Behavior of strong non-linear systems subjected to horizontal external excitation experiencing liquid sloshing impact

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The non-linear systems that involve strong non-linearity have attracted a great deal of attention because of their wide mathematical simulations. In the present study, the dynamic response of two-degree freedom impact system subjected to external excitation along the horizontal axis is considered. The intensity of the external forces is independent upon the response of the system, which is called external excitation. The system equations of motion include impact non-linearity and cubic structural geometric non-linearity. The multiple time scale method is used to construct a first order uniform expansion yielding two first order non-linear ordinary differential equations governing the modulations of the amplitude and phase angle for the two resonance modes of excitation. In the non-impact case, when the two modes are externally excited, the amplitudes response follow the chaotic behavior depend on the damping ratio as the control parameter. A small increase in the value of damping ratio will draw the system from the snap-through form of chaotic behavior to the quasi-period and hoph-bifurcation forms before reaching the steady state by another increase in damping ratio. In the presence of impact forces, the system possesses a steady state response amplitude response within a certain range of external detuning parameter. The study of the impact loading effect on response amplitude indicates that the impact suppresses the system response.

يخـتص هدا البحث بدراسة سلوك الأنظمة اللاخطية المعبرة عن الحركة التصادمية بفعل التحميل الخارجي الناشئ عن قوى أفقية خارجية غير بارا مترية. وقد استخدمت طريقة المقياس الزمني المتعدد كطريقة رياضية لإيجاد الحدود المعبرة عن هذا الرنين فى معادلات الحركة. وهذا الرنين ناشئ عن الحالة الرنينية الخاصة بالاهتزازات الرئيسية المعتمدة على وجود علاقة بيـن الـتردد الخارجي والترددين الحر الأول والثاني وكلاهما على الصورة الرئيسية. وقد تم حل هذه المعادلات لتعبر عن وضعين للمنظومة: إحداهما بإهمال حدود التصادم وفيها تم وصف الحركة اللاتصادمية والأخرى عند دراسة الحركة بوجود التصـادم وفـى كل حالة تم دراسة تأثير متغيرات المنظومة على الحركة وشكلها وأهمها تغيير قوى التصادم خاصة على الحـركة وكذلك تأثير معامل الامتصاص الخطى على الحركة اللاتصادمية. ولذلك تعتمد النتائج بقوة على الحلول العددية لهذه المعـادلات وذلـك باستعمال نطاق واسع لتغير معامل الحيود ولذلك نتوقع أن يتيح عدد كبير من المنحنيات الزمنية التى تعبر عن شكل الحركة سواء ذات السعة الثابته أو المتغيرة.

Keywords: Liquid sloshing modeling, Non-linear vibrations, Impact, External excitation

I. Introduction

An interesting feature of some non-linear systems is their capability of yielding chaotic response to a deterministic input even in the presence of damping. The strong non-linearity system is considered as the most important system, which gives rise to this phenomenon. The importance of these systems is due to their wide applications and representations for many practical and structural dynamic problems, which are encountered in several engineering applications. The design of these systems may create two or more nonlinear algebraic relationships between the system's normal mode frequencies. The problem of liquid sloshing involving an impact loading are the most important dynamical systems which can be simulated by these non-linear systems of equations.

The strongly non-linearity due to impact forces under parametric excitation are investigated by using the multiple scales method. The reader is referred to Sayad and Ibrahim [1,2]. The generalized coordinates are denoted $u_n(t)$, and the equations of motion are assumed to have the form:

$$Y_1'' + \omega_1^2 Y_1 = \varepsilon\{-2\bar{\zeta}_1\omega_1 Y_1' + (\Psi_{11})_{gn} + (\Psi_{11})_{impact} + (\Psi_{11})_{ex}\}, \qquad (1\text{-a})$$

$$\ddot{Y}_2 + \omega_2^2 Y_2 = \varepsilon \frac{m_{11}}{m_{22}} \{-2\bar{\zeta}_2 \omega_2 \dot{Y}_2 + (\Psi_{22})_{gn} + (\Psi_{22})_{impact} + (\Psi_{22})_{ex}\}. \qquad (1-b)$$

The parametric excitation of an elevated water tower experiencing liquid sloshing hydrodynamic impact is studied by Sayad and Ibrahim [1]. The impact loads were modeled based on a phenomenological representation in the form of a power function with higher exponent. When the first mode is parametrically excited the system experiences hard nonlinear behavior and the impact loading reduced the response amplitude. On the other hand, when the second mode is parametrically excited, the response switches from soft to hard nonlinear characteristics. Under combined parametric resonance, the system possesses a single steady state response in the absence and in the presence of impact. Sayad and Ibrahim [2] studied the nonlinear dynamic interaction between the impact of the first asymmetric liquid sloshing mode, represented by an equivalent pendulum and the elastic structural dynamics, the reader can be referred to Pillpchuk and Ibrahim [3] and Abramson[4]. Sayad and Ibrahim [2] considered the auto-parametric coupling in the neighborhood of parametric and internal resonance conditions. Depending on the initial conditions and internal detuning parameter, the response can be quasi-periodic or chaotic with irregular jumps between two unstable equilibriums.

Two degrees of freedom systems with quadratic non-linearity subjected to parametric and self excitation is studied by Asrar [5]. The method of multiple scales is used to study the response characteristics under the simultaneous effect of a harmonic parametric and self-excitation. The principle parametric resonance of the first and second mode is considered. Amplitudes and frequency response curves are presented with the characteristics of the system stability. Liquid sloshing in rectangular road tankers under going turning of braking maneuver is discussed by Popov, Sankar and Vatistas [6]. The steady state solution in terms of liquid forces and moments is derived analytically from the hydrostatic equations. The modified marker and-cell technique is used to study the sloshing problem and to obtain the damped frequencies and magnitudes of the sloshing parameters. The nonlinear interaction of liquid free surface motion with the dynamics of the elastic supporting structure of elevated water towers subjected to vertical sinusoidal ground motion was examined in the neighborhood of internal resonance by Ibrahim and Barr [7,8]. In the neighborhood of internal resonance conditions, the liquid structure system experienced complex response phenomena such as jump phenomena, multiple solution, and energy exchange. Resonance in non-linear structural vibrations involving two external periodic excitations is investigated by Haquang, Mook and Pluat [9]. The system includes weak quadratic and cubic non-linearity and is subjected to an external excitation with two harmonic components. Solutions are obtained using the method of multiple time scales. Steady state response amplitudes are determined and plotted as a function of detuning parameter and the excitation amplitudes. Ibrahim and Li [10,11] studied liquid-structure interaction under horizontal periodic motion. The system is examined in the neighborhood of the fourth order internal resonance. The method of multiple scales is used to obtain the amplitudes response for the first and second mode. Soundararajan and Ibrahim [12] examined more realistic cases, such as simultaneous random horizontal and vertical ground excitations. The mathematical model includes the interaction of one sloshing mode with one structural mode. The Averaging method solution to harmonic functions for strongly non-linear oscillators is examined by Xu and Cheung[13]. The method is applied to study the approximate solution of a strongly non-linear system in terms of generalized harmonic function. These functions are also periodic and are the exact solutions of strongly non-linear differential equations. Some phenomena considered, and include limit cycles of strongly non-linear oscillators subjected to weak harmonic excitation [14,15]. The present paper extends the work of Sayad and Ibrahim [1,2] in the horizontal direction by applying the harmonic external force $F_X(t)$ acting along the X-axis. The intensity of the external forces is independent upon the response of the system, which is called external excitation The impact forces will be represented by a high power model proposed by Hunt and Crossley [16]. The system response in the neighborhood of principal horizontal resonance conditions will be considered.

2. Formulation

For the given mathematical modeling presented by Pillpchuk Ibrahim [3]. The generalized coordinates are denoted $X_n(t)$, and the equation of motion are derived by the Lagrangian equations according to the structure shown in figs. 1-a, 1-b. This will introduce the equations of motions for this model as:

$$Y_1'' + \omega_1^2 Y_1 = \varepsilon\{-2\bar{\zeta}_1\omega_1 Y_1' + (\Psi_{11})_{gn} + (\Psi_{11})_{impact} + (\Psi_{11})_{ex}\}, \qquad (2\text{-a})$$

$$Y_2'' + \omega_2^2 Y_2 = \varepsilon \frac{m_{11}}{m_{22}}\{-2\bar{\zeta}_2\omega_2 Y_2' + (\Psi_{22})_{gn} + (\Psi_{22})_{impact} + (\Psi_{22})_{ex}\}, \qquad (2\text{-b})$$

where a linear viscous damping has been introduced to account for energy dissipation of the two modes, and the original damping factors $\zeta_i = \varepsilon\bar{\zeta}_i$, $\varepsilon = \mu\lambda^2 / m_{11}$.All other terms and constants are given in appendex A. The right-hand sides of these equations include inertia and stiffness nonlinearities of cubic-order which called geometric nonlinearities and are denoted by subscript "gn". They also include impact nonlinearities of fifth-order and are given by the expressions with subscript "impact".

According to the multiple scale method, the solution is expressed in a uniform approximate expansion form:

$$X_i = X_{i0}(T_0, T_1, T_2, \ldots) + \varepsilon X_{i1}(T_0, T_1, T_2, \ldots) + \ldots, \qquad (3)$$

where:

$T_0 = t, T_1 = \varepsilon t, T_2 = \varepsilon^2 t, \ldots \text{i.e.} T_n = \varepsilon^n t, = 0,1,2\ldots$

We note that the T_n represent different time scales because e is a small parameter. Using the Chain rule, we have:

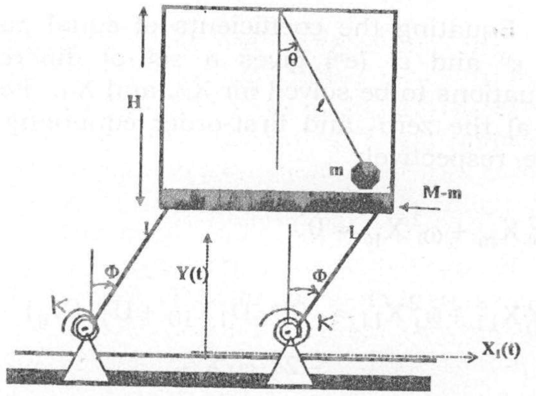

Fig. 1-a. The first mode shape for the amplitude X_1.

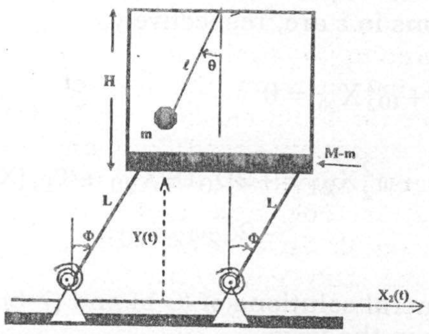

Fig. 1-b. The second mode shape for the amplitude X_2.

$$\frac{\varphi}{\theta} = \frac{\omega_1^2}{\lambda(1-\omega_1^2)} = \frac{1}{K_1} = (+)\text{sign}.$$

$$\frac{\varphi}{\theta} = \frac{\omega_1^2}{\lambda(1-\omega_2^2)} = \frac{1}{K2} = (-)\text{sign}.$$

$$\frac{d}{dt} = \frac{\partial}{\partial T_0} + \varepsilon\frac{\partial}{\partial T_1} + \varepsilon^2\frac{\partial}{\partial T_2} + \ldots = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \ldots,$$

where $D_n = \dfrac{\partial}{\partial T_n}$, (4-a)

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2(D_1^2 + 2D_0 D_2) + \ldots \qquad (4\text{-b})$$

Substituting the solution of eq. (3) into eq. (2) using the transformed time derivative, gives

$$\{D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 (D_1^2 + 2D_0 D_2) + \ldots\} X_i + \omega_i^2 X_i = \varepsilon \Psi_i . \qquad (5)$$

Equating the coefficients of equal powers of ε^0 and ε^1 (e^n) gives a set of differential equations to be solved for X_{i0}, and X_{i1}. For eq. (2-a) the zero- and first-order equations in ε are, respectively

$$D_0^2 X_{10} + \omega_1^2 X_{10} = 0 . \qquad (6\text{-a})$$

$$D_0^2 X_{11} + \omega_1^2 X_{11} = -2D_0 D_1 X_{10} + \Pi_{11}(X_{ij}) - 2\bar{\zeta}_1 \omega_1 X_{10}^{'} , \qquad (6\text{-b})$$

where Π_{ii} stands for nonlinear and excitation terms. For eq. (2-b) the zero- and first-order equations in ε are, respectively,

$$D_0^2 X_{20} + \omega_2^2 X_{20} = 0 \qquad (7\text{-a})$$

$$D_0^2 X_{21} + \omega_2^2 X_{21} = -2D_0 D_1 X_{20} + \Psi_{11}(X_{i0}) - 2\bar{\zeta}_2 \omega_2 \dot{X}_{20} . \qquad (7\text{-b})$$

The general solutions of (6-b) and (7-b) can be written in the form:

$$X_{10} = A(T_1)\exp(i\omega_1 T_0) + \bar{A}(T_1)\exp(-i\omega_1 T_0) . \qquad (8)$$

$$X_{20} = B(T_1)\exp(i\omega_2 T_0) + \bar{B}(T_1)\exp(-i\omega_2 T_0) \qquad (9)$$

Where the over-bar denotes conjugate and $i = \sqrt{-1}$. A (T_1) and B (T_1) are functions of the time scale T_1 and will be determined by eliminating the secular terms from eqs. (6-b) and (7-b). Substituting eqs. (8) and (9) into (6-b) and (7-b) gives:

$$D_0^2 X_{11} + \omega_1^2 X_{11} = -2D_0 D_1 (A(T_1)\exp(i\omega_1 T_0) + \bar{A}(T_1)\exp(-i\omega_1 T_0)) + \Phi_{11}(X_{i0}) - 2i\omega_1^2 \bar{\zeta}_1 (A(T_1)\exp(i\omega_1 T_0) + \ldots , \qquad (10)$$

$$D_0^2 X_{21} + \omega_2^2 X_{21} = -2D_0 D_1 (B(T_1)\exp(i\omega_2 T_0) + \bar{B}(T_1)\exp(-i\omega_2 T_0)) + \frac{m_{11}}{m_{22}} \Phi_{22}(X_{i0}) - 2i\omega_2^2 \zeta_2 (B(T_1)\exp(i\omega_2 T_0) + \ldots . \qquad (11)$$

Where F_{11} and F_{22} contain terms that produce secular terms in X_{i1} (i.e. terms with a small divisor). Obviously the exponents on the right-hand sides in eqs. (10) and (11) decide the resonance conditions associated with each equation.. For this excitation case, we will consider only the two relationships between the horizontal excitation frequency W_x in the external horizontal direction and the two natural frequencies of the system w_1 and w_2. Under this external excitation, the following resonance conditions will be considered:

1- Principal external resonance of the first mode $W_x = w_1$,

2- Principal external resonance of the second mode $W_x = w_2$.

The response characteristics corresponding to these resonance conditions are considered in the next sections.

3. First mode external excitation

For this case, and according to the multiple scale method, it is important to introduce a detuning parameter σ_x that measures the nearness to the exact external resonance:

$$\Omega_X = \omega_1 + \varepsilon \sigma_X$$

Extracting secular terms corresponding to this resonance condition from eq. (10) gives

$$D_0^2 X_{11} + \omega_1^2 X_{11} = - 2D_0 D_1 \{A \exp(i\omega_1 T_0)\} - 2i\omega_1^2 \zeta_1 A \exp(i\omega_1 T_0) - \{iG_{11} \exp\{i(\Omega_x T_0)\} \frac{X_0}{2} + (3G_{18} - 3G_{122}\omega_1^2) A^2 \bar{A} - (12iC_{15}\omega_2 - 12iC_{15}\omega_1 - 60C_{16}) A^2 \bar{A} B \bar{B} + (6iC_{15}\omega_1 - 24iC_{15}\omega_2 + 60C_{16}) B^2 A \bar{B}^2 + (2iC_{15}\omega_1 + 10C_{16}) A^3 \bar{A}^2)\} \exp(i\omega_1 T_0) . \qquad (12)$$

One can drop all terms including (B) in eq. (12), since the first mode is excited only.

Now, according to the multiple scale method, one can express the solution A (in the time scale T_1) in the complex polar form,

$$A = \frac{a}{2}\exp(i\alpha)\,,\quad \bar{A} = \frac{a}{2}\exp(-i\alpha)\,. \tag{13}$$

Substituting eq. (13) in eq. (12), setting all secular terms to zero, and separating real and imaginary parts gives the following two first-order differential equations:

$$-\omega_1 a \frac{\partial \alpha}{\partial T_1} = (G_{11})\frac{X_0}{4}\sin(\sigma_y T_1 - \alpha) - \frac{3}{8}(G_{122}\omega_1^2 - G_{18})a^3 + \frac{5}{16}C_{16}a^5\,, \tag{14-a}$$

$$\omega_1 \frac{\partial a}{\partial T_1} = -(G_{11})\frac{X_0}{4}\cos(\sigma_y T_1 - \alpha) - \omega_1^2\bar{\zeta}_1 a + \frac{\omega_1}{16}C_{15}a^5\,. \tag{14-b}$$

The non- autonomous form of expression for the eqs. (14-a) and (14-b) can be transformed into the following autonomous form by introducing the parameter $g_1 = s_x T_1$ - a

$$\frac{\omega_1}{2}a\frac{\partial \gamma_1}{\partial T_1} = \frac{\omega_1}{2}\sigma_y a + G_{13}a\frac{Y_0}{4}\sin\gamma_1 + G_3 a^3 + \frac{5}{16}C_{16}a^5\,. \tag{15}$$

$$\omega_1\frac{\partial a}{\partial T_1} = -G_{11}\frac{X_0}{4}\cos\gamma_1 - \omega_1^2\bar{\zeta}_1 a + \frac{\omega_1}{16}C_{15}a^5\,, \tag{16}$$

where $G_3 = (\frac{3}{8}G_{18} - \frac{3\omega_1^2}{8}G_{122})$, and the other constants are defined in Appendix A,B and C.

3.1. Non-impact response

In the absence of the strong non-linearity (C_{16} and C_{15} are equal to zero), The two eqs. (15,16) are integrated numerically using Runge-Kutta method (MACSYMA 2.3) for mass ratio μ=0.2, length ratio λ=0.2, local frequency ratio ν=0.5, excitation amplitude ratio X_0 = 0.1, s_x = -1.0, a_0 = 0.02. A sample of time history records of the amplitude and phase angle is shown in fig. 2-a, b. It is seen that the response reaches a stationary state after very short transient period for a value of damping ratio ζ_1 = 0.2. The response is independent upon the initial conditions for a_0 as shown in fig. 2-c, where a_0 = 1.0. Solving eqs. (14-a) and (14-b) for the steady state response, one gives polynomial as:

$$C_6^* a^6 + C_4 a^4 + C_2 a^2 + C_0 = 0\,. \tag{17}$$

The dependence of the steady-state amplitude on the detuning parameter s_x is shown in fig. 3, The dotted curve is belonging to non-impact excitation case. For the non-linear system, a small change in the control parameter of this system can lead to sudden changes in quantitative behavior of the whole system. For the given excitation case, the damping ratio ζ_1 is the control parameter which control the steady state solution of eqs. (16, 17) and the expected time history amplitudes and phase records. If the value of damping ratio ζ_1 is less than 0.1, it is clear that the response amplitudes do not reach any fixed points. Fig. 4 shows a sample of the time history record for damping ratio ζ_1 = 0.01; it is found that the amplitudes take the behavior of the chaotic response in the snap-through form. The detuning parameter s_x is limitting this behavior over a certain range as $-20 \le \sigma_x \le 20$. The amplitudes response behaves as random looking out of this domain as shown in fig. 5. Fig. 6 shows the Fourier Fast Transform (FFT) for the chaotic behavior of the the snap-through form after removing the transient period. This FFT plotting contains more than one frequency component. For a damping ratio ζ_1 = 0.05, the time history record displaying quasi-periodic behavior as shown in fig. 7. These forms of the chaotic behavior are independent upon the initial values of a_0 that are called non-strange attractors.

3.2. Impact response

It is clear that the presence of impact forces creates steady state response only for all the expected values of damping ratio ζ_1,

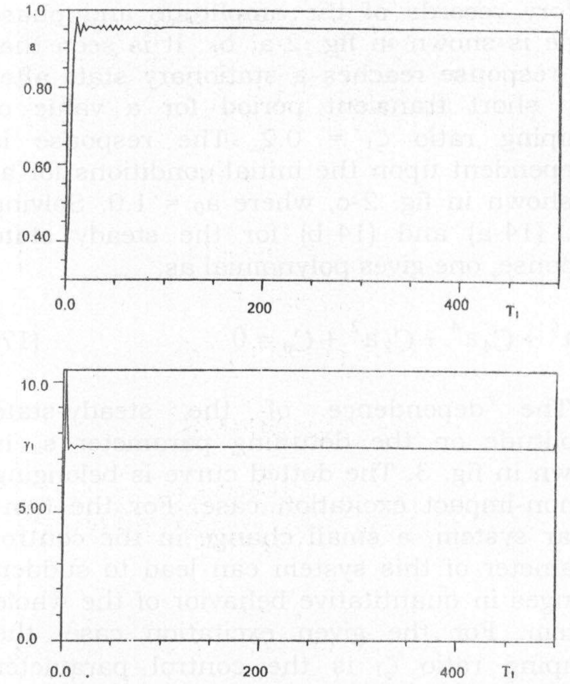

Fig. 2-a,b. Time history phase record for non-impact case under 1st mode external horizontal excitation (Xo=0.1, μ=0.2, λ = 0.2, σ_{x} = 1.0, ζ_1 = 0.2, a_o = 0.02, γ_o = 0.02).

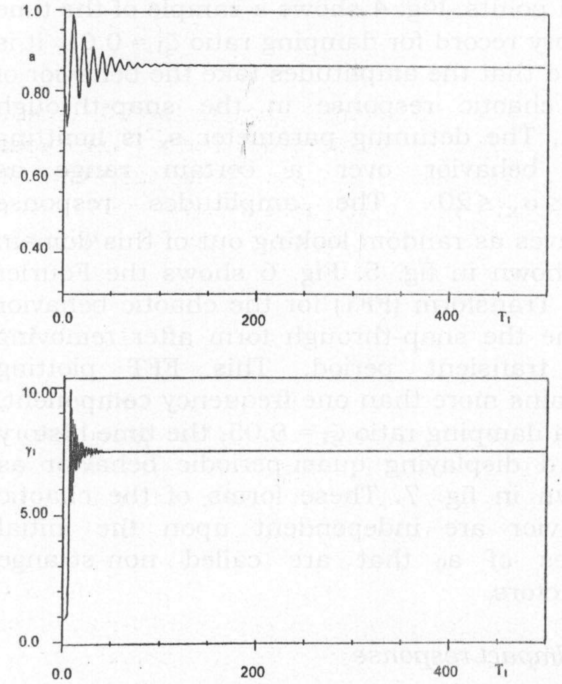

Fig. 2-c,d. Time history phase record for non-impact case under 1st mode external horizontal excitation (X_o = 0.1, μ = 0.2, λ = 0.2, σ_{xx} = 1.0, ζ_1 = 0.2, a_o = 0.08, γ_o = 0.8).

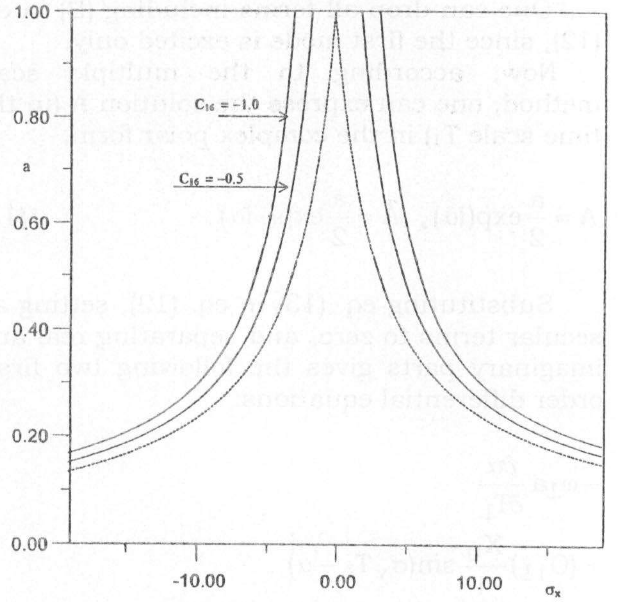

Fig. 3. Amplitude-frequency response curves under the horizontal external excitations for the first mode resonance case μ=0.2, λ = 0.2, ζ_1 = 0.2, X_o = 0.1)

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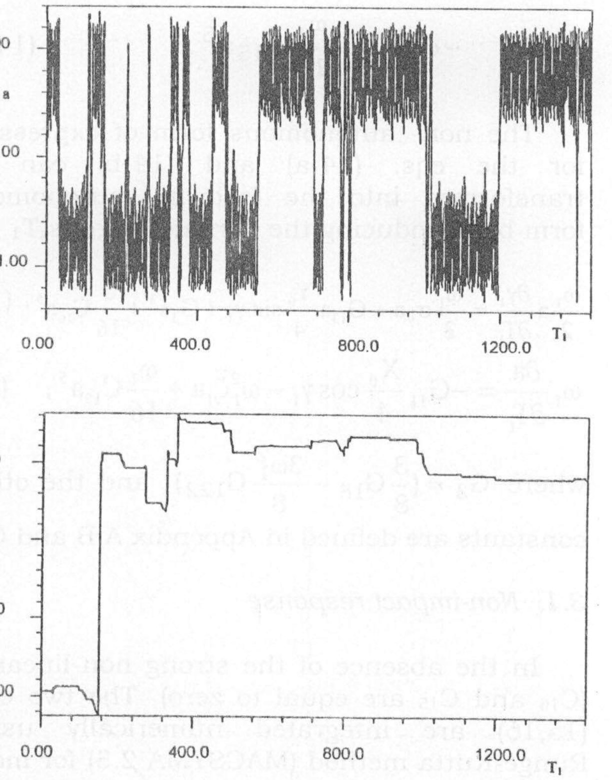

Fig. 4. Time history phase record for impact case under 1st mode external horizontal excitation (X_o=0.1, λ = 0.2, ζ_1 = 0.2, X_o = 0.1).

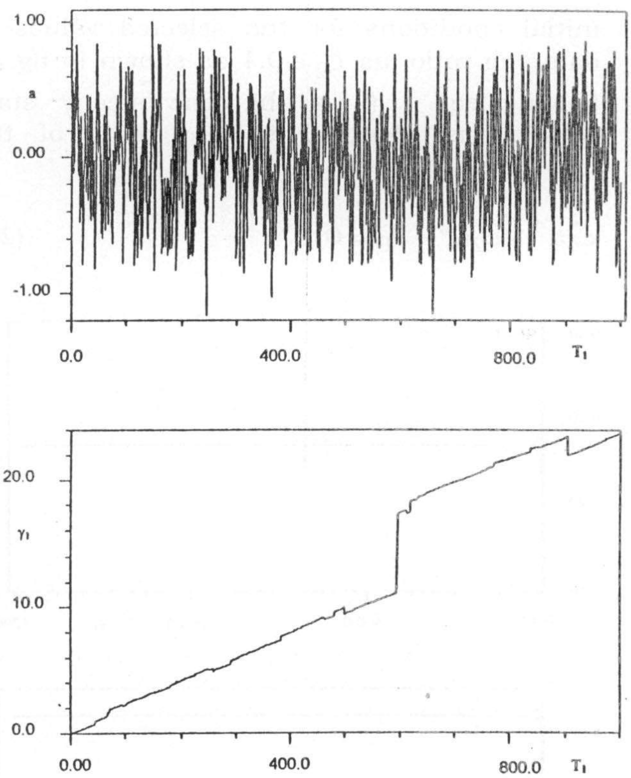

Fig. 5. Time history phase record for non-impact case under 1st mode external horizontal excitation (X_o=0.1, μ=0.2, λ= 0.2, σ_x = 25.0, ζ_{1o} = 0.05).

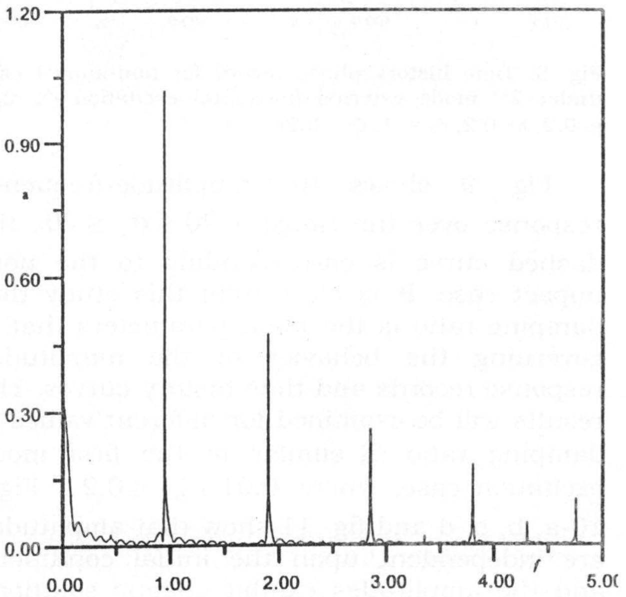

Fig. 6. The fourer fast transform for non-impact case under 1st mode of external horizontal excitation ((X_o=0.1, μ=0.2, λ= 0.2, σ_{xx} = 0.01.0, ζ_1= 0.05).

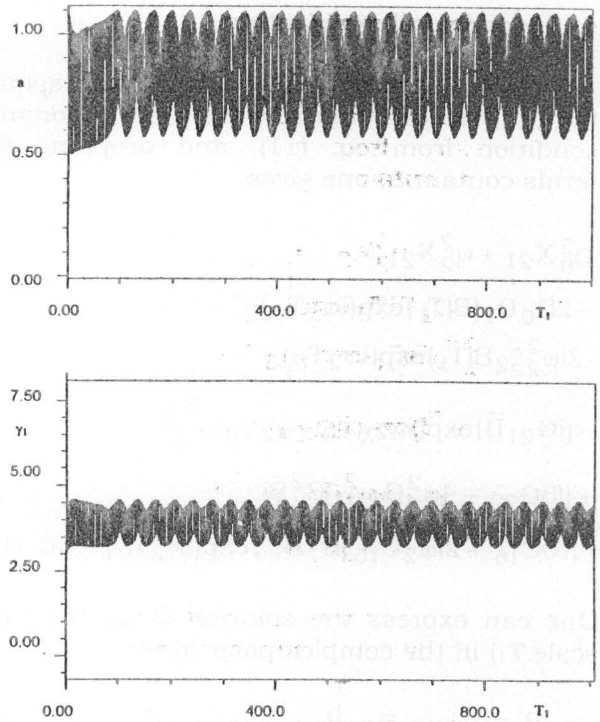

Fig. 7. Time history phase record for impact case under 1st mode external horizontal excitation (X_o=0.1, μ=0.2, λ= 0.2, σ_x = 0.01, ζ_1 = 0.05).

which is an important change for the case of weakly geometric nonlinearities (non-impact case). The steady - state response can be obtained by setting the left-hand side to zero in eqs. (15, 16). In this case the following equation is obtained:

$$C_{10}a^{10} + C_8 a^8 + C_8 a^8 + C_6 a^6 + C_4 a^4 + C_2 a^2 + C_0 = 0 . \quad (18)$$

In order to understand the influence of nonlinearity incurred by the presence of impact loading, fig. 3 shows the amplitude-frequency curves in the absence and in the presence of impact loading. The solid curve is belonging to impact manifold for impact parameters C_{15} = - 0.05 and C_{16}= - 0.5, -1.0, while the dotted one is the non-impact case. For fixed value of the detuning parameter s_x , it is seen that the impact force increases the response amplitudes by a considrable percent.

4. Second mode excitation

Extracting the secular terms corresponding to second mode external resonance condition from eq. (11), and dropping the terms contain A, one gives:

$$D_0^2 X_{21} + \omega_2^2 X_{21} =$$
$$-2D_0 D_1\{B(T_1)\exp(i\omega_2 T_0)\}$$
$$-2i\omega_2^2 \bar{\zeta}_2 B(T_1)\exp(i\omega_2 T_0) -$$
$$-(iG_{21}\bar{B})\exp\{i(\Omega_X T_0 - \omega_2 T_0)\}\frac{X_0}{2}$$
$$+\{(3G_{29} - 4\omega_2^2 G_{210})B^2\bar{B}$$
$$+(10C_{16} + 2i\omega_2 C_{15})B^3\bar{B}^2\}\exp(i\omega_2 T_0) + CC . \quad (19)$$

One can express the solution B (in the time scale T_1) in the complex polar form

$$B = \frac{b}{2}\exp(i\beta), \quad \bar{B} = \frac{b}{2}\exp(-i\beta) . \quad (20)$$

Substituting eq. (20) in eq. (19), setting all secular terms to zero, and separating real and imaginary parts, gives the following two first-order differential equations:

$$\frac{\omega_2}{2} b \frac{\partial \gamma_2}{\partial T_1} = \frac{\omega_2}{2} b\sigma_X + G_{21}\frac{X_0}{4}\sin(\gamma_2)$$
$$+ K_3 b^3 + \frac{5}{16}C_{16} b^5), \quad (21\text{-a})$$

$$\omega_2 \frac{\partial b}{\partial T_1} = -G_{21}\frac{X_0}{4}\cos(\gamma_2) - \omega_2^2 \bar{\zeta}_2 b + \frac{\omega_2}{16} C_{15} b^5 \quad (21\text{-b})$$

where $K_3 = (\frac{3}{8}G_{29} - \frac{\omega_2^2}{2}G_{210})$, g_2 = $s_x T_1$ -β., and the other constants are given in Appendix C.

4.1. Non-impact response

In the absence of impact forces $(C_{15} = C_{16} = 0)$, These two eqs. (21-a) and (21-b) are integrated numerically using Runge-Kutta method. Under any possible initial conditions the response is found to achieve a steady state independent upon the initial conditions for the selected values of damping ratio, as $\zeta_2 \geq 0.1$ as shown in fig. 8. Solving eqs. (21-a, b), The steady state solution is identical to the roots of the following equation:

$$c_4^* a^4 + c_2 a^2 + c_0 = 0 . \quad (22)$$

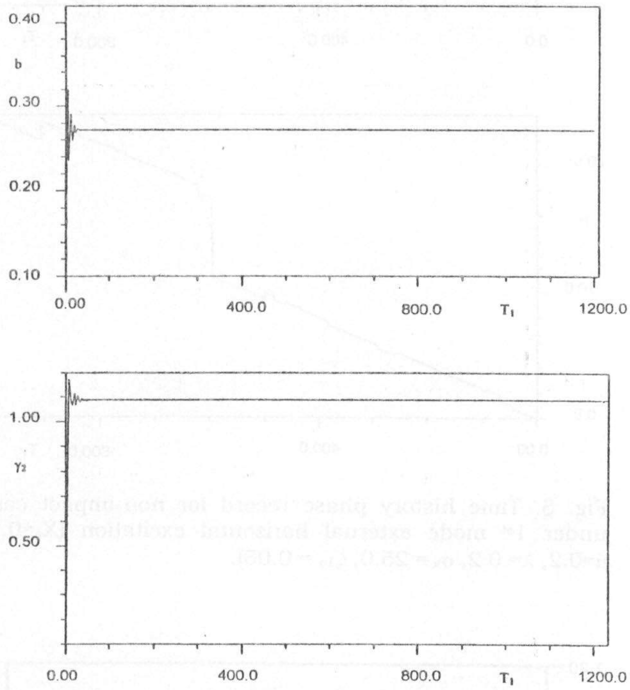

Fig. 8. Time history phase record for non-impact case under 2[nd] mode external horizontal excitation (X_o=0.1, μ=0.2, λ= 0.2, σ_x = -1, ζ_2 = 0.2).

Fig. 9 shows the amplitude-frequency response over the range $-20 \leq \sigma_x \leq 20$, the dashed curve is corresponding to the non-impact case. It is clear from this study that damping ratio is the main parameters that is governing the behavior of the amplitudes response records and time history curves. The results will be examined for different values of damping ratio ζ_2 similar to the first mode excitation case, where $0.01 \leq \zeta_2 \leq 0.2$. Figs. 10-a, b, c, d and fig. 11 show that amplitudes are independent upon the initial conditions and the amplitudes exhibit chaotic solutions for two different selected values of damping ratio ζ_2. These different scenarios introduce the form of snap-through oscillating about non-zero mean value in fig. 10, and hopf-bifurcation for two values of detuning parameter σ_x = -5.0, 0.01 as shown in figs. 11-a, b. The amplitudes are oscillating about non-zero mean value, and the trend continues for different values of detuning parameter σ_x over the range $-20 \le \sigma_x \le 20$. Fig. 12 show the Fourier Fast Transform of the response, after removing the transient period for the snap-through behavior which is identical to fig. 10.

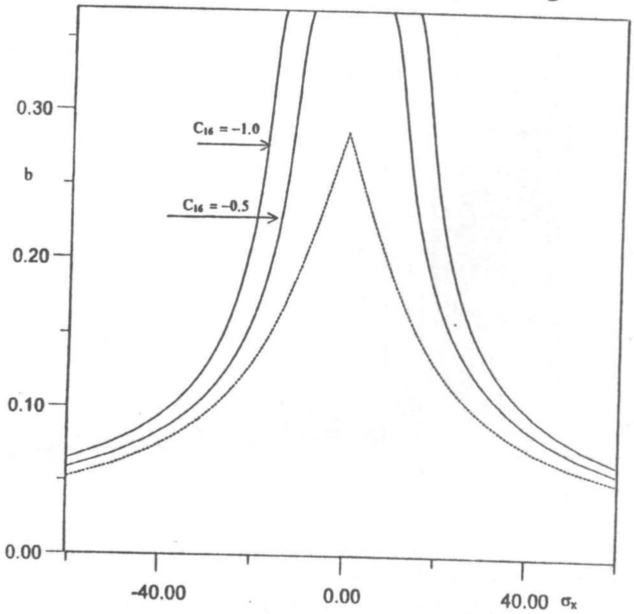

Fig. 9. Amplitude-frequency response curves under the horizontal external excitations for the second mode .resonance case ((X_o=0.2, λ= 0.2, Xo = 0.1, ζ_2 = 0.2)
__________ Impact ------------Non-impact.

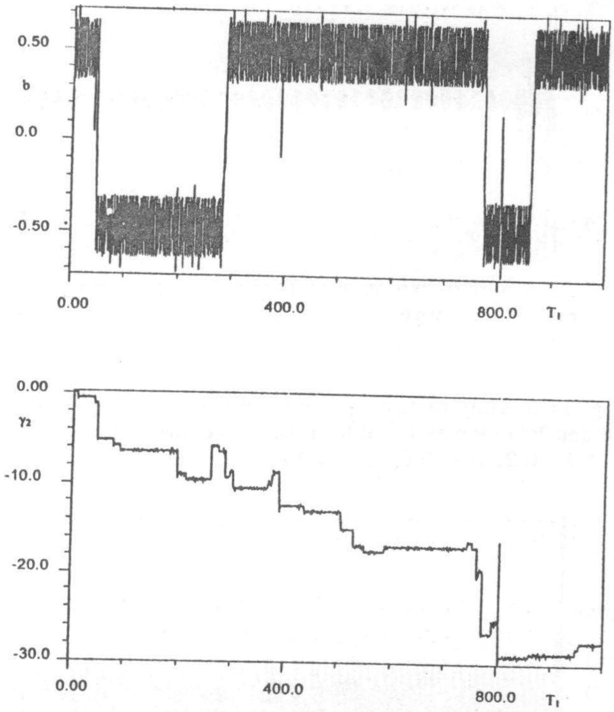

Fig. 10-a,b. Time history phase record for non-impact case under 2nd mode external horizontal excitation ((X_o=0.1, μ = 0.2, λ = 0.2, σ_x = 0.01, ζ_2 = 0.01, bo = 0.5, γ_o =0.02).

4.2. *Impact response*

For the impact case, eqs. (21-a, b) are numerically integrated using Runge-Kutta method (MACSYMA 2.3) for the amplitude b and phase angle g_2. The response records lead to steady state solutions and that are independent upon the damping ratio similar to the first excitation mode that implies that, the system possesses one stable fixed point due to impact forces. The steady-state response can be obtained by setting the left-hand side of eqs. (21-a, b) to zero. Squaring each side and adding gives the following polynomial in the amplitude b:

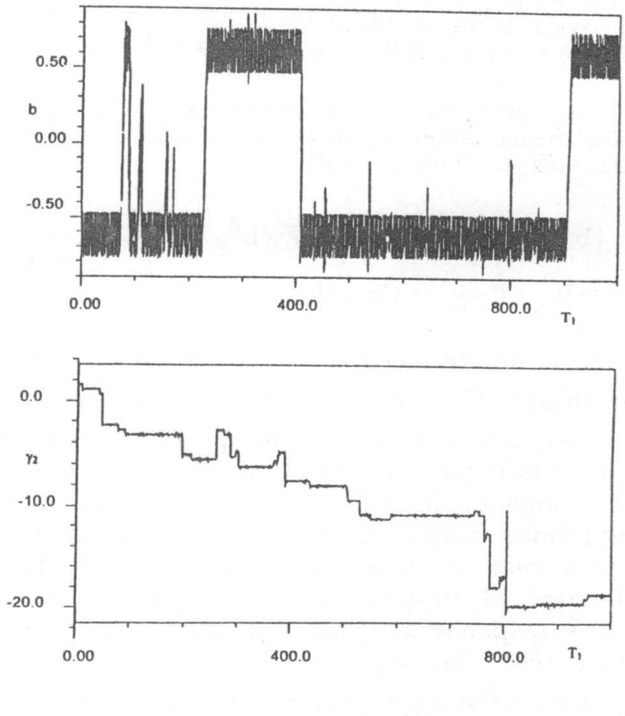

Fig. 10-c,d. Time history phase record for non-impact case under 2nd mode external horizontal excitation ((X_o=0.1, μ = 0.2, λ = 0.2, σ_x = 0.01, ζ_2 = 0.01, a_o = -0.5, γ_o =1.8).

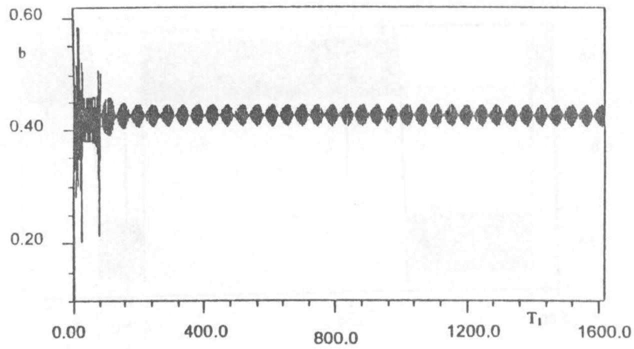

Fig. 11-a. Time history phase record for non-impact case under 2nd mode external horizontal excitation ((X_o=0.1, μ = 0.2, λ = 0.2, σ_x = -5.0, ζ_2 = 0.05).

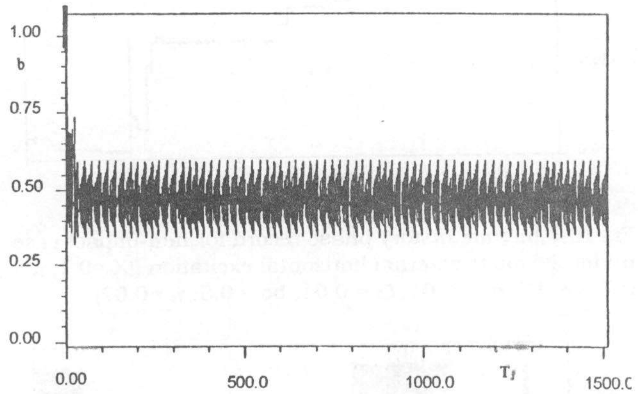

Fig. 11-b. Time history phase record for non-impact case under 2nd mode external horizontal excitation ((X_o=0.1, μ = 0.2, λ = 0.2, σ_x = 0.01, ζ_2 = 0.05).

$$\overline{C}_{10}\, b^{10} + \overline{C}_8\, b^8 + \overline{C}_8\, b^8 + \overline{C}_6\, b^6 + \overline{C}_4\, b^4 + \overline{C}_2\, b^2 + \overline{C}_0 = 0. \tag{23}$$

The amplitude-frequency response over the range $-20 \le \sigma_x \le 20$ is shown in fig. 9. The solid curve is corresponding to the impact case. It is important to note that presences of the impact forces are increasing the amplitudes response values than the non-impact case. It is important to examine the influence of impact coefficient C_{16} on the system response dynamic characteristics. It is noted that, for any selected value of the detuning parameter σ_x, the response amplitude increases as the impact parameter increases indicating that the impact suppresses the system response. Fig. 9 also shows the amplitude-frequency response for impact parameters C_{15} = - 0.05 and C_{16}= - 0.5, -1.0. However, the response amplitudes and their maximum recorded values for this excitation case is less than the values which is introduced for the first excitation mode.

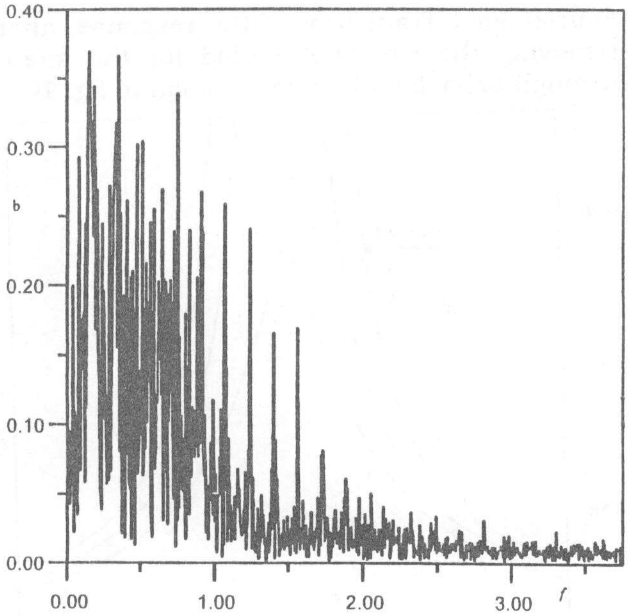

Fig. 12. The fourrer fast transform for non-impact case under 2 nd mode of external horizontal excitation (X_o=0.1, μ = 0.2, λ = 0.2, σx = 0.01, ζ_2 = 0.05).

5. Conclusions

The behavior of an impact system simulating liquid sloshing subjected to external horizontal non-parametric excitations was examined for two external resonance conditions. The impact loads were modeled based on a phenomenological representation in the form of a power function with a higher exponent. The system response has been examined in the neighborhood of two external resonance conditions. When the first mode is externally excited, the response of the amplitude behaved as quasi-periodic and chaotic response with irregular jumps between two unstable equilibrium positions. In the presence of impact forces, the system preserves fixed response amplitude response within a certain range depends upon the external detuning parameter. For the excitation of the second mode, the amplitude response mainly follows the same behaviors of the first mode but the resulting values for the amplitudes are lower than values recorded in the first mode. For the two resonance cases, the response amplitude increased as the impact parameter increased, indicating that the impact suppresses the system response. It is important to note that the different characteristics for the amplitude and phase angle were independent upon the initial conditions. It is also important to show that the linear damping ratio can play an important factor in the behaviors of the response characteristics.

Appendix A

$$\Theta = \frac{\theta}{\theta_0},\ F = \frac{\varphi}{\theta_0},\ t = w_\ell t,\ \omega_\ell^2 = \frac{g}{\ell},\ \omega_L^2 = \left(\frac{k}{ML^2} - \frac{g}{L}\right),$$

$$f_y(\tau) = \frac{F_y(\tau/\omega_\ell)}{\ell\omega_\ell^2\theta_0},\ f_x(\tau) = \frac{F_x(\tau/\omega_\ell)}{\ell\omega_\ell^2\theta_0},\ m = \frac{m}{M},\ l = \frac{\ell}{L},\ n = \frac{\omega_\ell}{\omega_L}$$

$$\omega_{1,2}^2 = \frac{(1+\nu^2) \mp \sqrt{(1-\nu^2)^2 + 4\mu\nu^2}}{2(1-\mu)},\qquad \left(\frac{A}{B}\right)_{1,2} = \frac{\omega_{1,2}^2}{\lambda(1-\omega_{1,2}^2)} = \frac{1}{\lambda(1-\omega_{1,2}^2)/\omega_{1,2}^2} = \frac{1}{K_{1,2}}$$

$$\begin{Bmatrix}\Theta\\ \Phi\end{Bmatrix} = [P]\begin{Bmatrix}X_1\\ X_2\end{Bmatrix},\qquad \text{The modal matrix } [P] = \begin{bmatrix}1 & 1\\ K_1 & K_2\end{bmatrix},\quad \varepsilon = \mu\frac{\lambda^2}{m_{11}}$$

$$m_{11} = K_1^2 + 2\mu\lambda K_1 + \mu\lambda^2,\ m_{22} = K_2^2 + 2\mu\lambda K_2 + \mu\lambda^2,\ k_{11} = \mu\lambda^2 + K_1^2\nu^2,\ k_{22} = \mu\lambda^2 + K_2^2\nu^2$$

Appendix B

$$\begin{aligned}(\Psi_{11})_{gn} = {} & G_{18}X_1^3 + G_{19}X_2^3 + G_{110}X_2^2\ddot{X}_2 + G_{111}X_2^2\ddot{X}_1 + G_{112}X_2X_1\ddot{X}_2 + \\ & G_{113}X_2X_1\ddot{X}_1 + G_{114}X_1^2\ddot{X}_2 + G_{115}X_2\dot{X}_2^2 + G_{116}X_1\dot{X}_2^2 + G_{117}X_2\dot{X}_2\dot{X}_1 + G_{118}X_1\dot{X}_2\dot{X}_1 + \\ & G_{119}X_1X_2^2 + G_{120}X_1^2X_2 + G_{121}X_2\dot{X}_1^2 + G_{122}X_1^2\ddot{X}_1 + G_{123}X_1\dot{X}_1^2\end{aligned}$$

$$\begin{aligned}(\Psi_{11})_{impact} = {} & C_{16}X_1^5 + C_{16}X_2^5 + 5C_{16}X_2X_1^4 + 5C_{16}X_2X_2^4 + 4C_{15}X_1\dot{X}_1X_2^3 + 4C_{15}X_1\dot{X}_2X_2^3 + \\ & 10C_{16}X_1^2X_2^3 + 6C_{15}\dot{X}_1X_1^2X_2^2 + 10C_{16}X_1^3X_2^2 + 6C_{15}\dot{X}_2X_1^2X_2^2 + 4C_{15}\dot{X}_2X_1^3X_2 + \\ & 4C_{15}X_1^3X_2X_1' + C_{15}X_1^4X_1' + C_{15}X_2^4X_1' + C_{15}X_2^4X_2' + C_{15}X_1^4X_2'\end{aligned}$$

$$(\Psi_{11})_{ex} = G_{11}f_X(t)$$

$$\begin{aligned}(\Psi_{22})_{gn} = {} & G_{28}X_1^3 + G_{29}X_2^3 + G_{210}X_2^2\ddot{X}_2 + G_{211}X_2^2\ddot{X}_1 + G_{212}X_2X_1\ddot{X}_2 + \\ & G_{213}X_2X_1\ddot{X}_1 + G_{214}X_1^2\ddot{X}_2 + G_{215}X_2\dot{X}_2^2 + G_{216}X_1\dot{X}_2^2 + G_{217}X_2\dot{X}_2\dot{X}_1 + G_{218}X_1\dot{X}_2\dot{X}_1 + \\ & G_{219}X_1X_2^2 + G_{220}X_1^2X_2 + G_{221}X_2\dot{X}_1^2 + G_{222}X_1^2\ddot{X}_1 + G_{223}X_1\dot{X}_1^2\end{aligned}$$

$$\begin{aligned}(\Psi_{22})_{impact} = {} & C_{16}X_1^5 + C_{16}X_2^5 + 5C_{16}X_2X_1^4 + 5C_{16}X_2X_2^4 + 4C_{15}X_1\dot{X}_1X_2^3 + 4C_{15}X_1\dot{X}_2X_2^3 + \\ & 10C_{16}X_1^2X_2^3 + 6C_{15}\dot{X}_1X_1^2X_2^2 + 10C_{16}X_1^3X_2^2 + 6C_{15}\dot{X}_2X_1^2X_2^2 + 4C_{15}\dot{X}_2X_1^3X_2 + \\ & 4C_{15}\dot{X}_1X_1^3X_2 + C_{15}\dot{X}_1X_1^4 + C_{15}\dot{X}_1X_2^4 + C_{15}\dot{X}_2X_2^4 + C_{15}\dot{X}_2X_1^4\end{aligned}$$

$$(\Psi_{22})_{ex} = G_{21}f_X(t)$$

Appendix C

$$G_{11} = \frac{1}{g\theta_0}\left(1 + \frac{K_1}{\mu\lambda^2}\right)\ G_{12} = \frac{1}{g}\left(1 - \frac{K_1^2}{\mu\lambda}\right),\ G_{13} = \frac{1}{g}\left(1 + \frac{K_1K_2}{\mu\lambda}\right),\quad G_{18} = \frac{\theta_0^2}{6}\left(1 - \frac{K_1^4}{\mu\lambda^2}\right),$$

$$G_{111} = \frac{\theta_0^2 K_1}{\lambda}(1+K_2)^2$$

$$G_{19} = \frac{\theta_0^2}{6}(1-\frac{K_1 K_2^3}{\mu\lambda}),\quad G_{110} = \frac{\theta_0^2}{2\lambda}(K_1+K_2)(1+K_2)^2,$$

$$G_{112} = \frac{\theta_0^2}{\lambda}(K_1+K_2)(1+K_2+K_1+K_1K_2),\ G_{113} = \frac{2\theta_0^2 K_1}{\lambda}(1+K_2+K_1+K_1K_2),$$

$$G_{114} = \frac{\theta_0^2}{2\lambda}(K_1+K_2)(1+K_1)^2,\ G_{115} = \frac{\theta_0^2}{\lambda}(1+K_2)(K_1+K_2^2),\ G_{116} = \frac{\theta_0^2}{\lambda}(1+K_1)(K_1+K_2^2),$$

$$G_{117} = \frac{2\theta_0^2 K_1}{\lambda}(1+K_2)^2,\ G_{118} = \frac{2\theta_0^2 K_1}{\lambda}(1+K_1)(K_1+K_2)\quad G_{119} = \frac{\theta_0^2}{2}(1-\frac{K_1^2 K_2^2}{\mu\lambda}),$$

$$G_{120} = \frac{\theta_0^2}{2}(1-\frac{K_1^3 K_2}{\mu\lambda}),G_{121} = \frac{1}{2}G_{118},G_{122} = \frac{\theta_0^2 K_1}{\lambda}(1+K_1)^2,\ G_{123} = G_{122}.$$

$$C_{15} = -\frac{d}{m\ell^2\omega_\ell},\quad C_{16} = \frac{b_0}{m\ell^2\omega_\ell^2\theta_0},\ G_{21} = \frac{1}{g\theta_0}(1+\frac{K_2}{\mu\lambda^2}),G_{22} = \frac{1}{g}(1-\frac{K_1K_2}{\mu\lambda}),$$

$$G_{23} = \frac{1}{g}(1-\frac{K_2^2}{\mu\lambda}),G_{28} = \frac{\theta_0^2}{6}(1-\frac{K_1^3K_2}{\mu\lambda}),\ G_{29} = \frac{\theta_0^2}{6}(1-\frac{K_2^4}{\mu\lambda})$$

$$G_{210} = \frac{\theta_0^2 K_2}{\lambda}(1+K_2)^2\ G_{211} = \frac{\theta_0^2}{2\lambda}(K_1+K_2)(1+K_2)^2,G_{212} = \frac{\theta_0^2 K_2}{\lambda}(1+K_1+K_2+K_1K_2),$$

$$G_{223} = G_{221}$$

$$G_{213} = \frac{\theta_0^2}{\lambda}(K_1+K_2)(1+K_1+K_2+K_1K_2),\ G_{214} = \frac{\theta_0^2 K_2}{\lambda}(1+K_1)^2,\ G_{215} = G_{210}$$

$$G_{216} = \frac{\theta_0^2 K_2}{\lambda}(1+K_1)(1+K_2),G_{217} = 2G_{216},G_{218} = 2G_{214},\ G_{219} = \frac{\theta_0^2}{2}(1-\frac{K_1K_2^3}{\mu\lambda})$$

$$G_{220} = \frac{\theta_0^2}{2}(1-\frac{K_1^2K_2^2}{\mu\lambda}),G_{221} = \frac{\theta_0^2}{\lambda}(1+K_2)(K_1^2+K_2),\ G_{222} = \frac{\theta_0^2}{2\lambda}(1+K_1)^2(K_1+K_2).$$

Appendix D

$$C_{10} = \frac{1}{256}(\omega_1^2 C_{15}^2 + 25C_{16}^2),C_8 = \frac{5}{8}G_3C_{16},\ C_6 = G_3^2,$$

$$C_6^* = G_3^2 - \frac{1}{8}\omega_1^3 C_{15}\zeta_1 + \frac{5}{16}\omega_1\sigma_x C_{16}\quad C_4 = \omega_1\sigma_X G_3, C_2 = \omega_1^4\zeta_1^2 + \frac{\omega_1^2}{4}\sigma_X^2,\ C_0 = -G_{11}^2X_0^2,$$

$$\overline{C}_{10} = \frac{1}{256}(\omega_1^2 C_{15}^2 + 25C_{16}^2),\overline{C}_8 = \frac{5}{8}K_3C_{16},\ \overline{C}_6 = K_3^2,$$

$$\overline{C}_6^* = K_3^2 - \frac{1}{8}\omega_1^3 C_{15}\overline{\zeta}_2 + \frac{5}{16}\omega_1\sigma_x C_{16}\quad \overline{C}_4 = \omega_1\sigma_X K_3,\overline{C}_2 = \omega_1^4\overline{\zeta}_2^2 + \frac{\omega_1^2}{4}\sigma_X^2,\ \overline{C}_0 = -G_{21}^2X_0^2$$

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