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Filter extensions of open nuclei

E. F. Lashein[a] and A. I. El-Maghrabi[b]
[a] *Mathematics and Physics Dept., Faculty of Engineering,Tanta University, Tanta, Egypt*
[b] *Mathematics Dept., Faculty of Education, Tanta University, Kafr El-Sheikh, Egypt*

The purpose of this paper is to introduce the concept of filter extensions of open nuclei and study some of its properties. Also, we discuss some separation axioms that are preserved under filter extensions of open nuclei. Moreover, we investigate the preservation of some weaker forms of compactness under filter extensions of open nuclei.

الهدف من هذا البحث هو تقديم ودراسة التوسع المثالى للأنبوية المفتوحة من النوع D. وأيضاً دراسة بعض خواص التوسع المثالى. بالاضافة الى ذلك ناقشنا المحافظة لبعض من مسلمات الانفصال من النوع D. وكذلك بعض من أنواع الاصماط من النوع D باستخدام التوسع المثالى للانوية المفتوحة من النوع D.

Keywords: Filter, Extension of topologies, Frames, Open nuclei

1. Introduction

All unexplained facts concerning frames can be found in P. T. Johnstone [1]. Recall that a frame is a complete lattice L in which the infinite distributive law, that is $a \wedge \vee S = \vee\{a \wedge s: s \in S\}$ holds for all $a \in L$, $S \subseteq L$. The set of all open sets of a topological space forms a frame. Frames can be viewed as generalized topological spaces.

A frame map [2] is a map from one fram to another that preserves the joins of arbitrary sets and the meets of finite sets.

A homomorphism [3] is a map, that is, both a meet-homomorphism ($\wedge$-homomorphism) and a join-homomorphism ($\vee$-homomorphism). Thus a homomorphism φ *of* the lattice L into the lattice L' is a map φ of L into L' satisfying both $(a \wedge b)\varphi = a\varphi \wedge b\varphi$ and $(a \vee b)\varphi = a\varphi \vee b\varphi$, for all $a, b \in L$.

A nucleus j on a frame L [2] that contains more than one element is defined as a map $j: L \to L$ satisfying: $a \le j(a)$, $j(a) = j(j(a))$, $j(a \wedge b) = j(a) \wedge j(b)$, for all $a, b \in L$.

Let a be an element of a frame L. Then the maps $c_a; u_a: L \to L$; $c_a(x) = a \vee x; u_a(x) = a \Rightarrow x$ (we put $x^* = x \Rightarrow 0$) are nuclei which for topologies correspond to a closed, open subspace, respectively. Nuclei of this form are therefore said to be closed, open respectively. We shall denote by *N (L)* the lattice of all nuclei, by *O(L)* the lattice of open nuclei and by *C(L)* the lattice of closed nuclei on a frame *L*. We denote the bottom and the top elements of *N (L)* by Δ, ∇. The interior and the closure of a nucleus j will be denoted by j°, j^-, respectively.

For $j \in N(L)$, we define the interior (resp. the closure) [2] of j as:

$j^\circ = \wedge\{k: k \in O(L), j \le k\}$ (resp. $j^- = \vee\{k: k \in C(L), k \le j\}$).

Let L be a frame, $j \in N(L)$. Then j [2] is said to be:

(i) semi-open, if there exists an open nucleus u of L such that $\bar{u} \le j \le u$,

(ii) α-open, if $j^{\circ-\circ} \le j$,

(iii) preopen, if $j^{-\circ} \le j$,

(iv) semi-preopen, if $j^{-\circ-} \le j$,

(v) dense, if $j^- = \Delta$.

The class of all semi-open, α-open, preopen, semi-preopen nuclei will be denoted by *SO (L), αO (L), PO (L), SPO (L)*, respectively.

Let L be a frame, $j \in N(L)$. Then j is said to be a regular-open (resp. a regular closed) [4] nucleus if $j^{-\circ} = j$ (resp. $j^{\circ-} = j$).

If x is an element of a frame L, then the element $\vee\{y\in L:y\wedge x=0\}$ will be denoted by x^* [5].

A sublattice [3] $< K,\wedge,\vee >$ of the lattice $< L,\wedge,\vee>$ is defined on a non void subset K of L with the property that $a,b\in K$ implies that $a\wedge b, a\vee b\in K$ ($\wedge,\vee$ taken in L) and the $\wedge$ and the $\vee$ of L are restrictions to K of the $\wedge$ and the $\vee$ of L. Let L be a lattice and f be a non void subset of L. Then f is called a filter [3] iff $a,b\in f$ implies that $a\wedge b\in f$ and $a\in f, x\in L; a\leq x$ implies that $x\in f$ [3].

A frame L is called submaximal [6] if all of its dense nuclei are open.

Lemma 1.1 [4]

Let L be a frame and $j,k\in N(L)$. Then

(i)If $j\leq k$, then $j^\circ\leq k^\circ$ and $j^-\leq k^-$.

(ii)j is an open (resp. closed) nucleus iff

(iii) $j=j^\circ$ (resp. $j=j^-$).

(iv) $(j\vee k)^\circ=j^\circ\vee k^\circ, (j\wedge k)^-=j^-\wedge k^-$.

Lemma 1.2 [4]

(i) A nucleus j is semi-open iff $j^{o-}\leq j$.

(ii) Every open nucleus is semi-open.

(iii) Every open nucleus is preopen.

(iv) An arbitrary meet of semi-open nuclei is semi-open.

(v) An arbitrary join of preopen nuclei is preopen.

Lemmal 1.3 [2]

Let L be a frame and $u\in O(L)$, $j\in N(L)>$. Then $u\vee j=u\vee j^-$ [4]. In an extremely disconnected frame, for u $u\vee j^-$ $O(L)$ and j $u\vee j^-$ $N(L)$, $u\vee j=u\vee j^-$ [6].

2. Filter extensions of open nuclei

In this section, we introduce the concept of filter extensions of open nuclei and we investigate some of its properties.

Definition 2.1

If L is a frame and j is a nucleus of L such that $j\notin O(L)$, then a frame $L(j)=\{u'\wedge(u\vee j):u,u'\in O(L)\}$ is called a simple extension of $O(L)$ by a nucleus j.

Lemma 2.1

In a frame L, j is a preopen nucleus of L iff there exists an open nucleus u of L such that $j^-\leq u\leq j$.

Proof

Obvious.

Lemma 2.2

If L is simple extension of $O(L)$ on a frame L, then $RO(O(L))$ = RO(L(j)) iff $\overline{G}_{L(j)}=\overline{G}_{L(j)}$, for every $G\in L(j)$.

Proof

Obvious.

Definition 2.2

Let L be a frame, j be a nucleus of L and $j\notin O(L)$. Then a frame $L(j)=\{u\vee k:u\in O(L), k<j\}$ is alocal discrete extension of $O(L)$ by a nucleus j.

We give further results on the concept of a preopen nucleus concept.

Lemma 2.3

A nucleus j of a frame L is preopen iff thre exists a regular open nucleus w and a dense nucleus k of L such that j = w $\vee k$.

Proof

Obvious.

Lemma 2.4

A frame $O(L)$ is submaximal iff $PO(L)=L$.

Proof

Obvious.

Theorem 2.1

If L is a frame and F is a family of principle filters F on L, then the family $L(F)=\{u\wedge f:u\in O(L), f\in F\}$ is a frame on L; $O(L)\subseteq L(F)$ and $f\in L(F)$ ($u\wedge f$ means that $u(L)\cap f$), for every $f\in F$.

Proof

We prove that $L(F) = \{u\wedge f : u \in O(L),\ f \in F\}$ forms a frame on L as follows: Firstly, we prove that $< L(F), \wedge, \vee >$ forms a lattice.

For $G, H, K \in L(F)$, there exists $u_1, u_2, u_3 \in O(L)$ and $f_1, f_2, f_3 \in F$ such that $G = u_1 \wedge f1$, $H = u_2 \wedge f_2$ and $k = u_3 \wedge f_3$. Since $O(L)$ is a frame, F is a sublattice of L [3], then both $\wedge$ and $\vee$ are idempotent, commutative, associative and they satisfy the two absorption identities on $L(F)$. Secondly, clearly, $L(F)$ is a complete lattice.

Thirdly, we prove that $< L(F), \wedge, \vee >$ satisfies $\vee$-infinite distributive law. For any $G \in L(F)$ and any subset H of $L(F)$, that is, $H = \{H_i : I \in I\}$ where $G = u \wedge f$ and $H_i = u_i \wedge f_i$, for $u, u_i \in O(L)$ and $f, fi \in F$, then we need to prove $G \wedge \underset{i\in I}{\vee} H_i = \underset{i\in I}{\vee}(G \wedge H_i)$. If $h \in G \wedge \underset{i\in I}{\vee} H_i$ implies that $h \in G$ and $h \in \underset{i\in I}{\vee} H_i$ implies that $h \in G$ and for all $i \in I$, $h \in Hi$ implies that, for all $I \in I$, $h \in G$ and $h \in H_i$, hence h $h \in \underset{i\in I}{\vee}(G \wedge H_i)$.

$$\text{Therefore } G \wedge \underset{i\in I}{\vee} H_i \le \underset{i\in I}{\vee}(G \wedge H_i) . \qquad (1)$$

Conversely, Let $h \in \underset{i\in I}{\vee}(G \wedge H_i)$. Then for all $i \in I$, $h \in I$, $h \in G \wedge H_i$ implies that for all $I \in I$, $h \in G$ and $h \in H_i$, hence $h \in G$ and $I \in I$, $h \in Hi$ implies that $h \in G$ and $h \underset{i\in I}{\vee} H_i$. thus $h \in \underset{i\in I}{\vee} H_i$.

Thus, $h \in G \wedge \underset{i\in I}{\vee} H_i$, then:

$$\underset{i\in I}{\vee}(G \wedge H_i) \le G \underset{i\in I}{\vee} H_i . \qquad (2)$$

Hence, from eqs. (1, 2) we have the result. Then $L(F)$ $\{u \wedge f : u \in O(L), f \in F\}$ forms a frame.

Also, if $G \in O(L)$, then $G = G \wedge \nabla$ implies that $G \in L(F)$. Hence $O(L) \subseteq L(F)$.

Furthermore, for any $f \in F$, then $f = \wedge \nabla f$, where $\nabla \in O(L)$. Hence $f \in L(f)$.

Remark 2.1

A frame which is constructed in theorem 2.1 called a filter extension of open nucli by a family of priniciple filters F on L, that is, if L is a frame and F is a family of principle filters on L, then frame $L(F) = \{u\wedge f : u \in O(L),\ f \in F\}$ is called the filter extension of $O(L)$ by a family of principle filters F on L.

Remark 2.2

Simple extensions of open nuclei, Iocal discrete extensions of open nuclei and filter extensions of open nuclei are independent as shown in example 2.1.

Example 2.1

Let $L = \{0, a,b,c,1\}$ be a frame one a chain $0 < a\ b < \ldots < 1$ on the closed unit interval. The set of all nuclei on L is $N(L) = \{j_0, j_a, j_b, j_c, j_1\}$, where $j_i(x) = \begin{cases} i, & x \le i \\ x, & x > i \end{cases}$. for each $i \in L$, the frame map $u_i(x) = \begin{cases} 1, & i = 0 \\ x, & i \ne 0 \end{cases}$ gives the class $O(L) = u_0, u_1 = u_a = u_b = u_c\}$.Hence, a simple extension of open nuclei by a nucleus j_b is $L(j_b) = u_0, u_1 = u_a = u_b = u_c, j_b\}$. Also, a local discrete extension of open nuclei by a nucleus j_b is $L(j_b) = u_0, u_1 = u_a = u_b = u_c, ja\}$.

Since the family of principle filters on L is $F = \{f_i : i = 0,a,b,c,1\}$, i.e. a filter extension of open nuclei (by a family of principle filters on L is $L(F) = \{u_o, u_1 = u_a = u_b = u_c, f_a, f_b, f_c\}$.

Example 2.2

A chain $0 \prec a \prec b \prec c \prec \cdots \prec 1$ on a closed [0,1] interval $f \in l(F)$ forms a frame. The set of all nuclei is $N(L) = \{j_i : i = 0, a, b, c, \cdots, 1\}$, the set of all open nuclei is $O(L) = \{u_0, u_1 = u_a = u_b = u_c = \cdots\}$ and the family of principle filters on L is $F = \{f_i : i = 0, a, b, c, \cdots, 1\}$. Then a filter extension of open nuclei is $L(F) = \{u_o, u_1 = u_a = u_b = u_c = \cdots, f_i : a \le i < 1\}$.

Lemma 2.5

If L is a frame and $L(F)$ is a filter extension of open nuclei by a family of principle filters F on L, then $O(L) = L(F)$ if $F = \{\nabla\}$, for every $f \in F$.

Proof

First. Let $G \in O(L)$. Then $G = G \wedge \nabla$, where $G \in O(L)$ and $F = \{\nabla\}$, for every $f \in F$. Hence, $G \in L(F)$. Therefore,

$O(L) \subseteq L(F)$. (3)

On the other hand. If $G \in L(F)$, then there exists $u \in O(L)$ and $f \in F$ such that $G = u \wedge f$ but $F = \{\nabla\}$, for every $f \in F$. Hence $G \in O(L)$. Thus;

$L(F) \subseteq O(L)$. (4)

According to eqs. (3, 4) the proof is complete.

Corollary 2.1

Let L, K be two frames; L(F),K(F) be a filter extension of *O(L)*, *O(K)*, respectively by a family of principle filters F on L,K and $\varphi: O(L) \to O(K)$ is homomorphism. Then $\varphi_F : L(F) \to K(F)$ is homomorphism if $F = \{\nabla\}$, for every $f \in F$.

Theorem 2.2

If L is a frame and *L(F)* is a filter extension of *O(L)* by a family of principle filters on L, then $\overline{G}_{L(F)} = \overline{G}_{O(L)}$, for every $G \in L(F)$.

Proof

For $G \in L(F)$, since $O(L) \subseteq L(F)$, then,

$\overline{G}_{L(F)} \leq \overline{G}_{O(L)}$. (5)

Conversely, let $x \in \overline{G}_{O(L)}$. Then there exists an open nucleus u of *O(L)* such that $u \wedge G \neq \Delta$. For $v \in L(F)$, there exists $w \in O(L)$ and $f_1 \in F$ such that $v = w \wedge f_1$, then $v \wedge G = (w \wedge f_1) \wedge (H \wedge f_2)$, where $G = H \wedge f_2$, for $H \in O(L), f_2 \in F$. Hence $v \wedge G \neq \Delta$ and $x \in \overline{G}_{L(F)}$, where $w \wedge H \in O(L)$ and $f_1 \wedge f_2 \in F$. Therefore $\overline{G}_{O(L)} \leq \overline{G}_{L(F)}$ (2). Then from eqs. (5, 6) we have $\overline{G}_{O(L)} = \overline{G}_{L(F)}$, for every $G \in L(F)$.

Theorem 2.3

Let L be a frame and *L(F)* be a filter extension of *O(L)* by a family of principle filters on L. Then *RO(O(L)) = RO(L(F))*.

Proof

Obvious from Lemma 2.2 and theorm 2.2.

Theorem 2.4

Let L be a frame and *L(F)* be a filter extension of *O(L)* by a family of principle filters on L, then $SO(O(L)) \subseteq SO(L(F))$.

Proof

Let G be a semi-open nucleus of *O(L)*. Then there exists an open nucleus u of *O(L)* such that $\overline{u} \leq G \leq u$. Hence G is a semi –open nucleus of *L(F)*by Theorem 1.4). Therefore $SO(O(L)) \subseteq SO(L(F))$.

Theorem 2.5

Let L is a frame and *L(F)* be a filter extension of *O(L)* by a family of principle filters on L and f is a semi-open nucleus of *O(L)*, for every $f \in F$ Then $SO(O(L)) \subseteq SO(L(F))$.

Proof

If G is a semi-open nucleus of *L(F)*, then there exists an open nucleus u of *L(F)* such that $\overline{u} \leq G \leq u$ where $u = H \wedge f$, for $H \in O(L)$, $f \in F$. Hence G is a semi-open nucleus of *O(L)* (by Theorem 2.2) and *f* is semi-open of *O(L)*, for every $f \in F$ Therefore $SO((L)F)) \leq SO((O(L))$.

Corollary 2.2

If L is a frame, *L(F)* is a filter extension of *O(L)* by a family of principle filters on L and $f \in SO(O(L))$, for every $f \in F$, then $SO(O(L)) = SO(L(F))$.

Proof

Obvious from theorems 2.4 and 2.5.

Corollary 2.3

If $\varphi: SO(O(L)) \to SO(O(K))$ is $\wedge$-homomorphism, then, $\varphi_F : SO(L(F)) \to SO(K(F))$ is $\wedge$-homomorphism if, f is semi-open of *O(L)*, *O(K)*, respectively, for every $f \in F$.

Therem 2.6

If *O(L)*is submaximal fram and *L(F)* is a filter extension of *O(L)* by a family of principle filters on L, then *PO(O(L))≤PO(L(F))*.

Proof

Since *O(L)* is submaximal frame, then *PO(O(L))=O(L)* (by Lemma 2.4). Hence *PO(OL)) = O(L)≤L(F)⊆PO(L(F))*. Therefore *PO(O(L)) ≤PO(L(F))*.

Theorem 2.7

If *O(L)* is submaximal frame and *L(F)* is a filter extension of *O(L)* by a family of principle filters on *L*, then $PO(O(L)) \subseteq P \quad O(L(F))$.

Proof

Since *O(L)* is submaximal frame, then *PO(O (L))=O(L)* (by Theorem 2.2). Hence $PO(O(L)) = O(L) \subseteq L(F) \subseteq P \quad O(L(F))$ .Therefore, $PO(O(L)) \subseteq P \quad O(L(F))$.

Theorem 2.8

Let L be a frame, *L(F)* be a filter extension of *O(L)* by a family of principle filters on *L* and f be a preopen nucleus of *O(L)*, for every $f \in F$. Then $PO(L(F)) \leq PO(O(L))$.

Proof

Let *G* be a preopen nucleus of *L(F)*. Then there exists an open nucleus u of *L(F)* such that $\overline{G} \leq u \leq G$ (by Lemma 2.1), where $u = w \wedge f$, for $w \in O(L)$, $f \in F$. Hence, G is a preopen nucleus of *O(L)* (by Theorem 2.4). Therefore,$Po(L(F)) \leq PO(O(L))$.

Corollary 2.4

If *L* is a frame and *L(F)* is a filter extension of *O(L)* by a family of principle filters on *L*, then $PO(O(L)) = P \quad O(L(F))$, if *O(L)* is a submaximal frame and $f \in PO(O(L))$, for every $f \in F$.

Proof

Obvious from theorems 2.6 and 2.7.

Corollary 2.5

If φ is $\wedge$-homomorphism from $PO(O(L))$ into $PO(O(K))$, then φ_F is $\wedge$-homomorphism from *PO(L(F))* into *PO(k(F))*, if $O(L), O(K)$ submaximal frames and f is a preopen nucleus of *O(L), O(K),* respectively, for every $f \in F$.

3. Preservation of some separation axioms for frames under filter extensions of open nuclei

In this section we discuss the preservation of some separation axioms for frames such as, T_i-frame $(i=1,2)$ [4]; regular, normal frame [5]; S-T_i frame $(i=1,2)$ [1] & *P*-T_i frame $(i = 1,2)$ [4] under filter extensions of open nuclei.

Proposition 3.1

Let *O(L)* be T_i-frame. Then *L(F)* is T_i-frame, if $F=\{\nabla\}$, for every $f \in F ; i = 1,2$.

Proof

We prove that the theorem only for a T_1-frame. Let *h, g* be a parallel pair of frame maps with domain $L(F)$ such that $h(x) \leq g(x)$, for every $x \in L(F)$. Then (by Lemma 2.5) *h, g* are parallel pair of frame maps of O(L) with domain *O(L)* such that $h(x) \leq g(x)$, for every $x \in O(L)$. Since *O(L)* is a T_1-frame, then $h = g$. Hence *L(F)* is a T_1-frame.

Proposition 3.2

If *O(L)* is a regular frame and $F=\{\nabla\}$, for every $f \in F$, then *L(F)* is regular.

Proof

Let $x \in L(F)$. Then (by Lemma 2.3) $x \in O(L)$. Since *O(L)* is a regular frame, then $x = \bigvee\{u \in L(F) : u^* \vee x = 1\}$. Hence *L(F)* is a regular frame.

Proposition 3.3

Let *O(L)* be a normal frame. Then *L(F)* is a normal frame, if $F=\{\nabla\}$, for every $f \in F$.

Proof

Assume that O(L) is a normal frame, then for every $x,y \in O(L) = L(F)$ satisfying $x \vee y = 1$, there exists $u \in o(L) = L(F)$ such that $x \vee u = y \vee u^* = 1$(by Lemma 2.5). Therefore, L(F) is normal.

Theorem 3.1

Let *O(L)* be a *S-T₁* frame and *d* fis emi-=open of SO)O(L)), for every $f \in F$, then L(F) is $S\text{-}T_i$ frame, where $I = 1,2$.

Proof

We prove that the theorem only for a $S-T_1$ frame. If for every two semi-open nuclei j_1, j_2 of *L(F)* such that $j_1(x) \le j_2(x)$, for $x \in L(F)$, then (by Corollary 2.2) j_1, j_2 are semi-open nuclei of *O(L)* such that $j_1(x) \le j_2(x)$, for $x \in O(L)$. Since *O(L) is* $S-T_1$ frame, then $j_1 = j_2$. Hence *L(F)* is a $S-T_1$ frame. The proof is analogous for $S\text{-}T_2$.

Therem 3.2

Let O*(L)* be $P-T_i$ frame. Then *L(F)* is $P-T_i$ frame, if *O(L)* is a submaximal frame and $f \in PO(O(L))$, for every $f \in F$, $i = 1,2$.

Proof

Assume that *O(L)* is a P-T_2 frame, then for every two preopen nuclei j_1,j_2 of *PO(O(L))= PO(L(F))* such that $j1(x) \wedge j2(y) = 0$ whenever $x \wedge y = 0$ in $O(L) = L(F)$ implies that $j1 = j2$ (by Corollary 2.4). Therefore *L(F)* is a *P-T2* frame. *If O(L)* is a *P– T1* frame, then the proof is similar.

4. Preservation of some weaker forms of compactness on frames under filter extensions of open nuclei

This section is devoted to study some weaker forms of compactness on frames such as, compactness [5]; semi-compactness [4] and strong compactness [4] frame which are preserved under filter extensions of open nuclei.

Lemma 4.1

If *O(L)* is a compact frame and *L(F)* is a filter extension of *O(L)* by a family of principle filters *F* on *L*, then *L(F)* is compact, if $F = \{\nabla\}$, for every $f \in F$.

Proof

Let $\{U_i : i \in I\}$ be any family of open nuclei of L(F) for which $\vee_{i \in I} U_i = \nabla$. Then $\{U_i : i \in I\}$ is a family of open nuclei of *O(L)* for which $\vee_{i \in I} U_i = \nabla$ (by Lemma 2.3). Since *O(L)* is a compact frame, then there exists a finite subfamily $\{U_{i1}, U_{i2}, \cdots, U_{in}\}$ of open nuclei of $L(F)$ for which $\vee_{k=1}^{n} U_{ik} = \nabla$. Hence *L(F)* is a compact frame.

Theorem 4.1

Let *O(L)* be a semi-compact frame, *L(F)* be a filter extension of *O(L)* by a family of principle filters *F* on *L* and $f \in SO(O(L))$, for every $f \in F$. Then L(F) is semi-compact.

Proof

Assume that $\{U_i : i \in I\}$ is a family of semi-open nuclei of *L(F)* for which $\bigvee_{i \in I} U_i = \nabla$, then $\{U_i : i \in I\}$ is a family of semi-open nuclei of *O(L)* for which $\bigvee_{i \in I} U_i = \nabla$ (by Corollary 2.2). Since *O(L)* is semi-compact, then there exists a finite subfamily $\{U_{i1}, U_{i2}, \cdots, U_{in}\}$ of semi-open nuclei of *L(F)* for which $\bigvee_{k=1}^{n} U_{ik} = \nabla$. Therefore *L(F)* is a semi-compact frame.

Theorem 4.2

If *O(L)* is a strongly compact frame and *L(F)* is a filter extension of *O(L)* by a family of principle filters *F* on *L*, then *L(F)* is strongly compact, if *O(L)* is a submaximal frame and $f \in PO(O(L))$, for every $f \in F$.

Proof

Let $\{U_i : i \in I\}$ be any family of preopen nuclei of *L(F)* for which $\vee_{i \in I} U_i = \nabla$. Then $\{U_i : i \in I\}$ is a family of preopen nuclei of *O(L)* for which $\vee_{i \in I} U_i = \nabla$ (by Corollary 2.4). Since *O(L)* is strongly compact, then there exists a finite subfamily $\{U_{i1}, U_{i2}, \cdots, U_{in}\}$ of preopen nuclei of $L(F)$ for which $\vee_{k=1}^{n} U_{ik} = \nabla$. Then *L(F)* is a strongly compact frame.

Work extension

What kind of probertiees of *L* carry over to subframes of *N(L)* containing *O(L)*?

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