# Characteristics of submerged hydraulic jumps in radial basins with a vertical drop in the bed 

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#### Abstract

In this paper, the submerged hydraulic jump formed in a radial stilling basin provided with sudden drop is theoretically and experimentally analyzed. The effects of various parameters, on the characteristics of the formed jump, such as submergence, height of vertical drop, the position of the drop measured from the beginning of the basin and supercritical flow Froude number are investigated. The experimental investigations were conducted on a 30 cm wide, 45 cm deep and 12 m long laboratory flume. In the theoretical study, the one-dimensional momentum and continuity equations were used to develop theoretical model for the relative depth ratio of the jump. Also, the energy equation is used to develop an expression for the energy loss ratio. The developed equations were verified using the obtained experimental data. Also, a model to predict the length of the submerged radial hydraulic jump was developed using both dimensional analysis and statistical methods. The model was calibrated and checked using the collected experimental data. Results of such statistical models were compared to the measured data and good agreement is existed. في هذا البحث أجريت در اسة نظرية ومعطلية للقفزة الهيدروليكية المغمورة الحادثة في أحو اض التهئئة القطرية علي قــاع ذي   المعطلية أجريت التجارب المعملية لاستخدام نتائجها في مراجعة وتحقيق العلاقات النظرية المستتجة وكذلك لتعين المعــاملات    النسبيين للقفزة الهيدروليكية المغمورة, في نفس الوقت تقل قيمة الفاقد النسبي في الطاقة. أما رقم فرويد فيعطــى تــأثنيرا لا  الإحصائي لإيجاد علاقة تجريبية لحساب الطول النسبي للقفزة الهيدروليكية المغمورة .


Keywords: Submerged hydraulic jump, Radial stilling basins, Drop, Energy dissipators

## 1. Introduction

The presence of a drop in the bed of the channel is sometimes necessary and can not be avoided based on the topography of the land where the channel is to be constructed. The effect of existing a vertical drop in a radial stilling basin, on the jump characteristics, has not been yet investigated to the best knowledge of the authors. However, the effect of the drop on the flow characteristics in rectangular prismatic channels was studied in horizontal channels by Hager [1], Hager and Bretez [2] and Ohtsu et al. [3]. Hager [1] studied B-jumps at an abrupt drop, he found that the relative energy dissipation becomes
insignificant when $F_{1}=8.0$, and maximum energy dissipation occur for relative drop height equal to 2.5. Hager and Bretez [2], studied the positive and the negative steps in a horizontal channel. They derived analytically the relationship for A -jump and B -jump. Ohtsu et al. [3] investigated the transition from supercritical to subcritical flow at an abrupt drop. Negm [4] studied the hydraulic jumps at positive and negative steps on sloping floors, and a predicting model for computing the sequent depth ratio of the hydraulic jump was developed. Armenio et al. [5] investigated the effects of vertical drop in pressure fluctuations at the bottom of a hydraulic jump.

The characteristics of submerged jump in rectangular channels were presented and discussed by many investigators e.g. Goivanda and Rajaratnam [6]. Long et al. [7] and Ma et al. [8] studied numerically submerged jump in rectangular channels. Ohtsu et al. [9] presented a systematic investigation on transition from supercritical to subcritical flow for submerged jump below an expanding outlet. The submerged radial jump was investigated theoretically and experimentally by McCorquodale and Khalifa [10]. They found that the dimensionless water depths and hydraulic grade line follow a fourth degree polynomial in Froude number $F_{1}$ and submergence ratio $S$. Also, they derived a theoretical equation for the submergence ratio. They found that the length of jump may be estimated as a function of $F_{1}$ and $S$ where,
$\frac{L_{j}}{y_{1}}=a\left(F_{1}+S\right)^{2}+b\left(F_{1}+S\right)+c$.

The results of their study are applicable for a total divergence angle of (13.5 $)$, $r_{o}$ of 1.7 to $2.0,2<S<10$ and $2.5<F_{1}<8$ where, $r_{o}$ is the dimensionless radius of the jump. It was concluded that the length of the radial submerged hydraulic jump is shorter than that of the rectangular one and may be longer than the free radial jump especially for lower submergence. This is due to the effect of the increased jump weight that causes the jet to take longer distance to decay. The length of hydraulic jump is needed to design the stilling basins. Shorter the length of jump means shorter length of basin and hence the basin will be more economic.

## 2. Development of the theoretical models

### 2.1. Relative depth of jump

Fig. 1 presents a definition sketch for submerged hydraulic jump in radial basin provided with a vertical drop.

Both the 1-D momentum and continuity equations are used to develop a theoretical design model for computing either the submergence ratio or the sequent depth ratio (in terms of the other one) for the submerged


Fig. 1. Definition sketch for radial stilling basin with vertical drop.
hydraulic jump formed in radial stilling basin provided with a drop. The present development is based on the following assumptions: (a) the flow is radial and steady (b) the liquid is incompressible (c) The channel is horizontal and has smooth boundaries, (d) hydrostatic pressure distribution along the considered reach of flow (e) uniform velocity distribution, i.e. the values of the kinetic energy correction factor and the momentum correction factor, $a$ and $\beta$ are considered unity, and ( $f$ ) the effects of air entrainment and turbulence are neglected.

In the present study, the control volume where the momentum equation is applied starts from the vena contracta (if a free flow occurs) and ends at the end of jump in the direction of the flow fig. 1. The momentum equation is written as follows:
$P_{1}-P_{2}+2 P_{s} \sin \frac{\theta}{2}+P_{s t}=\frac{V}{g} Q\left(V_{2}-V_{1}\right)$.

In which $P_{1}$ is the hydrostatic pressure force at vena contracta at a radial distance $r_{1}$ where the average velocity is $V_{1}$, the contracted depth of flow is $y_{1}$ and the depth just downstream the gate is $y_{3}, P_{2}$ is the hydrostatic pressure force at the end of jump at radial distance $r_{2}$ where the average velocity is $V_{2}$ and the depth of flow is $y_{4}, P_{s}$ is channel side pressure, $P_{\text {st }}$ pressure force acting on the drop of height $k$ at a radial distance $r_{s}$ and
depth of flow $y_{s}, Q$ is the discharge, $V$ is the specific weight of water, $g$ is the gravitational acceleration and $\theta$ is the angle of divergence of the radial basin.

$$
\begin{align*}
& P_{1}=\gamma r_{1} y_{3}^{2} \sin \frac{\theta}{2},  \tag{3}\\
& P_{2}=\gamma r_{2} y_{4}^{2} \sin \frac{\theta}{2},  \tag{4}\\
& P_{s}=0.5 \gamma\left[\left(r_{2}-r_{1}\right) \bar{y}^{2}-\left(r_{s}-r_{1}\right) k^{2}\right] . \tag{5}
\end{align*}
$$

In which $\bar{y}=\left(\sqrt{\left(y_{3}+k\right) y_{4}}\right)$ is the representative depth (geometric mean of $y_{3}+k$ and $y_{4}$ ).

The pressure force acting on the drop is,

$$
\begin{equation*}
P_{s t}=2 \gamma\left(y_{s} k+0.5 k^{2}\right) r_{s} \sin \frac{\theta}{2} . \tag{6}
\end{equation*}
$$

Keeping in mind that,
$V_{1}=Q /\left(2 r_{1} y_{1} \sin \frac{\theta}{2}\right)$, and
$V_{2}=Q /\left(2 r_{2} y_{4} \sin \frac{\theta}{2}\right)$.

Substituting eqs. (3-7) into eq. (2) and dividing it by $\left(\gamma r_{1} y_{1}^{2} \sin \theta / 2\right)$, yields:

$$
\begin{align*}
S^{2} & +S Y_{\circ}\left(r_{\circ}-1\right)+Y_{\circ}\left(r_{\circ}-1\right) K_{\circ} \\
& -(r-1) K_{\circ}^{2}-r_{\circ} Y_{\circ}^{2}+2\left(Y_{S} K_{\circ}+0.5 K_{\circ}^{2}\right) r \\
& -2 F_{1}^{2}\left(1 /\left(r_{\circ} Y_{\circ}\right)-1\right)=0 . \tag{8}
\end{align*}
$$

The solution of eq. (8) gives, $S=-0.5 A_{*}+$

$$
\sqrt{\begin{array}{l}
0.25 A_{*}^{2}-K_{\circ} A_{*}+(r-1) K_{\circ}^{2}+r_{\circ} Y_{\circ}^{2}  \tag{9}\\
-2\left(Y_{s} K_{\circ}+0.5 K_{\circ}^{2}\right) r+2 F_{1}^{2}\left(1 /\left(r_{\circ} Y_{\circ}\right)-1\right) .
\end{array}}
$$

In which $S=\frac{y_{3}}{y_{1}}, Y_{\circ}=\frac{y_{4}}{y_{1}}, r_{\circ}=\frac{r_{2}}{r_{1}}, r=\frac{r_{s}}{r_{1}}$, $K_{\circ}=\frac{k}{y_{1}}, Y_{s}=\frac{y_{s}}{y_{1}}$ and $A_{*}=Y_{\circ}\left(r_{\circ}-1\right)$

For stilling basin without drop ( $\mathrm{K}_{0}=0$ ), eq. (8) tends to the previously equation developed by McCorquodale and Khalifa [10].

Eq. (9) may be rearranged in another form to enable the trail and error computation of $Y_{o}$, as follow:
$r_{\circ} Y_{\circ}^{3}-\left(S+K_{\circ}\right)\left(r_{\circ}-1\right) Y_{\circ}^{2}$
$-\left[S^{2}+2 F_{1}^{2}+2\left(Y_{s} K_{\circ}+0.5 K_{\circ}^{2}\right) r-(r-1) K_{\circ}^{2}\right] Y_{\circ}$
$+2 F_{1}\left(1 / r_{\circ}\right)=0$.

### 2.2. Relative energy loss

Applying the energy and continuity equations between sections 1 and 2 , as shown in fig. 1, the energy loss through the radial hydraulic jump is obtained as follow,
$\frac{E_{L}}{E_{1}}=1-\frac{Y_{\circ}+0.5 F_{1}^{2} /\left(r_{\circ} Y_{\circ}\right)^{2}}{K_{\circ}+S+0.5 F_{1}^{2}}$.

When the drop is not existed, the term $K_{0}$ becomes zero, and eq. (11) reduces to the previously developed one by McCorquodale and Khalifa [10].

### 2.3. Relative length of jump

The use of the dimensional analysis showed that the length of jump ratio $\left(L_{j} / y_{1}\right)$ of the submerged radial hydraulic jump formed in radial basin with a vertical drop is a function of the $F_{1}, S, r$ and $K_{o}$ as follows,

$$
\begin{equation*}
\frac{L_{j}}{y_{1}}=f\left(F_{1}, S, K_{o}, r\right) . \tag{12}
\end{equation*}
$$

The function $f$ is to be determined using the experimental data.

## 3. Collection of experimental data

The experimental work of this study is conducted using a re-circulating adjustable flume of 12.0 m long, 45 cm deep and 30 cm wide. The discharges were measured using pre-calibrated orifice meter fixed on the feeding pipeline. The tailgate fixed at the end
of the flume was used to control the depth of flow for each run. The radial basin was made from a clear prespex to enable visual inspection of the phenomenon being under investigation. The model length was kept constant at 130 cm and the angle of the divergence was kept constant to $5.28^{\circ}$. The model was fixed in the middle third of the flume between its two side-walls fig. 1. A smooth block of wood was formed to fit well inside the basin model extending from upstream the gate by 5.0 cm to position where the drop was desired. The wood was painted very well by a waterproof material (plastic) to prevent wood from changing its volume by absorbing water. A fixed height of the drop of 2.5 cm was used. Different positions of the drop $\quad\left(r_{s}=r_{1}, \quad 1.16 r_{1}, \quad 1.33 r_{1}\right.$ and $\left.1.5 r_{1}\right)$ downstream from the gate opening were tested under the same flow conditions. The range of the experimental data were as follows: $Q$ (4 lit/s - $11 \mathrm{lit} / \mathrm{s}$ ), Froude numbers (2.0-7.0), submergence ratio $(4-9)$, $Y_{o}(4-12)$, $r_{o}(1.4-$ 1.6), relative position of the drop, $r(1.0-1.50)$ and relative height of the drop, $K_{o}(0.9-1.5)$.

Each model was tested using five different gate openings and five discharges for each gate opening. The measurements were recorded for several submergence ratios for each discharge. The total number of runs was 335. A typical test procedure consisted of (a) a gate opening was fixed and a selected discharge was allowed to pass. (b) the tailgate was adjusted until a submerged jump is formed. (c) once the stability conditions were reached, the flow rate, length of the jump, water depths upstream and just downstream of the gate in addition to the tail water depth and the depth of water above the drop were recorded. The length of jump was taken to be the section at which the flow depth becomes almost horizontal (d) the position of the tailgate was changed to obtain another submergence ratio and then step (c) is repeated. (e) step (d) was repeated several times then the procedure is repeated for another discharge till the required discharges were covered (f) the procedure was repeated for another gate opening and so on till the required ranges of the parameters being under investigation were covered.

## 4. Analysis and discussions of results

### 4.1. Water surface profile

Typical water surface profiles are presented in figs. 2-a and 2-b for the submerged hydraulic jump in radial stilling basins with and without drop at a relative distance of $r=1.33$. As shown in fig. 2-a, two typical water surface profiles for submerged radial hydraulic jump without drop as obtained by McCorquodale and Khalifa [10]. The discrepancies between the present profiles and those of McCorquodale and Khalifa are referred due to the difference in the divergence angle and the model setup details. In the present case, the divergence angle is $5.28^{\circ}$ while it was $13.5^{\circ}$ for the McCorquodale and Khalifa. They used a completely radial basin with a control radial gate while in the present study a radial basin preceded with a parallel contracted small length of the channel and the control vertical sluice gate is fixed 5.0 cm upstream the radial basin. The present setup of the radial basin simulates the actual conditions in the field because the stream is normally contracted for construction purposes and then enlarged again to restore the original dimensions of the channel.

The water surface profiles for submerged jump in radial basin with drop have a similar nature to those obtained for a basin without drop. It was noticed that the water surface profiles considering a drop are higher than the corresponding ones for basins without drop due to the effect of the drop height at particular location. Fig. 2-b shows that the location of the drop in the radial basin causes an increase in the water surface profiles due to the pressure forces acting on the drop and on the sides. This increase in the depth of the water surface profiles is easily interpreted with the help of the specific energy diagram. The flow away from the gate is subcritical (upper limb of the diagram) and just downstream (DS) of the drop the specific energy $E$ is higher than that just upstream (US) from the drop and hence the depth of flow is increased. The specific energy DS and US of the drop is given by $E_{U S}=E_{D S}+k-h_{L}$ where $k$ is the drop height and $h_{L}$ is the energy loss through the length


Fig. 2-a. Experimental water surface profiles for submerged radial jump for smooth case.


Fig. 2-b. Experimental water surface profiles for submerged radial jump for both smooth case.
from US to DS of the drop where the specific energy equation is applied. The rate of increase of the depth depends on the height of the drop and the rate of energy loss through the jump. If the rate of the energy loss is equal to the energy added by the height of the drop, no appreciable increase in the depth of water is observed.

### 4.2. The relative energy loss

The relative energy loss by the jump in radial basin without drop is presented in fig. 3-a for three ranges of submergence ratio $S$ with average values of 4.1, 6.5 and 8.5 for $r_{o}$ in the range 1.4-1.67. Eq. (11) is plotted for $S=4,6$ and $S=9$. Shown also in the figure the relative energy loss due to submerged jump formed in radial basin with divergence angle equals $13.5^{\circ}$ due to McCorquodale and Khalifa [10] at $S=2-4$, the relative energy loss by the submerged jump formed in rectangular basin at
$S=4.5$ due to Govinda and Rajaratnam [6], the relative energy loss by the free radial hydraulic jump due to Khalifa and McCorquodale [11] and the relative energy loss of the free rectangular hydraulic jump, Chow [12]. This figure shows that the energy loss by the submerged radial jump is more than that of the corresponding one in rectangular basin and similar observation is valid for the free jumps. These results corporate well with the results obtained by McCorquodale and Khalifa [10] regarding the same comparison. However, the small difference between present results and those of McCorquodale and Khalifa [10] is due to the difference in the value of the divergence angle. Moreover, the figure indicated that the equation is in good agreement with the experimental results and the lesser energy loss is associated with the greater submergence. The relative energy loss when a drop is present in the basin follow the same trend as that when the drop is absent as indicated in fig. 3-b. Also, eq. (11) is in good agreement with the experimental results of the submerged jump formed in radial basin provided with a drop of fixed height k (relative heights in the range $K_{o}=0.9-1.50$ ) at particular location ( $r=1.0$ ) and $r_{o}$ in the range 1.4-1.67. The position of the drop in the radial basin affects the relative energy loss as the rate of energy loss is increased at $r=1$ compared to the case of radial basin without drop and then it is decreased by the increase of the distance between the gate and the drop location (greater $r$ ) as indicated on fig. 3-c (the experimental data are not shown). The prediction of eq. (11) is plotted against all the experimental results for different values of $S$ ( $S=4-8.5, r(r=1-1.5)$ and Froude numbers, $F_{1}\left(F_{1}=2-7\right)$. The figure shows a good agreement between the theoretical eq. (11) and the experimental results (mean relative absolute error equals 0.04 and $R^{2}=0.97$ ). Figs. $4-\mathrm{a}, 4-\mathrm{b}$ and $4-\mathrm{c}$ shows theoretically the effects of the different parameters $S, r$ and $K_{o}$ on the relationship between the relative energy loss and the initial Froude number based on eq. (11). The relative energy loss increases with the increase of $F_{1}$ at constant values of $S$, $r$ and $K_{o}$, while it decreases with the increase of the submergence $S$, relative position of the


Fig. 3-a. The relationship between $F_{1} \& E_{L} / E_{1}$ for different $S$ for smooth case.


Fig. 3-b. The relationship between $F_{1} \& E_{L} / E_{1}$ for different $S$ at $r=1.0$.


Fig. 3-c. The relationship between $F_{1} \& E_{L} / E_{1}$ for different positions of step at $S=4.0-4.9$.


Fig. 4-a. Theoretical relationship between $E_{L} / E_{1} \& F_{1}$ for different submergence ratios at $r=1$ and $K_{0}=1.25$.


Fig. 4-b. Theoretical relationship between $F_{1} \& E_{L} / E_{1}$ for different positions of step at $S=5.0$ and $K_{0}=1.25$ according to eq.(11).


Fig. 4-c. Theoretical relationship between $E_{L} / E_{1} \& F_{1}$ for different relative heights of step at $S=4.0$ and $r=1.0$.
drop $r$ and relative height of the drop $K_{o}$ at constant $F_{1}$.

### 4.3. The relative depth ratio

The effect of submergence on the depth ratio of the submerged jump in radial basin without drop is indicated in fig. 5-a for values of $S=3.5-4.49,6.0-6.99$ and $8.0-8.99$. The values of $S=6-8$ obtained by McCorquodale and Khalifa [10] are also plotted on the figure. Eq. (10) is also plotted for $S=4,6$ and 8. Reasonable agreement between present and previous results is obtained. Eq. (10) shows good agreement with the observations (mean relative absolute error equals 0.05 and $\left.R^{2}=0.94\right)$. The figure indicates that the relative depth ratio increases nonlinearly with $F_{1}$ at fixed value of $S$ and increases with the increase of $S$ at fixed value of $F_{1}$. The depth ratio for the free radial hydraulic jump given by Khalifa and McCorquodale [11] is also shown in the figure.

Fig. 5-b presents the results of eq. (10), for $S=4,6$ and 8 , compared with the experimental results for $S$ in the range 3.5-4.0, 5.9-6.9 and 8.0-8.9 at $r=1.0, K_{o}$ in the range 0.9-1.50 and $r_{o}$ in the range $1.4-1.67$. The effect of the drop position on the depth ratio is indicated in fig. 5-c where only the fitting of the experimental data is shown. Clearly, the depth ratio is increased as the distance between the drop and the gate is increased (greater $r$ produces higher depth ratios) at the same $\mathrm{F}_{1}$. The effects of $S, r$ and $K_{o}$ on the relationship between $Y_{o}$ and $F_{1}$ obtained from eq. (10) are presented in figs. 6-a, 6-b and 6c, respectively. Clearly, $Y_{o}$ increases with the increase of $S, r$ and $K_{o}$ at fixed Froude number confirming the experimental results.

### 4.4 Relative length of jump

The results of the length of submerged jump formed in radial basin without drops are presented in fig. $7-\mathrm{a}$. The results due to McCorquodale and Khalifa [10] for submerged jump $(S=4,6)$, and due to Govinda and Rajaratnam [6] for submerged rectangular jump $(S=4)$ are also presented. Submerged radial jump produces shorter length of jump compared to the rectangular submerged


Fig. 5-a. The relationship between $Y_{0} \& F_{1}$ for different $S$ for smooth case.


Fig. 5-b. The relationship between $Y_{0} \& F_{1}$ for different $S$ at $r=1.0$.


Fig. 5-c. The relationship between $Y_{0} \& F_{1}$ for different step positions and $S=4.1-4.9$.
jumps. The general trend of the results of the present study corresponds well with those of the previous ones. Higher submergence yields longer jumps. This result was reached also by other authors such as Govinda and Rajaratnam [6] for rectangular submerged
jump and McCorquodale and Khalifa [10] for submerged radial jump.

The effect of submergence in case of a basin with a drop at $r=1.0$ is shown in fig. 7-b. Similar trend is observed as for the case of radial basin without drop. The drop positions affect the length of the jump as in fig. 7-c because as the distance between the gate and the drop is increased (greater $r$ ), the length of jump is longer at constant $S K_{o}$ in the range 0.9-1.50 and $r_{o}$ in the range 1.4-1.67). Experimental data are used to determine the function of eq. (12) in order to enable the prediction of the length of jump. Hence, the length of the basin can be designed for any drop position. Many prediction models based on eq. (12) were tested utilizing all the collected data. Once the most reasonable model is obtained, the Neural Connection [13] software was used to estimate the coefficient of the equation based on $70 \%$ of collected experimental data while $30 \%$ is kept for testing the prediction of the equation. Both data sets are selected randomly. The following model is found to be the best model representing the experimental data with $R^{2=}$ 0.75 and mean relative absolute error 0.07.

$$
\begin{align*}
\frac{L_{j}}{y_{1}}= & -24.286+5.101 F_{1}+3.37 S \\
& +35.79 r+4.558 K_{o} \tag{13}
\end{align*}
$$

It is possible to improve the prediction of eq. (13) by excluding the odd points (that may contain some experimental errors). In this case eq. (13) becomes (13-a) with $R^{2}=0.85$ and mean relative absolute error 0.051.

$$
\begin{align*}
\frac{L_{j}}{y_{1}}= & -23.75+5.95 F_{1}+3.27 \mathrm{~S} \\
& +37.92 r+0.579 K_{o} . \tag{13-a}
\end{align*}
$$

Figs. 8-a and 8-b show the comparison between the model results and the experimental results for the test data set. Fig. 8-a shows good agreement between prediction of eq. (13) and the experimental results. Fig. 8-b shows that the residuals are relatively small, distributed around the line of zero error and they are uncorrelated with $R=0.114$.


Fig. 6-a. Theoretical relationship between $Y_{0} \& F_{1}$ for different degrees of submergence at $r=1.0$ and $K_{o}=1.25$.


Fig. 6-b. Theoretical relationship between $Y_{0} \& F_{1}$ for different step positions at $S=5.0$ and $K_{0}=1.25$.


Fig. 6-c. Theoretical relation between $Y_{0} \& F_{1}$ for different relative heights of step at $S=5.0$ and $r=1.33$.


Fig. 7-a. The relationship between $L j / y_{1} \& F_{1}$ for different submergence for no step case.


Fig. 7-b. The relationship between $L j / y_{1} \& F_{1}$ for different submergence at $r=1.0$.


Fig. 7-c. The relationship between $L j / y_{1} \& F_{1}$ for different step positions and $S=5.0-5.9$.

The use of eq. (13) in the computation of the length of the hydraulic jump for different values of the flow and basin parameters is


Fig. 8-a. Predicted versus measured for $L j / y_{1}$ for test data set.


Fig. 8-b. Residuals versus predicted $L j / y_{1}$.


Fig. 9-a. Theoretical relationship between $L j / y_{1} \& F_{1}$ for different S at $r=1.0$ and $K_{0}=1.25$.
shown in figs. 9-a, 9-b and 9-c to indicate the effect of $S, r$ and $K_{o}$ on the relationship between $L_{j} / y_{1}$ and $F_{1}$.

## 5. Conclusions

Theoretical models were developed for computing the relative depth and relative energy loss of submerged hydraulic jump formed in non-prismatic channel reach in the


Fig. 9-b. Theoretical relationship between $L j / y_{1} \& F_{1}$ for different step positions at $S=5.0$ and $K_{o}=1.25$.


Fig. 9-c. Theoretical relationship between $L j / y_{1} \&_{*} F_{1}$ for different $K_{o}$ at $S=5.0$ and $r=1.33$.
form of radial stilling basin with a vertical drop. The developed and verified models were used to predict the jump relative depth and the relative energy loss of the hydraulic jump. The prediction of the models agreed well with the experimental results. The obtained results were used to investigate the effect of the different parameters as the submergence ratio and the relative position of the drop on the relative depth of the hydraulic jump and on its relative energy loss. The predicting equations may be recommended to be used in the design of the radial stilling basins provided with vertical drops. Both the dimensional and multiple linear regression analysis were employed to develop predicting equation for the length of the hydraulic jump. The experimental data were used to calibrate (70\%
of the data) and to test ( $30 \%$ of the data) the validity of the model.

Both the experimental results and the developed equations indicated that at a particular relative location of the drop, the relative water depth, relative energy loss and relative length of jump increase by increasing Froude number keeping the submergence unchanged. Also, it is proved that relative water depth and relative length of jump increase by increasing the submergence ratio (at a specific Froude number). Increasing the submergence reduces the relative energy loss keeping other factors unchanged. Also, moving the drop away from the gate (within the basin) increases the relative water depth and relative length of the submerged jump and decreases the energy loss ratio.

## Nomenclature

$b_{1}$ is the contracted width of the channel,
$B$ is the width of the channel,
$E_{1}$ is the total energy at the jump toe,
$E_{2}$ is the total energy at the jump heel,
$E_{L}$ is the relative energy loss,
$F_{1}$ is the Froude's number at the initial depth,
$G$ is the gate opening,
$k$ is the drop height,
$K_{o}$ is the ratio of k to $\mathrm{y}_{1}$,
$L_{j}$ is the length of the hydraulic jump,
$P_{1}$ is the hydrostatic pressure before the jump,
$P_{2}$ is the hydrostatic pressure after the jump,
$P_{s}$ is the channel side pressure force,
$P_{s t}$ is the pressure force due to drop,
$Q$ is the rate of flow,
$r_{1}$ is the radius at the beginning of the jump ,
$r_{2}$ is the radius at the end of the jump ,
$r_{0}$ is the ratio of $r_{2}$ to $r_{1}$,
$r_{s}$ is the radius at the end of the drop,
$r$ is the ratio of $r_{s}$ to $r_{1}$,
$R^{2}$ is the coefficient of determination:
$S$ is the degree of submergence, $y_{3} / y_{1}$,
$V_{1}$ is the average velocity at the initial depth,
$V_{2}$ is the average velocity at the sequent depth,
$y_{3}$ is the back up water depth just downstream the gate,
$y_{4}$ is the tail water depth at the end of the jump,
$Y_{o}$ is the relative tail water depth, $y_{4} / y_{1}$,
$y_{s}$ is the depth of water above the drop,
$Y_{s}$ is the ratio of $y_{s}$ to $y_{1}$,
$\bar{y}$ is the representative depth,
$Y$ is the specific weight, and
$\theta$ is the angle of divergence.

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