Limit analysis of ultimate strength of I-sections subjected to edge loading using genetic algorithms

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The behavior and strength of hot rolled beams or built up plate girder, subjected to localized edge loading in the plane of the web is analyzed. A modified failure mechanism is presented. Yield line method is used to determine the ultimate load and the corresponding failure mechanism by equating the internal work done by the yield lines and the external work done by applied loads then minimizing the virtual work equation with respect to the unknown parameters. Due to the eventual existence of an axial load through the web, a reduced plastic moment capacity for different yield lines positions is considered. An efficient modified optimization technique (Genetic Algorithms) is used to solve such complex problems.

تم تحليل ودراسة سلوك ومقاومة الكمرات المدرفلة على الساخن أو الكمرات اللوحية والمعرضة لأحمال موضعية خطية فى مستوى اتجاه عصب الكمرة. وقد تم تقديم شكل معدل للانهيار.تم استخدام طريقة خطوط الكسر لتعيين الحمل الأقصى وشكل الانهيار وذلك بمساواة الشغل الداخلي المبذول الناتج من تكون خطوط الكسر مع الشغل الخارجي المبذول بواسطة الأحمال المؤثرة ثم حل المعادلة الناتجة للحصول على أقل قيمة لحمل الانهيار. ومع تأثير الأحمال فى مستوى عصب القطاع فقد تم الأخذ فى الاعتبار التخفيض الناتج في قيمة العزم اللدن وذلك لمختلف مواضع خطوط الكسر. هذا وقد قدمت طريقة قوية مثلى لحل مثل هذا النوع من المعادلات ألا وهى الخوارزميات الجينية.

Keywords: Optimization, Genetic algorithms, Yield line, Plates, Local failure

1. Introduction

Girders often sustain loads on the top flange that produce a localized compression on the web near the applied load. For plate girder with slender webs, collapse occurs due to web crippling. Web crippling consists of local buckling in the zone close to the load it results in appreciable displacements (transverse to the web plane) in a small area of the web without significant deformation of the remainder of the cross-section. Numerous tests carried out over the past 50 years have indicated that failure occurs due to web crippling.

Roberts [1,2] Roberts and Rocky [3], Robert and Chang [4], Robert and Markovic [5] developed simple mechanism solution for predicating ultimate loads as shown in fig. 2. The mechanism solution for web buckling was based on the formation of plastic yield lines in the web plate and plastic hinges in the flange. This mechanism gives an ultimate load smaller than that obtained from the experimental tests. The ratio between the experimental load and the theoretical ultimate load was found to be 1.2 to 1.5.

Roberts [1] developed a simple closed form solution for ultimate load as follows:

$$P_u = \left[1.1 t_w^2 (E\sigma_{yw})^{0.5} (\frac{t_f}{t_w})^{0.25} (1+\frac{c_e t_w}{d_w t_f}) \right] \frac{1}{F} . \quad (1)$$

Where
E is the modulus of elasticity,
c_e is the effective loaded length,
d_w is the web height, and
F is the correction factor =1.45.

A semi-empirical formula for the ultimate knife load P_u can be deduced from a Von Karman analogy for normal plate girder geometry with slender unstiffened webs as follows:

$$P_u = 0.5 t_w^{0.5} \sqrt{E\sigma_{yw}} \sqrt{t_f / t_w} (b_f / 10 t_f)^{0.25} . \quad (2)$$

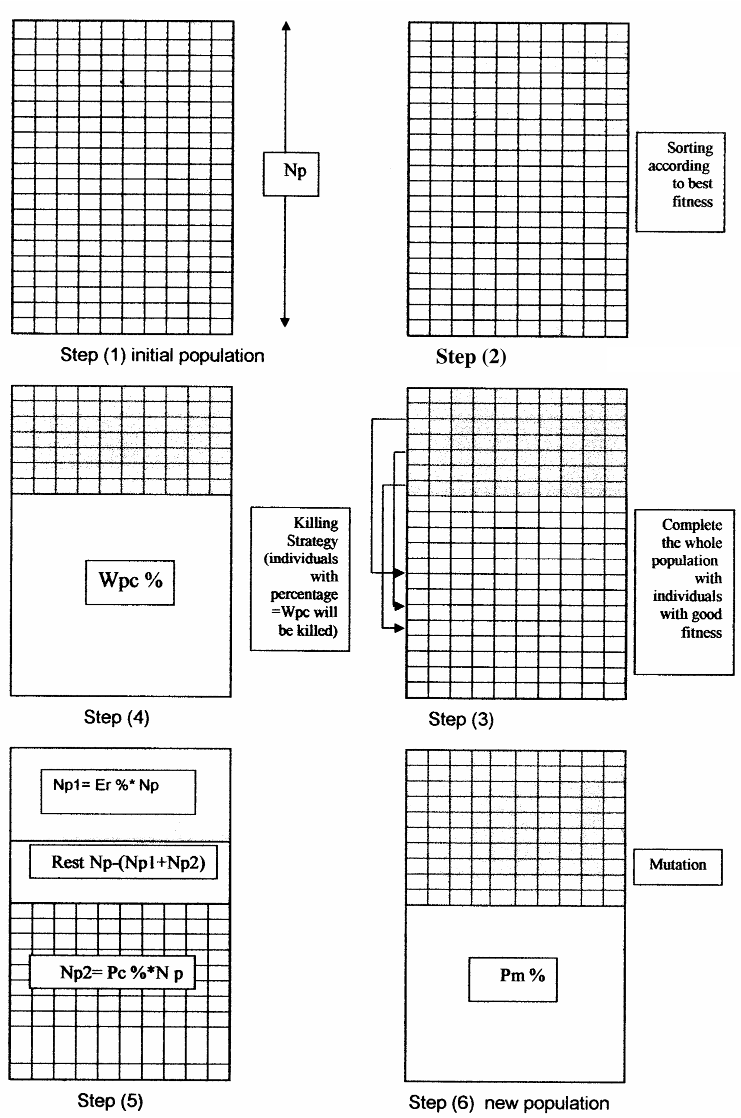

Fig. 1. Single generation step of the modified GA.

Section K1 of the Allowable Stress Design manual (ASD [6]), places limits on the compressive concentrated loads with respect to web crippling as follows:

$$P_u = 67.5 t_w^2 \left[1 + 3 (\frac{C_c}{d}) (\frac{t_w}{t_f})^{1.5} \right] \sqrt{\sigma_{yw} t_f / t_w} \ . \quad (3)$$

Where P_u is the maximum concentrated load (kips).

The Load and Resistance Factor Design method specification (LRFD-K1.4) [7] that is based on the work of Roberts gives the ultimate load that can be resisted by the web as follows:

$$R_n = 135 t_w^2 \left[1 + 3 (\frac{C_c}{d}) (\frac{t_w}{t_f})^{1.5} \right] \sqrt{(\sigma_{yw} t_f) / t_w} \ , \quad (4\text{-a})$$

and

$$P_U = \phi R_n \ . \quad (4\text{-b})$$

Where P_u is the required strength of the web (kips) and, $\phi = 0.75$.

LRFD specifications [8], give a modified form to find the ultimate load as follows:

$$R_n = 0.80 t_w^2 \left[1 + 3\left(\frac{C_c}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5} \right] \sqrt{\frac{E\sigma_{yw} t_f}{t_w}} , \tag{5-a}$$

and

$$P_u = \phi R_n . \tag{5-b}$$

A modified Genetic Algorithms technique is developed and presented. The ultimate load is determined using the yield line method for the simple classic failure mechanism, fig. 2, and the modified Genetic Algorithms to solve the virtual work equation. The ultimate load generated this way agrees with those obtained theoretically by other authors, but they are all less than the experimental values, Roberts [9]. In order to develop the actual failure mechanism and consequently the actual ultimate load, a modified failure mechanism is presented in this study. The yield line method is used to find the virtual work equation. The Genetic Algorithms is used as a better optimization technique to solve the obtained complex virtual work equations. The ultimate load based on the new modified failure mechanism agrees this time very well with the experimental results. The effect of some geometrical parameters on the ultimate load is presented as well.

2. Modified genetic algorithms technique

Genetic Algorithms (GAs) are computerized search methods based on the theories of genetics and natural selection developed by John Holland [10]. GAs use random techniques but exploit information from past experience to evolve solutions to real-world problems, once they are appropriately encoded. This adaptive search technique, which has powerful non-linear processing capabilities, can be used to solve multi-dimensional optimization problems with discrete variables and discontinuous functions. GAs has the ability to treat discrete variables and complex functions without derivatives. Further, GAs are less susceptible to becoming stuck at local optima compared to gradient search methods. GAs are different from traditional optimization methods such as the simplex method in the following aspects:

1. They work with a coded set of the variables and not with the variables themselves.
2. They search from a population of design variables rather than improving a single variable.
3. They use objective function information without any gradient information.
4. Their transition scheme is probabilistic, Goldberg [11], whereas traditional methods use gradient information.

The use of traditional simple Genetic Algorithm (GA) has indicated that the best individual of the population may fail to produce offspring for the next generation. The elitist strategy Davis [12] fixes this potential source of loss by copying the best individual of each generation into succeeding generation. Consequently, the elitist strategy increases the speed of domination of a population by a super individual. In other words, it appears to improve genetic algorithm performance. The elitist strategy keeps the best individuals with a certain percentage, termed elite ratio (E_r), of the population. Those individuals are called the elite.

In the present study a new technique, called killing strategy, is developed. The developed killing strategy kills the worst (weak) individuals with a certain percentage, termed weak percent (W_{pc}) of the population. The working procedure of the developed GA can be explained in the following steps, fig. 1:

1. Preparation of data files, which are the GA parameters population size N_p, elite ratio E_r, probability of crossover P_c, probability of mutation P_m, crossover operator, and seed number I, step 1.
2. Creation of population with number of individuals (chromosomes) equal to N_p.
3. Calculation of the objective function F_i (i = 1 ,2,...Np) for each individual.

The above three steps are included in step 1 of fig. 1.

4. Sorting the whole population N_p according to the value of fitness function of each individual where the smallest value of fitness

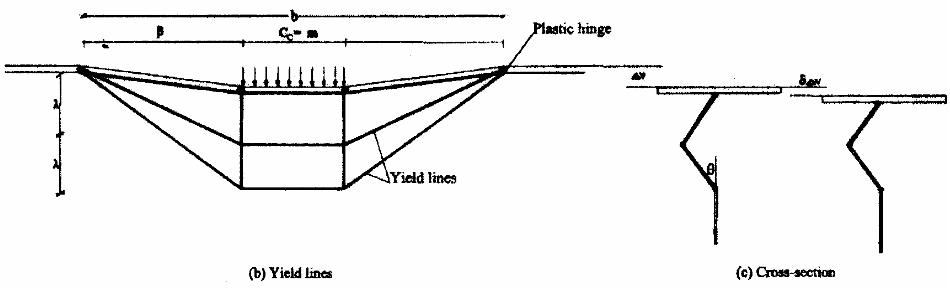

Fig. 2. Theoretical failure mechanism for web plate subjected to knife load.

function is the best value for function minimization, and the biggest value of fitness function is the best value for function maximization, step 2.
5. Killing all individuals with a percent equal to the weak percent Wpc, step 3.
6. Completion of the whole population with the survival individuals (arbitrary selection), step 4.
7. Applying of the elitist strategy and the crossover operator. At this, the population consists of the following three parts, step 5:
Part 1, contains the elite individuals. The number of these individuals $N_{p1}=E_r \bullet N_p$
Part 2, contains the number of individuals $N_{p2}=P_c \bullet N_p$
Part 3, contains the rest of population whose number of individuals equals: $Np-(N_{p1}+N_{p2})$.

The above three parts are included in fig. 1, step 5.
8. Performing the mutation stage, step 6.
9. The above steps are repeated until a converged solution is obtained or a prescribed maximum number of iteration has been performed.

The GA parameters were tested and tuned using 10 bar truss as a sizing problem. The following observations and concluding remarks can be deduced:
(a) The population size greater than 80 up to 200 gives good results for both one and two point crossover.
(b) The mutation probability should be very low, generally about 2 to 4 percent.
(c) The elite ratio should be low, generally about 10 to 30 percent.
(d) The crossover probability should be high, generally about 70 to 90 percent.
(e) The weak percentage should be high, generally about 60 to 80 percent.

3. Yield line method applied to the classic failure mechanism

The ultimate load can be determined using the yield line method for the proposed theoretical failure mechanism shown in fig.2. The dimensions λ and β define the position of the assumed yield lines and plastic hinges while the angle θ defines the deformation of the web just prior to collapse. If at the instant of collapse the applied load moves vertically through a small distance $\delta\Delta v$, the ultimate load can be determined by equating the internal work done by the yield lines and the external work done by the applied load as follows:

$$\left(\frac{P_u}{2}\right)(\delta\Delta v)$$

$$= m_w \left(\frac{3m+b}{4\lambda\lambda sin} + \frac{2\lambda\lambda cos}{(b-m)sin\theta} + \left(\frac{4\lambda^2 + \left(\frac{b-m}{2}\right)^2}{\lambda(b-m)sin\theta} \right) cos\theta + \frac{\left(\left(\frac{b-m}{2}\right)^2 + \sqrt{\left(\frac{b-m}{2}\right)^4 + 4\lambda^2\left(\frac{b-m}{2}\right)^2 4\lambda^4} \right) cos\theta}{\lambda(b-m)sin\theta} \right)$$

$$+ m_f b_f . \frac{\delta\Delta v}{\beta} . \qquad (6)$$

Where:

$b = (2\beta+m)$,

$$\sigma_{yf} = \sigma_{yw} = \sigma_y ,$$

$$m_w = \frac{\sigma_y . t_w^2}{4} , and \quad m_f = \frac{\sigma_y . t_f^2}{4} .$$

Eq. (6) is a nonlinear optimization problem. Solving this problem using the modified Genetic Algorithms (GA) technique leads to the ultimate load.

4. A modified failure mechanism for I-sections subjected to edge loading

For the new modified failure mechanism shown in fig. 3, the geometry of this mechanism is given in Fig. 4. Applying the yield line method, the internal work done by the web yield lines is as follows:

$$W_{in(web)} = W_{in(\overline{ab})} + W_{in(\overline{cd})} + W_{in(\overline{of})} + W_{in(\overline{oa})} + W_{in(\overline{oc})} + W_{in(\overline{ca})} + W_{in(\overline{cw})}$$

$$\begin{aligned}
W_{in(web)} = {} & mp_{ab}\left(\frac{b_3}{2}\right)\frac{\delta\Delta V}{2\lambda . \sin\phi} + mp_{cd}\,(b_2)\frac{\delta\Delta V}{2\lambda . \sin\phi} + mp_{of}\left(b_1 + \frac{m}{2}\right)\frac{\delta\Delta V}{2\lambda . \sin\phi} + mp_{oa}\left(\sqrt{4\lambda^2 + \left(b_1 + \frac{m}{2} - \frac{b_3}{2}\right)^2}\right)\times \\
& \left(\frac{1}{\sqrt{\lambda^2 + \left(b_1 + \frac{m}{2} - \frac{b_2}{2}\right)^2} . \sin\left(\tan^{-1}\left(\frac{2\lambda}{b_1 + \frac{m}{2} - \frac{b_3}{2}}\right) - \tan^{-1}\left(\frac{\lambda}{b_1 + \frac{m}{2} - \frac{b_2}{2}}\right)\right)}\right)\frac{\delta\Delta V . \cos\phi}{2\sin\phi} \\
& + mp_{oc}\left(\frac{1}{. \tan\left(\tan^{-1}\left(\frac{2\lambda}{b_1 + \frac{m}{2} - \frac{b_3}{2}}\right) - \tan^{-1}\left(\frac{\lambda}{b_1 + \frac{m}{2} - \frac{b_2}{2}}\right)\right)} + \frac{1}{taN\left(\tan^{-1}\left(\frac{\lambda}{b_1 + \frac{m}{2} - \frac{b_2}{2}}\right)\right)}\right)\frac{\delta\Delta V . \cos\phi}{2\sin\phi} \\
& + mp_{ca}\left(\frac{2}{\tan . \tan^{-1}\left(\frac{2\lambda}{b_3 - b_2}\right)}\right)\frac{\delta\Delta V\cos\phi}{2\sin\phi} + mp_{cy}\left(\sqrt{\lambda^2 + \left(\frac{b_2 - m}{2}\right)^2}\right)\times \\
& \left(\frac{\sin\left(\tan^{-1}\left(\frac{b_3 - b_2}{2\lambda}\right)\right)}{\lambda} + \frac{1}{\sqrt{\lambda^2 + \left(\frac{b_2 - m}{2}\right)^2} . \tan\left(\tan^{-1}\left(\frac{2\lambda}{b_2 - m}\right)\right)}\right)\frac{\delta\Delta\cos\phi}{2\sin\phi} ,
\end{aligned} \tag{7}$$

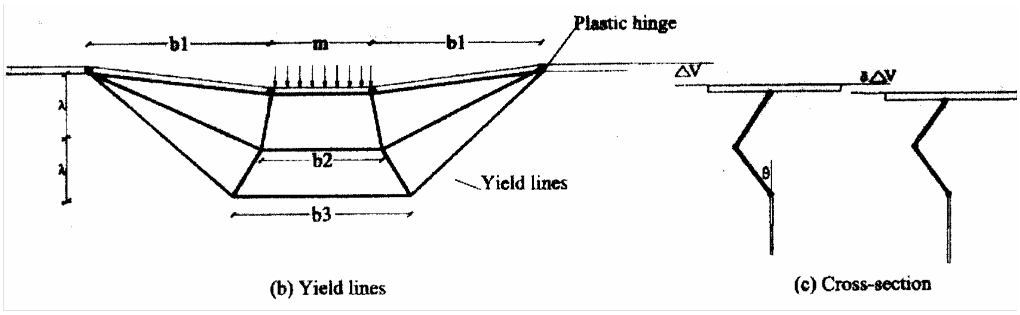

Fig. 3. Modified failure mechanism for web plate subjected to knife load.

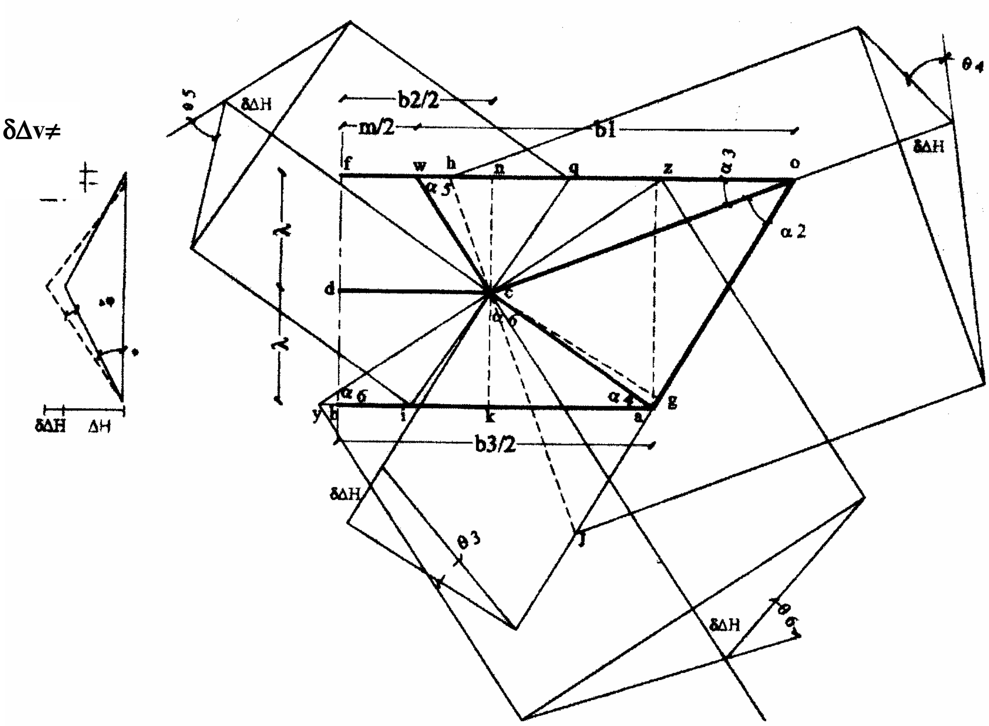

Fig. 4. Geometry of modified failure mechanism.

and the internal work done by the flange plastic hinges is as follows:

$$W_{in(flange)} = m_p^f \, .b_f \left(\frac{\delta \Delta V}{b_1} \right) (2). \tag{8}$$

The total internal work done by both yield lines in the web and plastic hinges in the flange.

$$W_{in(Total)} = W_{in(web)} + W_{in(flange)} \, . \tag{9}$$

The external work done by applied load is given by:

$$W_{ex} = \left(\frac{P_u}{2} \right) . \delta \Delta V, \tag{10}$$

$$W_{ex} = W_{in(Total)} \, . \tag{11}$$

Equating the internal work done $W_{in(\mathrm{total})}$ and the external work done W_{ex} ,then minimizing this equation with respect to the design variables defining the collapse mechanism leads to the ultimate loads.

5. Reduced plastic moments due to edge compression load

The existence of edge compression load on the web of an I-section generally reduces the plastic moment capacity of the yield line. This reduced plastic moment can be determined as follows:

The value of this moment depends on the orientation of the plastic line and on the magnitude of the applied normal stress. From the elementary theories of stress analysis, the stresses on transformed axes inclined an angle θ as shown in fig. 5-a are as follows:

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta, \quad (12)$$

$$\sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta, \quad (13)$$

$$\tau'_{xy} = -\frac{\sigma_{x-}\sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta . \quad (14)$$

The plastification of the web thickness is formed by the stresses σ_t and σ_c which are produced by considering the criterion of von Mises, eq. (15). The modified stress block is shown in fig. 5-b.

$$\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2 = \sigma_{yield}^2 . \quad (15)$$

Eq. (15) could be written in a simple form as $\sigma^2 + b\,\sigma + c = 0$.

There are two real roots for σ, one of them represents the compression stress σ_c and the other represents the tension stress σ_t. The reduced plastic moment of resistance m_p is given then as follows:

$$m'_p = \frac{\sigma_c t_c^2}{2} + \frac{\sigma_t t_t^2}{2}, \quad (16)$$

where,
$t = t_t + t_c$.

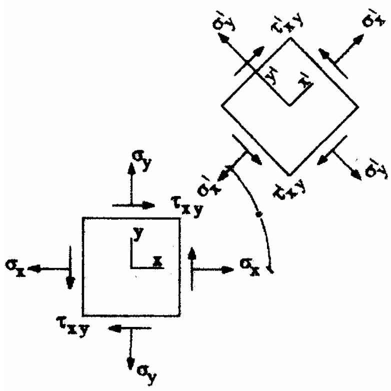

Fig. 5-a. Stress transformation to coordinate axes inclined angle θ.

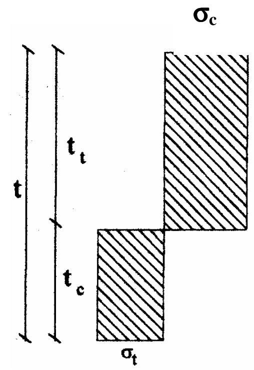

Fig. 5-b. Modified stress block.

6. Results

The ultimate load obtained by solving the optimization equation, eq. (11), with respect to the design parameters and taking into consideration the reduced plastic moment given by eq. (16) leads to the following results:

1. Figs. 6, 7 show the ultimate loads given by the modified mechanism taking into account the reduced plastic moment resistance compared to the results of 18 experimental tests by Roberts and Newark [9] and his theoretical results as well. This modified failure mechanism gives an ultimate load that agrees very well with the test results.
2. Studying the effect of changing some geometrical parameters on the ultimate load of some examples of the experimental tests by Roberts and Newark [9] leads to the following results:

- Flange width has no effect on the ultimate load capacity as shown in fig. 8

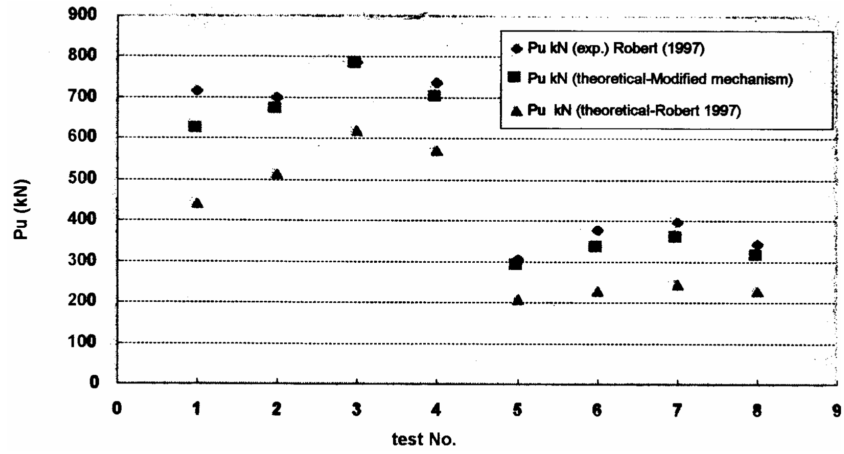

Fig. 6. Comparison of experimental, theoretical, and present study ultimate loads for tests (1-8).

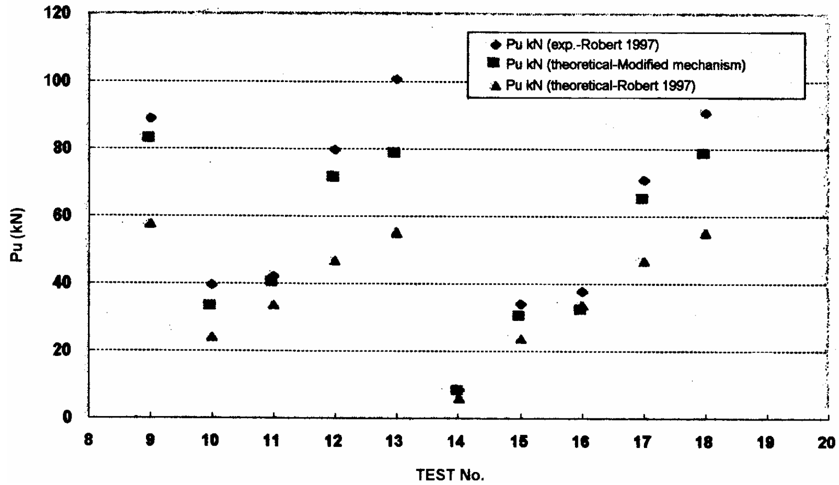

Fig. 7. Comparison of experimental, theoretical, and present study ultimate loads for tests (9-18).

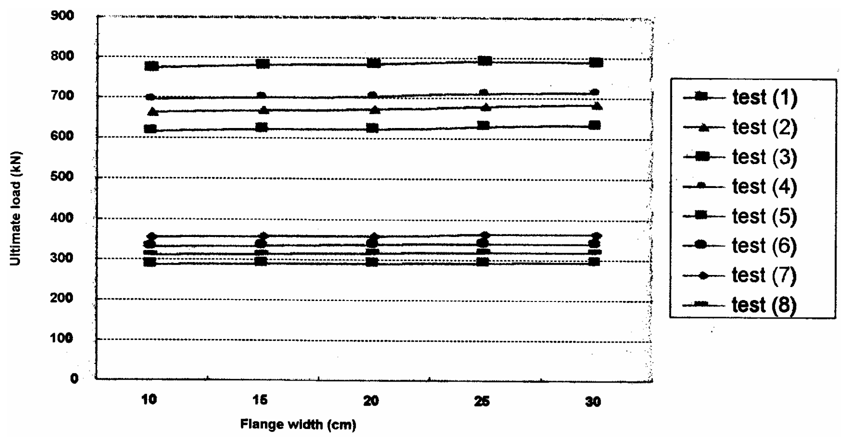

Fig. 8. Effect of flange width on ultimate load (modified mechanism).

• The ultimate load increases with the increase of its application length m as shown in fig. 9.
• The ultimate load increases slightly with the increase of flange thickness as shown in fig.10.
• The ultimate load increases rapidly with the increase of web thickness as shown in fig. 11.

7. Conclusions

The main conclusions can be summarized as follows:
• A numerical method based on the virtual work of the yield line theory has been described using modified genetic algorithm as an optimization technique. Since the method is numerical, it allows the yield line method to be applied for plates and steel connections with different shapes, complex failure mechanisms and loading.
• The modified GA technique developed using the elitist strategy in addition to the new killing strategy both improve the efficiency of the search process and tend to converge the problem faster, thus reducing the overall computational effort. The elitist strategy permits the algorithm to transfer the best members of a current population straight to the next population without changing them. Keeping the best part of the population is very important for speeding up the hill-climbing feature of genetic search where the fitness of the best member of population is to be improved from one generation to another. The killing strategy specifies how the less fit members are killed (removed) from the population after sorting the population depending upon its fitness. The genetic

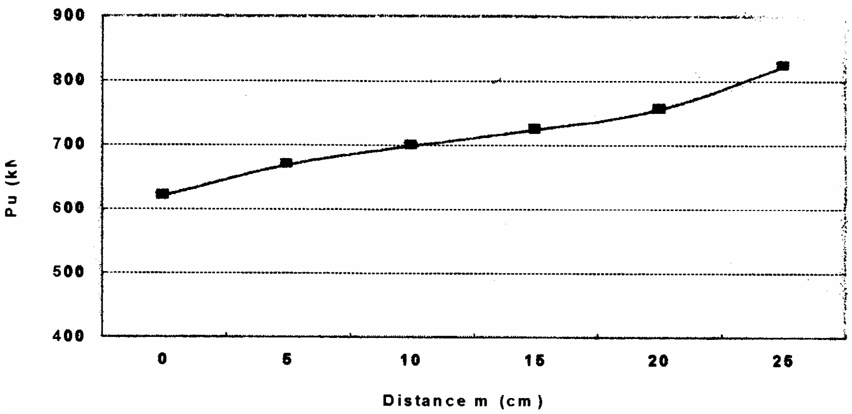

Fig. 9. Effect of loaded length on ultimate load (modified mechanism).

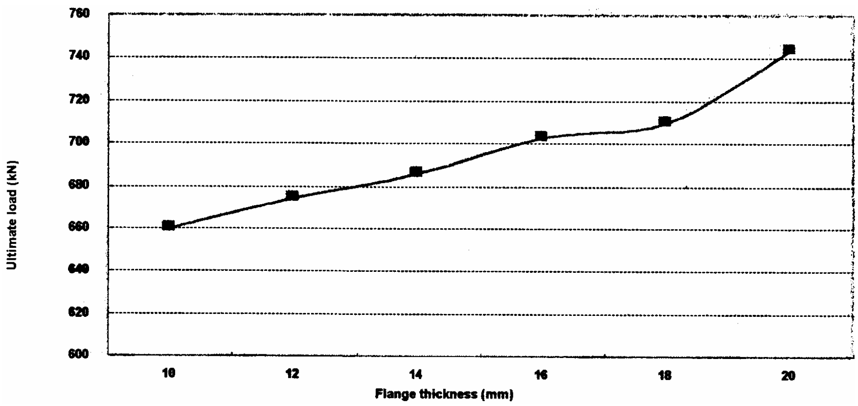

Fig. 10. Effect of flange thickness on ultimate load (modified mechanism).

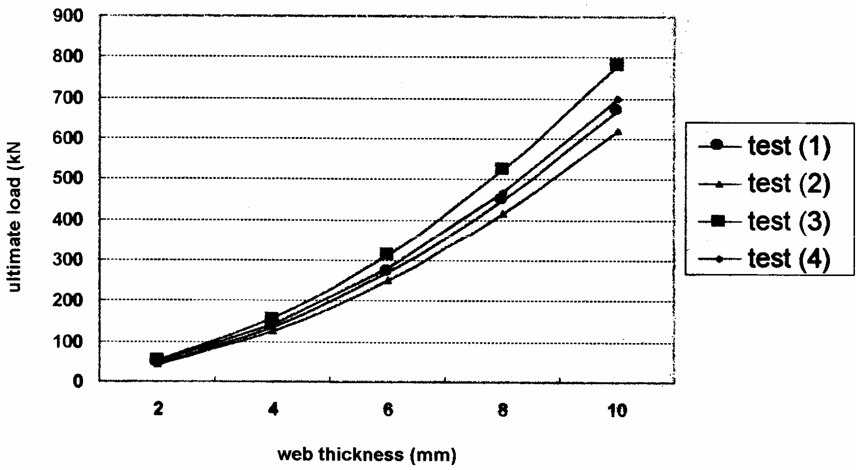

Fig. 11. Effect of web thickness on ultimate load (modified mechanism).

operator defining the number of chromosomes to be killed (removed) is called the weak percent.

• A modified failure mechanism is given for an I-sections subjected to knife loading. The presented theoretical ultimate loads based on this modified mechanism are close to those given by experimental tests.

• Since the plastic moment per unit length for each yield line is included in the work equation as variable parameters, the ultimate load and the corresponding failure mechanism can be determined for composite hybrid plate girders.

Notations

P_u	ultimate load,
t_w	web thickness,
E	modulus of elasticity ,
σ_{yw}	web yield stress,
σ_{yf}	flange yield stress,
t_f	flange thickness,
c_e	effective loaded length ,
d_w	web height,
d	overall depth of member,
b_f	flange width ,
N_p	population size,
P_c	probability of crossover,
P_m	probability of mutation,
E_r	elite percent and
Wpc	weak percent .

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