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Optimization of earthwork allocation with multiple soil types

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Earthmoving is the process of moving and processing soil from one location to another to alter an existing land surface into a desired configuration. Highways, dams, and airports are typical examples of heavy earthmoving projects. Over the years, construction managers have devised methods to determine the quantities of material to be moved from one place to another. Various types of soil (soft earth, sand, hard clay, ... etc.) create different levels of difficulty of the problem. The earthmoving problem has traditionally been solved using mass diagram or variety of operational research techniques. However, existing models do not present realistic solution for the problem. Multiple soil types are usually found in cut areas and specific types of soil are required in fill sections. Some soil types in cut areas are not suitable for use in fill sections and must be disposed-off. In this paper a new mathematical programming model is developed to find-out the optimum allocation of earthmoving materials. In developing the proposed model, different soil types are considered as well as variations of unit cost with earth quantities moved. Suggested borrow pits and/or disposal sites are introduced to minimize the overall earthmoving cost. The proposed model is entirely formulated using the programming capabilities of VB6 while LINDO is used to solve the formulated model. An example project is presented to show how the model can be implemented. A case study project is analyzed using the developed model and a sensitivity analysis is then performed.

تمثل أعمال نقل التربة جزءاً أساسياً من معظم مشروعات التشييد، كما تمثل تكلفة أعمال نقل التربة الجزء الأكبر من التكلفة الكلية في مشروعات الطرق والسدود والمطارات. ولمّا كانت التكلفة تعتمد على عوامل كثيرة فان هذه التكلفة من المسائل الهامة في إدارة المشروعات. تناول هذا البحث عرضاً لمختلف الطرق التي استخدمت في حساب التكلفة لأعمال نقل التربة والعوامل التي تؤثر على هذه التكلفة. تم تطوير النماذج الرياضية القديمة بحيث نستطيع التغلب على مشكلة تواجد أكثر من نوع من التربة في أماكن الحفر والردم. كما يعالج النموذج الجديد مشكلة نقل التربة عند تواجد أنواع من التربة في مناطق الحفر لابد من ترحيلها لأنها غير صالحة للردم. تم تكوين النموذج الرياضي للمشكلة حيث تم شرح كيفية تكوين دالة الهدف و القيود المفروضة بشكل مستفيض مع الأخذ بعين الاعتبار جميع الحالات التي يمكن أن تتواجد فيها المشكلة في الطبيعة.تم إعداد برنامج حاسب آلى باستخدام لغة (Visual Basic 6) لتكوين المشكلة فى الصورة القياسية وبرنامج (LINDO) للحصول على الحل الأمثل للمشكلة. كدراسة عملية للنموذج الرياضي الجديد تم تطبيق هذا النموذج على مشروع حقيقى. من أهم النتائج التي أكدها البحث أن التكلفة الكلية لنقل التربة تتناقص إذا تم الأخذ بعين الاعتبار المسافات الحقيقية بين مواقع الحفر و الردم المختلفة، كما أن التكلفة الكلية تتزايد بشكل خطي مع الزيادة الخطية لمعاملات النموذج الرياضي وهي وحدة الشراء للتربة ووحدة الحفر و النقل و الدمك ومعامل الانتفاش و بشكل معاكس فان التكلفة الكلية تتناقص بازدياد معامل التقلص.

Keywords: Construction management, Earthmoving problem, Optimization, Mixed-integer programming, Integrated programs

1. Introduction

Earthmoving operations represent major part of most construction projects. The construction of highways, dams, airports, buildings, and industrial sites are typical construction projects that require earthmoving. Over the years, construction managers have devised methods to determine the quantities of material to be moved from one place to another to alter an existing land surface into a desired configuration. Activities of earthmoving include excavation, loading, hauling, placing (dumping and spreading), compaction, grading, and finishing. Various types of soil (soft earth, sand, hard clay, etc.) create different levels of difficulty. Soil types such as crushed stones have high unit cost of purchase, excavation, and compaction, which may lead to high total cost when these soil types are required. Also many soil types in cut sections are not suitable for use in fill sections

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and must be disposed-off. Efficient management of earthmoving works requires accurate estimating of work quantities, unit cost of earthmoving materials, and proper selection of equipment.

The availability of borrow pits and landfills contributes in minimizing total earthmoving cost. Sometimes, alternative choices in the number and location of borrow and disposal sites are available. It may be economical and even necessary to establish borrow or disposal sites within proximity of the project. The alternative sites are called suggested borrow pits or suggested disposal sites. The use of these new alternative sites may reduce the total cost. Deciding whether to set up borrow pits and/or disposal sites depends upon some associated costs (Mayer and Stark [1]), which include: land acquisition, site preparation for excavation or for dumping, construction and maintenance of access roads, and refurbishing and cleanup. In most situations, the contractor does not pay for purchase of disposal sites. This is because the wasted materials may be needed in another place near the project or the excess materials may be disposed-off in random places near the project.

Earthmoving problem has traditionally been solved using different methods. These include mass diagram and a variety of operational research techniques, such as mathematical programming and queueing models. In this paper a new mathematical programming model was developed to find-out the optimum allocation of earthmoving materials. In developing the new model, multiple soil types are considered as well as variation of unit cost with earth quantities required. Suggested borrow pits and/or disposal sites are introduced. An example project is presented to show how the developed model can be implemented and a case study project is analyzed. A sensitivity analysis is performed to specify the most affected parameters.

2. Literature review

Several methods have been developed to find-out the most reasonable solution for the earthmoving problem. These methods could be classified into a number of categories. The first category is graphical methods, which include mainly mass diagram (Stark and Mayer [2]). The mass diagram is originally used to handle earthmoving problems, but it did not tell which material should be moved from one section to another. The mass diagram can not be used when hauling costs are not directly proportional to the haul distance, soil characteristics vary along the project sections (particularly swell and shrinkage), and additional quantities of soil are available in cut sections and are not required in fill sections.

The second category is mathematical programming methods, which employ linear (Stark and Mayer [2]), and mixed-integer programming (Stark and Mayer [2], Easa [3]). Mathematical programming methods provide optimal distribution of earth and consequently minimizes total cost. The third category is simulation-based methods. Simulation is used excessively to select the optimum number and capacity of trucks and loaders required for earthmoving works (Willenbrok [4], Smith et al. [5]). Other quantitative techniques are used to solve earthmoving problem such as queueing models (O'Shea et al. [6]), bunching theory (Ringwald [7]), sampling theory (Gates and Scrapa [8]), and linear regression and correlation methods (Smith [9]).

Linear programming formulation of earthmoving problem can be outlined as: Having a profile map of the proposed project, the problem is to determine how much earth must be moved and from where to where to obtain the desired profile at minimum cost. The objective is to minimize the total project cost. Stark and Mayer [1] formulated the problem as a linear programming model in a simplified way. Then, they introduced adjustments to include swell and shrinkage factors and suggested borrow pits and/or disposal sites. However, the Stark and Mayer model can not deal with situations where multiple soil types exist. Another drawback of the model is the assumption that unit cost elements are constant.

Easa [3] suggested a stepwise unit cost function to represent cost variation with quantity of earth purchased and another stepwise unit cost function for excavation. Easa incorporated unit cost of purchase and unit cost of excavation in a single unit cost function, and modified the Stark and Mayer [1] model in a mixed-integer programming form to accommodate for possible variations in unit cost of purchase and excavation for borrow pits. The model, however, did not account for multiple soil types, which are usually encountered in construction practice. Therefore, multiple soil type will be considered in developing the present model.

Nandgaonkar [10] handled earthmoving allocation as a transportation problem, which can be profitably used to achieve near optimality in earthwork problems. However, the model can be applied only when quantities of cut and fill are equal. Multiple soil types and variable unit cost are not permitted.

3. Mathematical formulation of the developed model

The operations involved in any earthmoving project could be classified into a number of groups such as cut, fill, waste earth, etc. The cut group, for instance, may contain the activities of cutting project areas, borrow pits, and suggested borrow pits. Fig. 1 shows a typical earthmoving roadway project which contains a number of cut and fill sections. Different soil types could be found and there may be borrow pits and/or disposal sites available for the project. The soil unit cost of excavation and purchase will be considered as a function of the required quantities. The step-wise unit cost function suggested by Easa [3] to represent cost variation is used and will be extended to allow for multiple soil types, as shown in fig. 2. In this figure, for example, if the earth quantity delivered from borrow pit (*b*) and soil type (*s*) is less than a certain limit; Q(*b*,*s*), the corresponding unit cost is C1(*b*,*s*). Q(*b*,*s*), R(*b*,*s*), and B(*b*,*s*) are earth quantities at which unit cost varies.

The primary decision variables required to formulate the problem are the quantity of earth moved from different project areas. The objective is to minimize the total earthmoving cost. The constraints of the problem are the earth quantities available in cut sections, earth quantities required in fill sections, and the capacities of different borrow and disposal sites.

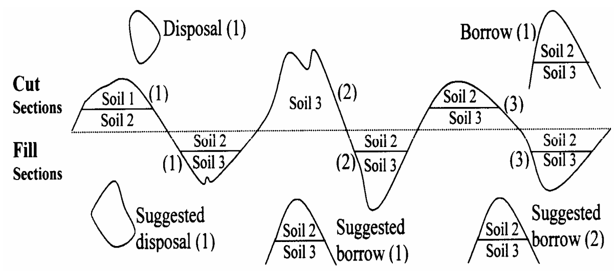


Fig. 1. Roadway profile with borrow and disposal sites.

3.1. Decision variables

Two types of decision variables are used: quantity decision variables and zero-one variables. Quantity decision variables are introduced to find-out the proper allocation of earth between different areas, and are denoted by *X* and *Y*. Subscripts are used to refer to various situations that may be encountered. For example, Xsb(*n*,*j*,*s*) is a quantity decision variable which represents quantity of earth moved from suggested borrow pit (*n*) to fill section (*j*) from soil type (*s*), while *Y2*(*b*,*s*) represents quantity of the second component of the step-wise function of borrow pit (*b*) and soil type (*s*) (fig. 2). Zero-one variables; $\lambda(b,s)$ and $\gamma(b,s)$, are introduced to interpret the variation of unit cost (purchase and excavation) for borrow pit (*b*) and soil type (*s*). From the practical point of view, single suggested borrow pit (and/or single suggested disposal site) is usually used. Because suggested sites are associated with set-up cost, the one which may result in minimum total cost should be selected. Another type of zero-one variables; δ, is used to insure that only one suggested borrow pit or suggested disposal site may be considered.

Decision variables of the problem are constrained by the earth quantities at cut sections, required quantities at fill sections, and capacities of borrow pits and disposal sites. Therefore, constraints of the problem must be formulated for cut sections, fill sections, borrow pits, disposal sites, suggested borrows, and suggested disposal sites.

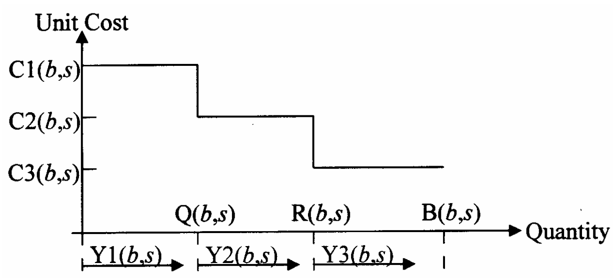


Fig. 2. Stepwise unit cost function of purchase and excavation {for borrow pit (b) and soil type (s)}.

3. 2. Mathematical statement of the developed model

The developed mixed-integer programming model is formulated as follows:

Minimize:

$$Z=\sum_{i=1}^{Nc}\sum_{j=1}^{Nf}\sum_{s\varepsilon(i,j)=1}^{Ns} C(i,j,s)X(i,j,s) + \sum_{i=1}^{Nc}\sum_{k=1}^{Nd}\sum_{s\varepsilon(i)=1}^{Ns} Cd(i,k,s)Xd(i,k,s) + \sum_{i=1}^{Nc}\sum_{m=1}^{Nsd}\sum_{s\varepsilon(i)=1}^{Ns} Csd(i,m,s)Xsd(i,m,s) + \sum_{b=1}^{Nb}\sum_{j=1}^{Nf}\sum_{s\varepsilon(b,j)=1}^{Ns} Cb(b,j,s)Xb(b,j,s) +$$

$$\sum_{b=1}^{Nb}\sum_{s\varepsilon(b,j)=1}^{Ns} C1(b,s)*Y1(b,s) + C2(b,s)*Y2(b,s) + C3(b,s)*Y3(b,s) + [C2(b,s)-C1(b,s)]*Q(b,s)*\lambda(b,s) + [\{C2(b,s)-C1(b,s)\}*Q(b,s) + \{C3(b,s)-C2(b,s)\}*R(b,s)]*\gamma(b,s) +$$

$$\sum_{n=1}^{Nsb}\sum_{j=1}^{Nf}\sum_{s\varepsilon(n,j)=1}^{Ns} Csb(n,j,s)Xsb(n,j,s) + \sum_{n=1}^{Nsb} Ksb(n)\delta sb(n) + \sum_{m=1}^{Nsd} Ksd(m)\delta sd(m) . \quad (1)$$

Subject to:

$$\sum_{j=1}^{Nf} X(i,j,s) + \sum_{k=1}^{Nd} Xd(i,k,s) + \sum_{m=1}^{Nsd} Xsd(i,m,s) = Qc(i,s)$$
$$s\varepsilon(i,j)=1,2,\ldots,Ns, \quad i=1,2,\ldots,Nc , \quad (2)$$

$$\sum_{i=1}^{Nc}\sum_{s\varepsilon(i)=1}^{Ns} Sh(s)X(i,j,s) + \sum_{b=1}^{Nb}\sum_{s\varepsilon(b)=1}^{Ns} Sh(s)Xb(b,j,s) + \sum_{n=1}^{Nsb}\sum_{s\varepsilon(n)=1}^{Ns} Sh(s)Xsb(n,j,s) = Qf(j) \quad j=1,2,\ldots,Nf , \quad (3\text{-a})$$

or,

$$\sum_{i=1}^{Nc} Sh(s)X(i,j,s) + \sum_{b=1}^{Nb} Sh(s)Xb(b,j,s) + \sum_{n=1}^{Nsb} Sh(s)Xsb(n,j,s) = Qf(j,s)$$
$$s\varepsilon(i,j),\ (b,j),\ \text{and}\ (n,j) = 1,2,\ldots,Ns, \quad j=1,2,\ldots,Nf, \quad (3\text{-b})$$

$$\sum_{j=1}^{Nf} Xb(b,j,s) \le Qb(b,s)$$
$$s\varepsilon(b,j) = 1,2,\ldots,Ns, \quad b=1,2,\ldots,Nb , \quad (4)$$

$$Y1(b,s) + Y2(b,s) + Y3(b,s) = \sum_{j=1}^{Nf} Xb(b,j,s)$$
$$s\varepsilon(b,j)=1,2,\ldots,Ns, \quad b=1,2,\ldots,Nb , \quad (5)$$

$$Y1(b,s) \le Q(b,s)$$
$$s\varepsilon(b,j)=1,2,\ldots,Ns, \quad b=1,2,\ldots,Nb , \quad (6)$$

$$Y1(b,s) - [\lambda(b,s) + \gamma(b,s)] * Q(b,s) \ge 0$$
$$s\varepsilon(b,j)=1,2,\ldots,Ns, \quad b=1,2,\ldots,Nb , \quad (7)$$

$$Y2(b,s) - [\lambda(b,s) + \gamma(b,s)] * [R(b,s) - Q(b,s)] \le 0$$
$$s\varepsilon(b,j)=1,2,\ldots,Ns, \quad b=1,2,\ldots,Nb , \quad (8)$$

$$Y2(b,s) - \gamma(b,s) * [R(b,s) - Q(b,s)] \ge 0$$
$$s\varepsilon(b,j)=1,2,\ldots,Ns, \quad b=1,2,\ldots,Nb , \quad (9)$$

$$Y3(b,s) - \gamma(b,s) * [B(b,s) - R(b,s)] \le 0$$
$$s\varepsilon(b,j)=1,2,\ldots,Ns, \quad b=1,2,\ldots,Nb , \quad (10)$$

$$\lambda(b,s) + \gamma(b,s) \le 0 ; \quad \lambda(b,s) \text{ and } \gamma(b,s)\ \varepsilon\ (0,1)$$
$$s\varepsilon(b,j)=1,2,\ldots,Ns, \quad b=1,2,\ldots,Nb, \quad (11)$$

$$\sum_{j=1}^{Nf} Xsb(n,j,s) - Qsb(n,s)\delta sb(n) \le 0$$
$$s\varepsilon(n,j)=1,2,\ldots,Ns, \quad n=1,2,\ldots,Nsb , (12)$$

$$\sum_{n=1}^{Nsb} \delta sb(n) \leq 1 , \quad (13)$$

$$\sum_{i=1}^{Nc} \sum_{S\varepsilon(i)=1}^{Ns} Sw(s)Xd(i,k,s) - Qd(k) \leq 0$$

$$k=1,2,\ldots,Nd \ , \quad (14)$$

$$\sum_{i=1}^{Nc} \sum_{s\varepsilon(i)=1}^{Ns} Sw(s)Xsd(i,m,s) - Qsd(m)\delta sd((m) \leq 0$$

$$m=1,2,\ldots,Nsd \ , \quad (15)$$

$$\sum_{m=1}^{Nsd} \delta sd(m) \leq 1 . \quad (16)$$

Where: *Nc* is number of cut sections, *Nf is* number of fill sections, *Ns is* number of soil types, *Nb is* number of borrow, *Nd is* number of disposal sites, *Nsb is* number of suggested borrow pits, *Ksb(n)* is set-up cost of suggested borrow pit (*n*), δsb(*n*) is zero–one variable for suggested borrow pit (*n*), *Ksd(m)* is set-up cost of suggested disposal (*m*), δsd(*m*) is zero–one variable for suggested disposal (*m*), and *λ(b,s)* and *γ(b,s)* are zero-one variables for borrow bit (*b*) which have a variable unit cost of purchase and excavation for soil type *(s).* sε() notations are used to insure soil type consistency between different sections. For example, if soil (2) is required at fill section (*j*), the objective function equation and constraints will contain only terms corresponding to cut sections, borrow pits, and suggested borrow pits having soil type (2) only. Therefore, redundant terms are eliminated.

Objective function (eq. (1)): The objective is to minimize the total earthmoving cost (Z), which can be obtained by multiplying earth quantities; *X(i,j,s), Xb(b,j,s), Xsb(n,j,s), Xd(i,k,s),* and *Xsd(i,m,s),* by the corresponding unit costs and adding them for all possible pairs of sections. Unit cost elements may include unit cost of: excavation, hauling, and compaction. If borrow pits are used, another unit cost element is used which is unit cost of purchase. The unit cost for different alternatives can be easily calculated and are given by Stark and Mayer [2]. The total cost can be calculated as follows:

Z = Cost of earth moved from cut sections to (fill sections + disposal + suggested disposal) + Cost of earth required to fill sections from (borrow pits + suggested borrow pits) + Set-up cost of suggested borrow pits and disposal sites.

Cut sections constraint (inequality (2)): This constraint is written for each cut section and various soil types. The quantities of earth from soil type (*s*) moved from cut section (*i*) to fill section (*j*), disposal (*k*), and suggested disposal (*m*) must be equal to the cut available in that section from the specified soil type; *Qc(i,s),* where *sε (i,j).*

Fill sections constraint (inequality (3-a) or (3-b)): This constraint is formulated for each fill section and soil types belong to that section. The quantities of earth delivered to fill section (*j*) from each cut section (*i*), borrow pit (*b*), and suggested borrow pit (*n*) must be equal to the required quantity at that section. Two cases may be encountered. If all soil types are suitable for use in fill sections, the transported earth quantities are constrained by the capacity of fill section (*j*); *Qf(j),* as given by eq. (3-a). On the other hand, if a soil type is unsuitable for use in a fill section because of its undesirable characteristics and must be disposed-off, eq. (3-a) is modified to account for this situation. The delivered earth to each fill section (*j*) from soil type *(s)* is restricted by the required quantity of that soil type; *Qf(j,s),* as given by eq. (3-b). It must be noted that delivered quantities form different soil types are multiplied by the corresponding shrinkage factor; *Sh(s)*, to account for compaction of earth at fill sections.

Borrow pits constraints (eqs. (4) through (11): A constraint of inequality type 4 is written for each borrow pit(*b*) to accommodate for its capacity from different soil types; *Qb(b,s)*, as given by eq. (4). For borrow pits having variable unit cost of purchase and excavation, another constraint is required to satisfy that the proper stepwise unit cost function is selected, as given by eq. (5). Zero-one variables; *λ(b,s)* and *γ(b,s)*, are introduced and the required constraints are given by eqs (6) through (10). The possible combinations of *λ(b,s)* and *γ(b,s),* as given by eq.(11), are: *[λ(b,s)* = 0, *γ(b,s)* = 0], *[λ(b,s)* = 1, *γ(b,s)* = 0], and *[λ(b,s)* = 0, *γ(b,s)* = 1]. These possible combinations correspond to cases in which the quantity delivered from borrow pit (*b*) is within the first, second, and third component of the stepwise unit cost function, respectively, as shown in fig. 2. For example, values of the third combination [$\lambda(b,s)$=0, $\gamma(b,s)$=1] are substituted into eqs. (6) to (10), which yields: $Y3(b,s) \leq [B(b,s) - R(b,s)]$ and the corre-sponding cost, as given by the underlined part in eq. (1)., is $C3(b,s)\{R(b,s)+Y3(b,s)\}$.

Suggested borrows constraints (inequalities (12) and (13)): Quantities of earth delivered form suggested borrow (*n*) to fill sections form soil type *(s)* are restricted by its capacity of that soil type; *Qsb(n,s),* and the required constraints are given by eq. (12). Because suggested borrows are costly (set-up cost), the one which results in minimum total cost should be selected. Zero-one variables*; δsb(n),* are used to insure that a single suggested borrow may be selected, as given by eq. (13).

Disposal sites constraint (inequality 14): Quantities of earth which can be transmitted to disposal site (*k*) are restricted by the capacity of that disposal; *Qd(k),* as given by eq. (14). The transported earth form different soil types are multiplied by the corresponding swelling factor; *Sw(s),* to account for volume increase of excavated soil.

Suggested disposals constraints (inequalities (15) and (16)): Similar to suggested borrow pits, quantities of earth transmitted to each suggested disposal site (*m*) are restricted by its capacity; *Qsd(m)*, as given by eq. (15). Zero one variable; *δsd(m);* is used to insure that a single disposal site which results in minimum total cost may be selected, as given by eq. (16).

4. Model implementation

In order to demonstrate model implementation and formulation, a simple example project (roadway) is considered (fig.1.). The roadway consists of 3 cut sections, 3 fill sections, one disposal site, one borrow pit, 2 suggested borrow pits, and one suggested disposal site, where 3 types of soil exist. Quantities of earth and other soil data are given in tables 1 and 2. Unit cost of excavation, hauling, and compaction for soil types 1, 2, and 3 are 2, 1.5, 2.5, respectively. The distances between different sections and sites are given in table 3. All distances are measured from center of mass of different sections and sites. Compaction is not required for disposed-off soils. Set-up cost for suggested borrows (1) and (2) and suggested disposal are LE2100, LE2200, and LE2000, respectively.

Table 1
Estimated quantities (m^3) and physical factors for different soil types

Soil	Cut sections			Fill sections			Borrow	S. borrows		Disposal	S. disposal	Factors	
type	1	2	3	1	2	3	1	1	2	1	1	Swell	Shrinkage
1	30	-	-	-	-	-	-	-	-			1.5	0.95
2	50	-	250	35	58	110	500	300	500	5000	6000	1.1	0.80
3	-	200	300	100	290	260	800	350	600			1.2	0.85

Table 2
Stepwise break-point of borrow pit (1)

Soil type	Break point 1	Break point 2	Stepwise unit cost (LE)		
			C1	C2	C3
2	50	100	5	4	3
3	200	300	7	6	4

Table 3
Distance (km) form fill and cut sections

Fill	Cut section			Borrow	Suggested borrow		Cut	Disposal	Suggested disposal
section	1	2	3	1	1	2	section	1	1

1	1	1	3	4	1	4	1	1	1
2	3	1	1	2	1	2	2	2	1
3	5	3	1	1	3	1	3	4	3

In this example project, *Nc* =3, *Nf* =3, *Nb* =1, *Nsb* =2, *Nd* =1, *Nsd* =1, and *Ns* =3. Examples of the required decision variables are: *X*(3,1,2) quantity of earth moved from cut section (3) to fill section (1) from soil type (2), *Xb*(1,2,3) quantity of earth to be purchased and excavated from borrow pit (1) for fill section (2) from soil type (3), *Xsb*(1,2,3) quantity of earth moved from suggested borrow pit (1) to fill section (2) from soil type (3), *Xd*(3,1,2) quantity of earth in cut section (3) to be wasted in disposal (1) from soil type (2), and *Xsd*(2,1,3) quantity of earth moved from cut section (2) to suggested disposal (1) from soil type (3). On the other hand, *Y1*(1,2), *Y2*(1,2), and *Y3*(1,2) are earth quantity (borrow pit (1) and soil type (2)) of the first, second, third component, of the step-wise function, respectively. δsb (2) and δsd (1) are the zero–one variables for suggested borrow pit (2) and suggested disposal (1), respectively. λ(1,2) and γ(1,2) are the zero-one variables for borrow pit (1) corresponding to soil type (2).

The complete mathematical mixed-integer programming model for the roadway project is given in Appendix (A). LINDO (1999) is used to solve the mathematical model given by (A1) through (A36). The optimal solution is given in table 4, in which the optimum total cost of the problem is LE8049.95.

5. Model automation

Implementing the developed model on commercial software simplifies the process and provides project managers with an automated tool to facilitate input data and incorporating results. Visual Basic 6.0 (VB6.0) is selected for coding the developed model because of its ease of use and programmability features. The developed program is named OPTEarth (OPTimization of Earthmoving) which has a user-friendly interface. A main menu is designed for input basic data as shown in fig. 3. A command button is arranged to facilitate file management. To demonstrate program capabilities, the example roadway project is input. The user first inputs basic data only. The designed computer program has the capability to handle earthmoving problem with varying conditions. The following functions are implemented by the program: check earthmoving activities, select unit cost type (constant or variable), specify unit cost of purchase and excavation as a step-wise function, etc. For more details about the capabilities of OPTEarth, refer to Jarad [11].

Having the basic data appropriate for a project entered, "INPUT Data" button is activated. On clicking INPUT Data, a sequence of message boxes appear which ask the user to input detailed data for each activity specified in the main menu. Sample of the message boxes are shown in fig. 4. When the user is satisfied with the input data, the "Model Formulation" button is activated. Such activation enables the user to formulate automatically project data as a mixed-integer programming model and stores it in a pre-specified file. The full formulated model is appeared in a sub-menu block in the main menu of the program, as shown in fig. 3. Using scroll buttons, the user can check (and modify if necessary) the formulated model on the screen.

Table 4
Optimal solution of the example project

Objective function value (Z) = LE8049.95					
Variable	Value	Variable	Value	Variable	Value
γ(1,3)	1.0	X(3,2,2)	72.50	Xd(1,1,1)	30.00
X(1,1,2)	3.75	X(3,2,3)	294.12	Xd(1,1,2)	46.25
X(2,1,3)	117.65	X(3,3,2)	137.50	Xd(2,1,3)	35.29
X(2,2,3)	47.06	X(3,3,3)	5.88	Y1(1,3)	200.00
X(3,1,2)	40.00	Xb(1,3,3)	300.00	Y2(1,3)	100.00

All other variables = 0

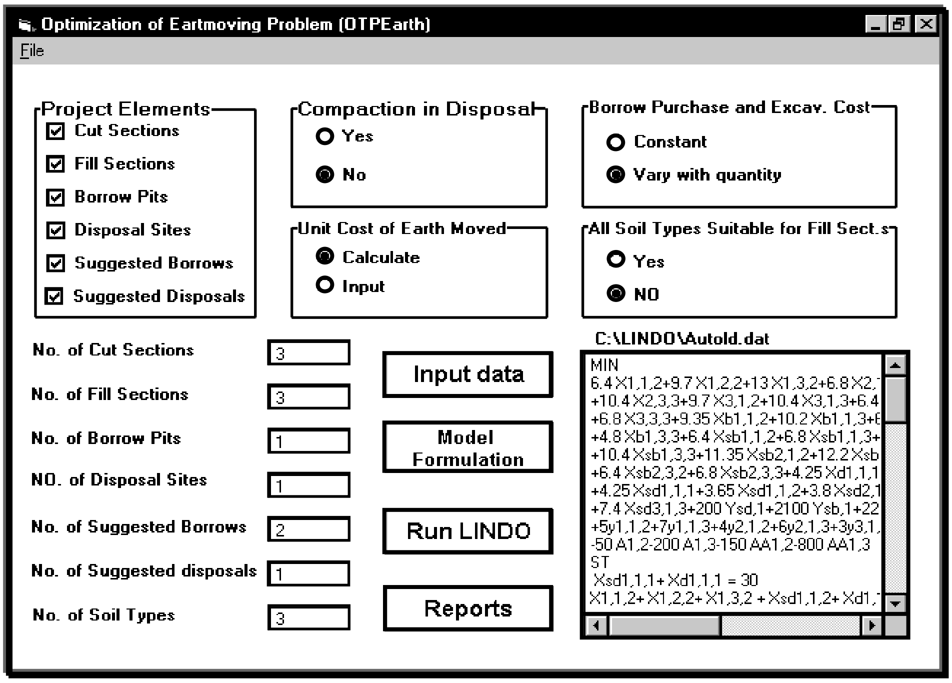


Fig. 3. Main menu of OPTEarth.

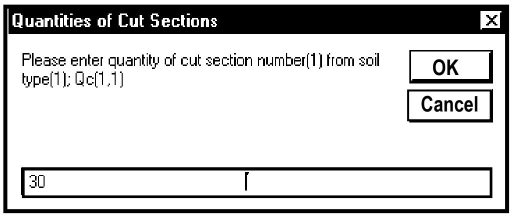


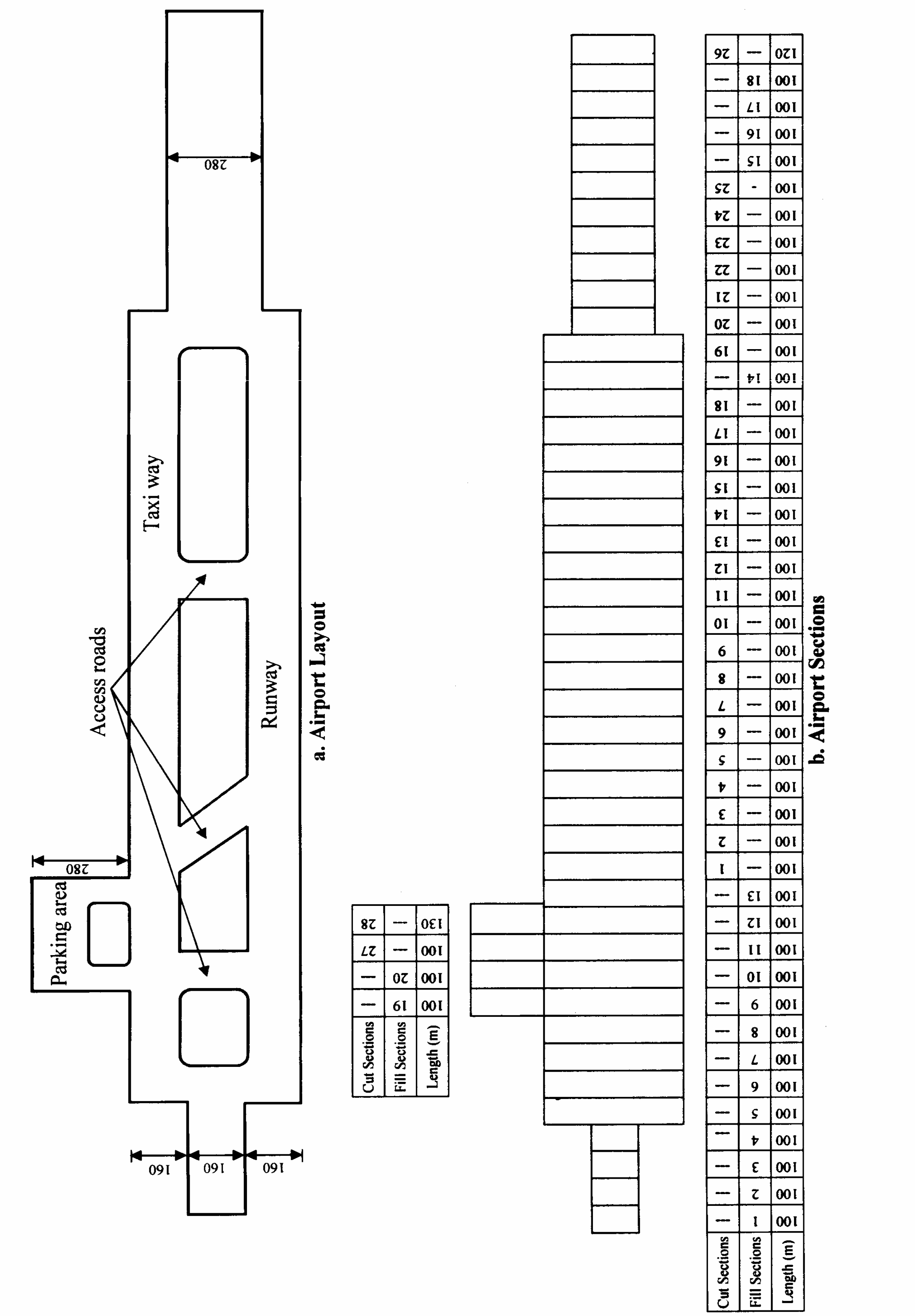
Fig. 4. Quantities of cut input screen.

Once the formulated model was completed, the "Run LINDO" button (fig. 3) is used to solve the automatically formulated model. The "Run LINDO" button, directs the formulated model to LINDO and the optimum solution (if exists) is then saved in a pre-specified file. “Reports” button enables the user to check the final report of the project. The program converts LINDO results into a suitable report format, which can be read easily. The program allows the user to obtain a hard copy of the final report.

6. Case study

The developed model and the designed computer program have been demonstrated to work effectively on the presented example project. Now, a case study will be presented in detail which is an airport project executed in Sinai, Egypt. The project is divided into 28 cut sections and 20 fill sections, as shown in fig.5. Cut sections 1 through 9 contain two types of soil, while other cut sections contain only soil type (1). This type of soil is suitable for use in fill sections and, hence, its unit cost of purchase is zero. The second type of soil is not suitable for fill sections and, therefore, will be disposed-off. The disposal site available for the project has unlimited capacity. Data of this case study are given in tables 5, and 6.

For this case study, the required number of decision variables is 597 while number of constraints is 58. LINDO takes about 1 second processing time to solve the problem on a PC (Penttium2 and 64.0 MB RAM). The final report shows that the resulting total cost is LE4,257,405. The contractor bid amounts LE4,727,504 (with the assumptions that swell factor = 1 for all soil types and considering only a mean distance for all transportations to project sections). As it is apparently seen when actual conditions of the problem and realistic parameters of soil types were



Cut Sections	—	—	27	28
Fill Sections	19	20	—	—
Length (m)	100	100	100	130

Cut Sections	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
Fill Sections	1	2	3	4	5	6	7	8	9	10	11	12	13	—	—
Length (m)	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Cut Sections	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Fill Sections	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Length (m)	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Cut Sections	18	—	19	20	21	22	23	24	25	—	—	—	—	26
Fill Sections	—	14	—	—	—	—	—	—	-	15	16	17	18	—
Length (m)	100	100	100	100	100	100	100	100	100	100	100	100	100	120

Fig. 5. Case study (airport).

considered, a cost saving of 11% is obtained. This will lead to realistic cost estimate and hence bid amount which increases the contractor chance to win the bid. Soil type (1) resulting from cut sections, is used in fill sections and the excess earth is moved to disposal site. Soil type (2), which is not suitable to fill sections, is also moved to disposal site.

Table 5
Estimated quantities (m^3) of cut and fill

Cut section	Quantity Soil (1)	Quantity Soil (2)	Fill section	Quantity soil (1)
1	8955	3700	1	18790
2	29571	9856	2	25003
3	64506	16126	3	37885
4	82874	14624	4	48786
5	84765	21191	5	99655
6	75073	18768	6	129758
7	52099	5788	7	114345
8	16272	1000	8	106476
9	5096	502	9	96028
10	4449	---	10	63031
11	12997	---	11	50579
12	34101	---	12	29524
13	98861	---	13	7643
14	119398	---	14	1170
15	144665	---	15	2202
16	127359	---	16	5435
17	76294	---	17	3204
18	15262	---	18	2604
19	7051	---	19	53555
20	24871	---	20	21763
21	40944	---		
22	31103	---		
23	20837	---		
24	5836	---		
25	94	---		
26	1150	---		
27	23779	---		
28	54184	---		

Table 6
Properties of soils

Property		Soil (1)	Soil (2)
Factor	Swell	1.1	1.0
	Shrinkage	0.9	1.0
Unit cost	Excavation	1.0	0.75
	Hauling	0.6	0.6
	Compaction	0.7	---

6. 1. Sensitivity analysis

In multi-variable problems, sensitivity analysis is usually done to indicate the relative effect of each parameter on a pre-specified criteria. Parameters of earthmoving problem include: unit cost (purchase, excavation, hauling and compaction), soil physical factors (swell and shrinkage), and distances between different project areas. In performing sensitivity analysis, a single parameter is usually allowed to vary while the other parameters are held constant. The parameters of each type of soil are varied as a percent of its initial value. Effect of parameters variations on the total cost are recorded.

To study the effect of variation of parameters of soil type (1), they are linearly increased at a step of 10%. The results showed that total cost is increased linearly except for the shrinkage factor as shown in fig.6. It can be noted that swell factor has the largest effect while shrinkage factor has the lowest effect. On the other hand, soil type (2) is not suitable to fill sections and consequently, both unit cost of compaction and shrinkage factor have no effect. When the distance of disposal site is linearly increased, the total cost is linearly increased for both types of soil.

6. 2. Comments

From the obtained results, it can be concluded that when actual characteristics of earthmoving problem are taken into consideration, considerable cost savings are obtained. To accomplish such cost saving, actual soil types and their physical properties should be used, unit cost parameters must be accurately calculated, and actual distances between different project sections must be considered. The sensitivity analysis showed that swell factor and unit cost of hauling have the largest effect on the total cost of earthmoving.

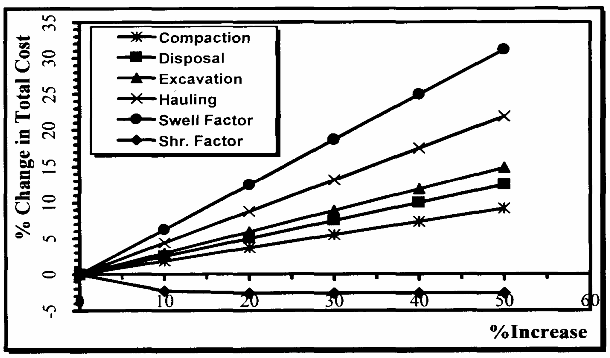


Fig.6 . Sensitivity analysis results of the case study (soil type).

7. Conclusions

In this paper, a new mixed-integer programming model was developed considering multiple soil types and other features of earthmoving problem. A computer program was designed using VB6.0 to handle automatically the problem with varying conditions. An example roadway project is then presented to show how the developed model can be implemented. A case study was analyzed and the results showed that considerable cost savings are obtained when actual characteristics are considered.

Based on the characteristics of the developed model and the performance of the deigned computer program, the main features of OPTEarth which make it an efficient tool for the analysis of earthmoving problem include:

- The problem is formulated in a standard form of mathematical programming; mixed-integer programming, in which the results guarantee the optimum solution;
- It considers the actual features of earthmoving problem in which multiple soil types usually exist;
- It considers step-wise function of unit cost of purchase and excavation where variation of unit cost with earth quantities are permitted;
- Suggested borrow and/or disposal sites are introduced to minimize the overall earthmoving cost; and
- The model is entirely formulated using a user-friendly computer program, which enables the user, with little computer knowledge, to work with efficient and powerful software.

Despite of the important benefits of the developed model, possible extensions can be made. The developed model can be extended and the designed computer program can be linked with an expert system to select appropriate earthmoving equipment type and their optimum size. The system can also be linked with a simulation model to determine the best matching between earthmoving equipment.

Appendix (A): Mathematical statement of the example project

Objective function: min.

$$Z = 6.4X(1,1,2) + 9.7X(1,2,2) + 13X(1,3,2) + 6.8X(2,1,3) + 6.8(X2,2,3) + 10.4X(2,3,3) + 9.7X(3,1,2) + 10.4X(3,1,3) + 6.4X(3,2,2) + 6.8X(3,2,3) + 6.4X(3,3,2) + 6.8X(3,3,3) + 9.35Xb(1,1,2) + 10.2Xb(1,1,3) + 6.05Xb(1,2,2) + 6.6Xb(1,2,3) + 4.4Xb(1,3,2) + 4.8Xb(1,3,3) + 5Y1(1,2) + 7Y1(1,3) + 4Y2(1,2) + 6Y2(1,3) + 3Y3(1,2) + 4Y3(1,3) - 50\lambda(1,2) - 200\lambda(1,3) - 150\gamma(1,2) - 800\gamma(1,3) + 6.4Xsb(1,1,2) + 6.8Xsb(1,1,3) + 6.4Xsb(1,2,2) + 6.8Xsb(1,2,3) + 9.7Xsb(1,3,2) + 10.4Xsb(1,3,3) + 11.35Xsb(2,1,2) + 12.2Xsb(2,1,3) + 8.05Xsb(2,2,2) + 8.6Xsb(2,2,3) + 6.4Xsb(2,3,2) + 6.8Xsb(2,3,3) + 4.25Xd(1,1,1) + 3.65Xd(1,1,2) + 5.6Xd(2,1,3) + 8.6Xd(3,1,2) + 9.2Xd(3,1,3) + 4.25Xsd(1,1,1) + 3.65Xsd(1,1,2) + 3.8Xsd(2,1,3) + 6.95Xsd(3,1,2) + 7.4Xsd(3,1,3) + 2100\delta sb(1) + 2100\delta sb(2) + 2000\delta sd(1) \quad (A1)$$

The coefficients which precede $X(1,1,2)$, $X(1,2,2)$, etc., are unit costs. The cost to move 1m^3 of soil type (2) from cut section (1) to fill section (1), for example, is the summation of costs of excavation, hauling, and compaction. Ue(2) =2, Uh(2) =1.5, Uc(2) =2.5, d(1,1) =1, and Sw(2) =1.1, then: C(1,1,2) = 2 + 1.1(1.5*1 + 2.5) = 6.4, where Ue(2), Uh(2), and Uc(2), are unit cost of excavation, hauling, and compaction of soil type (2), respectively, $Sw(2)$ is the shrinkage factor of soil type (2), and $d(1,1)$ is the distance between cut section (1) and fill section (1).

Constraints of the problem

Cut sections constraints: The quantity of earth removed from a cut section and transported to various fill sections and/or disposal sites must be equal to the required cut at that section. For example, 30 m^3 of cut (soil type 1) should be removed from section (1) to disposal (1) and suggested disposal (1). That is, for cut section (1) and soil type (1), the required constraint would be:

$$Xd(1,1,1) + Xsd(1,1,1) = 30 \quad (A2)$$

For other cut sections and different soil types, the required constraints are:

$$X(1,1,2) + X(1,2,2) + X(1,3,2) + Xd(1,1,2) + Xsd(1,1,2) = 50 \quad (A3)$$

X(2,1,3) + X(2,2,3) + X(2,3,3) + Xd(2,1,3) + Xsd(2,1,3) = 200 (A4)

X(3,1,2) + X(3,2,2) + X(3,3,2) + Xd(3,1,2) + Xsd(3,1,2) = 250 (A5)

X(3,1,3) + X(3,2,3) + X(3,3,3) + Xd(3,1,3) + Xsd(3,1,3) = 300 (A6)

Fill sections constraints: The quantity of earth transported to a fill section must be equal to the required fill at that section. For instance, 35 m^3 of fill from soil type (2) is required to fill section (1). Therefore, the required constraint would be:

0.80X(1,1,2) + 0.80X(3,1,2) + 0.80Xb(1,1,2) + 0.80Xsb(1,1,2) + 0.80Xsb(2,1,2) = 35 (A7)

Note that 0.80 is the shrinkage factor for soil type (2). For other fill sections and different soil types, the required constraints are:

0.80X(1,2,2) + 0.80X(3,2,2) + 0.80Xb(1,2,2) + 0.80Xsb(1,2,2) + 0.80Xsb(2,2,2) = 58 (A8)

0.80X(1,3,2) + 0.80X(3,3,2) + 0.80Xb(1,3,2) + 0.80Xsb(1,3,2) + 0.80Xsb(2,3,2) = 110 (A9)

0.85X(2,1,3) + 0.85X(3,1,3) + 0.85Xb(1,1,3) + 0.85Xsb(1,1,3) + 0.85Xsb(2,1,3) = 100 (A10)

0.85X(2,2,3) + 0.85X(3,2,3) + 0.85Xb(1,2,3) + 0.85Xsb(1,2,3) + 0.85Xsb(2,2,3) = 290 (A11)

0.85X(2,3,3) + 0.85X(3,3,3) + 0.85Xb(1,3,3) + 0.85Xsb(1,3,3) + 0.80Xsb(2,3,3) = 260 (A12)

Similarly, other types of constraints can be formulated as follows:

Borrow pit constraints:

Xb(1,1,2) + Xb(1,2,2) + Xb(1,3,2) ≤ 500 (A13)

Xb(1,1,3) + Xb(1,2,3) + Xb(1,3,3) ≤ 800 (A14)

Y1(1,2) + Y2(1,2) + Y3(1,2) - Xb(1,1,2) - Xb(1,2,2) - Xb(1,3,2) = 0 (A15)

Y1(1,3) + Y2(1,3) + Y3(1,3) - Xb(1,1,3) - Xb(1,2,3) - Xb(1,3,3) = 0 (A16)

Y1(1,2) ≤ 50 (A17)

Y1(1,3) ≤ 200 (A18)

Y1(1,2) - 50λ(1,2) - 50γ(1,2) ≥ 0 (A19)

Y1(1,3) - 200λ(1,3) - 200γ(1,3) ≥ 0 (A20)

Y2(1,2) - 50λ(1,2) - 50γ(1,2) ≤ 0 (A21)

Y2(1,3) - 100λ(1,3) - 100γ(1,3) ≤ 0 (A22)

Y2(1,2) - 50γ(1,2) ≥ 0 (A23)

Y2(1,3) - 100γ(1,3) ≥ 0 (A24)

Y3(1,2) - 400γ(1,2) ≤ 0 (A25)

Y3(1,3) - 500γ(1,3) ≤ 0 (A26)

Suggested borrow pit constraints:

Xsb(1,1,2) + Xsb(1,2,2) + Xsb(1,3,2) – 500δsb(1) ≤ 0 (A27)

Xsb(1,1,3) + Xsb(1,2,3) + Xsb(1,3,3) – 600δsb(1) ≤ 0 (A28)

Xsb(2,1,2) + Xsb(2,2,2) + Xsb(2,3,2) – 300δsb(2) ≤ 0 (A29)

Xsb(2,1,3) + Xsb(2,2,3) + Xsb(2,3,3) – 350δsb(2) ≤ 0 (A30)

Disposal site constraint:

1.5Xd(1,1,1) + 1.1Xd(1,1,2) + 1.2Xd(2,1,3) + 1.1Xd(3,1,2) + 1.2Xd(3,1,3) ≤ 5000 (A31)

Suggested disposal constraint:

1.5Xsd(1,1,1) + 1.1Xsd(1,1,2) + 1.2Xsd(2,1,3)+ 1.1Xsd(3,1,2) + 1.2Xsd(3,1,3) – 6000δsd(1) ≤ 0 (A32)

Zero-one variables constraints:

λ(1,2) + γ(1,2) ≤ 1 (A33)

λ(1,3) + γ(1,3) ≤ 1 (A34)

δsb(1) + δsb(2) ≤ 1 (A35)

δsd(1) ≤ 1 (A36)

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