Effect of multi-layers soil under hydraulic structures on seepage characteristics

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The present study is intended to investigate the effect of the sub-layers formation on seepage characteristics (uplift pressure, seepage discharge and hydraulic gradient). This objective is fulfilled through studying a hydraulic structure founded upon two horizontal layers with different arrangement and relative thickness. The study is carried out experimentally by using the sand model technique and numerically by using the boundary element method. The results indicate that, the decreasing of the permeability of the lower sub-layer causes an increasing in the values of the uplift pressure, an increasing in the hydraulic gradient, and a decreasing in the seepage discharge. Both the experimental and the numerical results show reasonable agreement in comparison with published results. A solved example is considered to illustrate the use of the obtained charts.

يقدم هذا البحث دراسة معملية لتأثير طبقات التربة المزدوجة أسفل المنشأت الهيدروليكية على خصائص التسرب ومقارنة النتائج المعملية بالنتائج النظرية التي تم الحصول عليها باستخدام نظرية العناصر الحدودية، و من خلال هذه الدراسة تم تحديد تأثير كلا من: فرق موازنة المياه، اختلاف ترتيب الطبقات تحت المنشأ واختلاف السمك النسبي للطبقات تحت المنشأ على خصائص التسرب (قوة الرفع- معدل السريان - الانحدار الهيدروليكي) وتم استخدام ثلاثة أنواع من الرمل (خشن، متوسط مناعم) وقد أوضحت نتائج الدراسة أن قوى الرفع والانحدار الهيدروليكي يتناسبا تناسبا عكسيا مع زيادة نفاذية الطبقة السفلي بالنسبة إلى الطبقة العليا بينما يزداد معدل السريان مع زيادة تلك النفاذية. أوضحت نتائج الدراسة أيضا أن زيادة عمق الطبقة الأقل نفاذية يؤدى إلى زيادة قوى الرفع في النصف الأول من فرشة المنشأ الهيدروليكي (الأمام) ونقص قيمتها في النصف الثاني (الخلف) ويؤدى أيضا إلى نقص في معدل السريان وزيادة في الانحدار الهيدروليكي .

Keywords: Uplift pressure, Multi-layers, Seepage, Boundary element method, Hydraulic structures

1. Introduction

The seepage is considered a very important phenomenon in the design of the heading-up hydraulic structures. The work presented herein is a trial for the better understanding of the seepage characteristics (uplift pressure, seepage discharge and hydraulic gradient) utilizing a multi-layers soil under the floor of the hydraulic structures. So that this paper concerned with studying the effect of the seepage head, H, the order of the sub-layers and the relative thickness of the sub-layers and the relative thickness of the sub-layer, D_U/D , on the seepage characteristics experimentally using sand model" tank. Three types of sand (coarse, medium and fine sand) of underneath permeable layer are used in the

experimental work. Sandy soil is used with depth equal to one and half of the floor length.

The order of sub-layers is changed two times to construct two categories. In the first category, the upper layer was less permeable than the lower one, for example "the upper layer is medium sand and the lower one is coarse sand", this category contains three cases No. 1,2 and 3. For case No. 1, the upper layer thickness is 15 cm and the lower one is 45 cm. For case No. 2, both the upper and the lower layer thickness are 30 cm. In case No. 3, the upper layer thickness is 45 cm and the lower one is 15 cm. In the second category, a homogenous soil with only medium sand with total depth equals 60 cm is used. The second category contains case No.

4. In the third category, the upper layer is more permeable than the lower one. This category contains three cases (No. 5, 6 and 7). For case No. 5, the upper layer thickness is 45 cm and the lower one is 15 cm. For case No. 6, the upper and lower layer thickness are 30 cm. For case No. 7, the upper layer thickness is 15 cm, while the lower one is 45 cm. For each order, the relative thickness, D_U/D, is changed three times to be 0.25, 0.5 and 0.75. For each relative thickness, the water head, H, is changed five times to be 3,6,9,12 and 15 cm. So, thirty five runs are conducted experimentally as shown in table 1.

A comparison between both the numerical and the experimental studies are carried out using Boundary Integral Equation 2-D Constant Potential (BIE2DCP). This program was obtained from United State Department of

the Interior, Bureau of Reclamation, and was developed by Amr [1] in FORTRAN 77L.

The main assumptions of the experimental work assumed to define the problem [2] are:

- 1. The flow is steady and confined.
- 2. The soil through each layer is homogenous and isotropic.
- 3. The differential equation governs the seepage flow in this case is described by Laplace's equation;

$$\nabla^2 \varphi = \nabla^2 \psi = 0.0 .$$

Both the physical model and the boundary conditions of this problem are illustrated in fig. 1 and table 2, alternatively. Fig. 2 shows the numerical model for case (1) and how the elements are numbered and the internal points locations.

Table 1 Experimental program

Category	Case	Upper thick (Cm)	Lower thick (Cm)	Effective head (H) (Cm)	No. of tests
1- The upper layer is medium	1	15	45	3, 6, 9,12,15	5
sand and the lower one is course	2	30	30	3, 6, 9,12,15	5
sand	3	45	15	3, 6, 9,12,15	5
2- Homogenous medium sand.	4	45	15	3, 6, 9,12,15	5
3- The upper layer is medium	5	30	30	3, 6, 9, 12, 15	5
sand and the lower one is fine	6	15	45	3, 6, 9, 12, 15	5
sand	7			3, 6, 9, 12, 15	5
					35 run

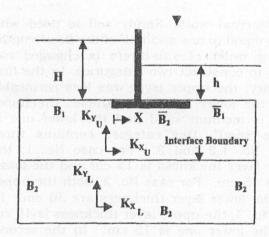


Fig. 1-a. Boundary conditions for under a floor of hydraulic structure (confined flow problem).

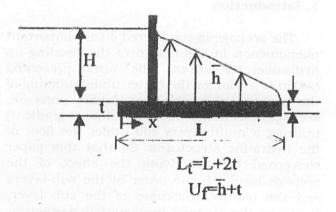


Fig. 1-b. Definition sketch.

Table 2
The boundary conditions for confined flow under floor of hydraulic structure

Face	For potential problem prescribed boundary condition	Face	For stream function problem prescribed boundary condition
В1	φ = H	В1	$\frac{\partial \psi}{\partial n} = \frac{\partial \psi}{\partial y} = 0$
$\overline{\mathrm{B_1}}$	or φ = h range Dangily again square and regard awl said state square	$\overline{\mathrm{B_1}}$	$\frac{\partial \psi}{\partial n} = \frac{\partial \psi}{\partial y} = 0$
В2	$\frac{\partial \phi}{\partial n} = 0$	В2	ψ = constant = C_2 = flow through the section or 100%
$\overline{B_2}$	$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial y} = 0$	$\overline{\mathrm{B}_2}$	$\psi = \text{constant} = C_1 = 0$

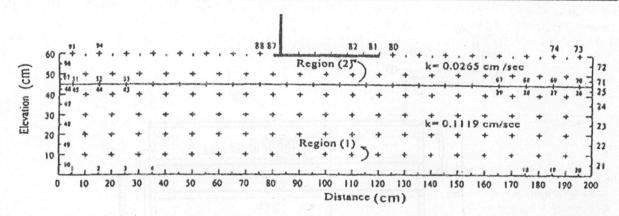


Fig. 2. The first case: the depth of the upper layer = 15.0 cm (DU/D=0.333) No. of boundary element of region (1) = 50, No. of boundary element of region (2) = 47 No. of Internal point of region (1) = 76, No. of Internal point of region (2) = 19.

2. Literature review

Most of the previous studies concerned with one layer soil under the hydraulic structure. El- Ganainy [2] studied the effect of both the upstream and the downstream reaches on the seepage characteristics using the conforaml mapping technique. From the results, El Ganainy concluded that, increasing of the upstream and downstream pervious reaches leads to an increasing in the values of the uplift pressure in the upstream part of the floor and a decrease in such values along its downstream part. For the same above condition, the exit gradient decreases and the seepage discharge increases. El Masry [3] studied the effect of the pervious sub-layer thickness beneath the floor on the seepage characteristics using the boundary element

technique. El Masry [3] showed that, the decreasing the thickness of the permeable layer underneath the structure decreases the uplift pressure, exit gradients and seepage flow. Mohamed [4] investigated the effect of the length of floor on the seepage characteristics numerically by using the finite element technique and experimentally by using the model tank. Mohamed S.M. concluded that the increasing of the floor length causes an increasing in the uplift pressure values through all the floor length, but decreases both the exit gradient and the seepage discharge.

Todd and Bear [5] used the electrical analogy method to show the effect of the soil non-homogeneity and anisotropy on the seepage characteristics. They concluded that, the seepage factor increases when the

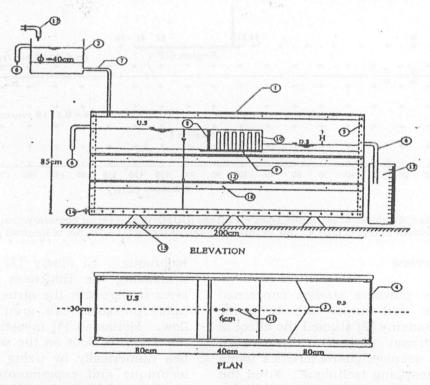
anisotropy ratio (greater horizontal permeability) decreases. Also, Amr [1] used the boundary element method to show the pervious effect. His main conclusion indicates that in case of two layers (the lower layer is more permeable than the upper one) the uplift pressure is less and the exit gradient is more than those existed in the case of homogenous one layer soil.

3. Experimental set-up

The problem of seepage through the multilayers soil under the floor of hydraulic structures is studied experimentally using a sand model tank. In order to be free form any end effects, the dimensions of the model are taken as follows,

$$L_1/L = L_2/L = 2 & D/L = 1.5$$

Fig. 3 demonstrates the experimental model arrangements, which consists of the seepage tank (1) and constant head tank (2). Seepage tank has two prespex faces (3) each of 200 x 85 x 1.5 cm, two steel end plates (4) each of 30 x 85 x 0.3 cm, and steel plate bottom 200 x 30 x 0.3 cm. The tank is provided with steel angles (5) 5 x 5x 0.5 cm to collect its components and provided with two tubes (6), 1.25cm diameter. The first one is used to escape the percolating water at the.



- 1-Seepage tank 200×85x 30cm.
- 2-Constant head tank.
- 3- Prespex sheets 1.5 cm thick
- 4- Steel side 0.3 cm thick.
- 5- Steel angles. 5.0x 5.0x 0,5 cm.
- 6- Over flow water pipe 0.5 inch.
- 7- Feeding pipe 0.5 inch.
- 8- Prespex floor 1.5 cm thick.
- 9- Prespex floor 1.5 cm thick.

- 10- 6 glass tubes 0.6 cm diam mounted Prespex board
- 11- Piezometer nozzles 0.2 cm diameter
- 12- permeable layer (sandy soil).
- 13- Graduated tube.
- 14- Drainage valve.
- 15- Support.
- 16- Bracing angles 5.0× 5.0× 0.5cm.
- 17- Control valve.

Fig. 3. Schematic diagram of the experimental model.

downstream side, while the second one is used to collect the overflowing water form the upstream side. The seepage tank is fed from the constant head tank through the pipe (7) of 1.25cm diameter. The constant head tank has internal diameter equals to 40 cm.

The structure model is made of prespex materials. It consists of prespex gate (8) and the floor (9), which has 40 cm length and 1.5 cm thickness. The uplift pressure under the floor is measured by six-glass tubes (10) of 0.6 cm diameter, which are installed vertically on the floor model. The floor has six piezometers, which are made by slotting the floor at distance X from the heel point, where X/L = 0.1977, 0.3372, 0.4767, 0.6163, 0.7558 and 0.8953. Each slot is 0.2 cm in diameter at the lower part of the floor and 0.6 cm diameter at the upper one, located at the centerline of the floor in the direction of the floor length.

The sand filling (12) is chosen as a porous medium sand for category No. 2, "medium/ coarse" sand for category No. 1 and medium/ fine sand for category No. 3. The overflowing water from the downstream tube is measured by graduated tube (13) to give the quantity of seepage. The seepage tank is provided with control valve (14) to drain the water when changing the soil under the floor. The seepage tank is adjusted horizontally using a set of supports (15). Bracing angles (16) 5.0 X 5.0 x 0.5 cm are used to avoid the buckling of the seepage tank during compaction of the soil. The inflow discharge can be controlled by 1.25 cm diameter control valve (17) located over the constant head thank. The water surface and bed level are measured by a point gauge which is supported on the two prespex plates (3).

The classification of the soil is carried out according to Massachusetts Institute of Technology (MIT) system. The different coefficients of permeability K_f , K_m , and K_c of the three type of sand are determined using the constant head permeability test as follows:

 $K_f = 0.0069 \text{ cm} / \text{sec for find sand,}$

 $k_m = 0.0265$ cm / sec for medium sand, and

 $k_c = 0.1119$ cm / sec for coarse sand.

4. Experimental procedures

The procedure of the experiments is carried out as follows:

- 1. Seepage tank is adjusted horizontally by using a set of supports.
- 2. Sand soil is compacted in layers each 15 cm thickness.
- 3. Total depth of soil has been saturated in order to avoid any settlement for the soil after fixing the structural model at the soil surface.
- 4. Structural model is fixed horizontally at the soil surface.
- 5. Water is supplied to the upstream side of the seepage tank from the constant head tank using the feeding pipe 1.25cm.
- 6. Upstream and downstream water levels are adjusted using the over flow pipes in order to obtain different effective values of head, H.
- 7. Steady state situation (i.e. constant seepage discharge) takes about 10 hours. After that, the readings of the piezometers "uplift force, U" and the graduated tube "seepage discharge, Q" are recorded.
- 8. The value of the water head, H, is changed five times (3,6,9,12 and 15 cm).
- 9. Soil inside the seepage tank is replaced by another and the steps form 2 to 8 are repeated.

5. Analysis and discussion of the results

Both the experimental and numerical results show that, the formation of the soil beneath the floor of the hydraulic structures has a noticeable influence on the seepage characteristics. This effect would be discussed in the following analysis of the experimental results.

5.1. Effect of the seepage head, H

The effect of the seepage head, H, on the relative values of uplift pressure U/H_0 is shown in fig. 4. From this figure, it is clear that, when H is constant, the values of U/H_0 are inversely proportional to the values of X/L_t . This trend agrees with the previous investigations such as Abou-Rehim [6], Kumar [7], and Mohamed [5]. Also, it can be noticed that, at constant value of X/L_t , the values of U/H_0 are directly proportional to the values of the seepage head, H. This result agrees with the result obtained by Mohamed S.M. [5].

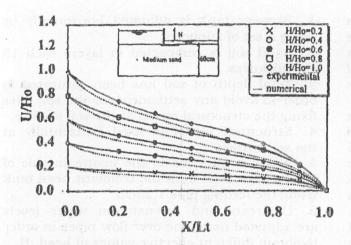


Fig. 4. Effect of the seepage head, H on U/Ho (case 4).

Figs. 5 and 6, show that the trend of case (4) is constant for the other cases have any sub-layers formation and relative thickness of

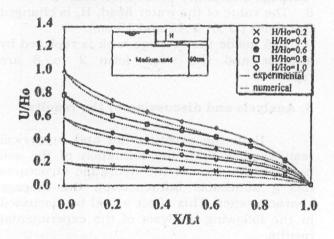


Fig. 5. Effect of the seepage head, H on U/Ho (case 7).

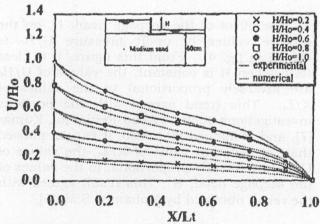


Fig. 6. Effect of the seepage head, H on U/Ho (case 1).

the sub-layers under the floor. Also, these figures show that, there is good agreement between both the numerical and experimental results.

Figs. 7, 8 and 9 show the effect of the seepage head, H, on the relative values of uplift pressure, U/H (cases No. 2,4, and 6). From these figures, it is seen that, there is no agreement between the trend of the numerical and experimental results. The experimental results indicate that, for any sub-layers formation and any value of the relative thickness, D_U/D , the values of U/H increase when the values of seepage head, H, decrease. But the numerical results show that, the seepage head, H, has no effect on the values of U/H, therefore, all curves lied over each other.

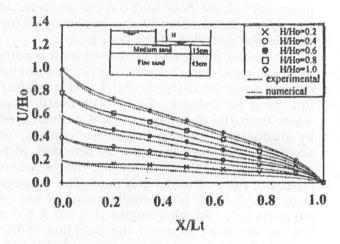


Fig. 7. Effect of the seepage head, H on U/H (case 2).

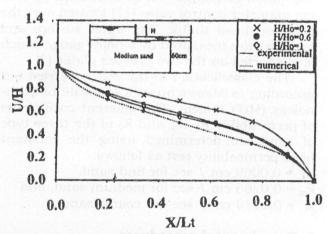


Fig. 8. Effect of the seepage head, H on U/H (case 4).

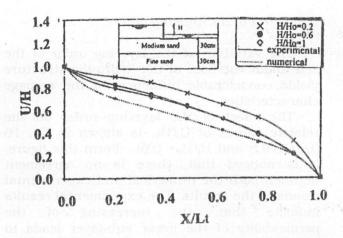


Fig. 9. Effect of the seepage head, H on U/H (case 6).

Fig. 10 shows the effect of the seepage head, H, on the values of seepage discharge, Q, when the soil under the floor is medium sand (case 4). From this figure, it can be seen that, the seepage discharge, Q, increases with increasing the effective seepage head, H. This trend agrees with the results of Mohammed S.M.[5]. Also figs. 11 and 12 show the same previous effect for multi sub-layers of soil (cases No. 1 and No. 7). From these figures, it can be concluded that the changing of the permeability of the sub-layers under the floor doesn't affect on the general trend but it affects on the discharge value at the same value of H/H₀.

The effect of the seepage head, H, on the hydraulic gradient, I, is shown in figs. 13 through 15 (cases No. 1, 4 and 7). From these

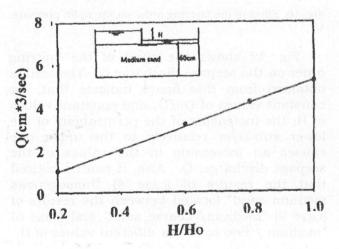


Fig. 10. Effect of the seepage head, H on the seepage discharge Q, (case 4).

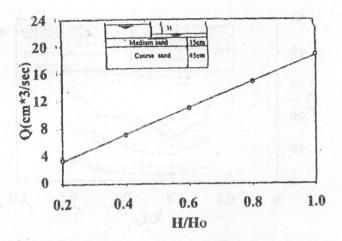


Fig. 11. Effect of the seepage head, H on the seepage discharge Q, (case 1).

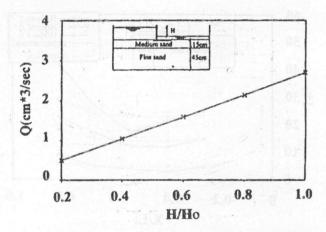


Fig. 12. Effect of the seepage head, H on the seepage discharge Q, (case 7).

figures, it is clear that, for any layering order and relative thickness, the values of the hydraulic gradient, I, increase when H increases. Also, it can be noticed that, at constant values of H, when the values of X/L increase, the values of the hydraulic gradient, I, decrease in the upstream part of the floor but increase in the downstream part of the floor.

From the experimental and numerical results it can be concluded that the seepage head, H has a remarkable effect on seepage characteristics. Also form these figures, it can be conclude that there is a good agreement between both the numerical and experimental results.

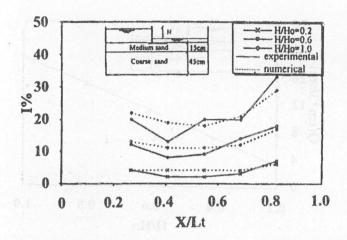


Fig. 13. Effect of the seepage head, H on the hydraulic gradient, I, (case 1).

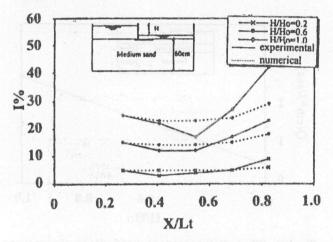


Fig. 14. Effect of the seepage head, H on the hydraulic gradient, I, (case 4).

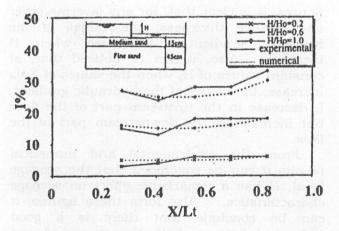


Fig. 15. Effect of the seepage head, H on the hydraulic gradient, I, (case 4).

5.2. Effect of the layering order

The variation of the layering order of the soil under the floor of the hydraulic structure yields considerable influence on the seepage characteristics.

The effect of the layering order on the relative values of U/H₀, is shown in fig. 16 $(D_U/D=0.5$ and $H/H_0=0.6$). Form this figure, it is noticed that, there is no agreement between both the numerical and experimental trends of the results. The experimental results indicate that. the increasing the permeability of the lower sub-layer leads to decreasing in the values of relative uplift pressure U/H₀. While the numerical results show that, the increasing of the layer sublayer permeability causes a decreasing in the values of U/H₀ in the upstream part of the floor and an increasing in such values in the downstream part.

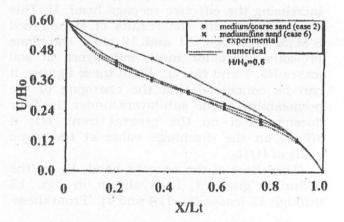


Fig. 16. Effect of the layering order on the uplift pressure, \boldsymbol{U} .

Fig. 17 shows the effect of the layering order on the seepage discharge Q. The results obtained from this figure indicate that, for constant values of D_U/D , and constant values of H, the increasing of the permeability of the lower sub-layer relatively to the upper one causes an increasing in the values of the seepage discharge, Q. Also, it can be noticed that the results of (case 4) "homogenous medium sand" located between the results of (case 2) "medium/ coarse sand" and (case 6) "medium/ fine sand" for different values of H.

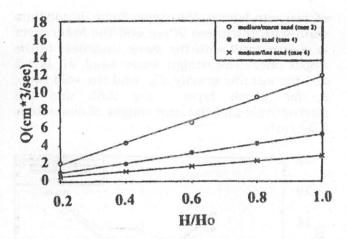


Fig. 17. Effect of the layering order on the seepage discharge, Q (case 4).

The effect of the layering order on the hydraulic gradient, I, of the seeping water under the floor of the hydraulic structure can be analyzed from fig. 18. From this figure, it is easy to notice that, there is a remarkable agreement between both the numerical and experimental results. This trend indicates that, the increasing of the permeability of the lower sub-layer relatively to the permeability of the upper sub-layer causes a decreasing in the values of the hydraulic gradient (I).

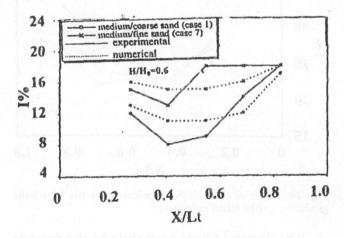


Fig. 18. Effect of the layering order on the hydraulic gradient, I.

5.3. Effect of the relative thickness

Figs. 19 and 20 illustrate the effect of relative thickness on the relative values of the uplift pressure U/H₀. It is noticed that, there is a good agreement between the trend of the

numerical and experimental results. These results state that, the increasing of the less permeable sub-layer depth (i.e medium sand in the first category and fine sand in the third category causes an increasing in the values of uplift pressure in the upstream part of the floor and a decreasing in such values in the downstream part.

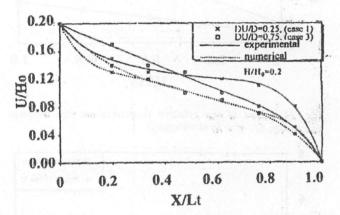


Fig. 19. Effect of the relative thickness on the uplift pressure, (The first category).

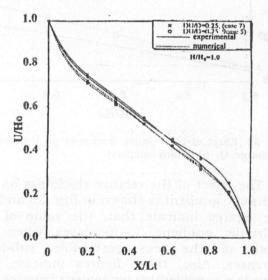


Fig. 20. Effect of the relative thickness on the uplift pressure, (the third category).

Figs. 21 and 22 show the effect of relative thickness, D_U/D , on the values of seepage discharge, Q. From these figures, it is easy to observe that, the increasing of the less permeable sub-layer depth leads to a decreasing in the values of the seepage discharge, Q.

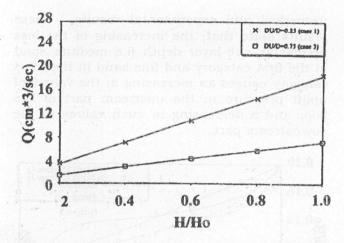


Fig. 21. Effect of the relative thickness on the seepage discharge, Q, (the first category).

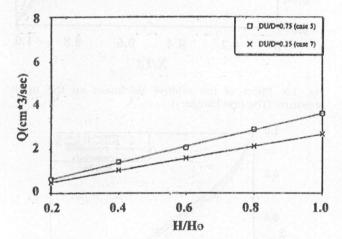


Fig. 22. Effect of the relative thickness on the seepage discharge, Q, (the third category).

The effect of the relative thickness on the hydraulic gradient is shown in figs. 23 and 24. The figures indicate that, the value of the hydraulic gradient, I, increases when the depth of the less permeable sub-layer increases. Also, these figures indicate that there is a considerable agreement between the trend of the numerical and experimental results.

5.4. Design example

The following example demonstrates the design procedures for the required save length, L, of floor and its thickness, t, for a hydraulic structure (weir). According to the present study, the soil under the floor consists

of two sub-layers, the upper layer is medium sand with thickness 20cm and the lower layer is coarse sand with the same thickness of the upper one. The design water head, H, is 1.5 m, the specific gravity, G_{s} and the void ratio, e, for which layer are 2.65 and 0.8 alternatively and the unit weight of concrete is 2.5 t/m^3 .

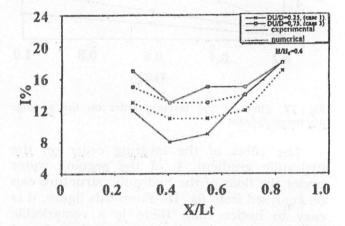


Fig. 23. Effect of the relative thickness on the hydraulic gradient, I, (the first category).

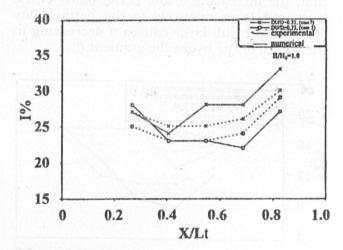


Fig. 24. Effect of the relative thickness on the hydraulic gradient, I, (the third category).

The design of weir according to the present study will be carried out as follows:

5.4.1. Floor length, L

Assume the floor thickness = 1.0m: The save exit gradient could be obtained from the following equations:

 $I_{critical}=(G_{s}-1)/(e+1)=(2.65-1)/(0.8+1)=0.92$

$$I_{\text{safe}} = \frac{I_{\text{critical}}}{4} = \frac{0.92}{4} = 0.23$$
, and $I_{\text{safe}}/H = 0.42 \text{ [D}^{0.12} \text{ / L}^{0.48}\text{]}.$

:. The floor length, L = 20.52 m

5.4.2. Floor thickness, t

The relative distance X/L_t at the critical section i-i = [8 / (20.52 + 2)] = (8/22.52) = 0.36.

To get the water head, H in the model, it must be substituted in the following equation:

$$\frac{H_{\text{m}}}{H_{\text{p}}} = \frac{L_{\text{t}_{\text{m}}}}{L_{\text{t}_{\text{p}}}}$$
 i.e. $\frac{H_{\text{m}}}{1.5} = \frac{0.43}{22.52}$,
 $\therefore H_{\text{m}} = \frac{1.5 \times 0.43}{22.52} = 0.03 \text{m} = 3 \text{cm}$.

From fig. 7, for X/L_t= 0.36 & H/H₀ =
$$\frac{3}{15}$$
 = 0.2

The relative uplift pressure U/H = 0.72Uplift pressure at sec i-i = $U = 0.72 \times 1.5 = 1.08m$

... The thickness of the floor at critical sec $i - i = \frac{U}{\gamma_C - 1} \times 1.3 = 0.95 \text{m}.$

6. Conclusions

On the basis of the present study it can be concluded that, the soil formation under the floor o f the hydraulic structures has a remarkable effect on the seepage characteristics. Also, the experimental results indicated that:

1. For constant layering order and relative thickness, D_U/D , the seepage head, H, is directly proportional to the relative uplift pressure (U/H_0) , the seepage discharge (Q) and the hydraulic gradient, I.

2. For constant values of both seepage head, (H) and relative thickness, (D_U/D) , the increasing of the lower layer permeability causes a decreasing in the values of the uplift pressure and the hydraulic gradient, but for the same above condition, the seepage discharge, Q, increases.

3. For constant values of the seepage head (H) and the layering order, the increase of the depth of the less permeable sub-layer increases the values of the uplift pressure in

the upstream part of the floor and decreased in the downstream part. For the same above condition, the values of seepage discharge, Q, decreased, while the values of the hydraulic gradient, I, increased.

4. The hydraulic gradient (I) is not constant along the floor, but it decreases in the upstream part of the floor and increases in the downstream part with increasing the value of $X/I_{\rm c}$.

5. The numerical results have good agreement with the experimental results except of the effect of the seepage head, H, on the relative values of U/H and effect of the layering order on the relative vales of U/H_0 .

Nomenclature

The following symbols are used in this paper:

- D Total depth of the permeable layer,
- D_U Depth of the upper layer,
- D_L Depth of the lower layer,
- g Gravitational constant,
- H Difference between upstream and downstream seepage head,
- H₀ Maximum seepage head,
- H_u Upstream seepage head,
- H_d Downstream seepage head,
- H Potential head at any point along the floor,
- I Hydraulic gradient,
- K Coefficient of permeability of permeable layer.
- L Horizontal length of floor,
- L_t Total length of floor ($L_t = L + 2t$),
- L₁ Upstream seepage face,
- L₂ Downstream seepage face,
- Q Seepage discharge,
- t Thickness of floor,
- U Uplift pressure measured from the
 - upper surface of the floor,
- Uf Uplift pressure measured from the
 - lower surface of the floor,
- x Longitudinal distance measured from
 - the toe of the floor,
- γ_c Unit weight of concrete,
- G_s Specific gravity of the soil,
- ϕ Potential function, and
- w Stream function.

$$\nabla$$
 Vector operator = $\frac{\partial}{\partial_x} \mathbf{i} + \frac{\partial}{\partial_y} \mathbf{j} + \frac{\partial}{\partial_z} \mathbf{k}$,

$$\nabla^2 \ \text{Laplace operator} = \frac{\partial^2}{\partial_{x^2}} \, i + \frac{\partial^2}{\partial_{y^2}} \, j + \frac{\partial^2}{\partial_{z^2}} \, k.$$

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