Experimental and theoretical study of curved rolled and castellated composite beams

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This paper is focused on the investigation of the behavior of curved composite rolled and castellated beams in which the web is encased in concrete. A series of experimental tests is carried out on six full-scale curved beams. A reference test of a straight composite castellated steel beam is also conducted. Moreover, a nonlinear finite element analysis is presented. The analytical models were verified by means of different comparisons. The deformations and ultimate loads were found to be in good agreement with the corresponding values predicted by using the present finite element models. A parametric study including 32-numerical applications including the main effective variables was carried out. The effects of the angle of curvature in plan as well as the type of beam cross-section (bare steel, composite rolled or composite castellated) were taken into consideration. From this research, the present finite element models were found to be suitable for representing real composite curved beams whether rolled or castellated. One of the most important results of the present parametric study is the classification of curved beams according to their angle of curvature. Moreover, significant improvement of the torsional rigidity of castellated beams due to composite action was distinctly observed. Finally, design guidelines for designers were achieved.

تم التركيز في هذا البحث علي دراسة تفصيلية لسلوك الكمرات المنحنية في مسقطها الأفقي و المكونة من قطاعات مركبة (قطاع فولاذي علي شكل حرف - I و يتم صب الخرسانة المسلحة حول عصبه). و القطاعات الفولاذية المستخدمة في هذا البحث إما قطاعات نمطية أو قطاعات كستيلاتيدية. وقد تمت هذه الدراسة من خلال ثلاث محاور، المحور الأول و هو الدراسة المعملية: وقد تمت علي ست كمرات منحنية ذات قطاع مركب بالحجم الطبيعي وكمرة واحدة مستقيمة (للمقارنة)، وأجريت هذه التجارب بمعمل المنشآت الثقيلة بكلية الهندسة– جامعة طنطا، أما المحور الثاني: فيتمثل في الدراسة النظرية باستخدام نظرية العناصر المحددة حيث تم عمل النماذج النظرية لحالات متغيرة حسب نوع القطاع، بينما تركز المحور الثالث والأخير علي عمل دراسة عددية تطبيقية علي عدد ٣٢ كمرة منحنية لدراسة تأثير التغير في نوع القطاع و زاوية انحناء الكمرة علي سلوكها و تحملها تحت تأثير الأحمال الرأسية. و قد تم التحقق من تطابق النتائج المعملية والنظرية و قد أظهرت الدراسة دقة النماذج النظرية المستخدمة. و من نتائج هذه الدراسة اتضح أن الكمرات الكستيلاتيدية قد اكتسبت جساءة عالية في اللي نتيجة وجود الخرسانة حول العصب عن مثيلاتها غير المركبة، وكذلك فإن زاوية الانحناء تلعب دوراً كبيراً في تحمل الكمرات، وعليه فقد تم وضع تصنيف للكمرات المركبة طبقاً لزاوية انحنائها. و من خلال هذه الدراسة أمكن وضع أسس و إرشادات لتصميم الكمرات المركبة المنحنية حيث تم فهم سلوكها في جميع مراحل التحميل.

Keywords: Composite, Beam, Curved, Rolled, Castellated

1. Introduction

Horizontally curved girders may be used in the construction of modern highway bridges, interchange facilities and balconies. To properly design a system of curved girders, the behavior of a single curved girder should be first understood. However, several approximate approaches have been proposed to overcome this problem [1-7]. The difficulty, that discouraged designers for using curved girders, is the complex mathematical manipulation that arises in describing their behavior. Therefore, it was essential that comprehensive experimental and theoretical investigations on beams with different in-plan curvatures should be carried out. The experimental studies will enable a good understanding of the real behavior of curved beams.

Early research works using composite rolled or castellated steel sections (concrete encasing the web) were focused on the behavior of straight beams with such section

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and subjected to static vertical loading [8]. Later, [9] studied experimentally and analytically the behavior of such straight composite beams subjected to torsional moments. The beam torsional rigidity due to composite action is clearly recognized. The last study by [9] on composite straight rolled and castellated beams drew attention to the importance of composite action and castellation process in increasing the ultimate capacity of straight beams subjected to torsion. On the other hand beams curved in plan are subjected to bending and torsional moments under vertical loading. Yet, there is no mature guidance for the design of curved composite beams with concrete encasing the web. This criticism is also applicable to Eurocode 4 [10].

This research aims to properly identify the effect of concrete encasing the web on the behavior of curved composite beams, using rolled or castellated sections. The experimental study was carried out on seven full-scale beams. All beams have been tested under the same end conditions. A concentrated load has been applied at mid-span of the beams. Finite element models using ANSYS program are also analysed.

Comparisons between the experimental and theoretical analyses show good agreement. More details about the research results can be found in [4].

2. Research significance

This research aims to: study the effect of composite action in enhancing the torsional rigidity of curved beams, investigate the influence of the castellation on the behavior of composite beams and understand the behavior of composite rolled and castellated I-beams with different angles of curvatures.

In order to achieve these goals, an experimental program was carried out. Also, finite element models for bare steel and composite curved beams were proposed. Then, a parametric study of different types of beam cross-sections with various angles of curvatures was conducted.

3. Experimental study

3.1. Beam specimens and test procedure

The seven tested beams were subjected to mid-span concentrated vertical load at the center of the top flange. table 1 and fig. 1 demonstrate the geometrical dimensions of the tested beams. These beams are classified as follows:

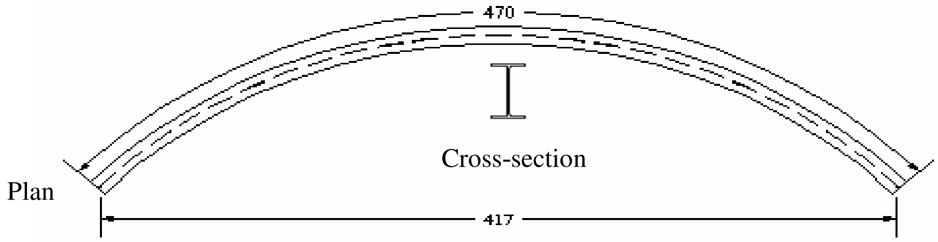
1. Castellated Composite Straight beam (CCS).
2. Rolled Bare steel Curved beam IPE200 with angle of curvature $\theta = 90^o$ (RBC90).
3. Rolled Composite Curved beam with $\theta = 90^o$ (RCC90).
4. Castellated Bare steel Curved beam with $\theta = 90^o$ (CBC90).
5. Castellated Composite Curved beam with $\theta = 90^o$ (CCC90).
6. Castellated Composite Curved beam with $\theta = 60^o$ (CCC60).
7. Rolled Composite Curved beam with $\theta = 60^o$ (RCC60).

Table 1
Dimensions of test specimens [mm]

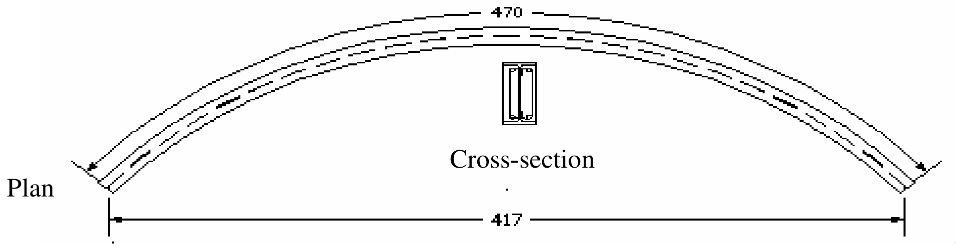
Beams	h	b_f	T_w	t_f	*L	** l	θ^o
B1 (CCS)	266	100	5.65	9.11	4283	4283	0
B2 (RBC90)	202.4	100	5.87	9.79	4700	4170	90
B3 (RCC90)	202.6	100	6.28	9.45	4700	4170	90
B4 (CBC90)	267.2	100	5.84	9.52	4730	4200	90
B5 (CCC90)	266.8	100	5.74	9.82	4730	4200	90
B6 (CCC60)	266.9	100	5.79	9.53	4550	4310	60
B7 (RCC60)	203.8	100	5.69	9.23	4550	4310	60

* L is curvature length of the beam.
** l is straight length of the beam.

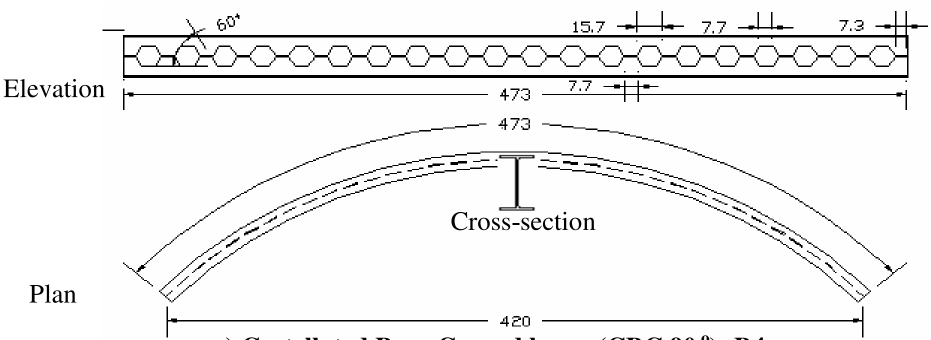
Moreover, the experimental study contains complementary control tests for steel and concrete materials. The tension tests performed on specimens of the steel beams gave actual experimental values for the ultimate and yield stresses of the steel material as 50.7 and 39.2 kN/cm^2, respectively. The cubic strength of the concrete after 28 days was 3.58 kN/cm^2.



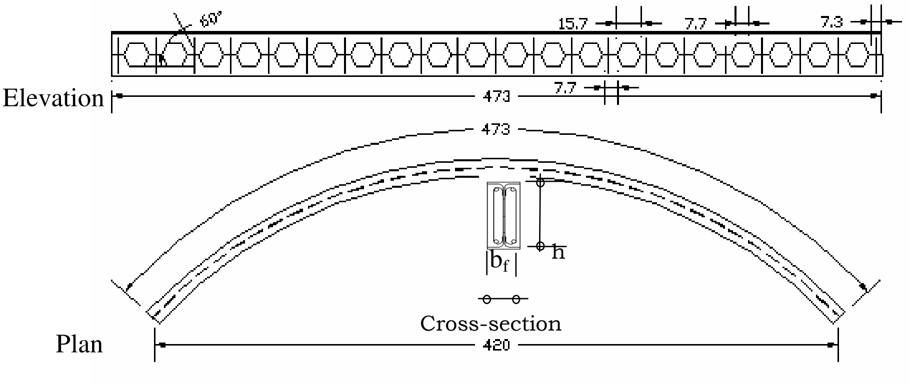
a) Rolled Bare Curved beam (RBC 90^{o}); B2



b) Rolled Composite Curved beam (RCC 60 or 90^{o}); B7 and B3



c) Castellated Bare Curved beam (CBC 90^{o}); B4



d) Castellated Composite Curved beam (CCC 60 or 90^{o}); B6 and B5

Fig. 1. Geometry of tested curved beams.

The supports of the tested beams are shown in Photo 1. The applied load was increased gradually up to failure. The vertical deflection and normal strain in the steel flanges were registered at each step of loading. Figs. 2 and 3 present the gauge locations.

3.2. Experimental results

In this paper, the experimental study focused on some important parameters. These parameters are:

1. the load deflection relationship till failure,
2. normal strain along the beam length,
3. profile of the beams (castellated or rolled),
4. the influence of concrete around the web on the behavior of the tested beams and
5. effect of change in the angle of curvature (θ°).

Table 2 shows the summary of the different parameters arranged in groups

Photo 1. Typical support for tested beams.

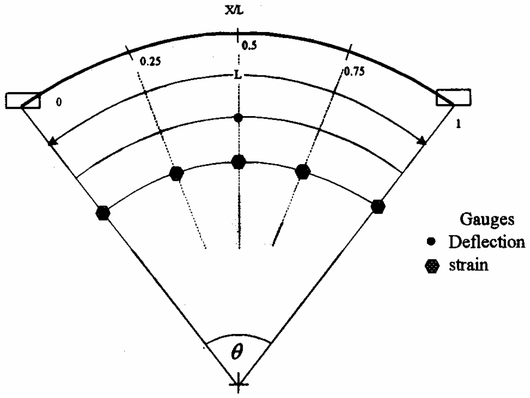


Fig. 2. Gauge locations for beams B4 and B5.

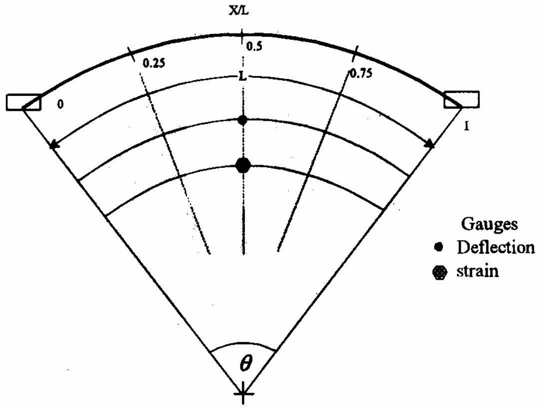


Fig. 3. Gauge locations for beams B1, B2, B3, B6 and B7.

Table 2
Summary of experimental variables

Groups	Beams	Beam types	θ°	Variable
1	B2, B3, B4 & B5	Rolled bare, Rolled composite Castellated bare, &Castellated composite	90	Beam type
2	B7 & B6	Rolled composite &Castellated composite	60	Beam type
3	B1, B6 & B5	Castellated composite	0, 60 & 90	Angle of curvature
4	B3 & B7	Rolled composite	90 & 60	Angle of curvature
5	B4	Castellated bare	90	Strain along length

3.2.1. Results of groups 1 and 2

From figs. 4 -6, it can be noticed that the load – deflection curves for curved composite rolled beams and the corresponding castellated ones are, more or less, the same in the elastic range; (B3 compared with B5 and B7 compared with B6).

From fig. 4, it can be recognized that the load- deflection curves, in the elastic range, of the curved bare steel rolled (B2) and curved castellated beams (B4) are identical. Also, there is a significant difference at the elastic and the plastic range, for load- deflection relationship curves, between the bare steel

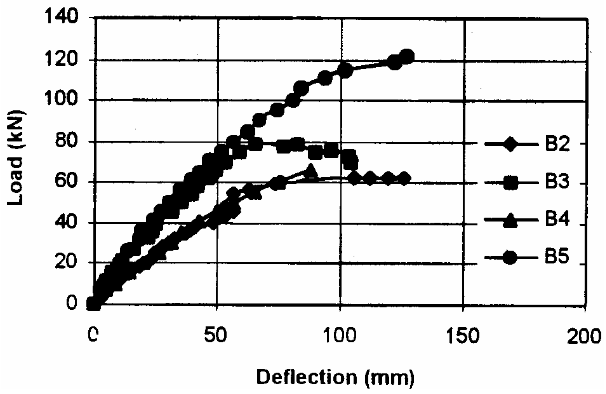


Fig. 4. Load-deflection relationship (groups 1 and 5).

beams (B2 and B4) and the corresponding composite ones (B3 and B5), respectively.

In other words, it can also be emphasized that the stiffness of the curved composite rolled beams (B3 or B7) is slightly less than the corresponding curved composite castellated beams (B5 or B6), respectively. Also, the stiffness of curved rolled bare steel beam (B2) is, more or less, the same as curved castellated steel beam (B4).

It can be observed from figs. 5-a and 7-a, for the same angle of curvature, that bare steel castellated beams have the biggest ultimate loads compared with rolled ones. Also, the composite beams have the biggest ultimate loads when compared with the bare steel beams. So, the ultimate loads of the composite castellated beams with web encased in reinforced concrete (B5 and B6) have the greatest ultimate capacities. From another point of view, the relative toughness ratio (TR= A_u / A_P) may be considered as a reasonable measurement to represent the ductility of the beam. A_u is the area under the load-deflection curve till the ultimate point and A_p is the area under the load-deflection curve till the proportional limit

Similarly, figs. 5-b and 7-b show different values of toughness ratios for tested beams groups 1 and 2. It can be distinctly seen that the ductility is higher for rolled beams (B2, B7) than the corresponding castellated ones (B4, B6) for both bare steel and composite beams, respectively. The ductility is higher for composite curved beams than that of bare steel curved ones. For beams curved in plan, the vertical load produces, in addition to vertical deflection, lateral displacement and twist. This introduces second order forces for open sections (bare steel beams), which cause further deformations. These further deformations cause quick failure to the bare steel beams after the proportional limit. This means that the bare steel curved beam develops less ductility than the composite beam.

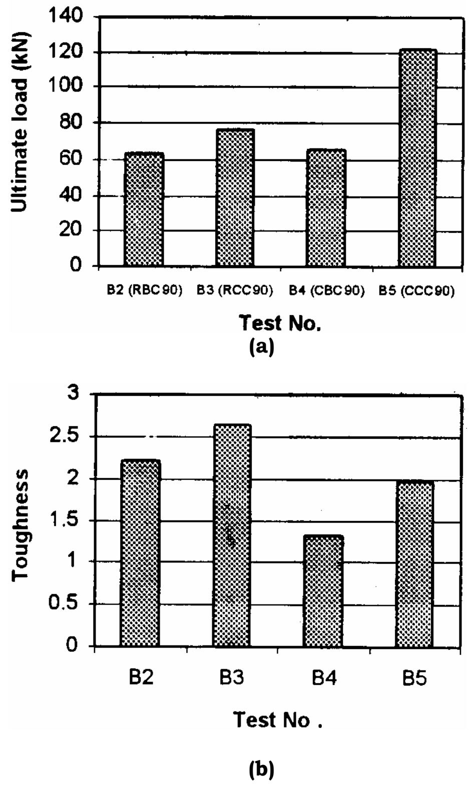


Fig. 5 Ultimate load and toughness (groups 1 and 5).

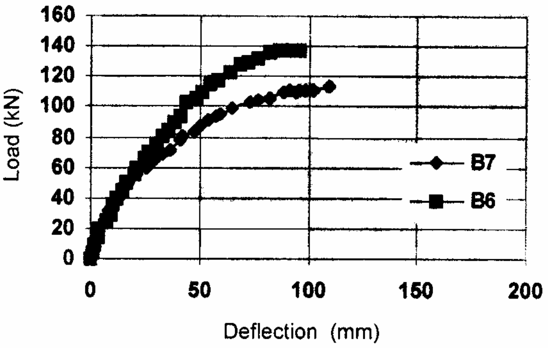
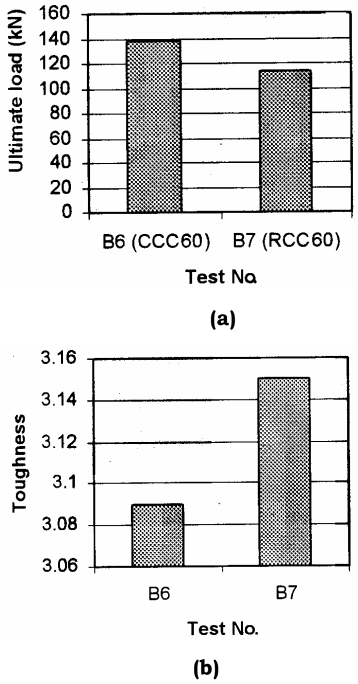


Fig. 6. Load-deflection relationship (group 2).



(a)

(b)

Fig. 7. Ultimate load and toughness (group 2).

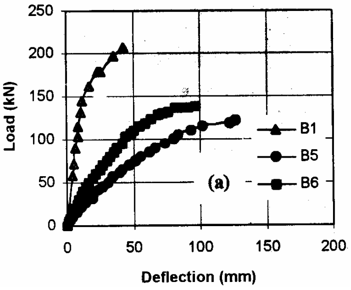


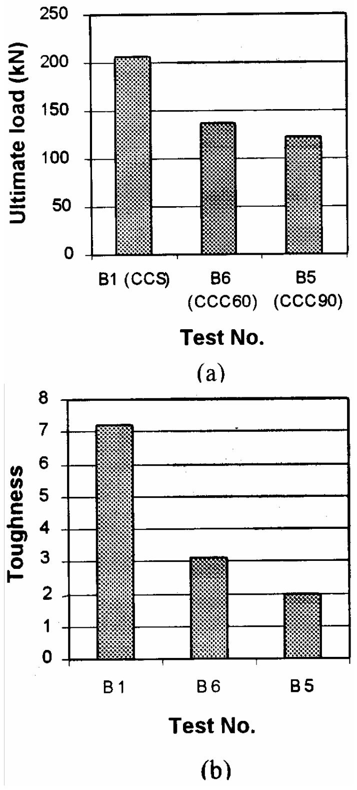
Fig. 8. Load-deflection relationship (group 3).

3.2.2. Results of groups 3 and 4

For groups 3 and 4, where the variable is the angle of curvature, Figs. 8 and 10 show that the value of the ultimate load is the maximum for composite castellated beam B1 followed by beams B6 and B5. Similarly, the bare steel castellated beam B7 has a higher value of ultimate load than beam B3. This means that the ultimate load decreases with the increase of the angle of curvature. This phenomenon should be expected since the geometrical nonlinearity is higher for members with large in – plan curvatures. Hence, these beams have lower ultimate strengths. From figs. 9-b and 11-b, it can be clearly seen that the ductility decreases with the increase of the angle of curvature. This is because the geometrical nonlinearity is higher for members with large curvature in the plan, which enforces these beams to rapid failure just after the proportional limit. Thus, these beams have lower ductility.

3.2.3. Longitudinal strain along the beam length

Fig. 12 shows the strain distribution along the beam length for beam B4. This shape of the distribution of the strain along the beam length is a typical one for all curved beams with the same end conditions. More details about the strain distribution for upper or lower flanges of curved beams can be found in ref. [8]. Experimental results also showed that the failure modes of all curved beams are governed by lateral-torsional deformations, where the lateral displacement increases rapidly once the maximum load is reached. This is particularly true in case of beams with larger in – plan curvatures.



(a)

(b)

Fig. 9. Ultimate load & toughness (group 3).

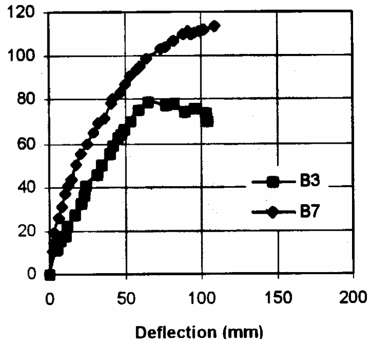
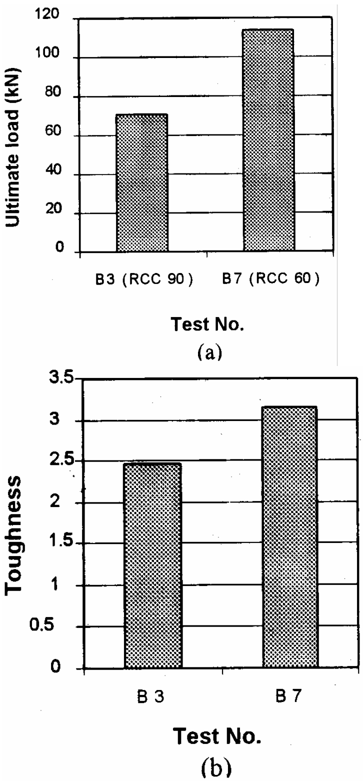


Fig. 10. Load-deflection relationship (group 4)



(a)

(b)

Fig. 11. Ultimate load & toughness (group 4).

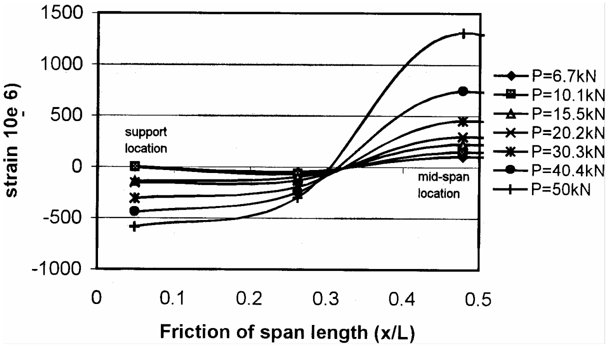


Fig. 12. Experimental data of internal upper strain gauges along B4.

4. Theoretical analysis

Finite element program ANSYS [11-13] was used to perform the analysis of curved composite rolled and castellated beams. Since the applied loads and support conditions are symmetrical, half of the beam length only is analyzed for both straight and curved beams. It was necessary to establish a local coordinate system whose orientation differs from that of the global system to reflect the actual behavior. A denser mesh is required to model the curved member, and it is particularly essential in case of castellated beam.

In addition, it is necessary to model geometrical nonlinearity besides material nonlinearity by using large deformation response which is available in ANSYS software. The Newton Iteration Technique is used to solve the nonlinear response in the loading bath. Several tools were furnished by ANSYS software to improve the convergence performance in the analysis such as use of automatic time stepping, solution control and bi-section.

The material nonlinearity for steel was modeled using two methods to define the stress strain relationship (BISO&MISO), while to give cracking and crushing capability to concrete; (CONCR) the concrete material data available in ANSYS was used [11-13]. Since the bolts stay in close contact with their connection plates through all load steps, they were defined as continuous with the end-plate nodes. This means that the bolts share their node with the end plate. One of the interesting and at the same time difficult aspect of treated beams is the unpredictability of the actual support condition at the end plate. Therefore, using the coupling equations available in the program as shown in fig. 13 solved the boundary at the end plate. The supports were modeled by using an end plate model as shown in fig. 13. The lower end plate is prevented from moving or rotation (fixed) and the upper end plate is coupled with the upper flange of the tested beam.

4.1. Description of theoretical models

Two types of models are presented:

1- Rolled and castellated bare steel beams.
2- Rolled and castellated composite beams.

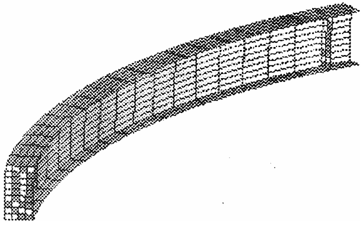
For bare steel beam model, plastic quadrilateral shell elements, so called shell 43 in the library of the program, were used to model the beam web, the flange, the end plates of the beam supports and the stiffeners. Bolts were idealized using the plastic straight pipe element called pipe20.

fig. 13 shows a typical finite element mesh used in the present analysis of bare steel beams. The elements used to model the composite beams are listed in table 3.

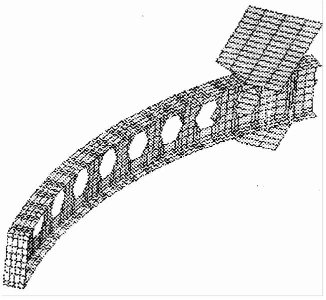
4.2. Verification of the theoretical models

The treated rolled and castellated composite beams are modeled in 3-D using the actual measured dimensions.

The resulting data from the experimental and theoretical studies are shown in figs. 15, 16 and 17. These figures represent the load versus deflection relationships. The ultimate loads obtained from the present experimental tests and those out of the predicted models are shown in table 4. The table shows that the agreement between the ultimate loads from the present finite element models and the test results is fairly good.



a) Finite element mesh for curved I-beam



b) Finite element mesh of curved castellated beam

Fig. 13. Typical finite element mesh used in the present analysis.

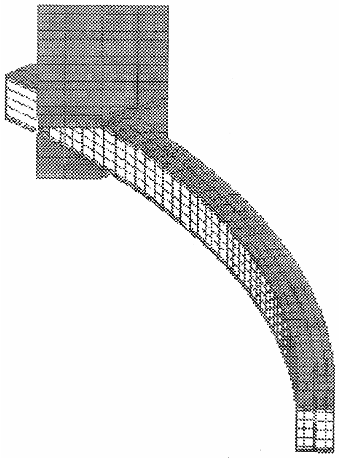


Fig.14. A typical finite element mesh for modeling of composite beams.

Table 3
Modeling of the composite beams

Component c a beam	Used element	Description
Steel web and flange	Solid 45	Plastic element with 3 degrees of freedom (ux, uy,uz)
Concrete around the web	Solid 65	Capable of crack in tension and crushing in compression.
Reinforced steel	Link 8	Axial tension-compression element with 3 degrees of freedom (ux, uy, uz)

5. Parametric study

The finite element model has been used to investigate the nonlinear inelastic behavior of composite rolled and castellated beams curved in plan. In studying the behavior of horizontally curved composite rolled and castellated I-beam, 32-numerical applications were carried out using ANSYS finite element simulation program. The nonlinear inelastic behavior of a group of curved beams with the included angle θ ranging from zero 0^o to 90^o was investigated using the present finite element model.

The geometry and the material properties of these curved beams are the same as those of the experimentally tested ones previously described. The developed length of each curved beam is L = 2.0 m, and the initial curvature increases from R = 57.3 m for $\theta = 2^o$ to R = 1.273 m for $\theta = 90^o$. These beams are simply supported at both ends with lateral displacement and twist prevented. They are subjected to vertical concentrated load at the center of the top flange.

Elastic-plastic load –deformation curves are usually used to demonstrate the relationship between the structural behavior and loading, in addition to access the strength of a structure. The variation of the dimensionless central vertical displacements of these curved beams (u_z/h) and load ratios (load of curved beam divided by load of corresponding straight one) P_c/P_s are shown in figs. 18 to 21.

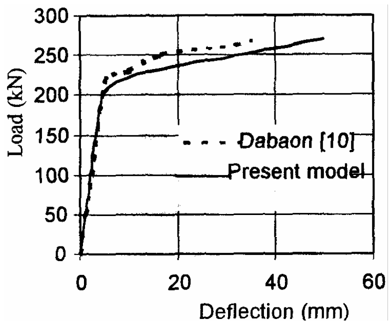


Fig. 15. Load - deflection relationship [14].

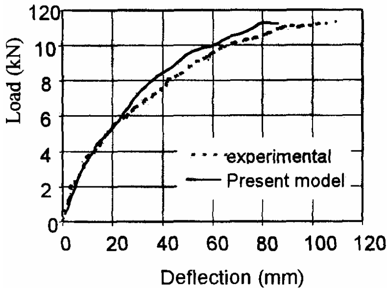


Fig. 16. Load-deflection relationship for tested beam RCC60 (B7).

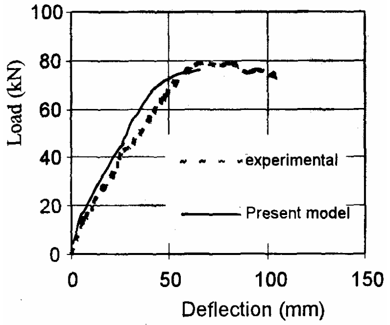


Fig. 17 Load-deflection relationship for tested beam RCC90 (B3).

Table 4
Comparison between the results of the present experimental and predicted theoretical results

Beam (1)	Pu (kN)		(2)/(3)
	Present experimental (2)	Present models (3)	
CCS (B1)	206	210	0.98
RBC90 (B2)	63.4	65.85	0.963
RCC90 (B3)	78.5	76	1.032
CBC90 (B4)	65.9	68	0.97
CCC90 (B5)	122	120	1.017
CCC60 (B6)	137.9	141.6	0.97
RCC60 (B7)	113.4	112.4	1.01

Table 5
List of 32 – cases studied

Groups	No.of beams	Type of section	Abbreviation
Group 1	1 7	- Rolled bare straight - Rolled bare curved	rbs rbc (θ)
Group 2	1 7	- Castellated bare straight - Castellated bare curved	cbs cbc(θ)
Group 3	1 7	- Rolled composite straight - Rolled composite curved	rcs rcc(θ)
Group 4	1 7	- Castellated composite straight - Castellated composite curved	ccs ccc(θ)

Where θ (angle of curvature) = 2, 5, 10, 20, 40, 60, 90°. Small letters are used, to describe the abbreviation for the analytical treated beams, instead of the capital letters used for the experimental tested ones.

Where P_c is the load of curved beam during the loading processes, P_s is the ultimate load of straight beam, u_z is the vertical deflection and h is the height of the beam.

Figs. 18 to 21 present the dimensionless load- deflection relationships for the studied beams.

For groups 1 and 2, it can be noticed that for curved beams with angle of curvature $\theta < 2°$ the ultimate load carrying capacities decrease with a small amount with the increase of the angle of curvature to reach nearly 80% of the straight beam carrying capacity. This is because the beams in this case are almost straight. For curved beams with angle of curvature 2° < θ < 20° the capacity of the beams has a big decrease with any small increase in the angle of curvature to reach nearly 50% of the straight beam carrying capacity at $\theta = 20°$.

The effect of torsion in this case increases and hence the deflection of the beams increases and the behavior of the beam becomes nonlinear. For curved beams with angle of curvature θ > 20° the capacity of the beam decreases with a smaller ratio compared with that of curved beams with angle of curvature 2° < θ < 20° since the torsion is the major effective factor in this stage and the failure of the beam is due to geometrical nonlinearity.

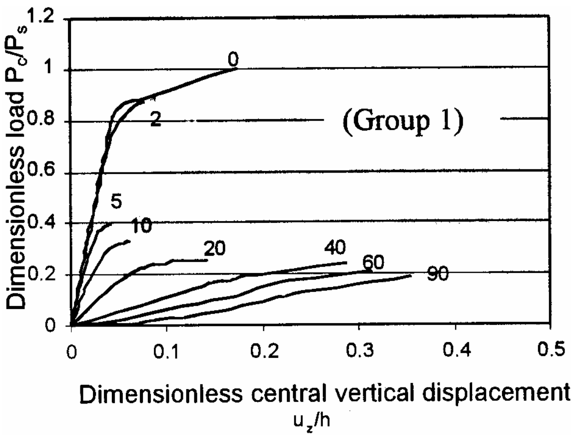


Fig. 18. Ultimate load and deflections for rolled bare steel beams: P_c/P_s versus u_z/h.

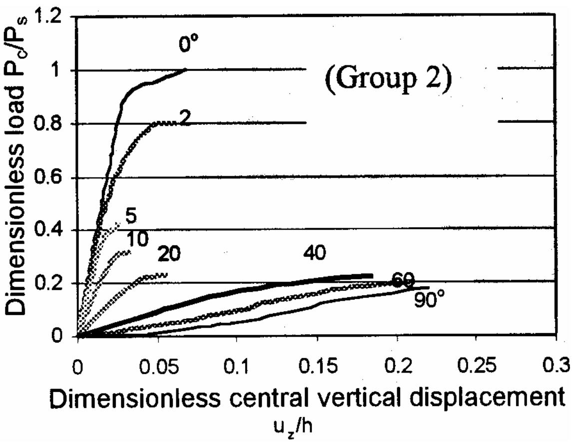


Fig. 19. Ultimate load and deflection ratios for castellated bare steel beams: P_c/P_s versus u_z/h.

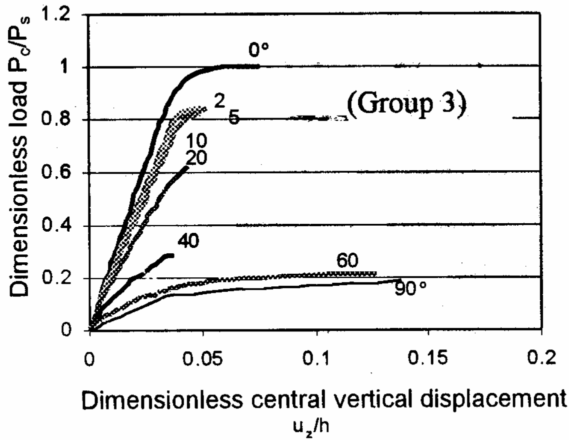


Fig. 20. Ultimate load and deflections ratios for rolled composite beams: P_c/P_s versus u_z/h.

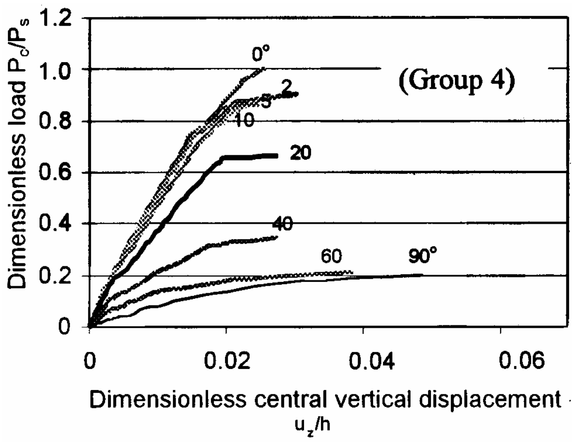


Fig. 21. Ultimate load and deflections ratios for composite composite beams: P_c/P_s versus u_z/h.

For groups 3 and 4, the ultimate load carrying capacity of curved beams with angle of curvature θ < 10° has a small decrease with the increase of the angle of curvature and there is a slight difference between the dimensionless load-deflection relationships. For curved beams with angle of curvature 10° < θ < 60° the small increase of the angle of curvature causes a big decrease in the ultimate capacity while curved beams with angle of curvature θ > 60° have small decrease in the ultimate load with the increase of the angle of curvature.

It can be also noticed that the ratio of the vertical deflection to the depth is small for castellated beams whether bare or composite compared with rolled beams. On the other hand, there is a significant difference between the ratio of vertical deformation to the depth for composite beams whether rolled or castellated compared with bare steel beams. Thus, geometric nonlinearity has a big effect on curved bare solid steel beams and has a small effect on composite castellated beams.

It can be noticed that the central lateral displacement is bigger for bare steel beams, (groups 1 and 2) compared with composite beams, (groups 3 and 4). For groups 3 and 4, the lateral displacement is almost small, it does not exceed 3% of the flange width, and thus the geometrical nonlinearity can be neglected for composite beams (see ref. [4]).

For groups 1 and 2, the central lateral displacement increases with the increases of angle of the curvature and reaches about 18% of the flange width. Geometrical nonlinearity cannot be neglected in this case. Moreover, the central lateral displacement decreases with the increase of the angle of curvature due to the coupling between the vertical displacement and the rotation of the section. This does not mean that the geometrical nonlinearity decreases because the vertical displacement and the rotation are large.

6. Summary and conclusions

An extensive parametric study was conducted on 32-bare and composite steel beam models analyzed using the finite element technique. These models were verified by test results from the present experimental work and from the results of previous researches. Based on this study the following conclusions are drawn:

- The behaviors, of curved beam whether bare steel or composite, depends mainly on the angle of curvature in plan.
- With respect to the angle of curvature, three categories of curved beams may be distinguished. The first one where the behavior of curved beams is similar to that of the straight ones as the bending is the major action. In the second category, both bending and torsion are important. In this case, as the angle of curvature increases significance of bending decreases and that of torsion increases. However, the inelastic lateral buckling phenomenon can be observed since the lateral displacements arise. In the third category, the torsion is the major action in which case the nonlinear inelastic behavior is developed early. Moreover, the vertical displacement and angle of twist are relatively large.
- The composite action enhances the ultimate capacity of the beam, which can be clearly observed in the first category. The range of this category is extended from 0° < θ < 2° for bare steel beams to 0° < θ < 10° for composite ones.
- For large angles of curvature (in category three) the deformations of composite beams are smaller compared with those of bare steel beams whether rolled or castellated.
- The castellated steel profiles, which are commonly used in the design of light-weight beams, are successfully suggested to be used in the design of typical composite beams in which the web is encased in reinforced concrete.
- The load carrying capacity of beams curved in plan increases with a small amount in case of angles of curvature θ > 10°. In this sense, the most efficient use of composite curved beams (web encased in concrete) is at small angles of curvature (θ < 10°). Despite of that, composite curved beams could be used in case of large angles of curvature where negligible deformations are required.
- Composite castellated beams with webs encased in reinforced concrete have the greatest ultimate capacity.
- The ductility is higher for rolled curved steel beams than for castellated ones. The same phenomenon acts in case of composite action.
- The ductility is higher for composite curved beams than for bare steel ones. This is due to the excessive rotation of curved steel beams.
- The ductility decreases with the increase of the angle of curvature.
- For large angles of curvature, a slight difference in the stiffness of composite curved rolled and castellated beams is observed as well as for bare steel rolled and castellated beams.

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