribution probabilities for all the random variables are known, there is a modeling technique for converting a problem that is an LP except for the random element into an equivalent deterministic linear programming.

Notations

- B_m is the amount invested now into investment section # m measured in face value amount,
 d_j is the amounts of aggregate selling in period j ,
 I_j is the amount of inventory left over at the end of period j ,
 L is the initial lump sum,
 P_j is the amount of production for period j , and
 S_I is the amount invested into account in year I .

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