New types of separation axioms for frames

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Frames is an important generalization to topological spaces. Several authors have tried to generalize many topological concepts by using frames. The aim of this paper is to generalized some topological concepts via frames. We introduced the concept of γ-open nucleus on frames. Also, we studied its relations with other nearly open nuclei on frames. We defined some separation axioms on frames such as $\gamma - T_i$ (resp. $cl - T_i$, $\alpha - T_i$, i=1,2). Moreover, we introduce some weaker forms of compactness on frames. Also, we studied the relations between them and other forms of compactness on frames.

تعتبر الهياكل من أهم التعميمات للفضاءات التوبولوجية ، ولقد حاول كثير من الباحثين تعميم مفاهيم توبولوجية عديدة باستخدام الهياكل. الهـدف الرئيسى لهذا البحث هو تعميم بعض المفاهيم التوبولوجية عن طريق الهياكل ، ولذلك قدمنا تعريفاً للنواه من الـنوع (γ) (γ – open nucleus)، ودرسـنا علاقة هذه النواه بالانوية قريبة الانفتاح ، وباستخدام هذا المفهوم عرفنا مسلمات الانفصال فى الهياكل مثل الانواع $\gamma - T_i$ ، $cl - T_i$ ، $\alpha - T_i$ حيث $i = 1, 2$ ودرسنا علاقتها بالانواع الأخرى ، كذلك أدخلنا أنواعاً جديدة من الاصمات فى الهياكل ودرسنا علاقة هذه الانواع بانواع الاصمات المعرفة قبل ذلك .

Keywords: Frames, Near open nucleui, Compactness on frames

1. Introduction

All unexplained facts concerning frames can be found in P. T. Johnstone [1]. Recall that a frame is a complete lattice L in which the infinite distributive law is fulfilled, that is, $x \wedge \bigvee_{i \in I} x_i = \bigvee_{i \in I} (x \wedge x_i)$, for every $x \in L$ and every subset $\{x_i\}_{i \in I}$ of L. The set of all open sets of a topological space forms a frame. Frames can be viewed as generalized topological spaces. These frames and frames isomorphic with them are called spatial or topologies. We shall call a map from one frame to another frame homomorphism if it preserves arbitrary joins and finite meets [2].

In [2] a nucleus on a frame L is defined as a map $j: L \longrightarrow L$ satisfying:

(i) $a \leq j(a)$,

(ii) $j(a) = j(j(a))$, and

(iii) $j(a \wedge b) = j(a) \wedge j(b)$, for all $a, b \in L$. Let a be an element of a frame L. Then the maps $c_a, u_a: L \longrightarrow L$; $c_a(x) = a \vee x, u_a(x) = a \Rightarrow x$ (we put $x^* = x \Rightarrow 0$) are nuclei which for topologies correspond to a closed and open subspace, respectively. Therefore, nuclei of this form are said to be closed and open, respectively. A nucleus which is both open and closed is said to be clopen. We shall denote by N(L) the lattice of all nuclei, by O(L) the lattice of open nuclei, by C(L) the lattice of closed nuclei and by CO(L)the lattice of clopen nuclei on a frame L. We shall define by Δ, ∇ the bottom and the top elements of N(L). For $j \in N(L)$, we define the interior (resp. the closure) of j [3] and denote j°, j^{-}, respectively.

$j^{\circ} = \wedge \{ k : k \in O(L), j \leq k \}$ (resp.

$j^{-} = \vee \{ k : k \in C(L), k \leq j \}$).

Let L be a frame, $j \in N(L)$. Then, j is said to be [2]:

(i) semi-open, if there exists an open nucleus U such that $\overline{u} \leq j \leq u$,

(ii) α-open, if $j^{\circ-\circ} \leq j$,

(iii) preopen, if $j^{-\circ} \leq j$,

(iv) semi-preopen (or β-open) if $j^{-\circ-} \leq j$.

The class of all semi-open, α-open, preopen and semi-preopen nuclei will be denoted by SO(L), αO(L), PO(L) and SPO(L) respectively. Their complements are semi-closed, α-closed, preclosed and semi-preclosed nuclei. The classes of these nuclei will be denoted by SC(L), αC(L), PC(L) and SPC(L), respectively.

Let L be a frame and $j \in N(L)$. Then, j is said to be:

(i) A dense nucleus if $j^- = \Delta$ [3].

(ii) A regular open (resp. a regular closed) nucleus [1], if $j^{-\circ} = j$ (resp. $j^{\circ -} = j$). The class of all regular open, regular closed nuclei of L will be denoted by RO(L), RC(L), respectively.

A frame L is said to be extremely disconnected [3], if the closure of every open nuclei on L is open.

Lemma 1.1 [4] :

Let L be a frame and $j, k \in N(L)$. Then:

(i) If $j \le k$, and $k \in O(L)$ then $j \le j^\circ \le k$.

(ii) If $k \le j$ and $k \in C(L)$, then $k \le j^- \le j$.

(iii) If $j \le k$, then $j^\circ \le k^\circ$ and $j^- \le k^-$..

(iv) j is an open nucleus iff $j = j^\circ$.

(v) $(j \vee k)^\circ = j^\circ \vee k^\circ ; (j \wedge k)^\circ \le j^\circ \wedge k^\circ$.

(vi) j is a closed nucleus iff $j = j^-$.

(vii) $(j \wedge k)^- = j^- \wedge k^- ; j^- \vee k^- \le (j \vee k)^-$

Lemma 1.2 [2] :

Let L be an extremely disconnected frame and $u \in O(L)$, $j \in N(L)$. Then, $u \vee j = u \vee j^-$.

Lemma 1.3 [2,4] :

(i) A nucleus j is semi-open iff $j^{\circ -} \le j$.

(ii) A nucleus j is semi-open iff $j^{\circ -} = j^-$.

(iii) Every open nucleus is semi-open [4].

(iv) Every open nucleus is preopen [4].

Lemma 1.4 [4]:

(i) An arbitrary meet of semi-open nuclei is semi-open.

(ii) An arbitrary meet of preopen nuclei is preopen.

(iii) An arbitrary meet of α-open nuclei is an α-open nucleus and the finite join of α-open nuclei is α-open.

In [5], if x is an element of a frame L, then the element $\vee\{y \in L : y \wedge x = 0\}$ is called the pseudocomplement of x and is denoted by x^*. A frame L is said to be T_1 (resp. T_2) [5], if for every parallel pair f, g of frame maps with domain L, the condition $f(x) \le g(x)$ for every $x \in L$ (resp. $f(x) \wedge g(y) = 0$ whenever $x \wedge y = 0$ in L) implies that $f = g$. A frame L is said to be regular (resp. normal) [5], if for every $x \in L, x = \vee\{u \in L : u_* \vee x = 1\}$ (resp. for every $x, y \in L$, satisfying $x \vee y = 1$, there exists $u \in L$ such that $x \vee u = y \vee u^* = 1$.

Let L be a frame; $a, b \in L$. Then, a covers b (b is covered by a)[5], in notation, $a \succ b$ ($(b \prec a)$), if $a > b$ and for no $x \in L$, $a > x > b$.

Theorem 1.1 [4] :

(i) Under a homomorphism, the image of a frame is also a frame.

(ii) Let L and f(L) be two frames, 1_L be a supremum of L and $1_{f(L)}$ be a supremum of f(L). Then $f(1_L) = 1_{f(L)}$.

A frame L is called compact [5] (resp. semi-compact, strongly compact, α-compact [4]), frame, if for every family $\{U_i : i \in I\}$ of open (resp. semi-open, preopen, α-open nuclei of *L* for which $\underset{i \in I}{\vee} U_i = \nabla$, there exists $\{U_{i_1}, \dots, U_{i_n}\}$ a finite subfamily of ;L for which $\overset{n}{\underset{i \in I}{\vee}} U_{i_k} = \nabla$.

2. γ-open and γ-interior of nuclei

In this section we introduce and study the notions of γ-open and γ-interior of a nucleus on a frame L.

Definition 2.1:

Let L be a frame and $j \in N(L)$. Then j is said to be γ-open if $j^{\circ -} \wedge j^{-\circ} \le j$. The complement of γ -open is γ -closed and we denote the set of all γ -open, γ -closed nuclei on a frame L by $\gamma O(L), \gamma C(L)$, respectively.

Example 2.1:

The chain $0 \prec a \prec b \prec c \prec d \prec e \prec \cdots \prec 1$ forms frame, then j_o, j_1 are γ -open, but j_a, j_b are not γ -open nuclei on a frame L.

Proposition 2.1:

The meet of an arbitrary number of γ -open nuclei is γ -open.

Proof:

Let j_i be γ -open nuclei of a frame L. Then $j_i^{\circ-} \wedge j_i^{-\circ} \leq j_i$ implies that $\bigwedge_{i\in I}(j_i^{\circ-} \wedge j_i^{-\circ}) \leq \bigwedge_{i\in I} j_i$,

then $(\bigwedge_{i\in I} j_i)^{\circ-} \wedge (\bigwedge_{i\in I} j_i)^{-\circ} \leq (\bigwedge_{i\in I} j_i^{\circ})^{-} \wedge (\bigwedge_{i\in I} j_i^{-})^{\circ} \leq \bigwedge_{i\in I} j_i$,

hence, $(\bigwedge_{i\in I} j_i)^{\circ-} \wedge (\bigwedge_{i\in I} j_i)^{-\circ} \leq \bigwedge_{i\in I} j_i$. Thus $\bigwedge_{i\in I} j_i$ is γ -open.

Lemma 2.1 :

(i)Let L be a frame and j a γ -open and u be an open nucleus of L. Then, $j \vee u$ is γ -open.

(ii)If L is a frame and j is an γ -open and α -closed nucleus of L, then j is a regular closed nucleus of L.

(iii) In a frame L, if j is a γ -open, semi-closed and preclosed nucleus of L, then $j = j^{\circ-} \wedge j^{-\circ}$.

Proof :

(i) Since j is γ -open, $j^{\circ-} \wedge j^{-\circ} \leq j$ holds and u is an open nucleus, $u^{\circ} = u$. Thus $(j^{\circ-} \wedge j^{-\circ}) \vee u^{\circ} \leq j \vee u$ implies that $(j^{\circ-} \vee u^{\circ}) \wedge (j^{-\circ} \vee u^{\circ}) \leq j \vee u$, then $(j \vee u)^{\circ-} \wedge (j \vee u)^{-\circ} \leq j \vee u$ (by Lemma 1.1, 1.3). Therefore $j \vee u$ is γ -open.

(ii) Follows directly by definitions and Lemma 1.1.

(iii) Obvious.

Corollary 2.1 :

If L is a frame and j is a γ -closed and α -open nucleus of L, then j is regular open.

Lemma 2.2 :

(i) If L is a frame, $j \in \gamma$ and j is a closed nucleus, then $j \in SO(L)$.

(ii) If L is a frame, k is a γ -closed and an open nucleus of L, then k is semi-closed.

(iii) In a frame L, if k is a γ -closed and c is a closed nucleus of L, then $c \wedge k$ is γ -closed.

Proof :

(i) Since, j is γ -open, $j^{\circ-} \wedge j^{-\circ} \leq j$ holds. But j is a closed nucleus, hence $j^{-} = j$, therefore $j^{\circ-} \wedge j^{\circ} \leq j$. Thus $j^{\circ-} \leq j$. Therefore, j is semi-open.

(ii) Let k be a γ -closed nucleus of L. Then $k \leq k^{-\circ} \vee k^{\circ-}$, but k is open, that is $k = k^{\circ}$, hence $k \leq k^{-\circ} \vee k^{-}$. Thus $k \leq k^{-\circ}$. Hence k is semi-closed.

(iii) Let j be a γ -closed nucleus of L. Then $k \leq k^{-\circ} \vee k^{\circ-}$ but c is closed, hence $c = \bar{c}$, thus $c \wedge k \leq \bar{c} \wedge (k^{-\circ} \vee k^{\circ-})$ implies that $c \wedge k \leq (\bar{c} \wedge k^{-\circ}) \vee (\bar{c} \wedge k^{\circ-})$. Then (by Lemma 1.1), $c \wedge k \leq (c^{-\circ} \wedge k^{-\circ}) \vee (c^{\circ-} \wedge k^{\circ-})$. Therefore $c \wedge k \leq (c \wedge k)^{-\circ} \vee (c \wedge k)^{\circ-}$. Hence $c \wedge k$ is γ -closed.

Lemma 2.3 :

Let L be a frame, $k \in \gamma O(L)$. Then:

(i) k is a preopen nucleus of L, if k^{c} is a dense nucleus of L (where, k^{c} is the complement of k).

(ii) k is a preopen nucleus of L, if L is an extremely disconnected frame

Proof :
(i) Let k be a γ-open nucleus of L. Then $k^{-\circ} \wedge k^{\circ-} \le k$, but k^{c} is dense, then $k^{c-} = \Delta$ implies that $k^{\circ} = \nabla$, hence $k^{\circ-} = \nabla$. Therefore $k^{-\circ} \le k$, thus k is preopen.
(ii) Since k is a γ-open nucleus of L; $k^{-\circ} \wedge k^{\circ-} \le k$, but L is extremely disconnected, k° is open, hence $k^{\circ-}$ is open, thus $k^{-\circ} \wedge k \le k$, then $k^{-\circ} \le k$. Therefore k is a preopen nucleus of L.

We introduce the notions of the interior (resp. the closure) of γ-open nucleus. The sets of these nuclei will be denoted by $\gamma(j^{\circ})$ (resp. $\gamma(j^{-})$).

Definition 2.2 :

Let L be a frame, $j \in N(L)$. Then the γ-interior and γ-closure is given by:

$\gamma(j^{\circ}) = \bigwedge\{u : u \in \gamma O(L), j \le u\}$,

$\gamma(j^{-}) = \bigvee\{c : c \in \gamma C(L), c \le j\}$.

From the above definition, we have the following :

Proposition 2.2:
(i) $\gamma(j^{\circ})$ (resp. $\gamma(j^{-})$) is γ-open (resp. γ-closed).
(ii) $\gamma(j^{\circ})$ (resp. $\gamma(j^{-})$) is the smallest (resp. largest) γ-open (resp. γ-closed) nucleus which contains (resp. contained in) j .
(iii) A nucleus j is γ-open iff $j = \gamma(j^{\circ})$.
(iv) A nucleus j is γ-closed iff $j = \gamma(j^{-})$.

Example 2.2 :

Let $j_{\circ}$ and j_1 be given as in Example 2.1. Then $j_{\circ}, j_1$ are $\gamma(j^{\circ})$ (resp. $\gamma(j^{-})$), but j_a, j_b are not $\gamma(j^{\circ})$ (resp. $\gamma(j^{-})$).

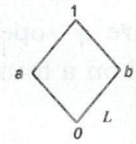
3. Separation axioms via γ-open nuclei

In [5], $S-T_i$, $P-T_i$ frames were defined. In the following we introduce $\gamma - T_i$ (resp. $\alpha - T_i$, clopen- T_i) frames, where i = 1, 2.

Proposition 3.1 [5] :
(i) A regular frame is T_2, (ii) a T_2-frame is T_1.

Example 3.1:

A frame L, with $L = \{0,a,b,1\}$, is regular and is therefore a T_2-frame, hence it also is T_1.



Definition 3.1 :

A frame L is called γ – T1 (resp. α-T_1), clopen – T1) if for every two γ-open (resp. α-open, clopen) nuclei j_1 , j_2 for which (x)≤ j_2 (x) holds for all x ε L, then $U_1 = y_2$ we have $j_1 = j_2$.

Definition 3.2 :

A frame L is said to be $\gamma - T_2$ (resp. $\alpha - T_2$, clopen $-T_2$) if, for every two γ-open (resp. α-open, clopen) nuclei j_1, j_2 for which $j_1(x) \wedge j_2(y) = 0$ holds whenever $x \wedge y = 0$ in *L* we have $j_1 = j_2$.

Theorem 3.1 :
(i) Every $\alpha - T_i$ frames is T_i, $i = 1,2$.
(ii) Every subframe of an $\alpha - T_i$ frame is $\alpha - T_i$, $u_1(x) \le u_2(x)$.
(iii) Every $\gamma - T_i$ frame is T_i , $i = 1,2$
(iv) Every $\gamma - T_i$ frame is $j_1(x) \le j_2(x)$, $i = 1,2$.

Proof:

We focus on the proff of T_1 frame and T_2 will be in a similar way.
(i) We prove the theorem only for T_1 . Let j_1, j_2 be open nuclei for which u_1, u_2 holds for all $x \in L$. Then j_1, j_2 are α-open nuclei for which $j_1(x) \le j_2(x)$ holds for all $x \in L$. Since L is an $\alpha - T_1$ frame, then $j_1 = j_2$. Hence L is a T_1-frame.
(ii) We shall prove the theorem for $\alpha - T_1$. Let L' be a subframe of $\alpha - T_1$ and j_1, j_2 are α-open nuclei of L' for which $j_1(x) \le j_2(x)$

holds for all $x \in L'$. Then $j_1 = L' \vee u_1$, $j_2 = L' \vee u_2$ where u_1, u_2 are α-open nuclei of L. Since $j_1(x) \le j_2(x)$, for $x \in L'$ we have $u_1(x) \le u_2(x)$ holds for all $x \in L$. Since L is an $\alpha - T_1$ frame, hence $u_1 = u_2$. Thus $j_1 = j_2$. Therefore L' is an $\alpha - T_1$ frame.

(iii) Similar to (i) and (ii), respectively.

(iv) Similar to (i) and (ii), respectively.

Corollary 3.1 :

(i) Every $\gamma - T_i$ frame is $P - T_i$, $i = 1,2$.

(ii) Every $\gamma - T_i$ frame is $\alpha - T_i$, $i = 1,2$.

Proof :

(i) Obvious from theorems 3.1,(iii),(iv).

(ii) Obvious by theorems 3.1,(i),(iii).

Theorem 3.2:

(i) Every subframe of a $\gamma - T_i$ frame is $\gamma - T_i$, $i = 1,2$.

(ii) Every $S - T_i$ frame is $\alpha - T_i$, $i = 1,2$.

Proof :

(i) We prove the theorem only for $\gamma - T_2$. Let L' be a subframe of $\gamma - T_2$ and j_1, j_2 are two γ-open nuclei of L' for which $j_1(x) \wedge j_2(y) = 0$ holds whenever $x \wedge y = 0$ in L'. Then $j_1 = L' \vee u_1$, $j_2 = L' \vee u_2$ where u_1, u_2 are γ-open nuclei of L. Since $j_1(x) \wedge j_2(y) = 0$, holds for all $x, y \in L'$ we have $u_1(x) \wedge u_2(y) = 0$ holds whenever $x \wedge y = 0$ in L. Since L is $\gamma - T_2$, then $u_1 = u_2$, . Thus $j_1 = j_2$. Therefore, L' is a $\gamma - T_2$ frame.

(ii) It is similar as (i)

Theorem 3.3 :

Every T_i-frame is clopen - T_i, $i = 1,2$.

Proof :

We shall prove the theorem only for clopen-T_1. Let j_1, j_2 be clopen nuclei for which $j_1(x) \le j_2(x)$ holds for all $x \in L$. Then j_1, j_2 are open nuclei for which $j_1(x) \le j_2(x)$ holds for all $x \in L$. Since L is T_1, then $j_1 = j_2$. Thus L is a clopen - T_1 frame.

Corollary 3.2 :

(i) Every subframe of a clopen-T_i frame is clopen - T_i, I = 1, 2.

(ii) Every $S - T_i$ frame is clopen - T_i, $i = 1,2$.

(iii) Every $P - T_i$ frame is clopen-T_i, $i = 1,2$.

(iv) Every $\alpha - T_i$ frame is clopen-T_i, $i = 1,2$.

Proof :

(i)Obvious from theorems 3.1 (ii) and 3.2 (i).

(ii), (iii) Obvious from theorem 3.3.

(iv) Obvious from theorem 3.3 (i).

4. Some weaker forms of compactness

In this section there are considered some notions of compactness of a frame defined by means of γ-open (resp. α-open; clopen) nuclei of this frame.

Definition 4.1 :

A frame L is called a γ-compact (resp. α-compact; clopen compact) frames, if for any family $\{U_i : i \in I\}$ of γ-open (resp. α-open ; clopen) nuclei of L for which $\vee_{i \in I} U_i = 1$, there exists a finite subfamily $\{U_{i_1}, \cdots, U_{i_n}\}$ of L for which $\vee_{k=1}^{n} U_{i_k} = 1$.

Lemma 4.1:

(i) if L is an α-compact frame, then L is compact.

(ii) Let L be a semi-compact frame. Then L is compact.

(iii) If L is a semi-compact frame, then L is α-compact.

Proof:

Obvious.

Theorem 4.1 :

If L is strongly compact frame, then L is a compact.

Proof :

Let $\{U_i : i \in I\}$ be any family of α-open nuclei of L for which $\underset{i \in I}{\vee} U_i = 1$. Then and L is a strongly compact frame, then there exists a finite subfamily $\{U_i : i \in I\}$ $\{U_{i_1}, \cdots, U_{i_n}\}$ of I such that $\overset{n}{\underset{k=1}{\vee}} U_{i_k} = 1$. Hence L is an α-compact frame.

Corollary 4.1:

A strongly compact frame L is compact.

Proof:

Ovbious by lemma 4.1 and theorem 4.1. The converse of theorem 4.1 is not true in general as shown by the following example.

Example 4.1 :

Let X be an infinite set and $L \equiv \tau(X) = \{\Delta, u : u^c \text{ is finite}\}$ a topology on X. Then L is a compact frame. If for any family $\{U_i : i \in I\}$ of open nuclei of L for which $\underset{i \in I}{\vee} U_i = 1$, then U_i^c is finite and contained in $\overset{n}{\underset{k=1}{\vee}} U_{i_k}^c = 1$. Thus L is not a strongly compact frame.

Lemma 4.2 :

If L is an γ-compact frame, then L is strongly compact.

Proof :

Let $\{U_i : i \in I\}$ be any family of preopen nuclei of L for which $\underset{i \in I}{\vee} U_i = 1$. Since L is γ-compact, then there exists a finite subfamily $\{U_{i_1}, \cdots, U_{i_n}\}$ of L for which $\overset{n}{\underset{k=1}{\vee}} U_{i_k} = 1$. Hence, L is a strongly compact frame.

Corollary 4.2:

(i) Let L be γ-compact frame. Then L is semi-compact.

(ii) If L is γ-compact frame. then L is compact.

Proof:

(i) Obvious from lemma 4.2.

(ii) Obvious from lemma 4.1, lemma 4.1. (ii)

Lemma 4.3 :

Every compact frame, is clopen compact.

Proof:

Let $\{U_i : i \in I\}$ be any family of clopen nuclei of L for which $\underset{i \in I}{\vee} U_i = 1$. And L is compact. Then there exiss a finite subfamily $\{U_{i_1}, \cdots, U_{i_n}\}$ of L for which $\overset{n}{\underset{k=1}{\vee}} U_{i_k} = 1$. Thus L is a clopen compact frame.

Theorem 4.2 :

Let L be a frame:

(i) If L is semi-compact, then L is a clopen compact frame.

(ii) If L is strongly compact, then L is clopen compact.

(iii) If L is α-compact, then L is a clopen compact frame.

Proof :

We prove the theorem only for (iii). Let $\{U_i : i \in I\}$ be any family of clopen nuclei of L for which $\underset{i \in I}{\vee} U_i = 1$. Since *L* is α-compact, then there exists a finite subfamily $\{U_{i_1}, \cdots, U_{i_n}\}$ of L for which $\overset{n}{\underset{k=1}{\vee}} U_{i_k} = 1$. Hence L is clopen compact.

Theorem 4.3 :

Each extremely disconnected strongly compact frame is γ-compact.

Proof :

Let $\{U_i : i \in I\}$ be any family of γ-open nuclei of L for which $\underset{i \in I}{\vee} U_i = 1$. Then $\{U_i : i \in I\}$ is a family of [reopen nucli of L for which $\underset{i \in I}{\vee} U_i = 1$. Since L is strongly compact, then there exists a finite subfamily $\{U_{i_1}, \cdots, U_{i_n}\}$ of L for which $\overset{n}{\underset{k=1}{\vee}} U_{i_k} = 1$. Hence L is γ - compact frame.

Theorem 4.5 :

Every closed nucleus of a semi-compact frame is γ -compact.

Proof :

Let $\{U_i : i \in I\}$ be any family of γ -open and closed nuclei of L for which $\underset{i \in I}{\vee} U_i = 1$. Then $\{U_i : i \in I\}$ be any family of semi-open nuclei of L for which $\underset{i \in I}{\vee} U_i = 1$. Since L is semi-compact, then there exists a finite subfamily $\{U_{i_1}, \cdots, U_{i_n}\}$ of L for which $\overset{n}{\underset{k=1}{\vee}} U_{i_k} = 1$. Thus, L is γ - compact frame.

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