# A new method for optimization of high voltage electrode surfaces using genetic algorithms

Mohamed Abd-Elsalam and M. M. Abouelsaad
*Electrical Eng. Dept., Faculty of Eng. at Shoubra, Zagazig University, Benha Branch, Cairo, Egypt*

Genetic Algorithms (GAs) have, recently, emerged as a powerful search and optimization technique for highly nonlinear problems. A new method for the optimization of high voltage electrode contours, based on Genetic Algorithms techniques, is presented. The approach seeks the optimization of the electrode shape in order to achieve a uniform field distribution along the electrode surface while maintaining the maximum field stress at a minimum value. This is accomplished through a properly designed fitness function. A practical example of an axisymmetric single phase GIS bus termination is considered. The shape of the electrode has been taken as a quarter-ellipse to allow more flexibility. The approach is also extended to the overall optimization of the GIS bus termination including its outer enclosure. The proposed method proved to be effective in reducing the effort usually required for accurate solution of such problems.

تعتبر الخوارزميات الجينية من الطرق الحديثة لمعالجة المشاكل الغير خطية. تعرض المقالة طريقة مبتكرة للتصميم الأمثل لأسطح موصلات الجهد العالي باستخدام الخوارزميات الجينية. وتهدف الطريقة إلي تحقيق مجال كهربي منتظم عل سطح الموصل بحيث يظل الحد الأقصى للمجال عند أصغر قيمة ممكنة وهو الذى كان يصعب تحقيقه بالطرق التقليدية. وتم تطبيق الطريقة المقترحة للحصول علي التصميم الأمثل لمثال عملي عبارة عن نهايات الموصلات المستخدمة في المحطات المعزولة بالغازات . كما تم أيضا تطبيق الطريقة للحصول علي التصميم الأمثل للاسطوانة الخارجية لنهايات الموصلات المستخدمة في المحطات المعزولة بالغازات. وقد وجد أن هذه الطريقة المبتكرة فعالة لتقليل الجهد والوقت اللازمين للحصول علي تصميم أمثل ودقيق لمثل تلك المشاكل العملية.

**Keywords:** High voltage electrodes, Electric field, Optimization, Genetic algorithms

## 1. Introduction

Recently, Artificial Neural Networks (ANN) have been introduced for optimization of electrode contours of high voltage apparatus [1,2]. Although these techniques offer some advantages, compared with traditional methods [3-5], they still suffer some limitations. Two main goals are usually sought when the design of electrode and insulator contour in a HV apparatus is considered. First, the maximum field stress along the contour should have a minimum limiting value. Second, the field distribution along the contour should be as uniform as possible. ANN techniques, while seeking to minimize the maximum field stress, do not meet the second requirement [1]. In addition, personal experience and judgement are repeatedly required during the optimization process and may result in reduced efficiency and accuracy. To avoid these limitations, a new method for the optimization of high voltage electrode contours, based on Genetic Algorithms techniques, is presented.

Genetic Algorithms are general-purpose stochastic search techniques that rely on the principle of evolution via survival of the fittest [6-11]. GAs computational schemes have been successfully applied in various areas of high voltage and electric power engineering including optimization of the charge simulation method [6], voltage profile optimization [7], optimal capacitor placement [8] and high voltage field assessment [9].

The objective of the present work is to use Genetic Algorithms for optimization of electric field distribution along high voltage electrode surfaces. The electrode contours have been taken as quarter ellipses because an elliptic shape is more flexible than a circular one. A practical example which describes the termination of a single phase GIS bus; an axisymmetric system, is presented. The

Charge Simulation Method (CSM) is used for field computation [12]. The role of the GAs is to determine the parameters which define the shape of the quarter ellipse, i.e. the shape of the electrode contour through the maximization of a relevant fitness function. The fitness function employed in the present work is the square root of the accumulated squared field deviations from a prespecified maximum field value. A genetic algorithm model is developed to achieve the above objectives. As the dimensions of the outer enclosure may influence the performance of the GIS bus termination, the approach is extended to the optimization of the GIS bus termination including its outer enclosure. The results of the optimization process demonstrate the effectiveness and accuracy of the proposed method.

## 2. Method of analysis

Fig. 1 shows a typical GIS bus termination. It is an axisymmetric system. The radii *r* and *R* of the HV conductor and the grounded external cylinder are fixed. The height *H* is also specified. The contour of the bus termination to be optimized is *G*. The major part of *G* is taken as a quarter ellipse, with semi-axes $x_1$ and $x_3$. When the distance $x_2$ is given, $x_3$ is determined automatically. The contour between the central axis and the beginning of the quarter ellipse is taken as a straight line perpendicular to the axis, that is, this surface is taken as a disc of radius $x_2$. The voltage of the conductor is assumed to be unity. $R$ = 2.75, $r$ =1 and $H$ = 1.5 units. N in fig. 1 is a Neumann plane on which the normal component of the flux density is zero. That is, equipotential lines are all perpendicular to N.

A typical stress distribution over contour *G*, obtained using the CSM, is shown in fig. 2 in per unit. It is observed that the stress distribution is non-uniform and is characterized by the presence of more than one peak along the electrode contour as also observed by [1]. Normally, the point of the maximum field stress lies on the contour G. The distance *S* of any point, over the electrode contour, is measured from the central axis and is expressed in per unit. At the optimum values of $x_1$ and $x_2$, the value of the maximum field

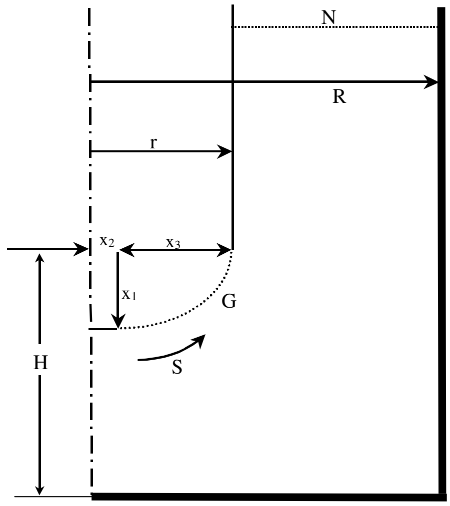



Fig. 1. Single phase GIS bus termination.

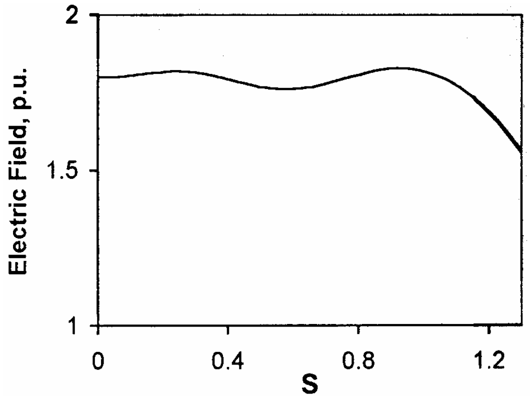



Fig. 2. A typical stress distribution on the single phase GIS bus termination shown in fig. 1.

stress F on the electrode contour is minimum.

The charge simulation method is used for field computation. For a given charge distribution, the potential $\phi_j$ at a point on the electrode surface is a summation of the potentials resulting from the individual charges, i.e.,

$$\varphi_j = \sum_{i=1}^{n} P_{ji} Q_i \ . \qquad (1)$$

where $n$ is the number of charges in the system. $P_{ji}$ are potential coefficients which are functions of the coordinates of a point on the electrode boundary ($r_j$ , $z_j$) and the coordinates of the source charge ($r_i$ , $z_i$). The vector of unknown charges, *Q,* is computed from the matrix equation:

$$\mathbf{P}\,\mathbf{Q} = \mathbf{V}. \tag{2}$$

and **V** is the vector of known potentials.

In order to simulate the GIS electrode, a set of *n* charges is chosen. The charges are arranged along the central axis. Two parameters $f_1$ and $f_2$ are used to determine both the initial location and axial distribution of the charges in order to obtain accurate potentials on the boundaries, using a suitable error function. The first parameter $f_1$ determines the location of the first charge which is usually located close to the center of the ellipse. The parameter $f_2$ is a constant, which controls the axial distribution of the charges in an exponential manner.

An exponential function in the form;

$$z(i) = z(i-1) + f_3\, Exp(f_2\; i / n)\,, \qquad i = 2 : n \tag{3}$$

is proposed, where $z(i)$ is the axial location of the $i^{th}$ charge, and $z(1)$ is the axial location of the first charge and is equal to $f_1$. The parameter $f_2$ is a real value equal or greater than zero. As the distance between the first and last charges may vary, the location of any other charge is adjusted using the factor $f_3$.

The error function used is simply the accumulated squared error which has the form [3,4,13]:

$$U = \sum_{j=1}^{m} \left[ V - \phi_j(r,z) \right]^2 . \tag{4}$$

Where *m* is the number of check points on the electrode boundary. The proper values of $f_1$ and $f_2$ are found when the value of *U* reaches a prespecified small value $\varepsilon$ ; usually in the order of $10^{-3}$ [13]. The location of the Neumann plane N is taken equal to approximately 3*H* so that the value of *U* is within the prespecified value of $\varepsilon$.

Examination of different stress distributions for the GIS bus termination, similar to the one shown earlier in fig. 2, revealed that the field distribution along the electrode surface is generally nonuniform and may contain more than one peak. GAs are well suited for the solution of such problems. The parameters $x_1$ and $x_2$ may be coded as chromosomes and the GAs will seek their optimum values through a properly designed fitness function to achieve uniform field distribution along the electrode contour. In addition, the fitness function may entail a term to maintain the maximum field stress at a minimum value. An algorithm, which satisfies the above requirements, is described in the following steps:

1. Determine the domain, in which the optimum dimensions of $x_1$ and $x_2$, using a few CSM trials.
2. Enter these ranges to the GAs as inputs to generate initial random values of $x_1$ and $x_2$.
3. For each call to the CSM routine by GAs, for a given $x_1$ and $x_2$, the proper values of $f_1$ and $f_2$ are determined locally within the CSM routine. The CSM will then, produce the field distribution along the electrode surface for a prespecified potential accuracy.
4. The GAs will evaluate a fitness function and modify $x_1$ and $x_2$ accordingly.
5. Repeat steps 3-4 for a prespecified number of times.

The fitness function should satisfy the required objectives which are the uniformity of the field distribution and the minimization of the maximum stress along the elliptic part of the electrode. A proposed expression is then;

$$F.F. = 1/(1+U_2), \tag{5}$$

where $U_2$ is given by;

$$U_2 = \sqrt{\sum_{j=1}^{m_1} \left[ E_j - E_1 \right]^2} . \tag{6}$$

Where $E_j$ is the electric field at point j on the elliptic part of the electrode, $m_1$ is the number of check points on that part, and $E_1$ is the minimum value of the maximum electric field. The value of $E_1$, within the domain of $x_1$ and $x_2$, is determined using genetic algorithms. This is accomplished using the same algorithm procedures described above with an alternate fitness function, *F.F.*, which takes the form:

$$F.F. = 1/(1+ E_{max}). \tag{7}$$

Where $E_{max}$ is the maximum electric field value on the elliptic part of the electrode surface for a given $x_1$ and $x_2$. The GAs will seek the maximization of eq. (7), i.e. the minimum value of $E_{max}$ . This minimum value is the required field stress $E_1$ appearing in eq. (6).

The problem is now reduced to the determination of the optimum values of the parameters $x_1$ and $x_2$ subject to the satisfaction of the fitness function given by eq. (5).

In the implementation procedure, the GAs will first seek the maximization of eq. (7) to determine $E_1$. Then, the GAs will seek the maximization of eq. (5) to determine the optimum values of $x_1$ and $x_2$. The following section describes the genetic algorithms procedures and operators implemented in the present work.

## 3. Genetic algorithms

Genetic Algorithms are search and optimization techniques based on the theory of natural selection [10,11]. An initial population of a constant size is created from a random selection of the parameters in the parameter space. Each parameter set represents the individual's chromosome. Each of the individuals is assigned a fitness value based on how well each individual chromosome allows it to perform in its environment.

Three basic operations occur in GAs to create the next generation: selection, crossover and mutation. More fit individuals are selected for mating while less fit ones diminish. Parents create a child, through the crossover operation with a chromosome set that is some mix of the parents' chromosomes. Then there is a small probability that one or more of the child's chromosomes will be mutated; thus introducing new individual into the population. The process of mating and child creation is continued until an entirely new population of the same size is generated. Improvement in the selection scheme can be achieved by introducing elitism into the selection process. The elitist strategy copies the best member of each generation into the succeeding generation. This strategy may increase the speed of domination of a population by a super individual and thus improve the search at the expense of a global perspective, but on balance it tends to improve the GA performance [10,11]. The genetic operators implemented in the present analysis are given in Appendix A.

## 4. Application and results

In order to demonstrate the merits of the proposed method, an axisymmetric single phase GIS bus termination is considered. The shape of the electrode has been taken as a quarter-ellipse. The number of simulating charges is 35 charges. The first is a point charge and the remaining 34 charges are semi-infinite line charges. The number of generations, $Ng$, used in the genetic model is 60 and the population size $Nc$ is 5. The crossover probability is 0.5 and the mutation rate is 0.02. Each of the parameters $x_1$ and $x_2$ is coded as a 15-bit binary string for a total chromosome length of 30 bits. The fitness function to be maximized is given by eq. (5) The parameter $x_1$ is selected to lie between 0.6 and 0.75, while $x_2$ varies between 0.01 and 0.2. The parameters of the CSM, $f_1$ and $f_2$, are selected to vary between -0.1 to +0.1 and 0 to 3, respectively. The minimum value of the maximum field stress, $E_1$, within the considered domain is found to be 1.75.

Different fitness functions are proposed and tested. The first one was given earlier by eq. (5). Another proposed function is based on the average of the electric field values, $Ea$, at the contour points and may be expressed by;

$$F.F. = 1 / ABS. ( Ea - E_1), \tag{8}$$

The last fitness function tested is given by;

$$F.F. = 1 / [ ABS. ( E_H - E_L) / 2 - E_1]. \tag{9}$$

Where $E_H$ is the highest field value of the distribution and $E_L$ is its lowest value.

The results for the electric field distributions along the electrode contour for the various fitness functions are shown in fig. 3 as A, B and C, respectively. It can be seen that distribution A obtained using eq. (5) is more uniform than both B and C obtained using eq. (8) and (9), respectively. Table 1 shows the results for the maximum and mean field values for each case as well as the standard deviation and the values of $x_1$ and $x_2$. The maximum value of distribution A is 1.767 (0.97% higher than $E_1$) and that of distribution B is 1.93 (10.29% higher than $E_1$) and finally for distribution C, the maximum field is 1.953 (11.6% higher than $E_1$). This, again, shows that the fitness function of eq. (5) is better in realizing the required objectives than the other two functions. The standard deviation of distribution A is smaller than that of B or C and results in a more uniform field distribution along the contour.

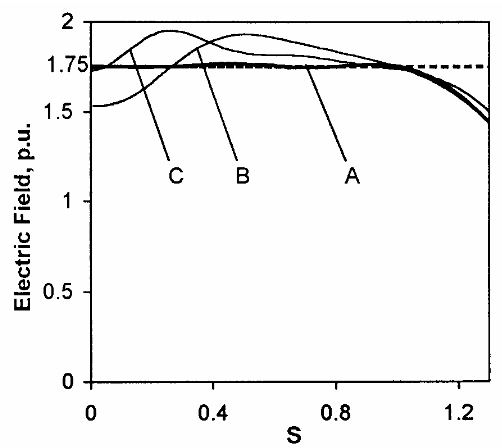



Fig. 3. Field distributions for different fitness functions, A: using eq.(5), B: using eq.(8), C: using eq.(9).

The computation time on a 350 MHz personal computer is 120s for 60 generations to obtain a complete solution. The above results demonstrates that the present approach is capable of achieving, both accurately and efficiently, the objective of a uniform stress distribution along the electrode contour while maintaining the maximum stress at a required minimum value.

An example of the convergence of the electric field distribution towards its final profile at three different generations $Ng$ = 5, 20, 60 and for $Nc$ = 3 and $Pc$ = 0.3 is shown in fig. 4. It can be seen that as the number of generations increases, the field distribution tends to become more uniform with its mean value approaching the required maximum stress $E_1$. The progress of convergence of the fitness function towards its final value is shown in fig. 5. The influence of the various genetic operators such as the population size $Nc$ and the crossover probability $Pc$ is studied and results are shown in fig. 5. The appropriate values of $Nc$ = 5 and $Pc$ = 0.5 which produce the highest value for the fitness function are used in the present work. It should be noted that convergence occurred at approximately 50 generations.

In order to demonstrate the generality of the method, other values for the required maximum stress $E_1$ on the electrode contour are considered. This is of practical importance as it may be possible to design the GIS bus termination at values of maximum field stress other than the minimum. Thus; three different values for the maximum field stress are selected which are 1.75, 1.79, 1.83. For each value, the algorithm is required to produce a uniform field distribution along the electrode contour while keeping the maximum field stress of the distribution at the required prespecified value. The values of $E_1$

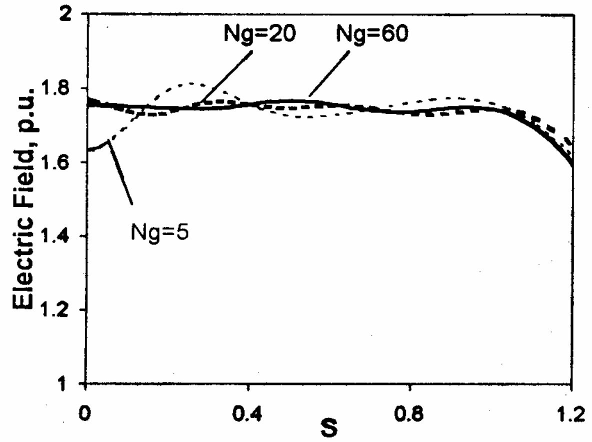



Fig. 4. Convergence of the field distribution at different number of generations

Table 1
Results for different fitness functions

| Distribution | Maximum stress, $F$ | Mean stress | Standard deviation(%) | $x_1$ | $x_2$ |
|---|---|---|---|---|---|
| A | 1.767 | 1.718 | 8.9 | 0.687 | 0.029 |
| B | 1.93 | 1.75 | 17.2 | 0.725 | 0.229 |
| C | 1.953 | 1.774 | 13.57 | 0.745 | 0.045 |

(1.79 and 1.83) are arbitrarily chosen to be higher than the minimum value of 1.75. These values are approximately 2.5% and 5% higher than the minimum value.

The results for the different distributions are shown in fig. 6 and tabulated in table 2. It can be seen that the proposed approach is successful in achieving the required goals. The deviations in the output results are 1.14%, 0.06% and 1.58% from the prespecified maximum stresses of 1.75, 1.79 and 1.83, respectively. The corresponding deviations in the mean values are 1.83%, 3.02% and 3.5%. The standard deviation, which can be looked upon as a measure of uniformity of the distributions, does not exceed 10% for the considered cases. The above results demonstrate clearly the accuracy and efficiency of the approach.

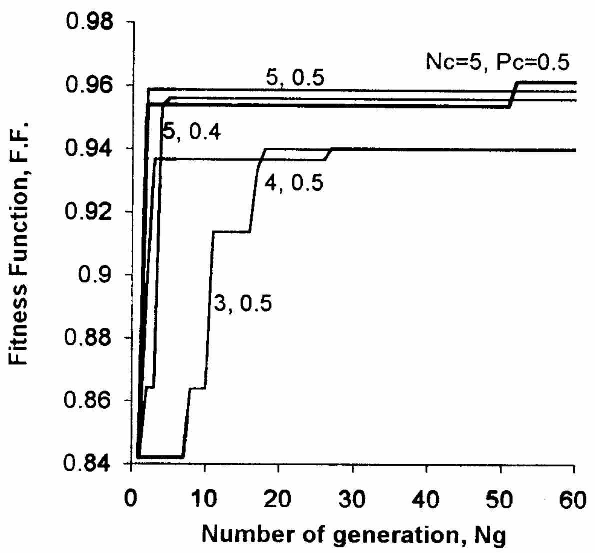



Fig. 5. Convergence of the fitness function for different values of $Nc$ and $Pc$.

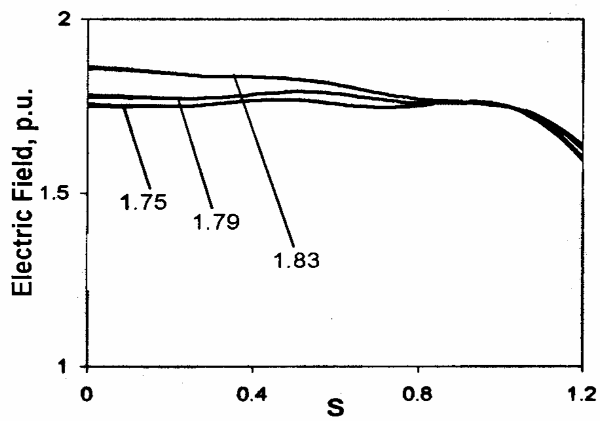



Fig. 6. Field distributions for different values of $E_1$.

The method can be further extended to examine the overall optimization of the GIS bus termination, i.e. the determination of the optimum dimensions of $x_1$, $x_2$ , $R$ and $H$ in order to achieve a uniform field distribution on the electrode with a minimum value for the maximum stress. The configuration, the domain for the parameters $x_1$, $x_2$ and the value of $E_1$ = 1.75 are the same as before. The fitness function is the one given earlier by eq. (5). $R$ is assumed to lie between 2.5 and 2.75 and $H$ is between 1.25 and 1.5. The genetic parameters are the same as before ($Nc$ = 0.5, $Pc$ = 0.5). Each of the four optimization parameters is coded as a 7-bit string for a total chromosome length of 28. The resulting field distribution is shown in fig. 7.

The maximum, mean and standard deviation of the distribution are 1.88, 1.78 and 7.08%, respectively.

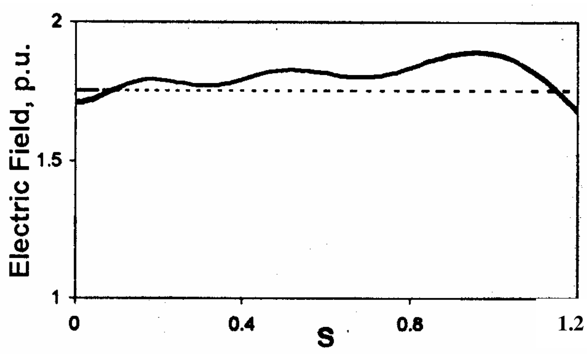



Fig. 7. An overall optimized field distribution along the electrode contour.

Table 2
Results for different values of the required maximum stress $E_1$

| Required stress, $E_1$ | Maximum stress, $F$ | Mean stress | Standard deviation(%) | $x_1$ | $x_2$ |
|---|---|---|---|---|---|
| 1.75 | 1.767 | 1.718 | 8.9 | 0.687 | 0.029 |
| 1.79 | 1.791 | 1.736 | 8.7 | 0.699 | 0.032 |
| 1.83 | 1.859 | 1.766 | 10.0 | 0.729 | 0.013 |

The deviation of the maximum stress from the required value is only 7.43%. The optimized parameters $x_1$, $x_2$, $R$ and $H$ are 0.600, 0.055, 2.64 and 1.36, respectively. This again demonstrates the effectiveness and capability of the method. It should be noted that practical and more realistic domain for the optimization parameters should be used when an overall optimization of the bus termination is to be considered which will be a subject for further research.

## 5. Conclusions

A new method for optimization of high voltage electrode contours, based on Genetic Algorithms techniques, has been presented. The method seeks the optimization of the electrode shape in order to achieve a uniform field distribution along the electrode surface while maintaining the maximum field stress at a minimum value. A practical example of an axisymmetric single-phase GIS bus termination is considered. The shape of the electrode has been taken as a quarter-ellipse to allow more flexibility.

Different fitness functions have been proposed and tested. The function that relies on the mean deviations from the required maximum field value proved to be effective and efficient in realizing the required objectives. The influence of the various genetic operators, such as the population size and the crossover probability, on the convergence process is also studied and assessed.

The generality of the method have been demonstrated using other values for the required maximum stress on the electrode contour. This is of practical importance as it may be possible to design the GIS bus termination at values of maximum field stress other than the minimum. The results demonstrate clearly the generality and accuracy of the approach.

The method has been further extended to examine the overall optimization of the GIS bus termination, i.e. the determination of the optimum dimensions of the electrode and the outer cylindrical enclosure in order to achieve a uniform field distribution on the electrode with a minimum value for the maximum stress. The method has, successfully, achieved the required objective and this demonstrates the effectiveness and capability of the method.

## Appendix A

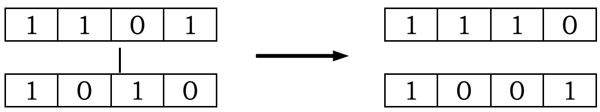
Three basic operations occur in GAs to create the next generation: Selection, Crossover and Mutation. These operators, implemented in the present analysis, are as follows:

*Parent selection (reproduction)*

Parent selection is a problem dependent operator and it is to intended to give more reproductive chances to those population members that are the fit. Several approaches exist. One of the most suitable approaches is the well-known "Tournament Selection". This works as follows: choose some number (tournament size) of individuals randomly from the population and copy the best individual, based on a fitness function, from this group into the rest of the population, and repeat $N$ times. Often tournaments are held between two individuals (binary tournament).

*Crossover*

Crossover is a random process of recombination of strings. Based on the probability of crossover, partial exchange of characters between two strings is performed. With the crossover operation, GAs are able to acquire more information with the generated individuals. The genetic search space is thus extended and is more complete. The crossover operator can be explained using the following schematic diagram, where each parent or child possess a string composed of 4 bits.




Crossover operation

*Mutation*

Mutation is an occasional random alteration of the bits in the string. With the binary representation, this simply means flipping the state of a bit from 1 to 0 or vice versa. The reason of applying the mutation operator in GAs is that mutation helps reproducing individuals that may be vital to the performance, through introducing new information into the bit level. The following schematic diagram illustrates the mutation operation for a four-bit string.

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | 0 |

↓

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | 1 |

Mutation operation

**References**

[1] P. K. Mukherjee, C. Trinitis, and H. Steinbigler "Optimization of HV Electrode Systems by Neural Networks Using a New Learning Method", IEEE Trans. on Dielectrics and Electr. Insulation, Vol. 3 (6), pp. 737-742 (1996).

[2] S. Chakravorti, and P. K. Mukherjee, "Application of Artificial Neural networks for Optimization of Electrode Contour", IEEE Trans. on Dielectrics and Elec. Insul., Vol. 1 (2), pp. 254-264 (1994).

[3] H. Singer, "Computation of Optimized Electrode Geometries", International Symposium on High Voltage Engineering (ISH), 11.06, Milan, Italy, pp. 1-4 (1979).

[4] M. Abdel-Salam and E.K. Stanek "Field Optimization of High Voltage Insulators", IEEE Trans. Industrial Application, Vol. 22 (4), pp. 594-601 (1986).

[5] M. Abdel-Salam and E. K. Stanek "Optimizing Field Stress on High Voltage Insulators", IEEE Trans. on Electric Insulation, Vol. 22 (1), pp. 47-56 (1987).

[6] M. M. Abouelsaad and M. M. El Bahy, "Application of Genetic Algorithms for Optimization of the Charge Simulation Method", Middle East Power Systems Conference, MEPCON'2000, Ain-Shams, University, Cairo, Egypt, March, pp. 551-555 (2000).

[7] T. Haida, et al., "Genetic Algorithms Approach to Voltage Optimization", Proc. First Int. Forum on Applications of Neural Networks to Power Systems, New York, USA, IEEE (1991).

[8] V. Ajjarapu, et al., "Application of Genetic based Algorithms to Optimal Capacitor Placement", Proc. First Int. Forum on Applications of Neural Networks to Power Systems, New York, USA, IEEE (1991).

[9] M. Zaki, M. Abdel-Salam, A. Abdel-Hakim, "The Use of Genetic Algorithms for Assessment of HV Fields", Computer and Electric Eng. Journal, Pergamon Press, Vol. 26, pp. 423-443 (2000).

[10] C. T Lin, C. S George Lee, Neural Fuzzy Systems, Prentice Hall, NJ (1995).

[11] D., Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley (1989).

[12] H. Singer, H. Steinbigler, and P. Weiss, "A Charge Simulation Method for the Calculation of High Voltage Fields", IEEE, Trans. on PAS, Vol. 93 (5), pp. 1660-1668 (1974).

[13] A. Yializis, E. Kuffel, and P. H. Alexander, "An Optimized Charge Simulation Method for the Calculation of High Voltage Fields", IEEE Trans. on PAS-97, pp. 2334 –2340 (1978).

Received September 12, 2002
Accepted February 18, 2003